

12

First, let’s find all the edges defined in the question.

I used isprime(n) function in MATLAB to check prime number.

The following is the code I wrote in MATLAB:

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%% HW3 for MATH381

% @Author: Baihan Lin, @Date: Oct 2016

clear all; close all; clc;

% initialization

max = 12;

V = [1:max];

E = [];

% get edges and the corresponding graph appears

for j = 1:max

for k = j+1:max

if (isprime(j^2+k^2+j\*k+4))

E = [E;[j k]];

end

end

end

% show edges

E

--------------------------------------------------------

Here is the output from MATLAB console:

--------------------------------------------------------

E =

1 2

1 3

1 6

1 7

1 11

2 3

2 5

2 7

2 9

2 11

3 4

3 5

3 6

3 7

3 8

3 11

3 12

4 7

4 9

5 7

5 10

5 12

6 7

6 11

7 8

7 9

7 10

7 11

7 12

8 11

9 12

11 12

--------------------------------------------------------

Then we do the IP formulation:

Now we have this graph *G* = (*V, E*) with 12 vertices.

Suppose we have a set of *n* colors to use to color the graph (we might not use all of them).

Define *yi* ∈{0, 1} *i* = 1, …, 12, with *yi* = 1 iff we use color *i*.

Define *xik* ∈{0, 1} *i* = 1, …, 12, with *xik* = 1 iff vertex *i* will have color *k*

Then our IP is this:

Minimize subject to



In order to generate necessary input for lp\_solve, under the inspiration of Dr. Conroy’s demo code on Knights’ Domination Problem, I wrote following code in Perl (lpGenerator.pl).

**Note:** Here I used another way to check prime in this Perl code: for each I divide it by all integers smaller than itself starting from 2, and check whether there is any divisible factor.

--------------------------------------------------------

#!/usr/local/bin/perl

# Baihan Lin, Oct 2016

#

# A perl script to generate an lpsolve-format IP (integer program)

$max=12;

## print out objective function

print "min:";

for($i=1; $i<$max+1; $i++) {

print "+y\_",$i;

}

print ";\n";

**## constraint 1**

for($j=1; $j<$max+1; $j++) {

for($i=1; $i<$max+1; $i++) {

print "+x\_",$j,"\_",$i;

}

print "=1;\n"

}

**## constraint 2**

for($j=1; $j<$max+1; $j++) {

for($i=1; $i<$max+1; $i++) {

print "x\_",$j,"\_",$i," -", "y\_",$i,"<=0;\n";

}

}

**## constraint 3**

for($j=1; $j<$max+1; $j++) {

for($i=$j+1; $i<$max+1; $i++) {

# prime number check

$n=$j\*$j+$i\*$i+$i\*$j+4;

$d=0; # 0 is prime, 1 is not

for($c=2;$c<$n;$c++) {

if($n%$c==0) {

$d=1;

break;

}

}

if($d==0){

for($k=1; $k<$max+1; $k++) {

print "x\_",$j,"\_",$k," +", "x\_",$i,"\_",$k,"<=1;\n";

}

}

}

}

**## constraint 4**

## specify that all variables are binary

print "bin ";

for($i=1; $i<$max+1; $i++) {

if ($i>1) { print ","; } ## prepend a comma first

print "y\_",$i;

}

for($j=1; $j<$max+1; $j++) {

for($i=1; $i<$max+1; $i++) {

print ",x\_",$j,"\_",$i;

}

}

print ";\n";

--------------------------------------------------------

Here is the output from this perl code I stored into lp3.txt:

**Note:** the output is realigned to be multiple column in Microsoft Word to avoid too many pages.

--------------------------------------------------------

min:+y\_1+y\_2+y\_3+y\_4+y\_5+y\_6+y\_7+y\_8+y\_9+y\_10+y\_11+y\_12;

+x\_1\_1+x\_1\_2+x\_1\_3+x\_1\_4+x\_1\_5+x\_1\_6+x\_1\_7+x\_1\_8+x\_1\_9+x\_1\_10+x\_1\_11+x\_1\_12=1;

+x\_2\_1+x\_2\_2+x\_2\_3+x\_2\_4+x\_2\_5+x\_2\_6+x\_2\_7+x\_2\_8+x\_2\_9+x\_2\_10+x\_2\_11+x\_2\_12=1;

+x\_3\_1+x\_3\_2+x\_3\_3+x\_3\_4+x\_3\_5+x\_3\_6+x\_3\_7+x\_3\_8+x\_3\_9+x\_3\_10+x\_3\_11+x\_3\_12=1;

