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Beyond Bayesian Accuracy: Skill, Abduction, and the Free Energy Principle in Normative Rationality

Original Research

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Abstract

This paper challenges traditional accuracy-centric accounts of rationality by synthesising the Free Energy Principle (FEP) with Charles Peirce's pragmatist epistemology. Whereas the FEP frames cognition as a biological imperative to minimise surprise through predictive models, we argue that its normative force emerges when integrated with Peircean abduction and skill-based metrics. By reinterpreting rationality through skill scores—Peirce's 1884 method for evaluating rare-event predictions—we demonstrate that survival-driven inference prioritises context-sensitive skill over abstract accuracy. The FEP's variational free-energy minimisation aligns with abduction's dynamic conjecture-making, revealing rationality as a pragmatic negotiation between organismic survival and environmental complexity. Critically, we show that Bayesian accuracy measures (e.g., Kullback–Leibler divergence) fail to capture the adequacy conditions for skillful forecasting, whereas Peirce's skill score satisfies constraints such as error weighting and directionality. This fusion of FEP and pragmatism advances a naturalistic-normative framework in which rationality is grounded in adaptive, enactive inference rather than idealised coherence, bridging computational neuroscience and theoretical biology with philosophical accounts of inquiry.

Keywords Normative rationality · Free energy principle · Pragmatism · Abduction · Accuracy · Skill scores

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1 Introduction

1.1 The Limits of Accuracy-Centric Rationality

Traditional views of rationality often emphasise logical consistency, optimisation, and accuracy as hallmarks of a rational agent (Hájek, 2008; Hartmann, 2020; Leitgeb & Pettigrew, 2010a, b). However, recent advancements in computational neuroscience, particularly through the Free Energy Principle (FEP), suggest that rationality emerges from biological imperatives. This paper elucidates the sense of rationality that arises in the context of these developments, examining the role of FEP in the process of probabilistic inference and its significance for maximising an agent's survival in a dynamically uncertain world (Beni & Pietarinen, 2021; Parr & Pezzulo, 2021; Pietarinen & Beni, 2021; Seddon, 2022). According to these proposals, to justify the norms of rationality, one must draw on the ability and skill of self-organising systems to maximise their survival by minimising the discrepancy between their generative models and the environment.

The Free Energy Principle (FEP), developed by Karl Friston and colleagues (Friston, 1994, 2010; Palm et al., 2015), has sparked lively philosophical and scientific debates. It has been applied to problems involving complex systems, the brain, cognition, life, and meaning and has even been applied in simple physical systems in the universe (Aguilera et al., 2022; Baltieri et al., 2020; Klein, 2018; Ramstead et al., 2020; van Es & Hipolito, 2020). Against this backdrop, a key contribution of this paper is to expose the reasons why FEP is operationalised in rational agents' probabilistic inferences and decision-making.

1.2 FEP and Pragmatism: A New Synthesis

Our proposal builds on prior engagements with accounts of the cognitive and neurophysiological mechanisms underlying FEP. We also consider the broader implications of this framework for understanding rational action (Beni & Pietarinen, 2021; Pietarinen & Beni, 2021; Seddon, 2022). We argue that FEP, when viewed through the lens of scientific discovery and prediction, aligns with Charles Peirce's distinction between skill and accuracy. Peirce's original work on skill scores in 1880 provides a framework for evaluating accuracy in successful predictions. Inspired by Peirce's work, this paper unearths an alternative accuracy measure to accompany norms of rationality, which is based on the score matrix that estimates the true skill of the forecasting agent—the guesser at the scientific hypotheses or the bet-maker in rational decisions.

Moreover, Peirce famously defended and justified the third mode of reasoning alongside deduction and induction, which he termed abduction. Abduction has since become a benchmark mode of inference, attesting that Bayesian explanations of rationality are insufficient and do not go into the roots of the matter. An interesting tension is thus seen to arise between the justification of probabilism, the origins of accuracy measures from their historical perspective, and the justification of abductive inferences. We argue that this tension only seemingly points to conflicting conclusions.

The paper's pragmatist perspective resonates with our application of the FEP, expanding the formal depiction of cognitive processes beyond the rigid boundaries of pure Bayesianism. This expansion is achieved through the adoption of active inference under the FEP for the analysis of rationality, allowing us a broad insight into the intricate biological mechanisms enabling survival within fluctuating environments. We cultivate this insight by tendering a yet more general perspective to rationality, namely one that sprouts from Peirce's pragmatist insights into the reasons why accuracy alone is not sufficient and why accuracy in fact fails to capture the full breadth of rational action. Rather, the ability to make skillful predictions, particularly in the context of uncertain and rare events, is of paramount importance.

The paper is structured as follows: Sect. 2 addresses the central question of how rationality can be justified. Section 3 introduces our pragmatist proposal. Section 4 explains the FEP, while Sect. 5 links it to abduction. Section 6 integrates Peirce's skill score metrics, highlighting how they complement the FEP. The conclusion in Sect. 7 ties these elements together, advancing a unified account of rationality.

2 The Justification of Rationality Question

2.1 Bayesianism's Survival Imperative

Bayes' theorem specifies the conditional probability of a hypothesis given the evidence on the basis of the unconditional probability of the hypothesis. Where H stands for the hypothesis and E represents the evidence, the rule is as follows:

$$\mathrm{p}(\mathrm{H} \mid \mathrm{E}) = \mathrm{p}(\mathrm{H})$$

The fundamental question posed by this simple formula in the context of rational behaviour is as follows: why is it that the degrees of belief of a rational agent *must* conform to probability functions modelled by Bayes' theorem?

Although the classification is not exhaustive, there are generally two kinds of arguments that may be invoked to justify Bayesianism in formal epistemology: the Dutch Book argument and the Expected (epistemic) Utility argument. The Dutch Book theorem, as stated by Bruno de Finetti ([1937] 1992), indicates that unless an agent conforms to the norms of probability calculus, there will be a Dutch Book against the agent. The Dutch Book consists of several bets, which, although those bets may individually be fair, taken collectively guarantee the agent's loss.

