

Take $I_1(t') = 10^{-4} t'$

$$V(t) = V(0)e^{-t/\tau} + \frac{1}{\tau} \int_0^t R I_1(t') e^{-(t-t')/\tau} dt'$$

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For simplicity, let $V(0) = 0$.

$\tau = RC = 0.01$ (10 ms - from problem set)

let $R = 10^4$, $C = 10^{-6}$ (can choose anything such that $R \cdot C = 0.01$)

$$V(t) = V(0)e^{-t/\tau} + \frac{1}{\tau} \int_0^t R I_1(t') e^{-(t-t')/\tau} dt'$$

$$= 0 + \frac{1}{\tau} \int_0^t R \cdot 10^{-4} t' e^{-(t-t')/\tau} dt'$$

$$= \frac{10^{-4}}{C} \int_0^t t' e^{-t/\tau} e^{t'/\tau} dt'$$

$$= \frac{10^{-4} e^{-t/\tau}}{C} \int_0^t t' e^{t'/\tau} dt'$$

(t is a constant, 30 ms, so
we can pull $e^{-t/\tau}$ out of
the integral)

integration by parts:

$$\int u dv = uv - \int v du$$

$$u = t'$$

$$v = \tau e^{t'/\tau}$$

$$du = 1$$

$$dv = e^{t'/\tau}$$

$$V(t) = \frac{10^{-4} e^{-t/\tau}}{C} \left[\tau t' e^{t'/\tau} - \tau \int_0^{t'} e^{t''/\tau} dt'' \right]_0^t$$

$$V(t) = \frac{10^{-4} e^{-t/\tau} \tau}{C} \left[t' e^{t'/\tau} - \tau e^{t'/\tau} \right]_0^t$$

$$V(t) = \frac{10^{-4} e^{-t/\tau} \tau}{C} \left[t e^{t/\tau} - \tau e^{t/\tau} - (0 - \tau) \right]$$

$$V(t) = \frac{10^{-4} \tau}{C} \left[t - \tau + \tau e^{-t/\tau} \right]$$

Now plug in $t = 0.03$ s

$$V(0.03) = \frac{10^{-4} \tau}{C} \left[0.03 - \tau + \tau e^{-0.03/\tau} \right] \approx 0.0205 \text{ V}$$

Now find the constant current input that gives $V(0.03) = 0.0205 \text{ V}$
(I_2)

$$\boxed{V(t) = (V(0) - V_{\infty}) e^{-t/\tau} + V_{\infty}} \quad \text{where } V_{\infty} = I_2 R$$

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Remember we took $V(0) = 0$.

$$V(t) = -I_2 R e^{-t/\tau} + I_2 R$$

$$= I_2 (-R e^{-t/\tau} + R) \Rightarrow I_2 = \frac{V(t)}{-R e^{-t/\tau} + R}$$

$$I_2 = \frac{0.0205}{-R e^{-0.03/\tau} + R} \approx 2.1574 \times 10^{-4} \text{ A}$$

We have:

$$I_1(t') = 10^{-4} t'$$

$$I_2(t') = 2.1574 \times 10^{-4}$$

Both of these current inputs give a voltage response of 0.0205 V at 30 msec.