+x\_4\_1+x\_4\_2+x\_4\_3+x\_4\_4+x\_4\_5+x\_4\_6+x\_4\_7+x\_4\_8+x\_4\_9+x\_4\_10+x\_4\_11+x\_4\_12=1;

+x\_5\_1+x\_5\_2+x\_5\_3+x\_5\_4+x\_5\_5+x\_5\_6+x\_5\_7+x\_5\_8+x\_5\_9+x\_5\_10+x\_5\_11+x\_5\_12=1;

+x\_6\_1+x\_6\_2+x\_6\_3+x\_6\_4+x\_6\_5+x\_6\_6+x\_6\_7+x\_6\_8+x\_6\_9+x\_6\_10+x\_6\_11+x\_6\_12=1;

+x\_7\_1+x\_7\_2+x\_7\_3+x\_7\_4+x\_7\_5+x\_7\_6+x\_7\_7+x\_7\_8+x\_7\_9+x\_7\_10+x\_7\_11+x\_7\_12=1;

+x\_8\_1+x\_8\_2+x\_8\_3+x\_8\_4+x\_8\_5+x\_8\_6+x\_8\_7+x\_8\_8+x\_8\_9+x\_8\_10+x\_8\_11+x\_8\_12=1;

+x\_9\_1+x\_9\_2+x\_9\_3+x\_9\_4+x\_9\_5+x\_9\_6+x\_9\_7+x\_9\_8+x\_9\_9+x\_9\_10+x\_9\_11+x\_9\_12=1;

+x\_10\_1+x\_10\_2+x\_10\_3+x\_10\_4+x\_10\_5+x\_10\_6+x\_10\_7+x\_10\_8+x\_10\_9+x\_10\_10+x\_10\_11+x\_10\_12=1;

+x\_11\_1+x\_11\_2+x\_11\_3+x\_11\_4+x\_11\_5+x\_11\_6+x\_11\_7+x\_11\_8+x\_11\_9+x\_11\_10+x\_11\_11+x\_11\_12=1;

+x\_12\_1+x\_12\_2+x\_12\_3+x\_12\_4+x\_12\_5+x\_12\_6+x\_12\_7+x\_12\_8+x\_12\_9+x\_12\_10+x\_12\_11+x\_12\_12=1;

x\_1\_1 -y\_1<=0;

x\_1\_2 -y\_2<=0;

x\_1\_3 -y\_3<=0;

x\_1\_4 -y\_4<=0;

x\_1\_5 -y\_5<=0;

x\_1\_6 -y\_6<=0;

x\_1\_7 -y\_7<=0;

x\_1\_8 -y\_8<=0;

x\_1\_9 -y\_9<=0;

x\_1\_10 -y\_10<=0;

x\_1\_11 -y\_11<=0;

x\_1\_12 -y\_12<=0;

x\_2\_1 -y\_1<=0;

x\_2\_2 -y\_2<=0;

x\_2\_3 -y\_3<=0;

x\_2\_4 -y\_4<=0;

x\_2\_5 -y\_5<=0;

x\_2\_6 -y\_6<=0;

x\_2\_7 -y\_7<=0;

x\_2\_8 -y\_8<=0;

x\_2\_9 -y\_9<=0;

x\_2\_10 -y\_10<=0;

x\_2\_11 -y\_11<=0;

x\_2\_12 -y\_12<=0;

x\_3\_1 -y\_1<=0;

x\_3\_2 -y\_2<=0;

x\_3\_3 -y\_3<=0;

x\_3\_4 -y\_4<=0;

x\_3\_5 -y\_5<=0;

x\_3\_6 -y\_6<=0;

x\_3\_7 -y\_7<=0;

x\_3\_8 -y\_8<=0;

x\_3\_9 -y\_9<=0;

x\_3\_10 -y\_10<=0;

x\_3\_11 -y\_11<=0;

x\_3\_12 -y\_12<=0;

x\_4\_1 -y\_1<=0;

x\_4\_2 -y\_2<=0;

x\_4\_3 -y\_3<=0;

x\_4\_4 -y\_4<=0;

x\_4\_5 -y\_5<=0;

x\_4\_6 -y\_6<=0;

x\_4\_7 -y\_7<=0;

x\_4\_8 -y\_8<=0;

x\_4\_9 -y\_9<=0;

x\_4\_10 -y\_10<=0;

x\_4\_11 -y\_11<=0;

x\_4\_12 -y\_12<=0;

x\_5\_1 -y\_1<=0;

x\_5\_2 -y\_2<=0;

x\_5\_3 -y\_3<=0;

x\_5\_4 -y\_4<=0;

x\_5\_5 -y\_5<=0;