2.2 Epistemic Utility Vs. Biological Pragmatism

In the present paper, we focus on justifications of Bayesianism in terms of the epistemic utility argument. Framing our argument in terms of expected utilities instead of the Dutch Book argument has no impact on the discussion, given Hajek's (2008) conclusions that all justifications for probability have the same form: *if the agents' credences do not conform to norms of probability*, *then their rationality will be impugned in some way or another.* The epistemic utility argument accounts for norms of rationality by representing functions of credence stated in terms of utility functions. Utility functions contribute to maximising the expected utility of the agent by assuming that any violation of norms of probability calculus would be irrational for an agent with a utility function that aims at maximising the expected utility (Pettigrew, 2016, p. 19). Specifically, according to Leitgeb & Pettigrew's (2010a, b) statement, the utility argument may be grounded upon norms of *accuracy*.[1]

Given Hajek's (2008) demonstration that all justifications for probability have the same form and assuming that any sound defence of norms of rationality must be indifferent to using expected utility or the Dutch book, we argue that Bayesianism is justifiable on the basis of the survival value of natural processes that ground the theory. Arguing that Bayesianism is justifiable on the basis of the survival value of natural processes is indifferent to the use of either the Dutch book argument or the expected utility argument because the survival value argument does not rely on any specific measure of epistemic utility but simply states that Bayesian belief updating is a natural process that has evolved because it has adaptive virtues. The proposal is more general than both the expected utility argument and the Dutch book argument, which both rely on specific measures of epistemic utility.

The general insight here is that the Dutch Book argument can be meaningfully integrated into our pragmatist framework by recasting its central demand for coherence through the lens of free-energy minimisation; the demand—that an agent's credences must be coherent to avoid a sure loss—is precisely what free-energy minimisation enforces. Any incoherent set of credences would result in unbounded variational free energy, a state an organism cannot afford metabolically. In this view, a Dutch Book scenario corresponds to a failure to minimise surprise, which would trigger a runaway energy expenditure. For example, consider an organism—for example, a predator—with an internal generative model of its environment. Its model incorrectly assigns a low probability to a common, energy-rich prey species being present in its hunting ground. This is an incoherent credence, as it contradicts actual environmental statistics. When a predator encounters this prey, its sensory data are highly surprising because its internal model does not predict it. This shows that incoherent credences are not merely a formal error but an immediate metabolic liability. This further demonstrates how bounded agents can adopt fast, frugal heuristics that approximate Bayesian coherence without incurring prohibitive computational costs, thereby sidestepping sure-loss gambits under real-world constraints, given that cognition is not about passively representing a pregiven world but about an organism's active, sensorimotor engagement with its environment. Therefore, within an embodied, ecologically embedded FEP, the Dutch Book criterion emerges as an energetic necessity and transcends a purely mental or logical inconsistency; it manifests as a failure of an embodied agent to successfully regulate its exchanges with its environment, where incoherent beliefs cause a breakdown in the agent's capacity for autopoiesis—its ability to actively maintain itself. Thus, the Dutch Book scenario becomes a dramatisation of a system being driven into unstable states from which it cannot recover, a fate that is fundamentally contrary to the FEP's core premise of a system striving to maintain its nonequilibrium steady state. Minimising free energy and maintaining coherent beliefs are two sides of the same survival-driven coin: when belief updating is cast as free-energy minimisation within such a system, the Dutch Book argument becomes an integral principle ensuring both coherence and resource-efficient inference.

[1] The Epistemic Utility argument is centred on the notion of accuracy. Epistemic agents desire accurate beliefs, because accurate beliefs are more likely to lead to success in achieving their epistemic goals.The argument assumes that "an epistemic agent ought to approximate the truth. In other words, she ought to minimize her inaccuracy" (Leitgeb & Pettigrew, 2010b, p. 209). To address normative aspects of Bayesianism, Leitgeb and Pettigrew speak about local and global *expected* accuracy, with synchronic and diachronic bifurcations.

In sum, we at this point gesture that Bayesianism is a finessed form of underpinning inference that organic systems invoke to minimise the divergence between states that they can actually occupy and those that they believe (or anticipate) that they must occupy to maximise their survival (Beni & Pietarinen, 2021; Pietarinen & Beni, 2021; Seddon, 2022). Let us turn to a more specific, abductive charge.

3 Naturalism, Pragmatism, and Scientific Reasoning

3.1 Abduction as a Cognitive Dynamic

To explicate norms of probabilism in a naturalistic framework is to remain broadly loyal to the spirit of Peirce's pragmatistic epistemology, which, from our perspective, scores well in answering the worries voiced on the presumed rationality of Bayesian models (Colombo et al., 2021).

Naturalism and pragmatism meet at the level of scientific reasoning. In the late 19th century, Peirce developed a theory of the logic of science, including nondeductive reasoning, one of which he termed *abduction*. Although his views on this topic matured over the decades, the core idea is the one that rests on the agent's faculties of conjecture-making that revises the system of preexisting conjectures in a given situation or model of its environment. Peirce's mature formulation of abduction's inferential schema captures that core idea: abduction is "reasoning from surprise to inquiry" (Ma & Pietarinen, 2017). As such, inference portrays retroduction: "Given a (surprising) fact C, if A (subjunctively) implies C, then it *is to be inquired whether* A plausibly holds." Like abduction elsewhere in Peirce's writings, this late schema also begins with an observation of a surprising fact or event (or, as in experimental research, is usually the case, an 'event class'), and through a conditional major premise imparts, in a *cohortative mood* that Peirce termed "the investigand", the conclusion that something ought to be done. Initially, aroused by curiosity, which leads to the formulation of the first premise of inquiry, the abductive schema encapsulates the dynamic process of reason, striving toward assertions of anticipated conjectures. Loyal to Peircean insights, the philosophical account of abduction can indeed be anchored to a scientific theory of cognitive inference.

