Lagrangian dynamics of dendritic microcircuits enables real-time backpropagation of errors

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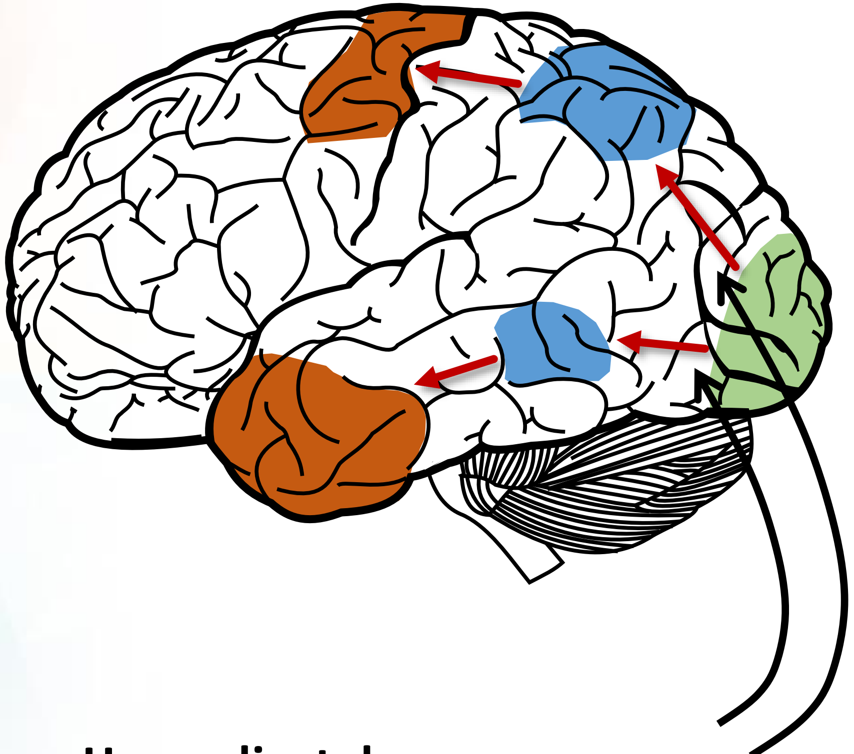
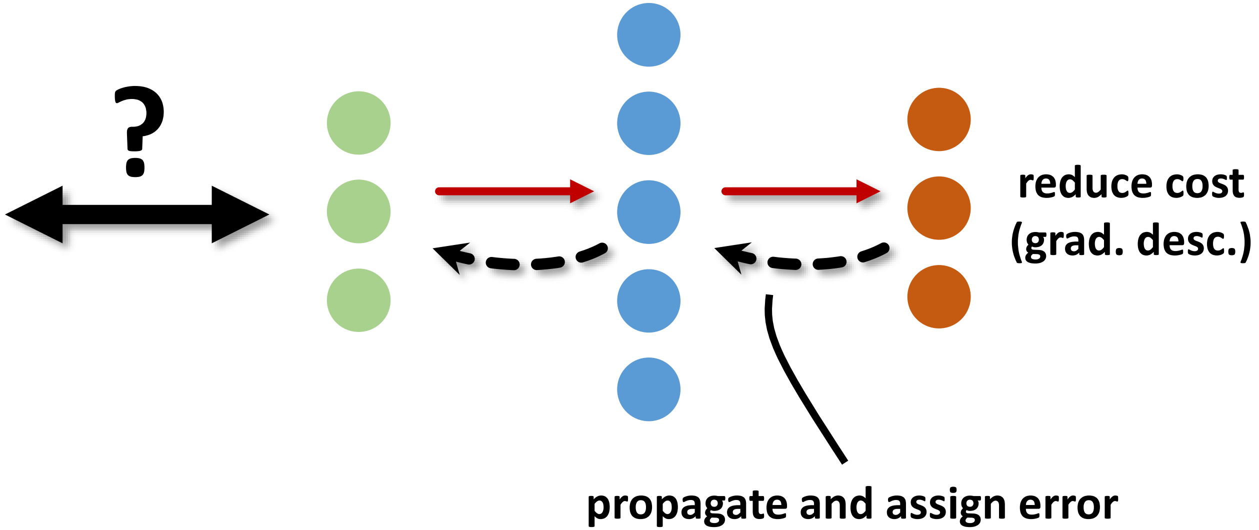
[5] MILA, Montreal.

