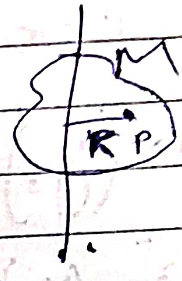
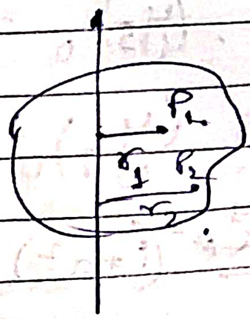
Date ..../..../........

Unit - 1

- MOI → Analog of mass $\boxed{(I) = MR^2}$



$$\boxed{COM = \frac{\int x\,dm}{\int dm}}$$



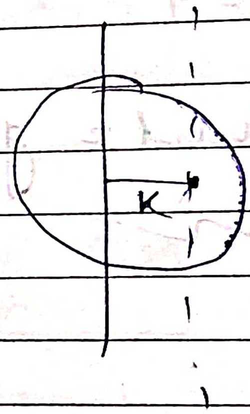
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$= \sum_i m_i r_i^2 \quad \Big\} \text{ discrete}$$

⇒ For continious distribution of mass :-

$$I = \int r^2\,dm$$

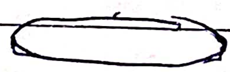
- Radius of gyration (K).



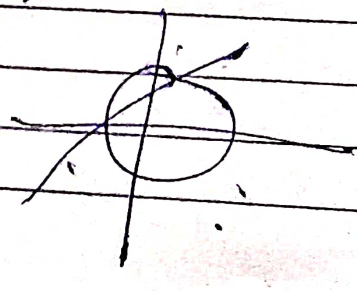
$$\boxed{I = Mk^2 \longrightarrow k = \sqrt{\frac{I_{axis}}{M}}}$$

Theorem

1. Perpendicular Axis Theorem.
2. Parallel Axis Theorem.



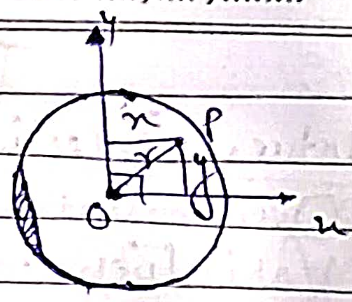
about an axis perpendicular to plane of laminar is equal to the sum of MOI about perpendicular to each other were perpend axis passes through it.



Spiral

Date ..../..../........

Q $I_z = I_x + I_y$



$I_x = \Sigma my^2$ —— (1)

$I_y = \Sigma mx^2$ —— (2)

$I = I_x + I_y = \Sigma mx^2 + \Sigma my^2$.

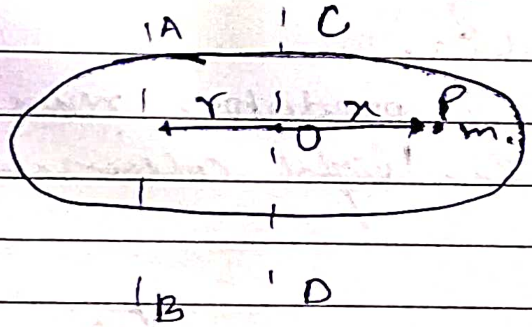
$= \Sigma m(x^2+y^2)$

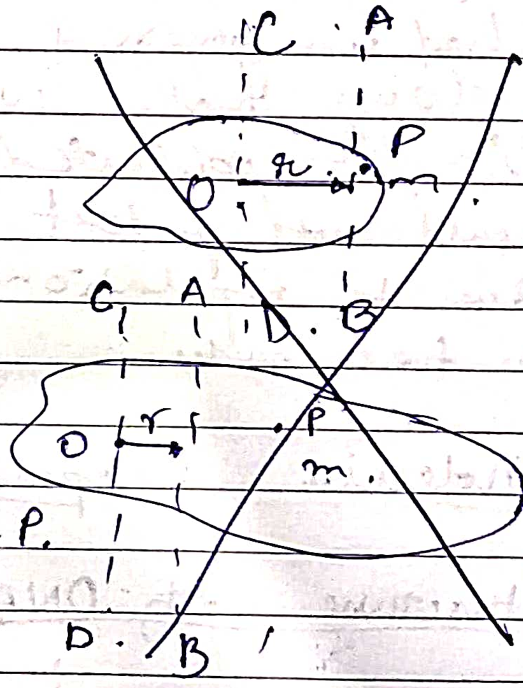
$$\boxed{I_z = \Sigma m r^2}$$

Hence, Proved

Q Parallel Axis Theorem

~~$I = mx^2 + Mr^2$.~~





MoI about axis AB of Smallelement P.

