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ISIMA

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QUBO for Track Finding

M2 Internship Project

Major: **Computer of Science**

(Course: ICS Master 2)

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UCA - 2024

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Chapter 1

Track particle finding problem

1.1 Problem definition

Track finding is a combinatorial optimization problem, given a set of signals in space, reconstructing particle trajectories subject to smoothness constraints [Peterson, 1989].

Idea:

Data:

- L: number of layers
- $l, 1 \leq l \leq L$: layer l, and generate plane $(x, y, z)^l$
- $d_{i,j}$: distance between 2 layers i, j
- $H_l, 0 \leq l \leq L$: number of hits for layer l
- h_l: set of hits for layer l
- T: number of tracks

Variables:

- $t, 1 \leq t \leq T$: track t
- $p_l^t = (x, y, z)_l^t = (x_l^t, y_l^t, z_l^t)$: particle p of track t in layer l with coordinates in 3D space corresponding to $(x, y, z)_l^t$

Let $\overrightarrow{v^{i,j}_{l,l+1}}$ is the vector from p^i_l to p^j_{l+1}:

$$
\begin{aligned}
\overrightarrow{v^{i,j}_{l,l+1}} &= \overrightarrow{(p^i_l, p^j_{l+1})} \\
&= \left((x^j_{l+1} - x^i_l), (y^j_{l+1} - y^i_l), (z^j_{l+1} - z^i_l)\right) \\
&= \left((x^j_{l+1} - x^i_l), (y^j_{l+1} - y^i_l), d_{l+1,l}\right) \\
&= \left(dx^{j,i}_{l+1,l}, dy^{j,i}_{l+1,l}, dz_{l+1,l}\right)
\end{aligned}
\tag{1.1}
$$

We want to calculate the angle of 2 vectors $\overrightarrow{v^{i,j}_{l-1,l}}, \overrightarrow{v^{i,j}_{l,l+1}}$.

Formulation:

$$
\begin{aligned}
&Min \sum_{i=1}^{T} \sum_{j=1}^{T} \sum_{k=1}^{T} \sum_{l=2}^{L-1} cos(\overrightarrow{v^{i,j}_{l-1,l}}, \overrightarrow{v^{j,k}_{l,l+1}}) \\
&= Min \sum_{i=1}^{T} \sum_{j=1}^{T} \sum_{k=1}^{T} \sum_{l=2}^{L-1} \frac{\overrightarrow{v^{i,j}_{l-1,l}} \bullet \overrightarrow{v^{j,k}_{l,l+1}}}{\left|\overrightarrow{v^{i,j}_{l-1,l}}\right| \cdot \left|\overrightarrow{v^{j,k}_{l,l+1}}\right|} \\
&= Min \sum_{i=1}^{T} \sum_{j=1}^{T} \sum_{k=1}^{T} \sum_{l=2}^{L-1} \frac{\overrightarrow{(p^i_{l-1}, p^j_l)} \bullet \overrightarrow{(p^j_l, p^k_{l+1})}}{\left|\overrightarrow{(p^i_{l-1}, p^j_l)}\right| \cdot \left|\overrightarrow{(p^j_l, p^k_{l+1})}\right|} \\
&= Min \sum_{i=1}^{T} \sum_{j=1}^{T} \sum_{k=1}^{T} \sum_{l=2}^{L-1} \frac{dx^{j,i}_{l,l-1} dx^{k,j}_{l+1,l} + dy^{j,i}_{l,l-1} dy^{k,j}_{l+1,l} + dz_{l,l-1} dz_{l+1,l}}{\sqrt{(dx^{j,i}_{l,l-1})^2 + (dy^{j,i}_{l,l-1})^2 + (dz_{l,l-1})^2} \sqrt{(dx^{k,j}_{l+1,l})^2 + (dy^{k,j}_{l+1,l})^2 + (dz_{l+1,l})^2}}
\end{aligned}
\tag{1.2}
$$

Where: $\bullet$ is scalar product

Constraints:

