1.24 If $x(t)$ is the signal shown in Figure 1.58, sketch

(a) $x(t-2)$

$\because x(t) = \begin{cases} 2-\frac{2}{3}t, & 0<t<3 \\ 0, & \text{otherwise} \end{cases}$

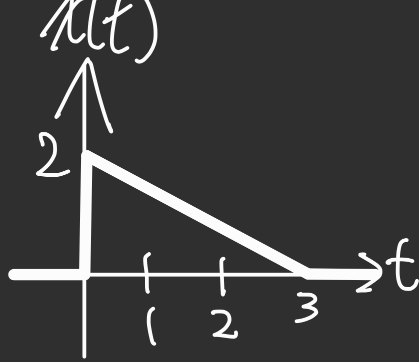


Figure 1.58

$\Rightarrow x(t-2) = \begin{cases} 2-\frac{2}{3}(t-2), & 0<t-2<3 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow x(t-2) = \begin{cases} \frac{2}{3}(5-t), & 2<t<5 \\ 0, & \text{otherwise} \end{cases}$

$\therefore$

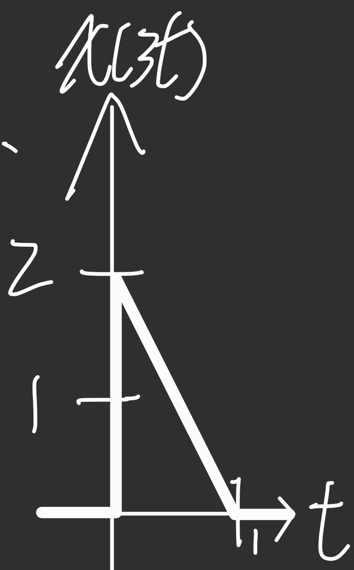
(shift $x(t)$ to the right by 2)

(b) $x(3t)$

$\because x(t) = \begin{cases} 2-\frac{2}{3}t, & 0<t<3 \\ 0, & \text{otherwise} \end{cases}$

$\Rightarrow x(3t) = \begin{cases} 2-2t, & 0<t<1 \\ 0, & \text{otherwise} \end{cases}$

$\therefore$



(compress $x(t)$ by 3)

, and (C) $y(t)=1+2x(t)$.

$$\because x(t)=\begin{cases} 2-\frac{2}{3}t, & 0<t<3 \\ 0, & \text{otherwise} \end{cases}$$