3.2 FEP's Role in Inquiry

One such framework is FEP, which provides a comprehensive perspective on how living systems, including cognitive processes, operate. This suggests that living systems, including the human mind, seek to minimise free energy or surprises in their interactions with the environment. This principle posits that organisms aim to reduce the discrepancy between their internal models of the world and the external sensory data they receive. In the context of abduction, this principle implies that the human mind, when faced with uncertain or novel situations, strives to generate hypotheses or conjectures that minimise the discrepancy between its current understanding and the observed data.

The issue addressed in this paper is that while the FEP offers a compelling framework for understanding cognitive processes and the minimisation of uncertainty, it does not, in and of itself, suffice as a justification for rationality. First, while the FEP provides a powerful mathematical framework—grounded in concepts such as nonequilibrium steady states and Markov blankets—critics often highlight the apparent inapplicability of these physics-derived formalisms to the disordered, context-sensitive reality of biological systems (Colombo & Palacios, 2021; Colombo & Wright, 2018). They argue that FEP's assumptions, such as ergodicity, may oversimplify the nongeneric properties of living organisms, creating a significant tension between its mathematical elegance and its biological realism.

This criticism, however, arises from a misconception of the FEP's general role in theoretical biology. It is not an empirical hypothesis about biology but a formal modelling language. Its scientific utility is not determined by the intrinsic valences of 'truth' or 'falsity' but by the way the language is applied to a specific domain (Andrews, 2021). The challenge, therefore, is not whether mathematical tools are applicable to cognition at all but how to correctly use them. As such, the FEP's explanatory power depends on careful calibration of its models to balance the trade-off between generality, realism, and precision. This approach, which Beni (2022) describes as achieving the 'right dosage,' ensures that FEP models are neither overly abstract nor so detailed that they lose their unifying explanatory force. In this view, the quantitative approach is not abandoned but refined, with model builders explicitly defining the level of abstraction and the empirical constraints needed for a plausible explanation (Beni, 2022).

The claim that an organism acts to minimise free energy does not necessarily account for why this should be considered a rational process in any normative or pragmatic sense. Minimising surprise or error might explain how organisms, including human agents, manage to stay within viable biological bounds and maintain homeostasis, but it does not explain why this tendency should be equated with rational decision-making. Thus, FEP should be considered a genuine foundation for rationality, particularly when the cognitive strategies involved in abduction and inference are considered. We address this issue in the next sections by drawing on Peirce's concept of "the investigand," which essentially communicates a scientific urge never to stop questioning and exploring: continue active inquiry, and one is guaranteed to reduce unwanted uncertainty.

Against this backdrop, in the remainder of this paper, we draw on theoretical and historical argumentation to flesh out the following insight: the ability to choose suitable information measures (including the prior probabilities) involves the capacity to make informed (intellectual, scientific, insightful, and skilful) guesses at unknown features of reality. These guesses are, in turn, informed by the guessing agent's ability to reduce uncertainty. Currently, active inference is a natural counterpart to what noninductive ampliative inferences do, especially those that can formally be characterised as abduction. Integral to our argumentation is that the notion of surprise (such as the strength of surprise in terms of novelty, unexpectedness, doubt and irritation, among other context-dependent influences and their varying degrees of intensity) plays a role in determining the pathways through which new information contributes to reasoners' probabilities conditioned on evidence.

4 Free Energy Minimisation as Biological Rationality

4.1 Predictive Processing and State Management

The brain hypothesises on the expectations of the likely causes of the data it receives. When applied to the brain, predictive processing (PP) theory indicates that the brain is in the business of using approximate Bayesian mechanisms to optimise its posterior beliefs on the basis of sensory information. The brain's prior beliefs about the world are registered in the brain's generative models. Generative models register the likelihood of data given their causes, as well as prior beliefs about the causes. The brain predicts the states of the world and modifies and updates its generative models on the basis of the received sensory inputs. PP specifies perceptions in terms of inverted models of the likelihood of causes of sensations—assuming that they are retroductive (i.e., formally abductive; Pietarinen & Beni, 2021) inferences or mappings from sensations to causes.

Similarly, FEP promises profound insights into the unified core of life, perception, cognition, and action. In summary, FEP provides a mathematical formulation of how adaptive systems can stay in nonequilibrium steady states. Under normal circumstances, the environment is constantly changing, and organic systems must resist the dispersing forces of the changing environment by keeping the amount of their sensory entropy low. Entropy is the average surprise, which is defined as the negative log probability of an outcome. Free energy is an information-theoretic measure that bounds the surprise given some generative model (Friston, 2010, p. 127). The agent must keep its internal entropy low relative to the states that it occupies in spacetime to maximise its survival.

The bearing on this paper's topic is that the FEP, as a guiding theory in computational neuroscience, posits that biological systems, including human cognitive agents, act in ways that minimise free energy or uncertainty about the world around them. By doing so, these systems maintain a form of internal coherence while actively engaging with their environments. Through the lens of FEP, rationality is reconceived not only as a sublime mathematical notion but also as a process deeply intertwined with survival. Agents are understood to form probabilistic inferences about the causal structure of their environments, enabling them to act adaptively in a manner that secures their continued existence. This process, although grounded in basic biological mechanisms, scales up to encompass sophisticated intellectual enterprises that require the agent to make predictions and decisions in complex and ambiguous circumstances.