x\_5\_6 -y\_6<=0;

x\_5\_7 -y\_7<=0;

x\_5\_8 -y\_8<=0;

x\_5\_9 -y\_9<=0;

x\_5\_10 -y\_10<=0;

x\_5\_11 -y\_11<=0;

x\_5\_12 -y\_12<=0;

x\_6\_1 -y\_1<=0;

x\_6\_2 -y\_2<=0;

x\_6\_3 -y\_3<=0;

x\_6\_4 -y\_4<=0;

x\_6\_5 -y\_5<=0;

x\_6\_6 -y\_6<=0;

x\_6\_7 -y\_7<=0;

x\_6\_8 -y\_8<=0;

x\_6\_9 -y\_9<=0;

x\_6\_10 -y\_10<=0;

x\_6\_11 -y\_11<=0;

x\_6\_12 -y\_12<=0;

x\_7\_1 -y\_1<=0;

x\_7\_2 -y\_2<=0;

x\_7\_3 -y\_3<=0;

x\_7\_4 -y\_4<=0;

x\_7\_5 -y\_5<=0;

x\_7\_6 -y\_6<=0;

x\_7\_7 -y\_7<=0;

x\_7\_8 -y\_8<=0;

x\_7\_9 -y\_9<=0;

x\_7\_10 -y\_10<=0;

x\_7\_11 -y\_11<=0;

x\_7\_12 -y\_12<=0;

x\_8\_1 -y\_1<=0;

x\_8\_2 -y\_2<=0;

x\_8\_3 -y\_3<=0;

x\_8\_4 -y\_4<=0;

x\_8\_5 -y\_5<=0;

x\_8\_6 -y\_6<=0;

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x\_8\_8 -y\_8<=0;

x\_8\_9 -y\_9<=0;

x\_8\_10 -y\_10<=0;

x\_8\_11 -y\_11<=0;

x\_8\_12 -y\_12<=0;

x\_9\_1 -y\_1<=0;

x\_9\_2 -y\_2<=0;

x\_9\_3 -y\_3<=0;

x\_9\_4 -y\_4<=0;

x\_9\_5 -y\_5<=0;

x\_9\_6 -y\_6<=0;

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x\_9\_8 -y\_8<=0;

x\_9\_9 -y\_9<=0;

x\_9\_10 -y\_10<=0;

x\_9\_11 -y\_11<=0;

x\_9\_12 -y\_12<=0;

x\_10\_1 -y\_1<=0;

x\_10\_2 -y\_2<=0;

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x\_12\_8 -y\_8<=0;

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x\_12\_11 -y\_11<=0;

x\_12\_12 -y\_12<=0;

x\_1\_1 +x\_2\_1<=1;

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x\_1\_7 +x\_11\_7<=1;

x\_1\_8 +x\_11\_8<=1;

x\_1\_9 +x\_11\_9<=1;

x\_1\_10 +x\_11\_10<=1;

x\_1\_11 +x\_11\_11<=1;

x\_1\_12 +x\_11\_12<=1;

x\_2\_1 +x\_3\_1<=1;

x\_2\_2 +x\_3\_2<=1;

x\_2\_3 +x\_3\_3<=1;

x\_2\_4 +x\_3\_4<=1;

x\_2\_5 +x\_3\_5<=1;

x\_2\_6 +x\_3\_6<=1;

x\_2\_7 +x\_3\_7<=1;

x\_2\_8 +x\_3\_8<=1;

x\_2\_9 +x\_3\_9<=1;

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x\_2\_11 +x\_3\_11<=1;

x\_2\_12 +x\_3\_12<=1;

x\_2\_1 +x\_5\_1<=1;

x\_2\_2 +x\_5\_2<=1;

x\_2\_3 +x\_5\_3<=1;

x\_2\_4 +x\_5\_4<=1;

x\_2\_5 +x\_5\_5<=1;

x\_2\_6 +x\_5\_6<=1;

x\_2\_7 +x\_5\_7<=1;

x\_2\_8 +x\_5\_8<=1;

x\_2\_9 +x\_5\_9<=1;

x\_2\_10 +x\_5\_10<=1;

x\_2\_11 +x\_5\_11<=1;

x\_2\_12 +x\_5\_12<=1;

x\_2\_1 +x\_7\_1<=1;

x\_2\_2 +x\_7\_2<=1;

x\_2\_3 +x\_7\_3<=1;

x\_2\_4 +x\_7\_4<=1;

x\_2\_5 +x\_7\_5<=1;

x\_2\_6 +x\_7\_6<=1;

x\_2\_7 +x\_7\_7<=1;