$= m(r+x)^2$

For whole body → $\Sigma m(r+x)^2$.

$I = \Sigma m(r+x)^2$.

$= \Sigma mr^2 + \underbrace{\Sigma mx^2} + 2\underbrace{\Sigma mxr}_{moment}$.

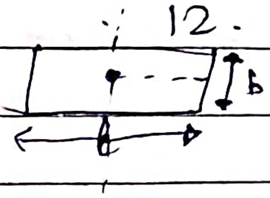
$\Sigma mx^2 = I_{CM}$.

↳ it is 0 at COM because body Spirals balances there.

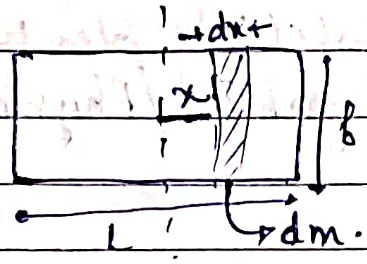
MI - rectangular lamina

$\frac{Ml^2}{12}$

$\frac{M(l^2+b^2)}{12}$



• about COM // to one side.



$\frac{M}{l \times b} = \sigma$

mass of small element $= \sigma (b\,dx)$.

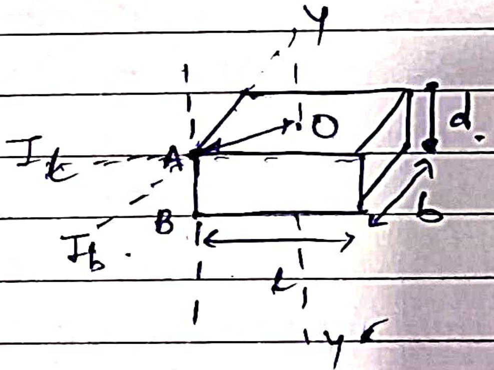
$\frac{M}{\cancel{lb}}\cancel{b}\,dx = \frac{M\,dx}{l}$

$$dI = \int_{-l/2}^{l/2} dm\,x^2 \rightarrow \frac{M\,dx}{l}x^2 \rightarrow \left[\frac{Mx^3}{3L}\right]_{-l/2}^{l/2}$$

$$\frac{Ml^3}{8 \times 3l} + \frac{Ml^3}{8 \times 3l} \rightarrow \frac{Ml^2}{12}$$

• M.I of rectangular lamina.

(ii)

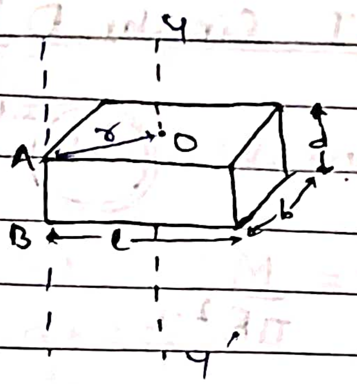


$$M\left(\frac{\sqrt{l^2+b^2}}{2}\right)^2 + \frac{M(l^2+b^2)}{12}$$

$$M\frac{(l^2+b^2)}{4} + M\frac{(l^2+b^2)}{12}$$

$$M \times 4\frac{(l^2+b^2)}{\cancel{12}\,3} = \frac{M(l^2+b^2)}{3} \text{ Ans}$$

MOI of rectangular lamina.



$$r = \frac{\sqrt{l^2+b^2}}{2}.$$

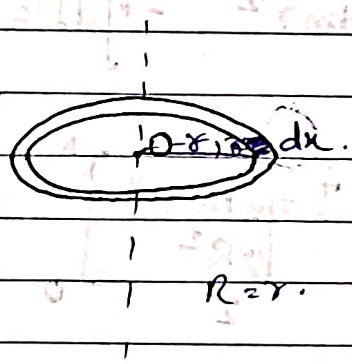
$$I_{AB} = I_{COM} + Mr^2.$$

$$= \frac{M(l^2+b^2)}{12} + M\frac{(l^2+b^2)}{4}.$$

$$= \frac{M(l^2+b^2)}{3}.$$

MOI of Circular Ring

(i) About COM $\perp$ to plane.