The FEP perspective introduces a novel way to integrate these two senses of rationality: the sublime and the mundane. The *sublime* mathematical formulation of free energy minimisation closely parallels probabilistic inference mechanisms. In contrast, the *mundane* aspect concerns the practical logic of optimising survival. These two forms of rationality are mutually reinforcing. The capacity for forming accurate probabilistic inferences about the environment provides a fundamental cognitive grip on causality. However, this grip is not an end in itself but serves the larger goal of survival optimisation, a form of rationality rooted in the practical demands of existence. We term the reinforcing situation "agential corationality."

Although the FEP depends on a variational Bayesian framework to express its theoretical content, the theoretical content itself cannot be reduced to mere Bayesian formalism. The content *designates* the contribution of minimising uncertainty to maximising survival. To integrate the concept of enhancing cognitive models with biological realities, the FEP suggests that organisms that have survived (and are fit) have succeeded in reducing the inaccuracy of their models of the environment. When considered from a normative standpoint, we propose that, to survive, organisms must develop accurate representations of the spatiotemporal structure by minimising predictive errors under the FEP. Some agents can estimate the precision of their expectations about the world as well as their inferences about the hidden states of the world. For such agents, relative entropy is decomposed into the divergence[2] between entropy and expected utility. This speaks to the distinction between intrinsic and extrinsic rewards in embodied cognition (Friston et al., 2013). The idea of the divergence or uncertainty that must be minimised is at the very heart of PP-FEP. The immaculate variational Bayesian framework developed by Friston and colleagues aims at capturing an organism's tendency to minimise its uncertainty.

4.2 Markov Blankets and Embodied Boundaries

The discrepancy between the active and sensory states of an organism increases the uncertainty in the cognitive system. This is not a purely formal notion but a theoretical one that, under a realist interpretation, may refer to the states of affairs in the real world. However, modern science usually employs mathematical tools to represent its theoretical content, and PP-FEP is no exception. As such, distinction or conditional independence is represented by Markovian models in the context of FEP (Hipólito et al., 2021; Kirchhoff, 2018; Ramstead et al., 2020). Markovian blankets are Bayesian network models: the Markov blanket of a node consists of the node's parents, children, and coparents of its children. In the context of FEP, Markov blankets are introduced to show how the internal states of all kinds of self-organising systems—from macromolecules to organisms and societies—can be distinguished from their external states. Markov blankets provide models of biophysical boundaries in embodied systems. These boundaries consist of sensory states and active states whose patterns of dependencies are identifiable based on their spatial locations, provided that Markov blankets are embodied in biophysical systems (Palacios et al., 2020). Because the inside and outside of self-organising structures are statistically independent, they can regulate the amount of their sensory entropy through manipulation of their internal states.

We observe that the conditional independence between internal and external states is *regimented* into a Bayesian account of the maximisation of model evidence or marginal likelihood. However, the theoretical content is not Markov blankets or any mathematical Bayesian fact. In realist reading, theoretical content refers to natural facts (Beni, 2021; Bruinberg et al., 2022). The formalism of Markov blankets is used to regiment representations of the way that mechanisms of perception/cognition are related to fundamental biological mechanisms of self-reorganising complex systems.

The core idea of constructors is similar to the models arising from the agential perspective to active inferences, as both accommodate *conditional expectations* (i.e., likelihoods of the hypothesis on evidence) of what the future states would or could be like, given the evidence accumulated in agential generative models. Not only are there expectations about future states; organisms can actually find improved ways of choosing policies that satisfy the fulfilment of their expectations in the future. At any event, the conclusion of active inference is a conjecture about the feasibility of adopting certain strategy profiles that the organism entertains over time integrals. Expressed in the form of our natural language, the anticipatory conditional states that, "If I *were* to exist in the next state, then I *would* display certain classes of properties…" Such statements involve reference to real possibilities of what would, could, or might be the case in the states of affairs that are conceivable for the

[2] This is called Kullback–Leibler (KL) divergence, which is a measure of the difference between the approximate posterior distribution and the true posterior distribution.

agent. This, in other words, is a paraphrase of the kernel of Peirce's pragmatist perspective to rationality, as we argue in the final section of this paper.

First, let us recap. The mechanisms of Bayesian inference that an organism applies to represent the structure of the world to itself are grounded in the dynamics of the organism's informational interaction with the environment through its statistically definitive borders. In this fashion, via PP-FEP, the mechanisms of *bounded rationality* can be grounded in existential facts about the survival of the organism. Bounded rationality is defined in terms of variational Bayesian inferences that optimise free energy based on model evidence. Theory can be represented formally, but the content needs not be confused with the formalism that is used for representational purposes.

In the next section, we argue that the expected utility defence of norms of rationality can be effectively grounded in the FEP. This is because the FEP suggests that living systems, including cognitive processes, aim to minimise surprise or free energy, which can be seen as a way to minimise uncertainty or inaccuracy in their internal models of the world. In the context of rational decision-making, this aligns with the notion that rational agents should strive to make choices that maximise their expected epistemic utility. In this context, epistemic utility can be viewed as a measure of how beneficial a belief or action is for the acquisition of knowledge.

5 Abduction and the Pragmatist Turn

The normative aspects of probabilism can be explained in terms of the organism's natural tendency to frequently occupy the same state. The capacity to stay in nonequilibrium steady states underlies the coherence of the organism's beliefs and behaviours. The theoretical content of PP-FEP indicates that by believing and behaving coherently, agents whose relative entropy levels are constrained can maximise their survival. (This assumes that relative entropy is decomposable into the divergence between entropy and expected utility; see Friston et al., 2013.)