Can backprop be implemented in the brain?





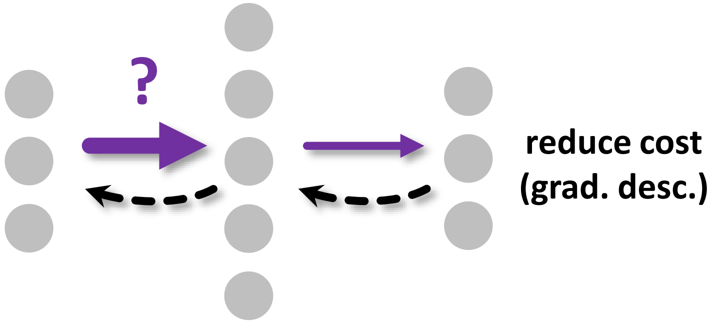
How adjust deep synapses to improve final output…?

<u>Towards bio-plausible backprop?</u>

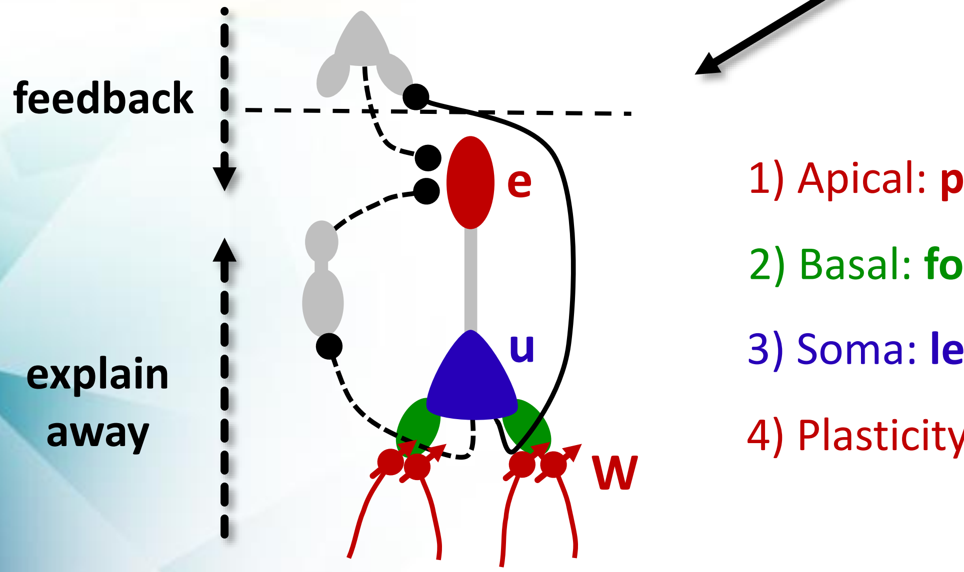
- **A theoretical framework for backprop**, *Y. le Cun (1988)*
- **Feedback alignment**, *T. Lillicrap et al. (2016)*
- **Equilibrium Propagation**, *B. Scellier & Y. Bengio (2016)*
- **Dendritic backprop**, *J. Sacramento et al. (2018)*
- **Review** *by Whittington & Bogacz (2019)*
- **…**

Image adjusted from https://commons.wikimedia.org/wiki/File:Brain_Surface_Gyri.SVG

Apical dendrite carries local prediction error



error locally accessible?



1) Apical: **prediction error**[1]

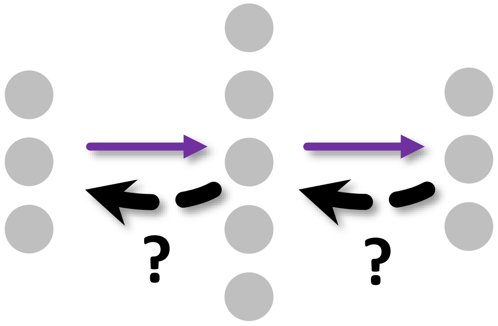
2) Basal: **forward input**

3) Soma: **leaky integration**

4) Plasticity: **dendritic prediction of soma**[2]