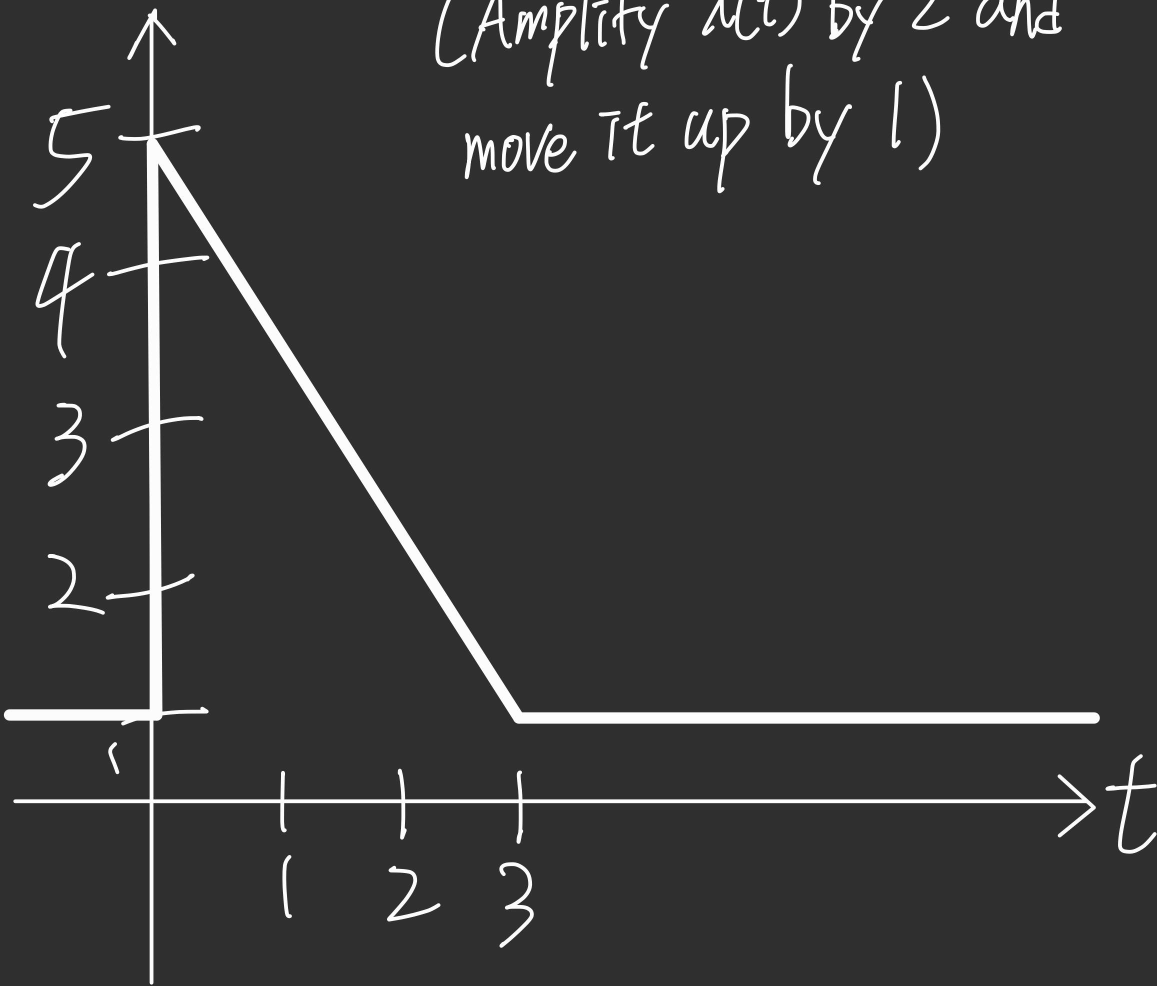
$$\Rightarrow 2x(t)=\begin{cases} 4-\frac{4}{3}t, & 0<t<3 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow y(t)=1+2x(t)$$

$$=\begin{cases} 5-\frac{4}{3}t, & 0<t<3 \\ 1, & \text{otherwise} \end{cases}$$

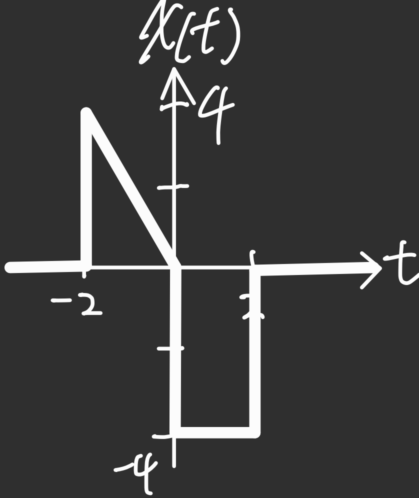
$y(t) = 1 + 2x(t)$

(Amplify $x(t)$ by 2 and move it up by 1)



1.29 Given $x(t)$ in Figure 1.59, sketch

(a) $y(t) = -x(t-1)$

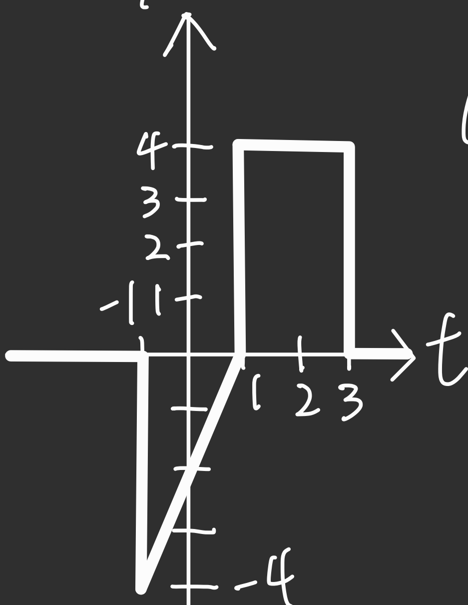


$$\because x(t) = \begin{cases} -2t, & -2<t<0 \\ -4, & 0<t<2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow x(t-1) = \begin{cases} -2t+2, & -1<t<1 \\ -4, & 1<t<3 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow y(t) = -x(t-1) = \begin{cases} 2t-2, & -1<t<1 \\ 4, & 1<t<3 \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$ $y(t)$



(Shift $x(t)$ to the right by 1 and reflect it about the t-axis.)