5.1 From Surprise To Hypothesis

In our pragmatist approach, we deconstruct the elevated conception of rationality's norms into the fundamental aspects of an organism's state occupancy. This mildly reductionist perspective centers on the assumption that the organism *must* occupy some states more than others to maximise its survival. This means that the organism *must* decrease the divergence between the set of distributions of probabilities in states that it *should* occupy (the sense of 'should' is here evolutionary, cashed out in terms of a descent in adaptive behaviour, see Beni, 2020). Therefore, conforming to probabilistic calculus is just a manifestation of the natural tendency to minimise uncertainty by fixing coherent conjectures (Peirce's 'investigands'), resulting in generalizable habits of acting in certain ways in certain kinds of circumstances (Peirce's pragmaticism). The main insight here is the grounding of norms of rationality in the cognitive mechanisms delineated by the FEP, which underscores that living systems, including cognitive entities, are inherently driven to minimise free energy or surprise, which is synonymous with reducing uncertainty.

In the context of rationality norms, the imperative of minimising uncertainty is paramount. This is because high levels of uncertainty can lead to inaccurate judgments and decisions, which, in turn, can have adverse consequences for an organism's ability to navigate its environment and attain its goals, which explains how, by forming coherent beliefs and utilising them in decision-making, individuals reduce uncertainty and, consequently, enhance their capacity for rational and effective action. To support this general claim, we build upon recent attempts at grounding inferential thinking in the FEP framework. For example, Seddon (2022) proposed psychology and neuroscience to reinforce Lipton's (2004) account of inference to the best explanation (IBE). An outline of Seddon's argument is as follows (Seddon, 2022, p. 3):

> P1. Best-guess theory aims to explain how the brain interprets information. The brain uses a "process very similar to abduction to make best-guess interpretations of sensory information."
> P2. "There is considerable empirical support from both psychology and neuroscience for Best-guess theory."
> P3. "[H]uman sensory systems are the result of hundreds of millions of years of evolution, they are very good at interpreting sensory information.

From P1, P2, and P3, it follows that:

> C1. Without conscious effort, our brains use a process remarkably similar to abduction as a powerful way of interpreting sensory information.
> P4. "Lipton's (2004) IBE theory argues that scientists use abduction when creating theories to explain phenomena in the real world."

From C1 and P4, it follows that

> C2. Lipton is right to claim that scientists use abduction and explanatory inferences.

Overall, the argument purports to indicate that "IBE theory is likely to be valid because it proposes that theory building relies on abduction, and brain research shows that a process very similar to abduction is a powerful way of interpreting sensory information." Seddon's application of scientific evidence from best-guess theory (drawn from the context of predictive coding theory, see Friston, 2012; Rao & Ballard, 1999) meshes with Lipton's (2004) Bayesian account of explanatory inferences. The fact that versions of best-guess theory have recently been proven useful in shedding light on similar philosophical discussions may reinforce this optimism. (For a similar scientifically informed defence of Lipton's account, see Beni, 2019, p. 135 ff., and for an application of Bayesian accounts of cognition to Peirce's account of abduction, see Beni & Pietarinen, 2021).

5.2 Skill Vs. Accuracy in Inference

Similarly, it is worth exploring how the FEP's account of living systems, including cognitive entities, which are inherently driven to minimise free energy or surprise, aligns with traditional justifications of rationality in terms of accuracy. The FEP posits that organisms aim to reduce the divergence between their internal models and the external world, thereby enhancing their cognitive grip on the environment. This is achieved through the minimisation of relative entropy, a measure of the discrepancy between desired states and attainable states over time (Friston et al., 2013). This approach closely resembles theories of bounded rationality, where an organism, utilising Bayesian inference, can optimise the free energy bound on model evidence. By minimising this bound, the organism effectively enhances the accuracy of its predictions, ensuring that its cognitive models better align with environmental contingencies.

However, while this framework provides a sophisticated account of how living systems increase accuracy and reduce uncertainty, it does not fully answer the primary question posed by this paper: why should rationality, in a normative sense, be justified by the processes through which self-organising systems increase the accuracy of their models? Although the FEP explains how organisms act to optimise their survival by refining their internal models, this account remains semi-mechanistic and does not, by itself, offer a compelling rationale for why these processes should be considered rational in a human sense. In the following section, we strive to develop a pragmatist response to this question, drawing on Peirce's insights into abduction, skill, and the nature of the inquiry to suggest that rationality, when understood pragmatically, involves more than mere accuracy—it requires skillful and context-sensitive engagement with the environment, aimed at achieving successful outcomes in a complex and uncertain world.

6 Skill Scores Over Accuracy: Adaptive Norms of Rationality

We are now ready to address the main question of the paper: What compelling reasons do we have that agents actually use FEP in the process of forming probabilistic inferences that maximise their survival in the world? In the pursuit of an answer, we draw on the works of Peirce, whose contributions to the philosophy of abduction and scientific reasoning offer crucial insights into how we might ground FEP's account of minimising free energy in a broader, pragmatic theory of rationality. Our general insight is that Peirce's concept of abductive reasoning provides a practical complement to the FEP. Abduction is the process of generating explanatory hypotheses in the face of novel or unexpected phenomena. We argue that rationality emerges through the generation of cognitive models that not only strive for accuracy but also reflect deeper engagement with uncertainty and error correction.

6.1 From Accuracy to Skill

Abduction does not guarantee truth or accuracy in any immediate or pressing senses. The abductive mode of inference serves as a guiding principle for further inquiry. Previous work has developed this insight into a pragmatic defence of abduction reinforced by theoretical findings from the FEP literature. Here, we argue that Peirce's approach offers a critical refinement: rationality involves not only forming hypotheses that reduce discrepancy but also developing the skill to do so in ways that are sensitive to the complexity of the environment and the consequences of error (Beni & Pietarinen, 2021). This idea speaks directly to the scientific idea that the minimisation of free energy, as posited by FEP, can be understood as a biological imperative that drives the organism to maintain stable conditions for survival. However, from a rational standpoint, it is not merely the reduction in free energy that justifies an agent's actions; rather, it is the ability to adapt those actions to the complexity and unpredictability of the world. This, too, can be explained in terms of the pragmatic assessment of which hypotheses are worth pursuing, which errors can be tolerated, and how the trade-offs between accuracy and other cognitive goals (such as efficiency and adaptability) are negotiated (Pietarinen & Beni, 2021).