x\_2\_8 +x\_7\_8<=1;

x\_2\_9 +x\_7\_9<=1;

x\_2\_10 +x\_7\_10<=1;

x\_2\_11 +x\_7\_11<=1;

x\_2\_12 +x\_7\_12<=1;

x\_2\_1 +x\_9\_1<=1;

x\_2\_2 +x\_9\_2<=1;

x\_2\_3 +x\_9\_3<=1;

x\_2\_4 +x\_9\_4<=1;

x\_2\_5 +x\_9\_5<=1;

x\_2\_6 +x\_9\_6<=1;

x\_2\_7 +x\_9\_7<=1;

x\_2\_8 +x\_9\_8<=1;

x\_2\_9 +x\_9\_9<=1;

x\_2\_10 +x\_9\_10<=1;

x\_2\_11 +x\_9\_11<=1;

x\_2\_12 +x\_9\_12<=1;

x\_2\_1 +x\_11\_1<=1;

x\_2\_2 +x\_11\_2<=1;

x\_2\_3 +x\_11\_3<=1;

x\_2\_4 +x\_11\_4<=1;

x\_2\_5 +x\_11\_5<=1;

x\_2\_6 +x\_11\_6<=1;

x\_2\_7 +x\_11\_7<=1;

x\_2\_8 +x\_11\_8<=1;

x\_2\_9 +x\_11\_9<=1;

x\_2\_10 +x\_11\_10<=1;

x\_2\_11 +x\_11\_11<=1;

x\_2\_12 +x\_11\_12<=1;

x\_3\_1 +x\_4\_1<=1;

x\_3\_2 +x\_4\_2<=1;

x\_3\_3 +x\_4\_3<=1;

x\_3\_4 +x\_4\_4<=1;

x\_3\_5 +x\_4\_5<=1;

x\_3\_6 +x\_4\_6<=1;

x\_3\_7 +x\_4\_7<=1;

x\_3\_8 +x\_4\_8<=1;

x\_3\_9 +x\_4\_9<=1;

x\_3\_10 +x\_4\_10<=1;

x\_3\_11 +x\_4\_11<=1;

x\_3\_12 +x\_4\_12<=1;

x\_3\_1 +x\_5\_1<=1;

x\_3\_2 +x\_5\_2<=1;

x\_3\_3 +x\_5\_3<=1;

x\_3\_4 +x\_5\_4<=1;

x\_3\_5 +x\_5\_5<=1;

x\_3\_6 +x\_5\_6<=1;

x\_3\_7 +x\_5\_7<=1;

x\_3\_8 +x\_5\_8<=1;

x\_3\_9 +x\_5\_9<=1;

x\_3\_10 +x\_5\_10<=1;

x\_3\_11 +x\_5\_11<=1;

x\_3\_12 +x\_5\_12<=1;

x\_3\_1 +x\_6\_1<=1;

x\_3\_2 +x\_6\_2<=1;

x\_3\_3 +x\_6\_3<=1;

x\_3\_4 +x\_6\_4<=1;

x\_3\_5 +x\_6\_5<=1;

x\_3\_6 +x\_6\_6<=1;

x\_3\_7 +x\_6\_7<=1;

x\_3\_8 +x\_6\_8<=1;

x\_3\_9 +x\_6\_9<=1;

x\_3\_10 +x\_6\_10<=1;

x\_3\_11 +x\_6\_11<=1;

x\_3\_12 +x\_6\_12<=1;

x\_3\_1 +x\_7\_1<=1;

x\_3\_2 +x\_7\_2<=1;

x\_3\_3 +x\_7\_3<=1;

x\_3\_4 +x\_7\_4<=1;

x\_3\_5 +x\_7\_5<=1;

x\_3\_6 +x\_7\_6<=1;

x\_3\_7 +x\_7\_7<=1;

x\_3\_8 +x\_7\_8<=1;

x\_3\_9 +x\_7\_9<=1;

x\_3\_10 +x\_7\_10<=1;

x\_3\_11 +x\_7\_11<=1;

x\_3\_12 +x\_7\_12<=1;

x\_3\_1 +x\_8\_1<=1;

x\_3\_2 +x\_8\_2<=1;

x\_3\_3 +x\_8\_3<=1;

x\_3\_4 +x\_8\_4<=1;

x\_3\_5 +x\_8\_5<=1;

x\_3\_6 +x\_8\_6<=1;

x\_3\_7 +x\_8\_7<=1;

x\_3\_8 +x\_8\_8<=1;

x\_3\_9 +x\_8\_9<=1;

x\_3\_10 +x\_8\_10<=1;

x\_3\_11 +x\_8\_11<=1;