$$\lambda = \frac{M}{2\pi r}$$

$$dm = \frac{M}{2\pi r}dx. \qquad \boxed{dx = r\,d\theta}$$

$$dI = dm\,R^2$$

$$I = \int dm\,R^2 \Rightarrow \left[\frac{M}{2\pi r}dx\,R^2\right]_0^{2\pi r} = \frac{M}{2\pi r}\cancel{x}R^2.$$

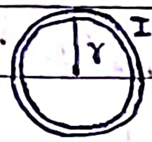
$$\Rightarrow \left[\frac{M}{2\pi r}r\,d\theta \cdot R^2\right] \rightarrow \left[\frac{M}{2\pi r}r\,\theta R^2\right]_0^{2\pi}$$

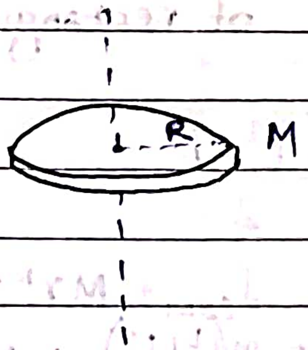
$$\rightarrow \frac{M(2\cancel{\pi})R^2}{2\cancel{\pi}} \Rightarrow MR^2 = I_{COM}.$$

(ii) About COM $\parallel$ to diameter

$$I_z = I_x + I_y \Rightarrow I_z = 2I_x \Rightarrow \frac{I_z}{2} \Rightarrow \frac{MR^2}{2} = I_{\parallel dia}.$$

M.O.I Circular Disk

element =  Idr



$\sigma = \frac{M}{\pi R^2}$.

$dm = \frac{M}{\pi R^2}(2\pi r\, dr)$

$\int dI = \int dm\, r^2 \rightarrow \int dI = \int \frac{M}{\cancel{\pi} R^2} 2\cancel{\pi} r^3\, dr$.

$I = \left[\frac{\cancel{2}M r^4}{\cancel{4}R^2}\right]_0^R \Rightarrow \left[\frac{MR^4}{2R^2}\right] = \frac{MR^2}{2}$. Ans

(ii) Along the diameter

$I_z = I_x + I_y \Rightarrow I_z = 2I_x \Rightarrow \frac{MR^2}{2} = 2I_x \rightarrow \frac{MR^2}{4} = I_x$.

(iii) Along the tangent.

(a) Perpendicular to plane.

$I_A = \frac{mR^2}{2} + mR^2 = \frac{3MR^2}{2}$.

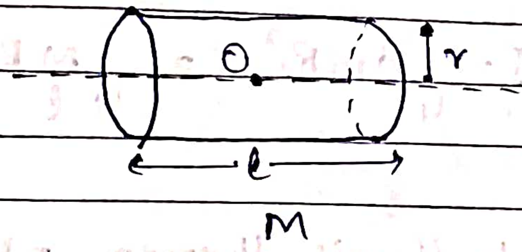
(b) Parallel to diameter

$I_A = \frac{mR^2}{4} + mR^2 = \frac{5}{4} MR^2$

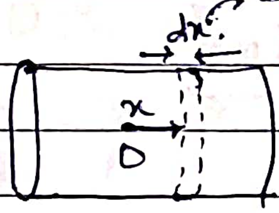
Date ..../..../........

Moment of Inertia of Solid Cylinder

- About its geometrical axis



$$I = \frac{MR^2}{2}.$$



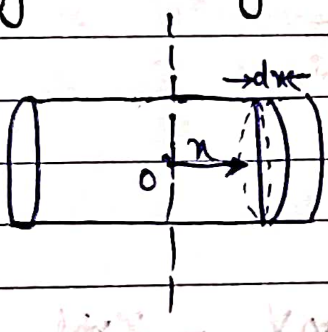
$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 L}.$$

$$dm = \rho \times \pi R^2 dx \Rightarrow \frac{M}{l} dx.$$

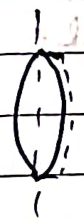
$$M.I \text{ of disk} = \frac{1}{2} dm R^2 .$$

$$\int \frac{1}{2} dm R^2 = \left[\frac{1}{2} R^2 \frac{M}{l} x \right]_{-l/2}^{l/2} \qquad \frac{1}{2} R^2 \frac{M}{\cancel{l}} \times \cancel{l} \Rightarrow \frac{MR^2}{2}.$$