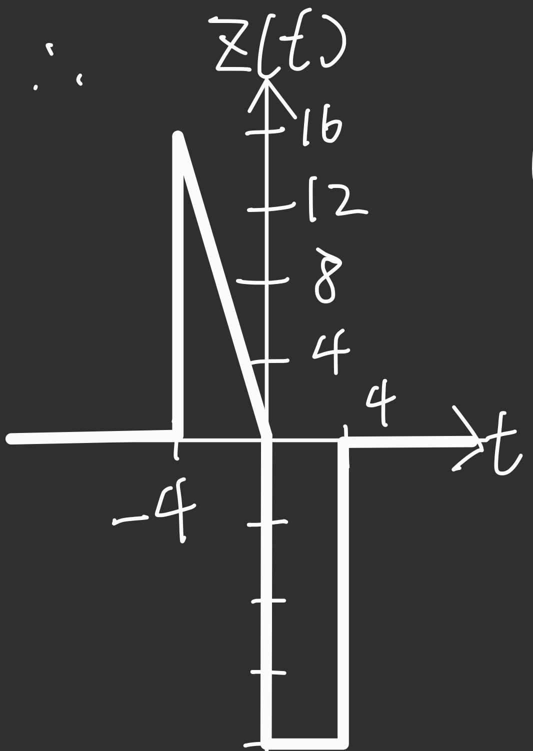
(b) $z(t) = 4x(\frac{t}{2})$

$$\because x(t) = \begin{cases} -2t, & -2<t<0 \\ -4, & 0<t<2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow x(\tfrac{t}{2}) = \begin{cases} -t, & -4<t<0 \\ -4, & 0<t<4 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow z(t) = 4x(\tfrac{t}{2}) = \begin{cases} -4t, & -4<t<0 \\ -16, & 0<t<4 \\ 0, & \text{otherwise} \end{cases}$$

$\therefore$



(Expand $x(t)$ by 2 and amplify it by 4.)

(c) $h(t) = x(2-t)$

$$\because x(t) = \begin{cases} -2t, & -2<t<0 \\ -4, & 0<t<2 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow x(t-2) = \begin{cases} -2(t-2), & -2<(t-2)<0 \\ -4, & 0<(t-2)<2 \\ 0, & \text{otherwise} \end{cases}$$

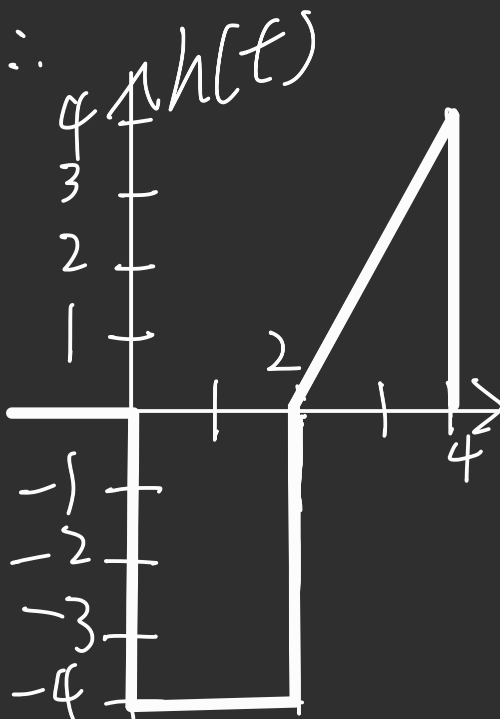
$$\Rightarrow h(t) = x(2-t) = x(-(t-2))$$

$$= \begin{cases} 2(t-2), & -2<-(t-2)<0 \\ -4, & 0<-(t-2)<2 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} 2t-4, & 2<t<4 \\ -4, & 0<t<2 \\ 0, & \text{otherwise} \end{cases}$$

(Shift $x(t)$ right by 2 and reflect it about $t=2$)

$\therefore$



1.32 Consider the discrete-time signal in Figure 1.62.

Sketch the following signals:

