Ryan Lovrien (rcl73)
Phelps Williams (pww36)
12-Feb-2008

CS569 Assignment 1 Written Part

1. Warm Up:

$$w = p - o$$

$$Q' = o + td$$

The projection of w onto the function o + td equals the following:

$$\frac{w \cdot d}{||d||^2} d$$

$$Q' = o + \frac{w \cdot d}{||d||^2} d$$

$$t = \frac{\frac{w \cdot d}{||d||^2} d}{d}$$

2. Tangent space:

i)

$$P = M * T$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$x = au + bv$$

$$y = cu + dv$$

$$z = eu + fv$$

With two known points and texture coordinates, relative to P_0 it is possible to solve matrix M. Where M is composed of the Tangent (T) and Binormal(B) vectors.

$$p_i = u_i * T + v_i * B$$

$$T = \begin{bmatrix} a \\ c \\ e \end{bmatrix}, \qquad B = \begin{bmatrix} b \\ d \\ f \end{bmatrix}$$

$$P_a = P_1 - P_0$$

$$P_b = P_2 - P_0$$

$$u_a = u_1 - u_0$$

$$v_a = v_1 - v_0$$

$$u_b = u_2 - u_0$$

$$v_b = v_2 - v_0$$

Relative to P_0 we can write equations relating the relative positions of P_1 and P_2 to the relative positions of texture coordinates uv_1 and uv_2.

$$P_a = u_a * T + v_a * B$$

$$P_b = u_b * T + v_b * B$$

We now have a linear function which maps *uv* texture coordinates to points on the triangle relative to position P_0.

ii) With the work from part i, we can now solve for equations of T and B, the Tangent and Binormal vectors respectively.

$$T = \frac{(v_b) * (p_a) - (v_a) * (p_b)}{(u_a) * (v_b) - (v_a) * (u_b)}$$

Or expanded as:

$$T = \frac{(v_2 - v_0) * (p_1 - p_0) - (v_1 - v_0) * (p_2 - p_0)}{(u_1 - u_0) * (v_2 - v_0) - (v_1 - v_0) * (u_2 - u_0)}$$

Similarly B in expanded form:

$$B = \frac{(u_2 - u_0) * (p_1 - p_0) - (u_1 - u_0) * (p_2 - p_0)}{(v_1 - v_0) * (u_2 - u_0) - (u_1 - u_0) * (v_2 - v_0)}$$

Because T, B and the Normal vector N should be a coordinate system base, we can calculate N by the cross product of T and B:

$$N = T \times B$$

Using Gram-Schmidt Orthogonalization it is possible to ensure these vectors are orthogonal.

$$T = T - N * (N \cdot T)$$

References:

http://jerome.jouvie.free.fr/OpenGl/Lessons/Lesson8.php

http://www.blacksmith-studios.dk/projects/downloads/tangent_matrix_derivation.php