- About an axis perpendicular to geometrical axis and passing through center of mass.



element =



$$\frac{M}{V} = \frac{M}{\pi R^2 l}. \qquad dm = \frac{M}{l} dx.$$

M.O.I of circular disk about diameter of disk.

$$dI = \frac{1}{4} dm R^2 = \frac{1}{4}\frac{M}{l} R^2 dx.$$

I of element

shifting.

A/c to // axis theorem. → $\underbrace{I_{AB} = I_0 + \int dm x^2}$

$$I_{AB} = I_0 + \int \frac{M x^2 dx}{l}$$

$$I_{AB} = \int \frac{1}{4}\frac{M}{l} R^2 dx + \int \frac{M}{l} x^2 dx.$$

$$= \frac{MR^2}{4} + \frac{M}{3l}\left[\frac{2l^3}{8}\right]$$

$$= \frac{MR^2}{4} + \frac{Ml^2}{12} \rightarrow \frac{M}{4}\left[R^2 + \frac{l^2}{3}\right] \text{ Ans.}$$

Q Find l/R ratio when I is minimum.

$$I = \frac{MR^2}{4} + \frac{Ml^2}{12}.$$

$$V = \pi r^2 l \rightarrow r^2 = \frac{V}{\pi l}.$$

$$I = \frac{MV}{4\pi l} + \frac{Ml^2}{12}$$

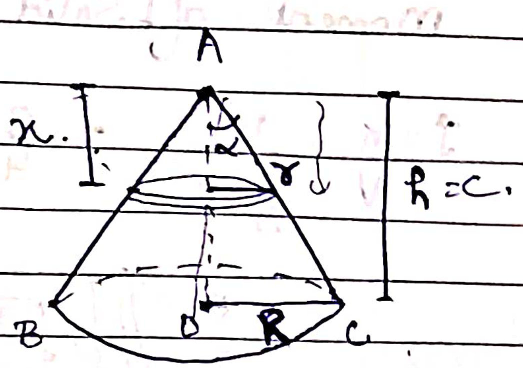
$$\frac{dI}{dl} = \frac{-MV}{4\pi l^2} + \frac{2Ml}{12}. \Rightarrow \frac{MV}{4\pi l^2} = \frac{2Ml}{12}$$

$$\frac{\pi r^2 l}{\pi l^3} = \frac{2}{3} \rightarrow \frac{l}{r} = \sqrt{\frac{3}{2}}$$

Spiral

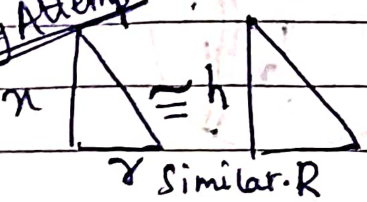
Date ..../..../........

Moment of Inertia of Solid Cone.



• About its Vertical axis.

My Attempt



$$\frac{x}{h} = \frac{r}{R} \Rightarrow r = \frac{Rx}{h}.$$

$$\frac{M}{V} = \frac{3M}{\pi R^2 h}.$$

$$dm = \rho dV = \pi r^2 dx \Rightarrow \cancel{\pi}\left(\frac{R^2 x^2}{h^2}\right) dx \times \frac{3M}{\cancel{\pi R^2} h} = \frac{3M x^2 dx}{h^3}.$$

$$dI = \cancel{\frac{1}{2} dm R^2} \quad \frac{1}{2} dm\, r^2.$$

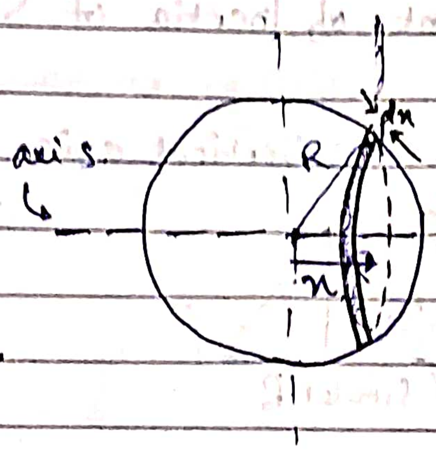
$$= \frac{1}{2}\left[\frac{3M}{h^3} x^2 dx\right]\left[\frac{R^2 x^2}{h^2}\right].$$