[1] *Sacramento et al., NIPS 2018;* [2] *Urbanczik & Senn, Neuron 2014*

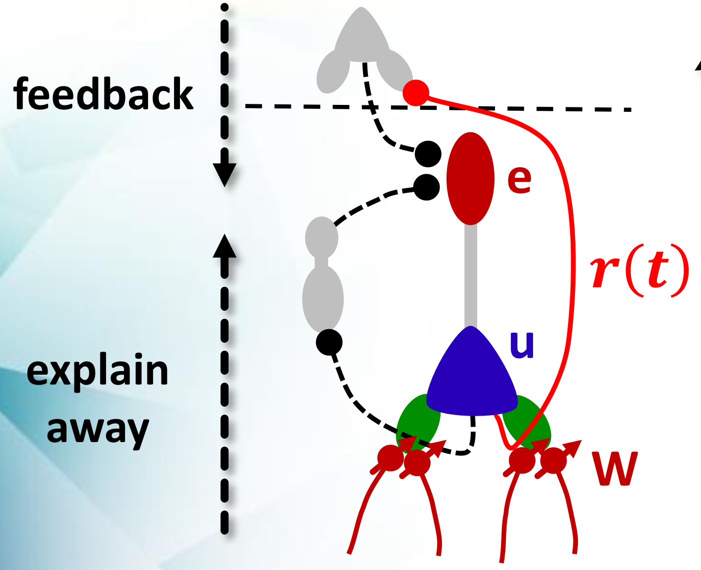
Prospective response enables just-in-time error



reduce cost (grad. desc.)

error locally accessible?

error delayed?



1) Apical: **prediction error**[1]

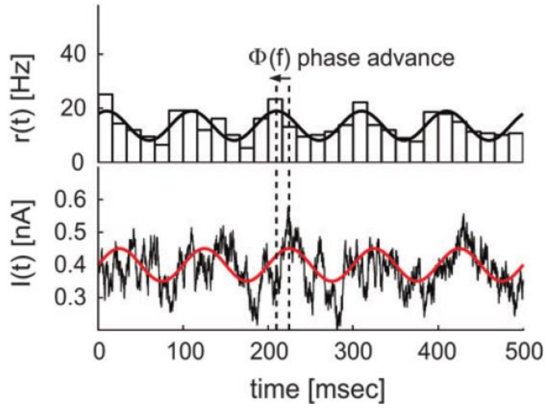
2) Basal: **forward input**

3) Soma: **leaky integration**

4) Plasticity: **dendritic prediction of soma**[2]

use **prospective response**[3]

$$r(t) \approx \varphi(u(t+\tau))$$



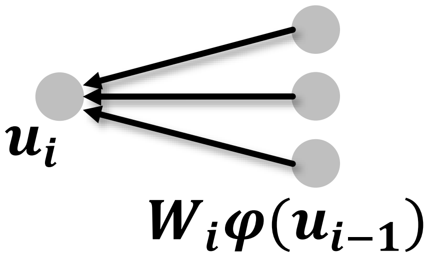
[1] *Sacramento et al., NIPS 2018;* [2] *Urbanczik & Senn, Neuron 2014;* [3] *Köndgen et al., Cereb Cortex 2008*

Prospective coding and least-action principle

Energy function

teacher strength

$$E(u) = \sum_i \| \underbrace{u_i - W_i \varphi(u_{i-1})}_{\text{prediction error } e_i} \|^2 + \beta \cdot cost$$



Lagrange function

$$L = -E(\underbrace{\tilde{u}, \dot{\tilde{u}}}_{\text{prospective response}}, \mathrm{t})$$

Physics!

$$\delta \int L\, dt = 0 \qquad \dot{W} \propto \nabla_W L$$

$$\frac{\partial L}{\partial \tilde{u}} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\tilde{u}}} = 0$$

<u>**Theorem 1**</u> *(real-time backprop)*

$$e_i = W_{i+1}^{\mathrm{T}} e_{i+1}\,, \quad \dot{W}_i \propto e_i \varphi(u_i)^{\mathrm{T}}$$

