StatR 101: Fall 2012
Homework 6
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Due Tuesday, November 6, 6:00 pm

Instructions: Please submit a SINGLE DOCUMENT with all the R code, short answers and figures. Upload the completed homework assignment into the course webpage drop-box. In my experience, probability problems cause a lot of confusion - hence our emphasis on simulation. Don't hesitate to discuss the problems amongst yourselves on the forum.

1. **Card playing:** A standard playing deck of 52 cards consists of 13 unique valued cards (2 to 10, and four *face cards*, the Ace, Jack, Queen and King) each in 4 different *suits* (hearts - ♡, diamonds - ♢, clubs - ♣, and spades - ♠). You can generate a deck of cards in R using something like the following code:

```
CardNames <- c(2:10, "Jack", "Queen", "King", "Ace")
Deck <- c(paste(CardNames, "of Diamonds"),
          paste(CardNames, "of Hearts"),
          paste(CardNames, "of Clubs"),
          paste(CardNames, "of Spades"))
```

   And then "draw" five cards simply with: `sample(Deck, 5, replace=FALSE)`.

   (a) How many possible hands of five unique cards can be drawn from a deck of cards? (Recall that when you draw cards, you are not replacing them in the deck.)

   (b) In poker, a *flush* is defined as five cards of the same suit, regardless of the value of the card. Write a function called `IsFlush` that draws five cards from a deck and determines (`TRUE/FALSE`) whether or not it is a flush. (Hint: you probably don't want to use the completely defined deck above, but create a simpler vector.)

   (c) Simulate this process 10,000 times and count how many times you draw a flush. What is the approximate probability of a flush based on this experiment? Note there are two ways that you can do this: with a loop, or by creating a matrix of decks and using `apply()`.

   (d) A *straight* is a sequence of five cards in numerical order.[1] Repeat exercises (b) and (c) for the straight, i.e. creating a function called `IsStraight()` and repeating it 10,000 times.

   (e) Based on these results, which is the more likely hand to be dealt in a game of poker?

   Bonus: Compute the exact probabilities of a flush and a straight. This is most easily done by counting the number of possible flushes and straights and dividing by the answer to problem (a).

[1] For simplicity, we give the Ace card a value of 13 and assume that the Ace-2-3-4-5 is not considered straight

While you're at it, count the number of straight flushes (this, you can do with a friend on your fingers and toes).

2. **Airplane functioning:** You are planning on taking a flight on Epsilon Airlines across the Pacific Ocean. To have a successful flight, 100 different components on the plane must ALL function correctly. Each of these components has a probability 0.001 of failing.

   (a) Use the `rbinom()` command to simulate a flight on Epsilon airline by creating a vector of 100 elements which succeed or fail with the appropriate probability.

   (b) Use the `rbinom()` command in a slightly different to way to produce a vector of length 1,000,000, in which each element is the number of components that fail in each flight. Use this vector to produce an estimate of the probability of failure for the plane.

   (c) Calculate the exact probability that the flight will not be successful. What important assumption are you making in this calculation (and in the simulations above)?

   (d) Would you take a flight on Epsilon airlines?

3. **Elections:** The U.S. has a somewhat strange electoral system in which the president is elected by an "electoral college" composed of designated electors representing each state. There are between 3 and 55 electors per state depending on its population, and in almost all states, all the electors from a given state pledge to vote for the candidate that wins the popular vote. Thus, in states where the victor squeezes by on 50.1% of the populat vote, all the electoral votes go to the victor.

   Let's imagine a slightly different (but no less strange) variation in which each elector for each state votes for a candidate with probability p equal to the proportion of votes each candidate receives. Consider the following (approximate) results from the 2008 election for three states:

State	Electors	Obama (%)	McCain (%)
Alaska	3	39	61
Indiana	11	50.5	49.5
California	55	62	38

   (a) Illustrate, under this system, the discrete probability distribution of the number of electoral votes Obama would receive from each of these three states.

   (b) Calculate for each state the probability that McCain would have won each of these states according to this voting model.

   (c) Qualitatively compare these three distributions, commenting on their width and symmetry.