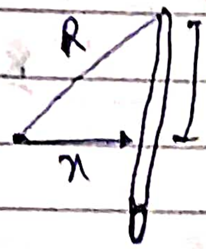
$$dI \Rightarrow \frac{3}{2}\frac{MR^2}{h^5} x^4 dx \Rightarrow \frac{3}{2}\frac{MR^2}{h^5}\left[\frac{x^5}{5}\right]_0^h \Rightarrow \frac{3}{2}\frac{MR^2}{\cancel{h^5}} \times \frac{\cancel{h^5}}{5}$$

$$\Rightarrow \frac{3}{10} MR^2$$ Correct

Moment of I Solid Sphere.

$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{3M}{4\pi R^3}$





$= \sqrt{R^2 + x^2} = r$

$r = \sqrt{R^2 - x^2} \Rightarrow dm = \rho \times \pi r^2 dx$

$dm = \frac{3M}{4\pi R^3} \times \pi (R^2 - x^2) dx \Rightarrow \left(\frac{3M}{4R} - \frac{3Mx^2}{4R^3}\right) dx$

$dI = \frac{1}{2} mR^2 = \frac{1}{2} dm\, r^2$

$dI = \frac{1}{2} \times \left[\frac{3m}{4R} - \frac{3mx^2}{4R^3}\right]\left[R^2 - x^2\right] dx$

$\frac{1}{2} \frac{3m}{4R^3} (R^2 - x^2)(R^2 - x^2) dx$

$dI = \frac{1}{2}\left[\frac{3m}{4R^3}\right](R^2 - x^2)^2 dx$

$\frac{1}{2}\left[\frac{3M}{4R^3}\right]\int (R^4 + x^4 - 2R^2x^2) dx$

$\left[xR^4 + \frac{x^5}{5} - \frac{2R^2x^3}{3}\right]_{-R}^{R}$

Date ..../..../........

$$2\left[R^5+\frac{R^5}{5}-\cancel{\frac{2R^5}{3}}\right]+\left[+R^5+\frac{R^5}{5}+\frac{2R^5}{3}\right].$$

$$\frac{15R^5+3R^5+10R^5}{15}$$

$$2\times\frac{8R^5}{15}=\frac{\cancelto{2}{16}\,\cancelto{2}{R^5}}{\cancelto{5}{15}}\times\frac{1}{2}\,\frac{\cancel{3}M}{\cancel{4}\pi \cancel{R^3}}=\frac{2}{5}MR^2$$

Correct

Date ..../..../........

Oscillations (SHM).

Hooke's law

Force $\propto -x$.

Freq $\propto -x$.

$F = -kx$ or $-Sx$.

force constant.

$F = ma$.

$ma = -kx$.

$a = \frac{-k}{m}x$.

$a = \propto -x$

$k = \frac{ma}{x}$.

$\frac{\cancel{kg}\ kg\,m/s^2}{\cancel{m}}$

$\frac{k}{m} = \frac{m/s^2}{m} = [T^{-2}]$

$\omega^2 = (T^{-2})$

$\frac{k}{m} = (T^{-2})$ $\Rightarrow$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Eqⁿ of motion. for SHM oscillator

$a \propto -x$.

$a = -\frac{S}{m}x$.

$\frac{d^2x}{dt^2} = -\frac{S}{m}x$.

$$\boxed{\frac{d^2x}{dt^2} + \frac{Sx}{m} = 0}$$

$$\boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0}$$

Spiral

Date ..../..../........

let the general solⁿ: $x = Ae^{\alpha t}$. $A\alpha \to$ arbitary constant.