x\_3\_12 +x\_8\_12<=1;

x\_3\_1 +x\_11\_1<=1;

x\_3\_2 +x\_11\_2<=1;

x\_3\_3 +x\_11\_3<=1;

x\_3\_4 +x\_11\_4<=1;

x\_3\_5 +x\_11\_5<=1;

x\_3\_6 +x\_11\_6<=1;

x\_3\_7 +x\_11\_7<=1;

x\_3\_8 +x\_11\_8<=1;

x\_3\_9 +x\_11\_9<=1;

x\_3\_10 +x\_11\_10<=1;

x\_3\_11 +x\_11\_11<=1;

x\_3\_12 +x\_11\_12<=1;

x\_3\_1 +x\_12\_1<=1;

x\_3\_2 +x\_12\_2<=1;

x\_3\_3 +x\_12\_3<=1;

x\_3\_4 +x\_12\_4<=1;

x\_3\_5 +x\_12\_5<=1;

x\_3\_6 +x\_12\_6<=1;

x\_3\_7 +x\_12\_7<=1;

x\_3\_8 +x\_12\_8<=1;

x\_3\_9 +x\_12\_9<=1;

x\_3\_10 +x\_12\_10<=1;

x\_3\_11 +x\_12\_11<=1;

x\_3\_12 +x\_12\_12<=1;

x\_4\_1 +x\_7\_1<=1;

x\_4\_2 +x\_7\_2<=1;

x\_4\_3 +x\_7\_3<=1;

x\_4\_4 +x\_7\_4<=1;

x\_4\_5 +x\_7\_5<=1;

x\_4\_6 +x\_7\_6<=1;

x\_4\_7 +x\_7\_7<=1;

x\_4\_8 +x\_7\_8<=1;

x\_4\_9 +x\_7\_9<=1;

x\_4\_10 +x\_7\_10<=1;

x\_4\_11 +x\_7\_11<=1;

x\_4\_12 +x\_7\_12<=1;

x\_4\_1 +x\_9\_1<=1;

x\_4\_2 +x\_9\_2<=1;

x\_4\_3 +x\_9\_3<=1;

x\_4\_4 +x\_9\_4<=1;

x\_4\_5 +x\_9\_5<=1;

x\_4\_6 +x\_9\_6<=1;

x\_4\_7 +x\_9\_7<=1;

x\_4\_8 +x\_9\_8<=1;

x\_4\_9 +x\_9\_9<=1;

x\_4\_10 +x\_9\_10<=1;

x\_4\_11 +x\_9\_11<=1;

x\_4\_12 +x\_9\_12<=1;

x\_5\_1 +x\_7\_1<=1;

x\_5\_2 +x\_7\_2<=1;

x\_5\_3 +x\_7\_3<=1;

x\_5\_4 +x\_7\_4<=1;

x\_5\_5 +x\_7\_5<=1;

x\_5\_6 +x\_7\_6<=1;

x\_5\_7 +x\_7\_7<=1;

x\_5\_8 +x\_7\_8<=1;

x\_5\_9 +x\_7\_9<=1;

x\_5\_10 +x\_7\_10<=1;

x\_5\_11 +x\_7\_11<=1;

x\_5\_12 +x\_7\_12<=1;

x\_5\_1 +x\_10\_1<=1;

x\_5\_2 +x\_10\_2<=1;

x\_5\_3 +x\_10\_3<=1;

x\_5\_4 +x\_10\_4<=1;

x\_5\_5 +x\_10\_5<=1;

x\_5\_6 +x\_10\_6<=1;

x\_5\_7 +x\_10\_7<=1;

x\_5\_8 +x\_10\_8<=1;

x\_5\_9 +x\_10\_9<=1;

x\_5\_10 +x\_10\_10<=1;

x\_5\_11 +x\_10\_11<=1;

x\_5\_12 +x\_10\_12<=1;

x\_5\_1 +x\_12\_1<=1;

x\_5\_2 +x\_12\_2<=1;

x\_5\_3 +x\_12\_3<=1;

x\_5\_4 +x\_12\_4<=1;

x\_5\_5 +x\_12\_5<=1;

x\_5\_6 +x\_12\_6<=1;

x\_5\_7 +x\_12\_7<=1;

x\_5\_8 +x\_12\_8<=1;

x\_5\_9 +x\_12\_9<=1;

x\_5\_10 +x\_12\_10<=1;

x\_5\_11 +x\_12\_11<=1;

x\_5\_12 +x\_12\_12<=1;

x\_6\_1 +x\_7\_1<=1;

x\_6\_2 +x\_7\_2<=1;