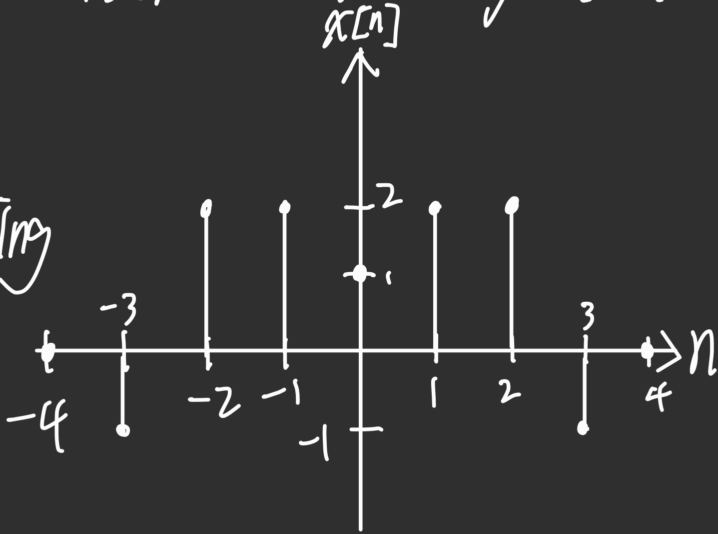


Figure 1.62

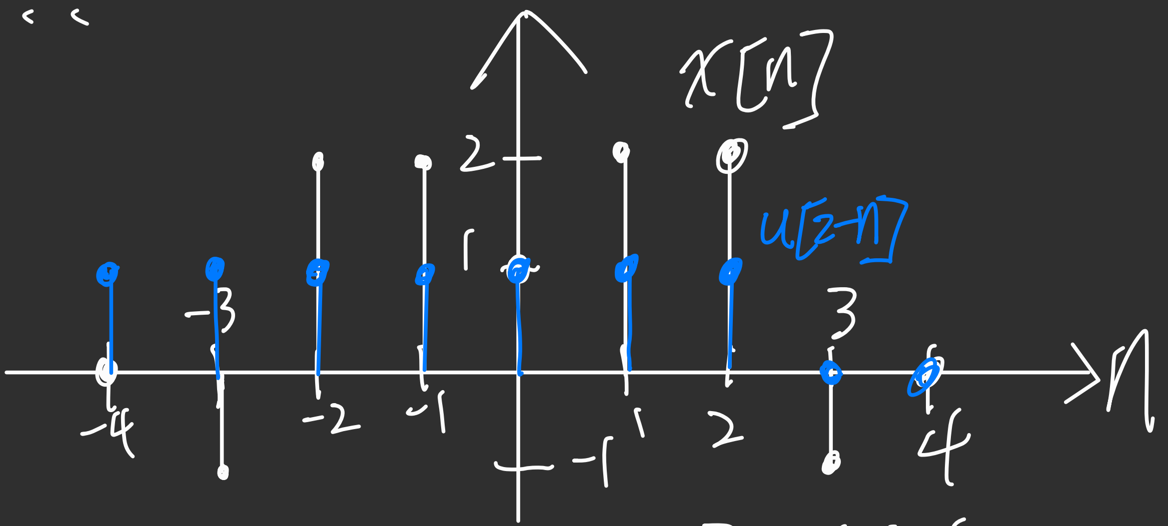
(a) $x[n]u[2-n]$

$$\because x[n] = \begin{cases} 1, & n=0 \\ 2, & |n|=1 \text{ or } 2 \\ -1, & |n|=3 \\ 0, & \text{otherwise} \end{cases}$$

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n<0 \end{cases}$$

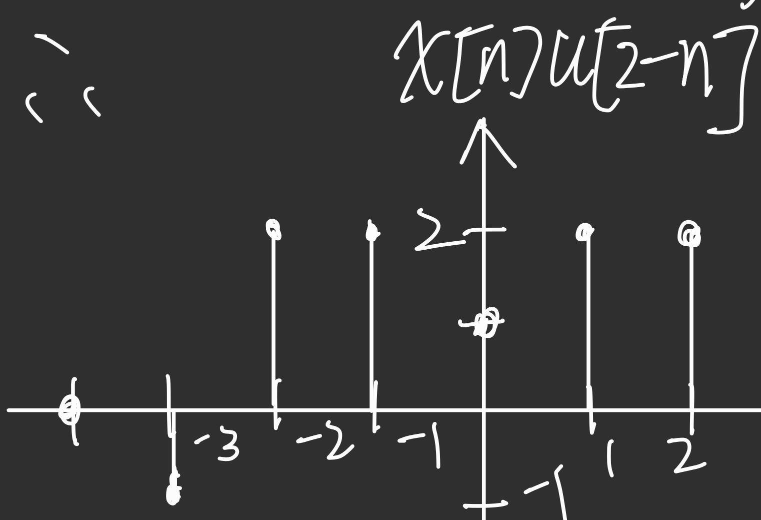
$$\Rightarrow u[n-2] = \begin{cases} 1, & n \geq 2 \\ 0, & n<2 \end{cases} \Rightarrow u[2-n] = \begin{cases} 0, & n>2 \\ 1, & n \leq 2 \end{cases}$$