This insight provides leverage for arguing that the FEP's measure of accuracy is supplemented by a deeper account of how organisms achieve rationality through their capacity for skilled inference. It is not enough for an agent to minimise free energy in a mechanistic sense; the agent must also demonstrate the capacity to navigate the uncertainties of the environment in a way that reflects a deeper cognitive engagement with the world. This engagement is precisely what Peirce's account of abduction and skill elucidates: rationality is not about reducing surprise but about doing so in a way that is sufficiently context sensitive, error correct, and pragmatically justified in wider and strategic senses of justification. This justification of rationality within the FEP framework demands recognition of the role that skill plays in managing uncertainty and guiding logical inferences.

Our proposal brings together a sublime mathematical (probabilistic) notion of rationality and a mundane notion that concerns the logic of optimising survival. From a pragmaticist point of view, these two senses of rationality are interdependent and noncircular. On the one hand, the mechanism of minimising free energy is similar to processes of forming probabilistic inferences that allow the organism to obtain a cognitive grip on the causal structure of its environment. As rudimentary as this capacity for cognitive grip on causes and effects may be, it also provides the basis for a sophisticated form of probabilistic inference applied in complicated forms of intellectual enterprises.

6.2 Peirce Skill Score: Metrics and Adequacy

Here follows our plot twist. Accuracy measures that were adopted much later to justify the rationality of Bayesian reasoning originated from Peirce's paper published in *Science* in 1884. The Kullback–Leibler (KL) divergence measure is actually among many alternative accuracy measures from the late 19th century, including one that Peirce proposed in his 1884 note, which is now known as the Peirce skill score (PSS), and variously also known as the true skill statistic, Hanssen–Kuiper discriminant, or, in medical diagnostics, Youden's index. The threshold that maximises relevant statistical indices is in the PSS calculated as the coincidence of the event's posterior probability with its prior probability. The highest PSS point can be used as a measure of the classifier's skill as the point that maximises the value of the prediction:

$$\mathrm{PSS} = \frac{\text{Hits}}{\text{Total events}} - \frac{\text{False alarms}}{\text{Total nulls}} = \text{Hit rate} - \text{False alarm rate} \tag{1}$$

As a 2×2 matrix, PSS is calculated with four values: $(ad - bc)/[(a+c)\,(b+d)]$, where a is the number of positive forecasts and positive occurrences of an event (hits), b positive forecasts and negative occurrences (false positives), *c* negative forecasts and positive occurrences (false negatives), and *d* negative forecasts and negative occurrences of the event (hits).[3]

The PSS is still considered to be the best scoring table method for measuring, among other factors, the forecast accuracies of significant weather events (Manzato, 2007). Ebert & Milne (2021) argue that "the Peirce Skill Score is the only score that meets all three adequacy constraints" for rare events. These adequacy constraints include the following three requirements.

1. Skilled predictions are *better than chance*; that is, the set of predictions is an improvement in the levels of predictions obtained from the source of random chance. Consequently, the quality of skill leads to a reduction in both Type 1 (false positive) and Type 2 (false negative) statistical error classes.[4]
2. Skilled predictions have the right *direction of fit*, namely, predictions are evaluated against the standard of events that actually occurred, not by having events match predictions.
3. Skilled prediction tables *are sensitive to error weighting*. Hasty prediction has consequences and, occasionally, even severe consequences. The scoring method that mitigates Type 2 errors compared with the method that makes equally proportionate mitigation of Type 1 errors can be argued to be more valuable and have less repair cost associated with it.

These constraints, one needs to emphasise, also hold for rare and severe events. This is not generally the case for the other accuracy measures. We submit that the essential epistemological element in significant scientific discoveries is t ocome up with hypotheses that are both hard to come by and impactful. The purpose of scientific inquiry is not to extol

[3] As we argue, justifications of rationality should revoke inadequate accuracy measures and look for scores that represent the *skill* of the forecasting agent, not merely the accuracy of various types of expectations. This desideratum is missing from Leitgeb & Pettigrew's (2010a, b) account but is imperative to block unwanted results that follow from adopting an ultraconservative strategy for achieving any proportion-correct score – as Peirce famously noticed a long time ago, "an ignoramus in tornado studies can predict no tornadoes for a whole season, and obtain an average of fully ninety-five per cent" (G, 1884, p. 126). The converse can be noticed to obtain for any such strategy to minimise inaccuracy (see fn 4 below). No risk, no suprisal, no FEP to be applied.

[4] Therefore, PSS is immune to the paradoxes of maximising accuracy and minimising inaccuracy. Charitably assuming that synchronic and diachronic expected local forms of accuracy, respectively, suffice to justify probabilism and conditionalization, accuracy would indicate that the agent is compelled to update its beliefs by valid rules of inference. Leitgeb and Pettigrew (2010a, b) did not validate the main assumption of the detailed justification of Bayesianism. Their justification draws on an assumption about the agent's tendency to approximate the truth. Approximating truth has been operationalised in terms of minimising inaccuracy. Why does the agent need to do that? This question was not addressed by Leitgeb & Pettigrew (2010b). Nor do they justify their basic assumption (Leitgeb & Pettigrew, 2010a, p. 215). However, to paraphrase the previous quip from Peirce, the opposite of the paradox of accuracy also arises: pending further constraints on minimising inaccuracy; "an ignoramus in tornado studies can predict tornadoes for a whole season, and obtain an average of meagre five per cent" inaccuracy, as one now minimises the inaccuracy of one's predictions well below the level of an expert who correctly predicts rare events more often than not. The ability to justify assumptions about agents' tendencies to approximate the truth is assessed outside the domain of Leitgeb and Pettigrew's particular piece of formal reasoning. This means that the particular piece of formalism adopted is too narrow to offer a full story about norms of rationality. We argue that this proposal indeed leaves much to be desired since the Brier score, which Leitgeb and Pettigrew adopted as the preferred accuracy measure, turns out to be an inadequate measure for rare events owing to its low resolution for small differences in predicting rare and unexpected occurrences.

competent but largely random guesses; the purpose is to match conjectures to the way the world may be while catching errors by systematic probing. Good statistical inference subjects propositions to severe testing: a properly conducted method of efficient and organised experimentation follows a protocol that *would* find a claim false if it indeed were false, with high probability. Such methods can be trusted to catch false claims: they attempt to probe enough of the ways in which *we could be wrong* about the proposition put forth. Good protocols and research agendas are those that expedite discovery and thus economise the process by following such a method of error correction.