x\_6\_3 +x\_7\_3<=1;

x\_6\_4 +x\_7\_4<=1;

x\_6\_5 +x\_7\_5<=1;

x\_6\_6 +x\_7\_6<=1;

x\_6\_7 +x\_7\_7<=1;

x\_6\_8 +x\_7\_8<=1;

x\_6\_9 +x\_7\_9<=1;

x\_6\_10 +x\_7\_10<=1;

x\_6\_11 +x\_7\_11<=1;

x\_6\_12 +x\_7\_12<=1;

x\_6\_1 +x\_11\_1<=1;

x\_6\_2 +x\_11\_2<=1;

x\_6\_3 +x\_11\_3<=1;

x\_6\_4 +x\_11\_4<=1;

x\_6\_5 +x\_11\_5<=1;

x\_6\_6 +x\_11\_6<=1;

x\_6\_7 +x\_11\_7<=1;

x\_6\_8 +x\_11\_8<=1;

x\_6\_9 +x\_11\_9<=1;

x\_6\_10 +x\_11\_10<=1;

x\_6\_11 +x\_11\_11<=1;

x\_6\_12 +x\_11\_12<=1;

x\_7\_1 +x\_8\_1<=1;

x\_7\_2 +x\_8\_2<=1;

x\_7\_3 +x\_8\_3<=1;

x\_7\_4 +x\_8\_4<=1;

x\_7\_5 +x\_8\_5<=1;

x\_7\_6 +x\_8\_6<=1;

x\_7\_7 +x\_8\_7<=1;

x\_7\_8 +x\_8\_8<=1;

x\_7\_9 +x\_8\_9<=1;

x\_7\_10 +x\_8\_10<=1;

x\_7\_11 +x\_8\_11<=1;

x\_7\_12 +x\_8\_12<=1;

x\_7\_1 +x\_9\_1<=1;

x\_7\_2 +x\_9\_2<=1;

x\_7\_3 +x\_9\_3<=1;

x\_7\_4 +x\_9\_4<=1;

x\_7\_5 +x\_9\_5<=1;

x\_7\_6 +x\_9\_6<=1;

x\_7\_7 +x\_9\_7<=1;

x\_7\_8 +x\_9\_8<=1;

x\_7\_9 +x\_9\_9<=1;

x\_7\_10 +x\_9\_10<=1;

x\_7\_11 +x\_9\_11<=1;

x\_7\_12 +x\_9\_12<=1;

x\_7\_1 +x\_10\_1<=1;

x\_7\_2 +x\_10\_2<=1;

x\_7\_3 +x\_10\_3<=1;

x\_7\_4 +x\_10\_4<=1;

x\_7\_5 +x\_10\_5<=1;

x\_7\_6 +x\_10\_6<=1;

x\_7\_7 +x\_10\_7<=1;

x\_7\_8 +x\_10\_8<=1;

x\_7\_9 +x\_10\_9<=1;

x\_7\_10 +x\_10\_10<=1;

x\_7\_11 +x\_10\_11<=1;

x\_7\_12 +x\_10\_12<=1;

x\_7\_1 +x\_11\_1<=1;

x\_7\_2 +x\_11\_2<=1;

x\_7\_3 +x\_11\_3<=1;

x\_7\_4 +x\_11\_4<=1;

x\_7\_5 +x\_11\_5<=1;

x\_7\_6 +x\_11\_6<=1;

x\_7\_7 +x\_11\_7<=1;

x\_7\_8 +x\_11\_8<=1;

x\_7\_9 +x\_11\_9<=1;

x\_7\_10 +x\_11\_10<=1;

x\_7\_11 +x\_11\_11<=1;

x\_7\_12 +x\_11\_12<=1;

x\_7\_1 +x\_12\_1<=1;

x\_7\_2 +x\_12\_2<=1;

x\_7\_3 +x\_12\_3<=1;

x\_7\_4 +x\_12\_4<=1;

x\_7\_5 +x\_12\_5<=1;

x\_7\_6 +x\_12\_6<=1;

x\_7\_7 +x\_12\_7<=1;

x\_7\_8 +x\_12\_8<=1;

x\_7\_9 +x\_12\_9<=1;

x\_7\_10 +x\_12\_10<=1;

x\_7\_11 +x\_12\_11<=1;

x\_7\_12 +x\_12\_12<=1;

x\_8\_1 +x\_11\_1<=1;

x\_8\_2 +x\_11\_2<=1;

x\_8\_3 +x\_11\_3<=1;

x\_8\_4 +x\_11\_4<=1;

x\_8\_5 +x\_11\_5<=1;