∴



$$\therefore\ x[n]u[2-n] = \begin{cases} 2, & |n| = 1 \text{ or } 2 \\ 1, & n = 0 \\ -1, & n = -3 \\ 0, & \text{otherwise} \end{cases}$$

∴



(Remain values of $x[n]$ with $n \leq 2$)

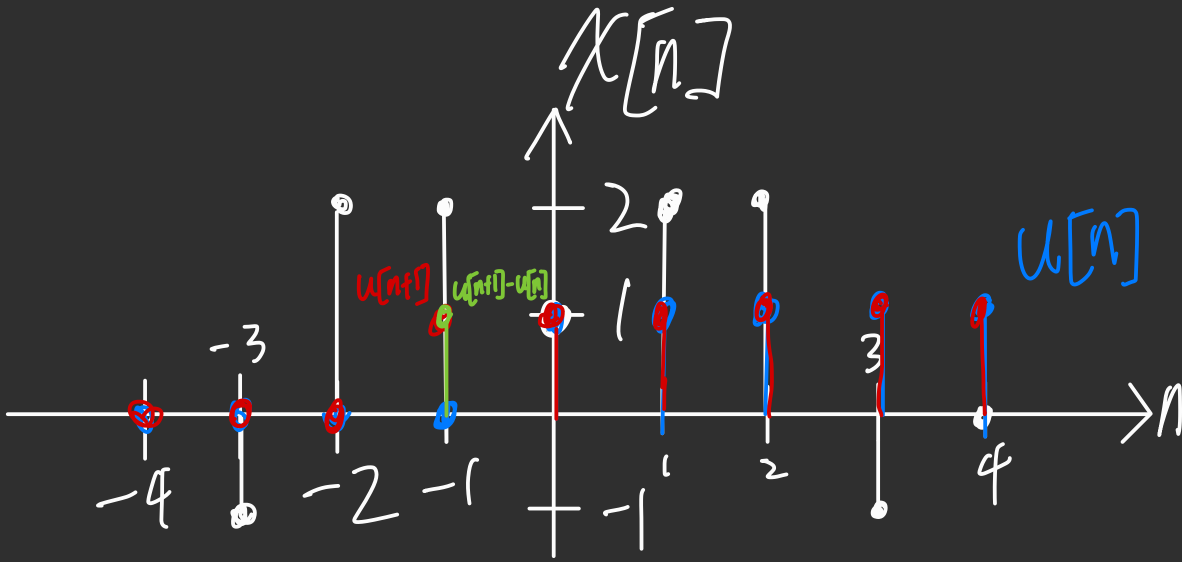
(b) $x[n]\left[u[n+1]-u[n]\right]$

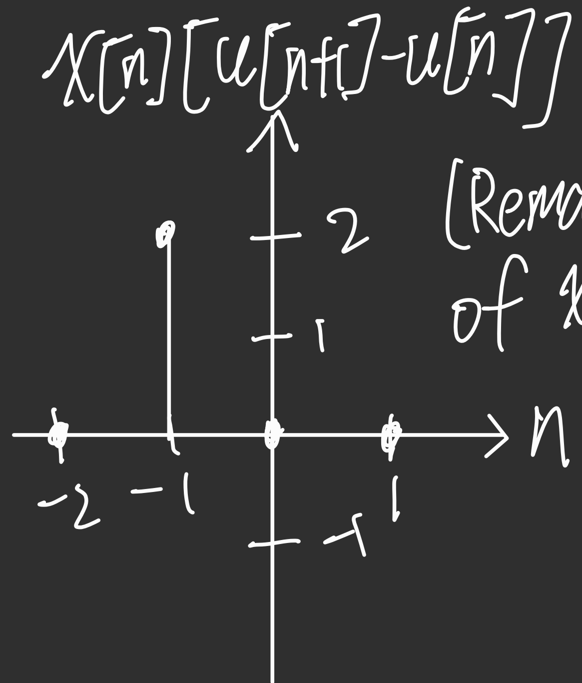
$\because\ x[n]=\begin{cases}1, & n=0\\ 2, & |n|=1 \text{ or } 2\\ -1, & |n|=3\\ 0, & \text{otherwise}\end{cases}$