<u>**Theorem 2**</u> *(real-time gradient descent)*

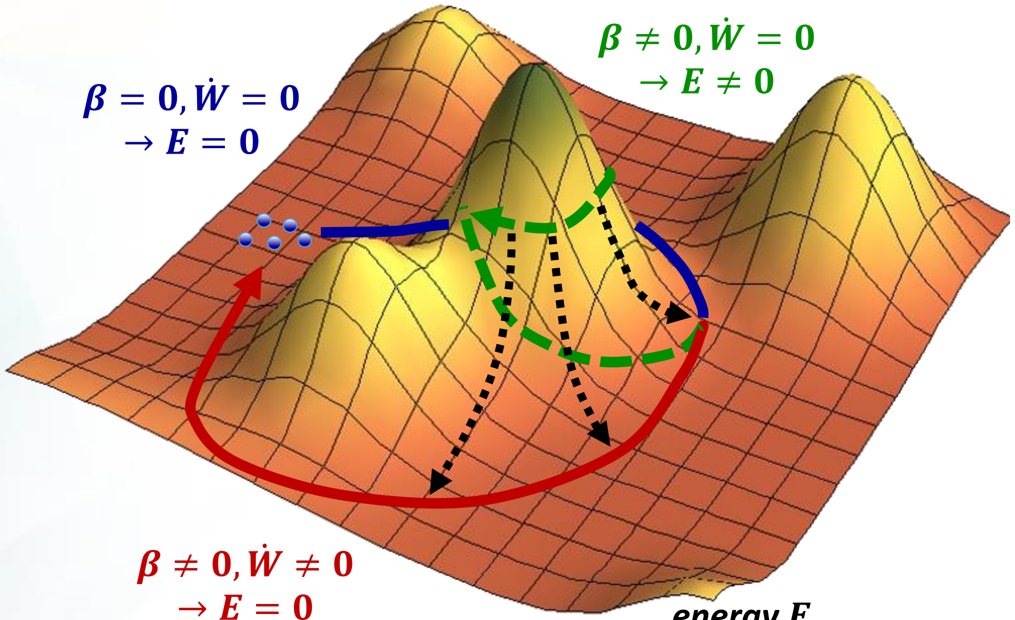
$$\frac{\mathbf{d}}{\mathbf{d}W_i} cost = \lim_{\beta \to 0} \frac{1}{\beta} e_i^{\beta} \varphi^{\beta}(u_i)^{\mathrm{T}}$$

Prospective coding and least-action principle

Energy function

teacher strength

$$\boldsymbol{E}(\boldsymbol{u}) = \sum_i \| \boldsymbol{u}_{\boldsymbol{i}} - \boldsymbol{W}_{\boldsymbol{i}} \boldsymbol{\varphi}(\boldsymbol{u}_{\boldsymbol{i-1}}) \|^2 + \beta \cdot \boldsymbol{cost}$$