x\_8\_6 +x\_11\_6<=1;

x\_8\_7 +x\_11\_7<=1;

x\_8\_8 +x\_11\_8<=1;

x\_8\_9 +x\_11\_9<=1;

x\_8\_10 +x\_11\_10<=1;

x\_8\_11 +x\_11\_11<=1;

x\_8\_12 +x\_11\_12<=1;

x\_9\_1 +x\_12\_1<=1;

x\_9\_2 +x\_12\_2<=1;

x\_9\_3 +x\_12\_3<=1;

x\_9\_4 +x\_12\_4<=1;

x\_9\_5 +x\_12\_5<=1;

x\_9\_6 +x\_12\_6<=1;

x\_9\_7 +x\_12\_7<=1;

x\_9\_8 +x\_12\_8<=1;

x\_9\_9 +x\_12\_9<=1;

x\_9\_10 +x\_12\_10<=1;

x\_9\_11 +x\_12\_11<=1;

x\_9\_12 +x\_12\_12<=1;

x\_11\_1 +x\_12\_1<=1;

x\_11\_2 +x\_12\_2<=1;

x\_11\_3 +x\_12\_3<=1;

x\_11\_4 +x\_12\_4<=1;

x\_11\_5 +x\_12\_5<=1;

x\_11\_6 +x\_12\_6<=1;

x\_11\_7 +x\_12\_7<=1;

x\_11\_8 +x\_12\_8<=1;

x\_11\_9 +x\_12\_9<=1;

x\_11\_10 +x\_12\_10<=1;

x\_11\_11 +x\_12\_11<=1;

x\_11\_12 +x\_12\_12<=1;

bin y\_1,y\_2,y\_3,y\_4,y\_5,y\_6,y\_7,y\_8,y\_9,y\_10,y\_11,y\_12,x\_1\_1,x\_1\_2,x\_1\_3,x\_1\_4,x\_1\_5,x\_1\_6,x\_1\_7,x\_1\_8,x\_1\_9,x\_1\_10,x\_1\_11,x\_1\_12,x\_2\_1,x\_2\_2,x\_2\_3,x\_2\_4,x\_2\_5,x\_2\_6,x\_2\_7,x\_2\_8,x\_2\_9,x\_2\_10,x\_2\_11,x\_2\_12,x\_3\_1,x\_3\_2,x\_3\_3,x\_3\_4,x\_3\_5,x\_3\_6,x\_3\_7,x\_3\_8,x\_3\_9,x\_3\_10,x\_3\_11,x\_3\_12,x\_4\_1,x\_4\_2,x\_4\_3,x\_4\_4,x\_4\_5,x\_4\_6,x\_4\_7,x\_4\_8,x\_4\_9,x\_4\_10,x\_4\_11,x\_4\_12,x\_5\_1,x\_5\_2,x\_5\_3,x\_5\_4,x\_5\_5,x\_5\_6,x\_5\_7,x\_5\_8,x\_5\_9,x\_5\_10,x\_5\_11,x\_5\_12,x\_6\_1,x\_6\_2,x\_6\_3,x\_6\_4,x\_6\_5,x\_6\_6,x\_6\_7,x\_6\_8,x\_6\_9,x\_6\_10,x\_6\_11,x\_6\_12,x\_7\_1,x\_7\_2,x\_7\_3,x\_7\_4,x\_7\_5,x\_7\_6,x\_7\_7,x\_7\_8,x\_7\_9,x\_7\_10,x\_7\_11,x\_7\_12,x\_8\_1,x\_8\_2,x\_8\_3,x\_8\_4,x\_8\_5,x\_8\_6,x\_8\_7,x\_8\_8,x\_8\_9,x\_8\_10,x\_8\_11,x\_8\_12,x\_9\_1,x\_9\_2,x\_9\_3,x\_9\_4,x\_9\_5,x\_9\_6,x\_9\_7,x\_9\_8,x\_9\_9,x\_9\_10,x\_9\_11,x\_9\_12,x\_10\_1,x\_10\_2,x\_10\_3,x\_10\_4,x\_10\_5,x\_10\_6,x\_10\_7,x\_10\_8,x\_10\_9,x\_10\_10,x\_10\_11,x\_10\_12,x\_11\_1,x\_11\_2,x\_11\_3,x\_11\_4,x\_11\_5,x\_11\_6,x\_11\_7,x\_11\_8,x\_11\_9,x\_11\_10,x\_11\_11,x\_11\_12,x\_12\_1,x\_12\_2,x\_12\_3,x\_12\_4,x\_12\_5,x\_12\_6,x\_12\_7,x\_12\_8,x\_12\_9,x\_12\_10,x\_12\_11,x\_12\_12;