6.3 Foundations of Skill: from History to Neurophysiology

On a historical note, Peirce did not explicitly connect his 1884 skill score metrics with the qualities of abductive reasoning. However, the mere fact that both have stood the test of the development of modern scientific methods so well is evidence that the PSS is the right tool for evaluating the quality of guessing in scientific abduction. After all, abduction strives to conclude that there is reason for further action to be taken regarding what is asserted in its conclusion. This is provoked whenever scientists encounter surprising phenomena, anomalies, unforeseen events, or other cerebral frictions and cognitive irritations (aided by non-epistemic principles such as the *economy of research*, Chiffi et al., 2020; Chiffi & Pietarinen, 2019; Mukhopadhyay, 2022).

From this pragmatist perspective, the norms that justify rationality when rare and cognitively weighty ideas and hypotheses are being guessed at the bottom cannot be the kinds of norms derived from measures of (expected) *accuracy.* They have to be norms derived from the predictor's *skill* and must be allied with adequacy conditions such as those framed above by (1)–(3). Accuracy alone achieves better results than chance does; it has no preferred direction of fit, and it assigns no weights to its error classes, which manifestly do have nonsymmetric reward structures.

In contrast, among the measures that are well calibrated to compute rare-event accuracies, PSS is not only the original measurement metric provided for such cases but also represents the *only* calculation that satisfies all three adequacy constraints.

This insight is particularly compatible with an enactivist interpretation of the FEP and its measure of accuracy. Enactivism posits that cognition is not merely a matter of internal representation but is rooted in the organism's dynamic interaction with its environment Kirchhoff & Robertson, 2018; Korbak, 2021). When applied to the context of the present discussion, enactivism is congenial to the idea that rationality cannot be justified solely by abstract accuracy metrics. Rather than treating accuracy as an isolated criterion, enactivism emphasises how organisms skillfully engage with their surroundings, using embodied practices that allow them to actively generate meaning and adapt to novel situations. This active, embodied engagement mirrors Peirce's emphasis on predictor skill, where rationality is not simply a matter of matching predictions to outcomes but of skilfully navigating uncertainty in a way that reflects the organism's ongoing involvement with its environment.

To return to the context of Peirce's work, a common limitation of measures of accuracy and skill is that they are typically binary: either the prediction matches the event, or it does not: the hypothesis turns out to be true in the guise of the theory or it is rejected by systematic probing of falsification. In reality, conjectures are asserted in their promissory and indefinite dressing, where 'strictly true' and 'strictly false' are the extremal values assigned to a future state of those conjectured assertions.

Three remarks on this charge are in order. First, hypotheses that have made it through an abductive reasoning filter do sufficiently attract the inquirer's mind such that scientists are typically invited to give them a try: even though their probabtive conjectures may well be wrong, something might remain from the process of deriving them for later uses and the progress of inquiry. Such hypotheses possess, in Peirce's terms, "young truth," cryptic qualities that ought to attract our careful attention beyond their status as possibilities or mere *may-be*s (Pietarinen, 2021).

Second, multicategory extensions of scoring methods are available for nonbinary events (Rodwell, 2011). This is good news: their fundamental qualities tend to remain the same as those of binary classifiers. PSS, for example, is among those metrics that can indeed be expanded to cover more than two categories, the fact that Peirce stated in his 1884 paper and while he did not prove that assertion in the published paper, a consistent nonbinary extension was developed in Rodwell (2011). On the other hand --- and here is another crucial point of our criticism of accuracy --- those other measures of accuracy, such as the Brier score or Kullback–Leibler (KL) divergence, do not have this property and hence remain inadequate measures for assessing rare-event occurrences when generalised to three or more categories. For such limitative reasons, accuracy measures, despite having prominently been proposed in the recent literature to justify norms of rationality, ought to be approached with caution as proper measures of accuracy when evaluating the inquirer's true skill as a discoverer.

Third, the generalisation of the PSS to three-category cases is sufficient to cover all multicategory cases since the third category can be taken to tally all those guesses that have an 'undefined' or 'undetermined' value.[5] In general, extensions to multicategory cases apply *equitability constraints* in skill scoring (Gandin & Murphy, 1992; Joyce, 1998). Equitablity constraints guarantee fair differentiation between the scoring of two unskilled (random, static, noisy) systems on the one hand and the cases in which one of the unskilled systems receives a certain amount of skill and thus should fairly and equitably score above the other, on the other hand. In brief, Peirce's original result both justifies and is justified using equitability constraints in numerical estimations of skilful predictions.