$\begin{cases} u[n]=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases} \\ u[n+1]=\begin{cases}1, & n\geq -1\\ 0, & n<-1\end{cases}\end{cases}$

$\Rightarrow u[n+1]-u[n]=\begin{cases}1, & n=-1\\ 0, & \text{otherwise}\end{cases}$

$\therefore\ x[n]\left[u[n+1]-u[n]\right]=\begin{cases}2, & n=-1\\ 0, & \text{otherwise}\end{cases}$





(Remain the value of x[n] with n=-1)

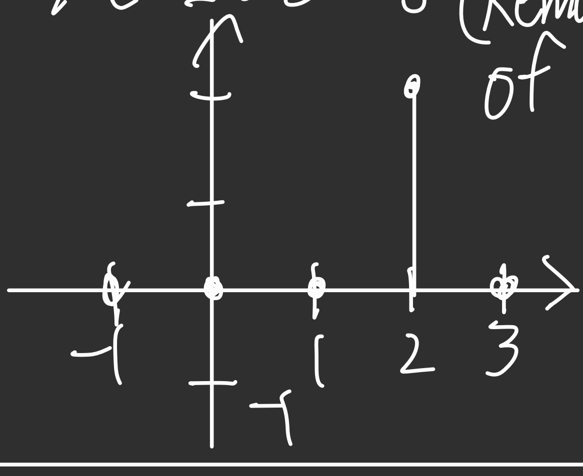
(c) $x[n]\delta[n-2]$

$$\because x[n] = \begin{cases} 2, & |n| = 1 \text{ or } 2 \\ 1, & n = 0 \\ -1, & |n| = 3 \\ 0, & \text{otherwise} \end{cases}$$

$$\delta[n-2] = \begin{cases} 1, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

$$\therefore x[n]\delta[n-2] = \begin{cases} 2, & n = 2 \\ 0, & \text{otherwise} \end{cases}$$

$x[n]\delta[n-2]$ (Remain the value of $x[n]$ with $n=2$)



1.36 Determine which of the following systems is linear:

(a) $y(t) = e^{x(t)}$

$\because e^{kx(t)} = \left(e^{x(t)}\right)^k$

$= (y(t))^k \neq k\,y(t)$

$\therefore$ $y(t)$ doesn't satisfy homogeneity.

$\therefore$ $y(t)$ isn't linear. #

(b) $y(t) = \cos(x(t))$

$\because \cos(kx(t))$

$\neq k\cos(x(t)) = ky(t)$

$\therefore y(t)$ doesn't satisfy homogeneity.

$\therefore$ <u>$y(t)$ isn't linear.</u> #

(C) $y(t)=t^2x(t)$

① $\because$ $t^2 \cdot kx(t) = k(t^2x(t))$

$= ky(t)$

$\therefore$ $y(t)=t^2x(t)$ satisfies homogeneity.

② $\because$ $t^2x_1(t) + t^2x_2(t)$

$= y_1(t) + y_2(t)$

$\therefore$ $y(t)=t^2x(t)$ satisfies additivity.

③ By ① & ②,

<u>$y(t)$ is linear.</u> #

1.39 Determine whether the following systems are causal or noncausal, memoryless or with memory.

(a) $y(t) = e^{x(t)} \sin t$

$\because$ $y(t)$ depends only on the present values and doesn't depend on the future values of the input $x(t)$.

$\therefore$ <u>$y(t)$ is causal and memoryless.</u> #

(b) $y(t) = \int_0^t x(\tau)\tau \, d\tau$

By fundamental theorem of calculus, $\forall x(t), \exists X(t)$ s.t. $X'(t) = x(t)\tau$ i.e.

$$y(t) = \int_0^t x(\tau)\tau d\tau = X(t) - X(0) = X(t) - C.$$

$\because$ $y(t)$ depends on neither the future nor the past values but the present values of the antiderivative of the input $x(t)$.

$\therefore$ <u>$y(t)$ is causal and memoryless.</u> #