1. The angle between 2 vectors must be greater than 0 and less than or equal to π:

$$-1 \leq cos(\overrightarrow{v^{i,j}_{l-1,l}}, \overrightarrow{v^{j,k}_{l,l+1}}) < 1, \forall l = 2, ..L-1, \forall i,j,k = 1,...,T$$

2. Only one vector from a particle of track i in layer l to a particle of track j in layer $l+1$:

$$\sum_{j=1}^{T} |\overrightarrow{v^{i,j}_{l,l+1}}| = 1, \forall i = 1,...,T, \forall l = 1,..L-1$$

3. Only one particle for a track:

$$\sum_{l=1}^{L} p_l^i = 1, \forall i = 1, ..., T$$

4. Number of particles for a layer:

$$p_l^i \in h_l, \sum_{i=1}^{T} p_l^i = H_l, \forall l = 1, ..., L$$

Idea 2:

Data:

- L: number of layers
- $l, 1 \leq l \leq L$: layer l, and generate plane $(x,y,z)^l$
- $d_{i,j}$: distance between 2 layers i, j
- $H_l, 0 \leq l \leq L$: number of hits for layer l
- h_l: set of hits for layer l
- T: number of tracks

Variables:

- $t, 1 \leq t \leq T$: track t
- $p_l^t = (x,y,z)_l^t = (x_l^t, y_l^t, z_l^t)$: particle p of track t in layer l with coordinates in 3D space corresponding to $(x,y,z)_l^t$

Let:

- $d(p_l^i, p_{l+1}^j)$ is a distance from particle p_l^i to p_{l+1}^j.
- $(p_l^i + 1)^*$ is a projection of p_l^i onto layer $l+1$.

- $\alpha_{l,l+1}^{i,j}$ is an angle between vector from p_l^i to p_{l+1}^j and vector from p_l^i to $(p_l^i)^*$.

$$\begin{aligned}
\alpha_{l,l+1}^{i,j} &= arccos\left(\frac{d(p_l^i,(p_{l+1}^i)^*)}{d(p_l^i,p_{l+1}^j)}\right) \\
&= arccos\left(\frac{z_{l+1}^i - z_l^j}{d(p_l^i,p_{l+1}^j)}\right) \\
&= arccos\left(\frac{z_{l+1} - z_l}{d(p_l^i,p_{l+1}^j)}\right) \\
&= arccos\left(\frac{z_{l+1} - z_l}{\sqrt{(x_l^i - x_{l+1}^j)^2 + (y_l^i - y_{l+1}^j)^2 + (z_l^i - z_{l+1}^j)^2}}\right)
\end{aligned} \tag{1.3}$$

Target: Minimize all angles $\alpha_{l,l+1}^{i,j}$:

$$Min \sum_{i=1}^{T}\sum_{j=1}^{T}\sum_{l=2}^{L-1} arccos\left(\frac{z_{l+1} - z_l}{d(p_l^i,p_{l+1}^j)}\right) \tag{1.4}$$

Constraints:

1. The angle between 2 vectors must be greater than 0 and less than or equal to $\frac{\pi}{2}$:

$$0 <\leq arccos\left(\frac{z_{l+1} - z_l}{d(p_l^i,p_{l+1}^j)}\right) \leq \frac{\pi}{2}, \forall l = 1,..L-1, \forall i,j = 1,...,T$$

2. Only one vector $(\overrightarrow{p_l^i,p_{l+1}^j})$ from a particle of track i in layer l to a particle of track j in layer $l+1$:

$$\sum_{j=1}^{T} |(\overrightarrow{p_l^i,p_{l+1}^j})| = 1, \forall i = 1,...,T, \forall l = 1,..L-1$$

3. Only one particle for a track:

$$\sum_{l=1}^{L} p_l^i = 1, \forall i = 1,...,T$$

4. Number of particles for a layer:

$$p_l^i \in h_l, \sum_{i=1}^{T} p_l^i = H_l, \forall l = 1,...,L$$

Bibliography

[Peterson, 1989] Peterson, C. (1989). Track finding with neural networks. *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, 279(3):537–545.