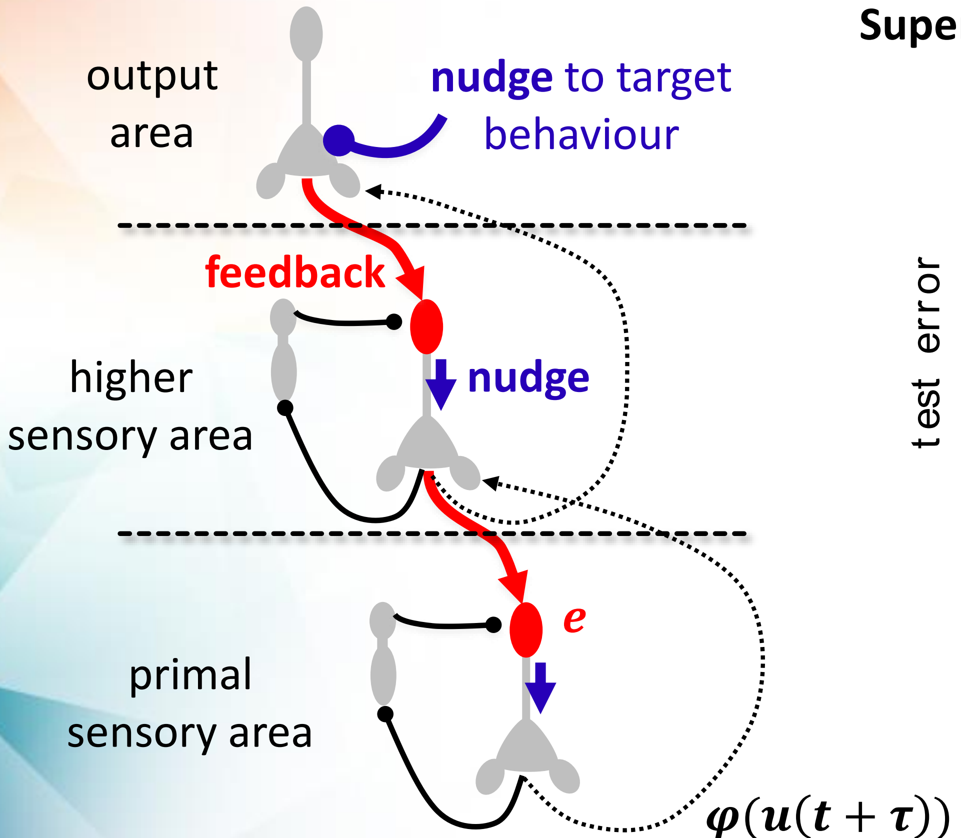
Theorem 1 *(real-time backprop)*

$$\boldsymbol{e}_{\boldsymbol{i}} = W_{i+1}^{\mathrm{T}} \boldsymbol{e}_{\boldsymbol{i+1}}\,, \quad \dot{\boldsymbol{W}}_{\boldsymbol{i}} \propto \boldsymbol{e}_{\boldsymbol{i}} \varphi(u_i)^{\mathrm{T}}$$

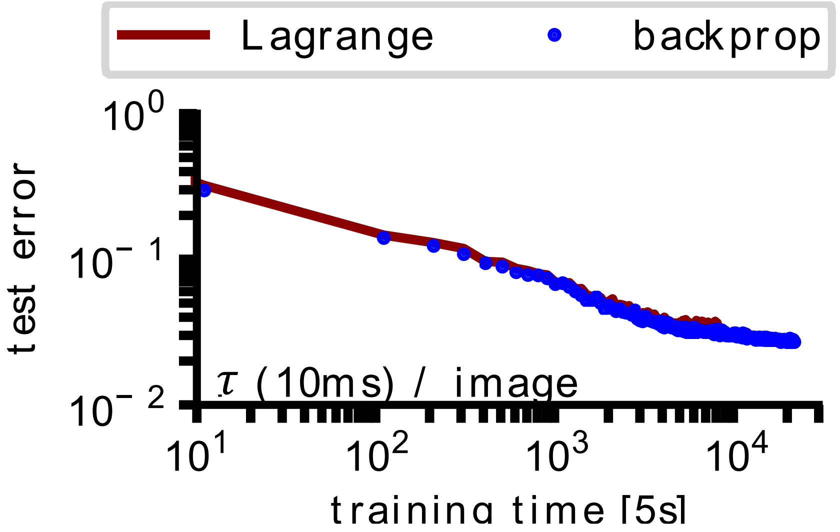
Theorem 2 *(real-time gradient descent)*

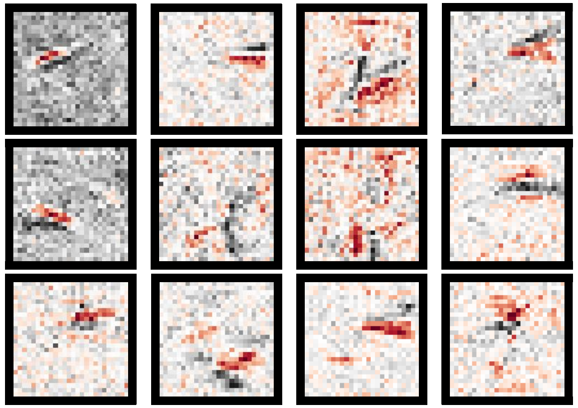
$$\frac{\mathbf{d}}{\mathbf{d}\boldsymbol{W}_{\boldsymbol{i}}} \boldsymbol{cost} = \lim_{\beta \to 0} \frac{1}{\beta} \boldsymbol{e}_{\boldsymbol{i}}^{\boldsymbol{\beta}} \varphi^{\beta}(u_i)^{\mathrm{T}}$$

