Approximation of data

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1 Goal

The goal of this project is to find a polynomial approximation of some data. Several approximation can be made. First, if the goal of the user is to find a trend for the data, the program will return a polynomial with a degree lower than the number of points minus one. Then, the program can compute an interpolation of data. Several possibilities are available to interpolate. The first one is a simple polynomial interpolation, where the degree is equal to the number of points minus 1. Then, a piecewise interpolation can be made. Depending on the needs of the user this interpolation can be spline or not.

2 Requirements

A gcc compiler (version 4.7.2 or above) a verifier??? and the CMake software (version 2.6 or above) to link the files are necessary to compile the program. An external library is used to solve linear systems, that is the Eigen library.

3 Method

To approximate the data, the main method that have been used is the least squares method. The method aim to minimize the error between the true value and the estimated one, that is:

$$min\sum_{i} (y_i - p(x_i))^2 \tag{1}$$

 $p(x_i) = a_0 + a_1 \cdot x_i + \ldots + a_1 \cdot x_i^m$ being the interpolant polynomial at x_i of degree m. By deriving this equation with respect with all the coefficient of the polynomial, the following linear system is obtained:

$$\begin{bmatrix} \sum_{i} x_{i}^{0} & \sum_{i} x_{i}^{1} & \cdots & \sum_{i} x_{i}^{m} \\ \sum_{i} x_{i}^{1} & \sum_{i} x_{i}^{2} & \cdots & \sum_{i} x_{i}^{m+1} \\ \vdots & & \ddots & \vdots \\ \sum_{i} x_{i}^{1} & \cdots & \sum_{i} x_{i}^{2m-1} & \sum_{i} x_{i}^{2m} \end{bmatrix} \begin{bmatrix} a_{0} \\ a_{1} \\ \vdots \\ a_{m} \end{bmatrix} = \begin{bmatrix} \sum_{i} y_{i} \\ \sum_{i} y_{i} x_{i} \\ \vdots \\ \sum_{i} y_{i} x_{i}^{m} \end{bmatrix}$$
(2)

The linear system Xa = b can be solved by computing $a = X^{-1}b$. For the spline interpolation, https://www.math.uh.edu/jingqiu/math4364/spline.pdf the following paper has been used.

4 Usage of the program

The program developed allows the user to give a .csv file with some input inside. Depending on what is in the file, the program will compute what the user has asked. The .csv should have the following form:

data.csv Type	Approximation Degree
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The data.csv file contains the points coordinate. The type is the approximation wanted by the user; it can be either Data fitting, Interpolation, Piece-wise Interpolation or spline. Finally, the approximation degree is the degree of the polynomial which approximate the data.

To execute the program to complete

5 Class description

READFILE APPROXIMATION

- 6 Tests
- 7 Remarks