--------------------------------------------------------

Then I took this lp3.txt as input into lp\_solve, here is the output from lp\_solve in terminal:

**Note:** the output is realigned to be multiple column in Microsoft Word to avoid too many pages.

--------------------------------------------------------

Value of objective function: 5.00000000

Actual values of the variables:

y\_1 1

y\_2 1

y\_3 1

y\_4 1

y\_5 1

y\_6 0

y\_7 0

y\_8 0

y\_9 0

y\_10 0

y\_11 0

y\_12 0

x\_1\_1 1

x\_1\_2 0

x\_1\_3 0

x\_1\_4 0

x\_1\_5 0

x\_1\_6 0

x\_1\_7 0

x\_1\_8 0

x\_1\_9 0

x\_1\_10 0

x\_1\_11 0

x\_1\_12 0

x\_2\_1 0

x\_2\_2 1

x\_2\_3 0

x\_2\_4 0

x\_2\_5 0

x\_2\_6 0

x\_2\_7 0

x\_2\_8 0

x\_2\_9 0

x\_2\_10 0

x\_2\_11 0

x\_2\_12 0

x\_3\_1 0

x\_3\_2 0

x\_3\_3 1

x\_3\_4 0

x\_3\_5 0

x\_3\_6 0

x\_3\_7 0

x\_3\_8 0

x\_3\_9 0

x\_3\_10 0

x\_3\_11 0

x\_3\_12 0

x\_4\_1 1

x\_4\_2 0

x\_4\_3 0

x\_4\_4 0

x\_4\_5 0

x\_4\_6 0

x\_4\_7 0

x\_4\_8 0

x\_4\_9 0

x\_4\_10 0

x\_4\_11 0

x\_4\_12 0

x\_5\_1 1

x\_5\_2 0

x\_5\_3 0

x\_5\_4 0

x\_5\_5 0

x\_5\_6 0

x\_5\_7 0

x\_5\_8 0

x\_5\_9 0

x\_5\_10 0

x\_5\_11 0

x\_5\_12 0

x\_6\_1 0

x\_6\_2 1

x\_6\_3 0

x\_6\_4 0

x\_6\_5 0

x\_6\_6 0

x\_6\_7 0

x\_6\_8 0

x\_6\_9 0

x\_6\_10 0

x\_6\_11 0

x\_6\_12 0

x\_7\_1 0

x\_7\_2 0

x\_7\_3 0

x\_7\_4 1

x\_7\_5 0

x\_7\_6 0

x\_7\_7 0

x\_7\_8 0

x\_7\_9 0

x\_7\_10 0

x\_7\_11 0

x\_7\_12 0

x\_8\_1 0

x\_8\_2 1

x\_8\_3 0

x\_8\_4 0

x\_8\_5 0

x\_8\_6 0

x\_8\_7 0

x\_8\_8 0

x\_8\_9 0

x\_8\_10 0

x\_8\_11 0

x\_8\_12 0

x\_9\_1 0

x\_9\_2 0

x\_9\_3 0

x\_9\_4 0

x\_9\_5 1

x\_9\_6 0

x\_9\_7 0

x\_9\_8 0

x\_9\_9 0

x\_9\_10 0

x\_9\_11 0

x\_9\_12 0

x\_10\_1 0

x\_10\_2 0

x\_10\_3 1

x\_10\_4 0

x\_10\_5 0

x\_10\_6 0

x\_10\_7 0

x\_10\_8 0

x\_10\_9 0

x\_10\_10 0

x\_10\_11 0

x\_10\_12 0

x\_11\_1 0

x\_11\_2 0

x\_11\_3 0

x\_11\_4 0

x\_11\_5 1

x\_11\_6 0

x\_11\_7 0

x\_11\_8 0

x\_11\_9 0

x\_11\_10 0

x\_11\_11 0

x\_11\_12 0

x\_12\_1 0

x\_12\_2 1

x\_12\_3 0

x\_12\_4 0

x\_12\_5 0

x\_12\_6 0

x\_12\_7 0

x\_12\_8 0

x\_12\_9 0

x\_12\_10 0

x\_12\_11 0

x\_12\_12 0

--------------------------------------------------------

Therefore, from this solution, we know that the minimum number of colors required to color the graph when no two connected vertices are the same color is 5, in another word, the smallest number of needed colors, the chromatic number of the graph *G* is 5.

For example:

**Color 1 —** 1, 4, 5

**Color 2 —** 2, 6, 8, 12

**Color 3 —** 3, 10

**Color 4 —** 7

**Color 5 —** 9, 11