Real-time error backpropagation in cortical circuits



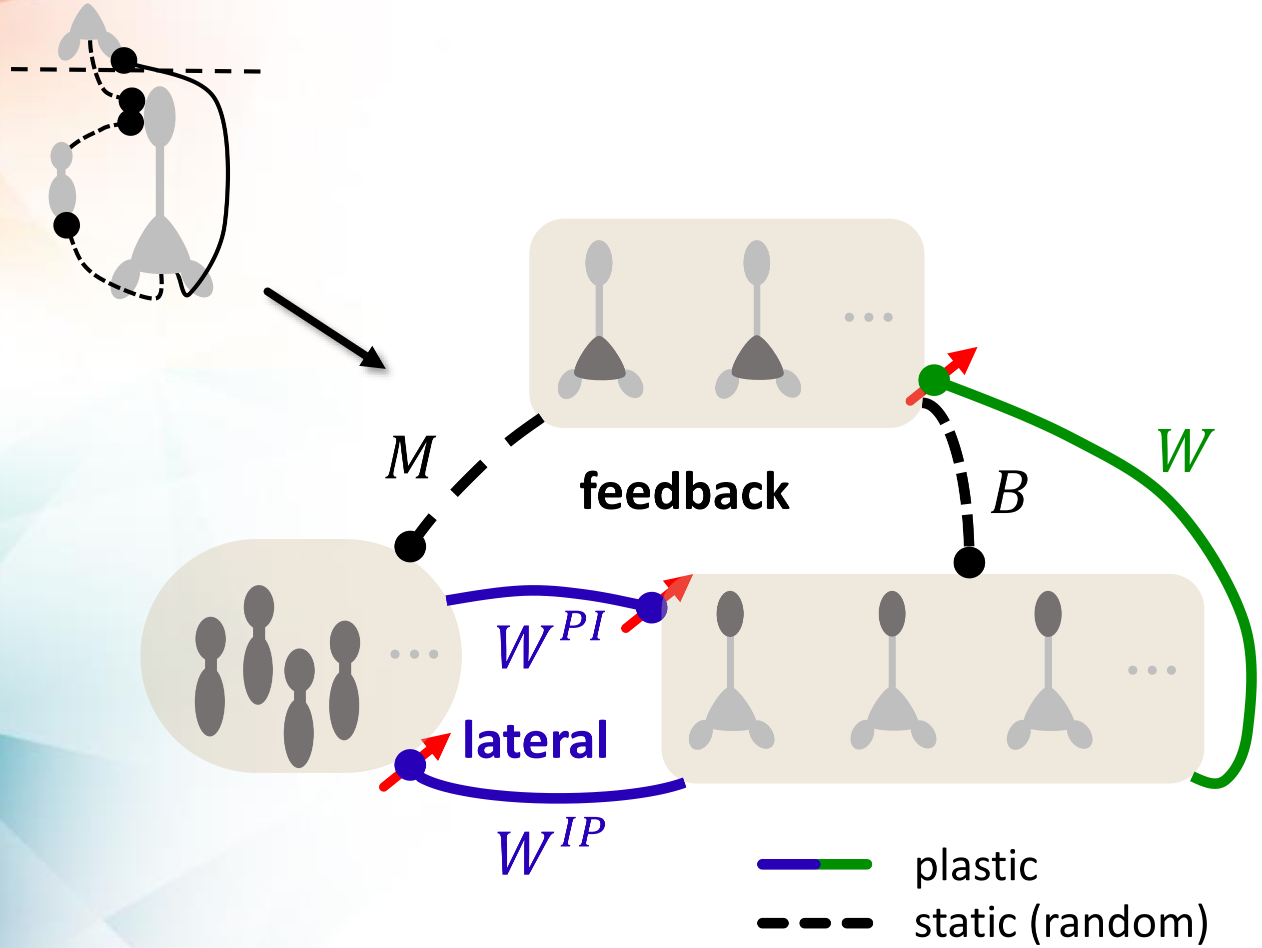
Supervised learning (MNIST)