Recalling the preceding section, whatever qualities and constraints a good notion of skill we ultimately take the predictors and guessers to possess, the faculties required for genuinely skilful performance have much to do with the development and performance of agents' neurophysiological mechanisms. As explained, such mechanisms are the omphalos of the project of naturalising probabilistic reasoning (Seddon, 2022, p. 3) (On recent work specifically on abduction, predictions, and neuroscience, see e.g. Cevolani, 2023; Coraci et al., 2022; Costa et al., 2022; Vallverdú & Sans Pinillos, 2022). Moreover, neurophysiological facts are at play in meeting the three adequacy constraints outlined above. For example, fine-grained synaptic plasticity addresses uncertainty by forecasting the error bars in addition to weights, which is conducive to succeeding with predictions above chance levels. Second, these mechanisms serve to fit predictions to the world rather than the other way around, which other proposed measures do not accomplish. Third, evolution has ensured that the brain errs on the side of false negatives. Above all, fine-graining guarantees sensitivity to small differences in noisy data when predicting weak perceptual phenomena, as

[5] The technique is the same as in partial models in logic where the third category is not strictly speaking a separate value but the residual that the binary model leaves undetermined.

supported by theories of neuronal group voting, the 'thousand-brains' hypothesis, and other similar and recent findings (Hawkins et al., 2019).

The moral from Peirce's 1884 schema is that distinguishing successful forecasts, accurate predictions, and correct conjectures that may result from statistical chance, ill directions of fit, or insensitivity to error-weighing, from genuinely skilful performances that produce such results, is of perennial importance to the long-term success of anticipations, predictions, and optimisations. Accuracy measures alone meet none of the natural adequacy constraints for truly skilful scoring. Among the many skill scores proposed during the century, only the PSS has been shown to meet these constraints. To repeat the lesson in the context of the present paper once more, it is not the hit-rate success, accuracy, or correctness but skill, coupled with the relevant adequacy constraints (1)–(3), that justify the adoption of probabilities in rational decision making.

One last point. A verification of the goodness of the classifier is the business of signal-detection theory (SDT), which dates back to Swets et al. (1961) and Green and Swets (1966). SDT aims at verifying the goodness of a classifier, a problem with no shortage of applications ranging from deep learning, image processing, medical diagnostics and psychological testing to forecasting and forensic sciences. SDT has proven to be a promising methodology for computing optimal inferential thresholds (McNicol, 1972). The early goal was to statistically understand the mechanisms by which the human perceptual system is able to decide and report upon the reception of signals amidst insignificant noise. Interestingly, the origins of SDT also date back to Peirce's early experimental work on perception with Jastrow (Peirce & Jastrow, 1884) and their famous rebuttal of the Weber–Fechner discrete threshold principle in psychophysics. They argued that the threshold principle relies on unviable Laplacian principles of probability, notably the principle of indifference. In its place, proper procedures of statistical inference must be adopted to evaluate claims of non-continuous threshold functions that aim at detecting real signals from background noise. In SDT, the procedure involves the development of the notion of an *ideal observer*. It is this insight that SDT shares with the Peirce skill score (PSS): a skilful inquirer aims at matching predictions to the world; a task in which an ideal or omniscient being, free from all bias and noise, would succeed without failure. This involves setting up counterfactuals, as in abduction. In the PSS, the perfect predictor has the expected value of 1, and the score is rewritten as $1 - [(c/a+c)] - [b/b+d]]$. (To recap, a is the number of predicted and observed events; b is predicted but not observed; c is not predicted but observed; d is the number of not predicted and not observed events; a generalisation of this formalisation to nonbinary category cases is straightforward.) The conceptual connection between the two methods should come as no surprise, given that Peirce wrote and published his skill score paper in the same year (1884), no less than a month apart when his pioneering psychophysical experiments with his student Joseph Jastrow on small differences in sensations were reported for the first time.

Accuracy measures fail to justify probabilism. Whether we can do so, we are first drawn to the late 19th century fountain of ideas that had tapped into the relevant insights early on. Moreover, the FEP itself has known antecedents in Hermann von Helmholtz's work (Helmholtz, 1866), which inspired Peirce's critical appraisal of discovery through the development of statistical and logical methods. The theory of abduction matured in Peirce's work throughout the late 19th century. What abductive reasoning does, and what the PSS-type accuracy matrices that track the forecaster's skills add, to the question of the justification of prior probabilities in probabilistic foundations for rationality is that Bayes's compressed formula actually signifies multiple convergent historical and systematic storylines that strive to make explicit the dynamics of how the priors arise and how they undergo change into posterity.

7 Rationality Recast: Pragmatist-Naturalist Synthesis

This paper was concerned with the justification of probabilism. We developed a general pragmatist-naturalist perspective on the question of norms of rationality. To reinforce our pragmatist-naturalist justification of rationality, we consolidated the justification of rationality on the basis of the survival value of concrete evolutionary mechanisms captured by FEP formalism. The shift in view now becoming non-Bayesian, naturalist justification, from coupling priors and their update with abductive reasoning, we are led to a skill measure originally defined as the Peirce skill score to supplant faulty measures of accuracy previously proposed to justify the rationality of probabilism. The result is a historically informed, pragmatist–naturalist justification of norms of rationality, in which skill and not accuracy takes priority.

The proposed synthesis of the FEP and Peircean pragmatism reorients rationality from static coherence to dynamic, adaptive skill. By grounding norms in survival-driven inference and contextualised skill metrics, we resolve the tension between biological imperatives and normative epistemology. The FEP's variational mechanisms, when coupled with abduction's conjecture-making and the Peirce skill score, reveal rationality as an enactive negotiation between environmental complexity and organismic constraints. This framework not only challenges Bayesian accuracy-centric models but also invites interdisciplinary dialogue: cognitive science gains a biologically grounded account of inference, whereas philosophy reclaims rationality as a pragmatic, evolutionary achievement. Future work could operationalise this approach in AI ethics, decision neuroscience, or cross-cultural epistemology—domains where skill, uncertainty, and adaptive forecaster success converge. Ultimately, rationality emerges not as a rulebook but as the lived capacity to thrive in a world of surprises.

Author Contributions Authorship statement
•All authors whose names appear on the submission made substantial contributions to the conception or design of the work; the acquisition, analysis, and interpretation of data; drafted the work and revised it critically for important intellectual content; approved the version to be published; and agree to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

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Declarations

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