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Finite Markov Chains

With a New Appendix
"Generalization of a Fundamental Matrix"

With 12 Illustrations

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