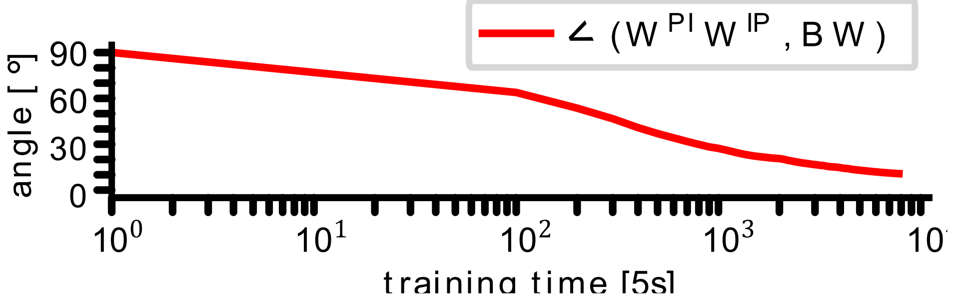




also applicable to
unsupervised learning and
reinforcement learning

Plastic microcircuit – no need for weight transport



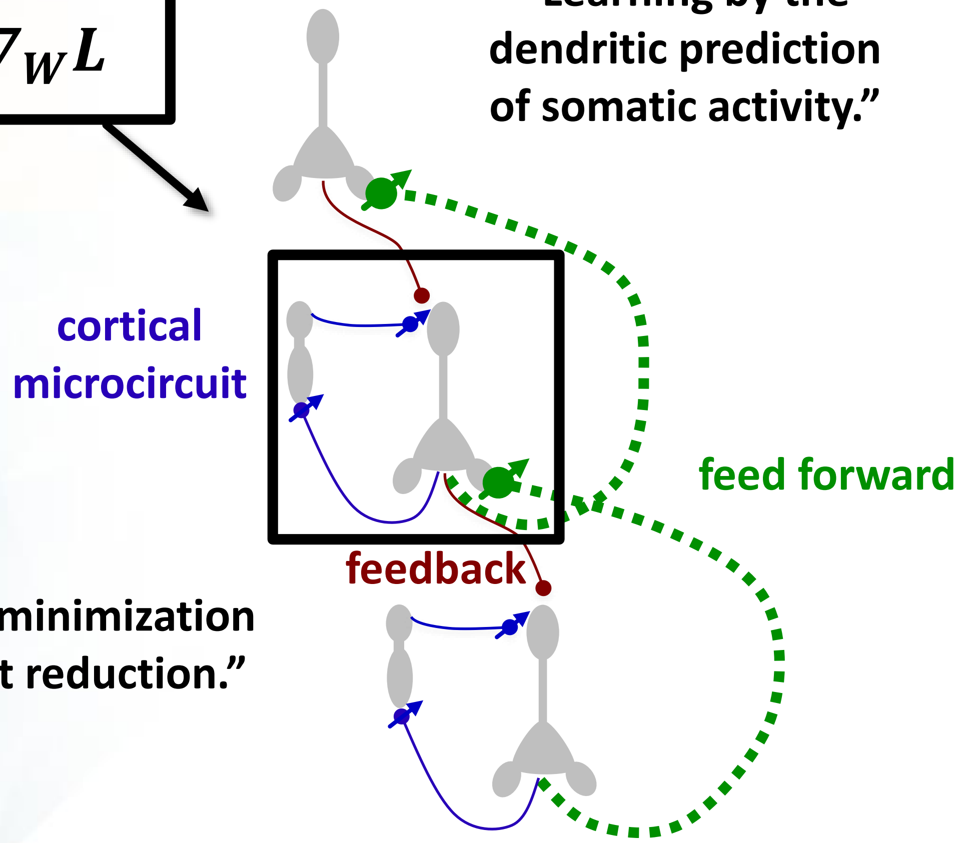


Neurosynaptic dynamics implement backprop

$$\delta \int L \, dt = 0$$

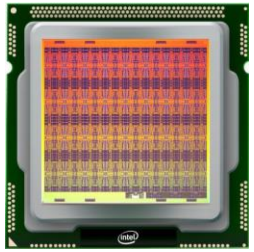
$$\dot{W} \propto \nabla_W L$$

"Learning by the dendritic prediction of somatic activity."



"Local error minimization is global cost reduction."

Efficient learning for neuromorphic hardware



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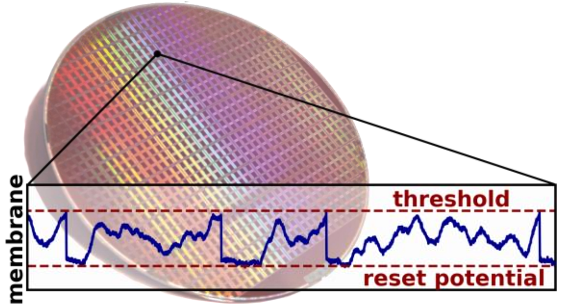

Uni Heidelberg BrainScaleS system

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