$$\frac{dx}{dt} = Ae^{\alpha t}\alpha.$$

$$\frac{d^2x}{dt^2} = A\alpha^2 e^{\alpha t} \quad \text{--- (III)}$$

$$A\alpha^2 e^{\alpha t} + \omega^2 Ae^{\alpha t} = 0 \quad \text{--- (IV)}$$

$$Ae^{\alpha t}\underbrace{(\alpha^2+\omega^2)}_{=0} = 0; \because e^{\alpha t} \neq 0$$

$$\alpha^2+\omega^2 = 0 \to \alpha^2 = -\omega^2.$$

$$\alpha = \sqrt{-\omega^2} = \pm i\omega.$$

$$\alpha_1 = i\omega \qquad \alpha_2 = -i\omega.$$

$$x_1 = A_1 e^{i\omega t} \qquad x_2 = A_2 e^{-i\omega}.$$

General solution will be linear combination of displace

$$x = x_1 + x_2 \qquad e^{i\theta} = \cos\theta + i\sin\theta.$$

$$x = A_1[\cos\omega t + i\sin\omega t] + A_2[\cos\omega t - i\sin\omega t].$$

$$= \cos(\omega t)(A_1+A_2) + \sin(\omega t)(iA_1 - iA_2)$$

let $A_1 + A_2 = x_m \sin\phi \Rightarrow iA_1 - iA_2 = x_m\cos\phi$

$$= x_m\cos\omega t\sin\phi + x_m\sin\omega t\cos\phi$$

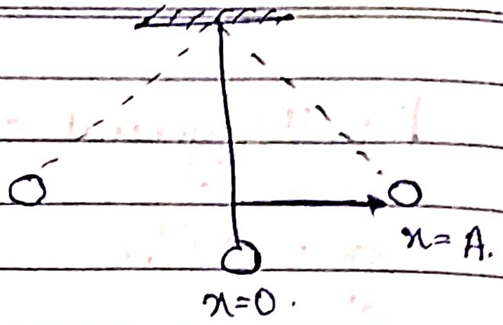
$$\boxed{x = x_m\sin(\omega t + \phi)}$$ How? TF.

Phase. initial phase angle.

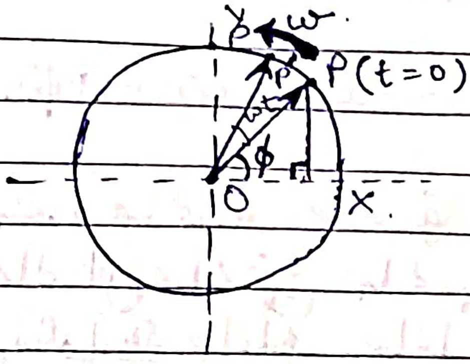
Spiral

$$\omega t + \phi = \frac{\pi}{2} \Rightarrow \left[\omega t = \frac{\pi}{2} - \phi\right]$$

$$x(t) = A \sin(\omega t + \phi)$$

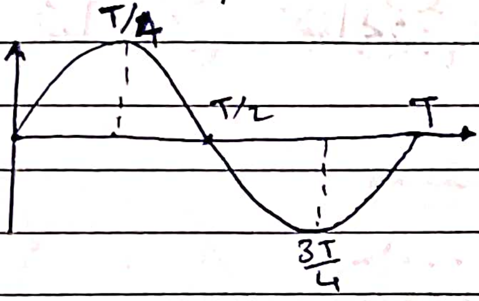


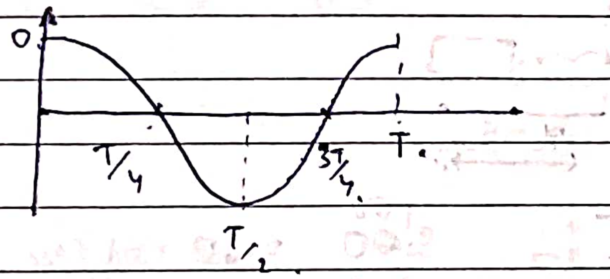
$x = x_m \sin(\omega t + \phi)$



$x(t) = x \sin(\omega t + \phi)$

$x = 0, t = 0, \phi = 0, 2\pi, \pi, \ldots$





Velocity of SHM oscilator

$$\frac{V^2}{(a\omega)^2} + \frac{x^2}{a^2} = 0 \quad \Big\} \text{ eclipse/ellipse.}$$

$x = a \sin(\omega t + \phi)$

$V = a\omega \cos(\omega t + \phi)$

$V = \omega \sqrt{a^2 - x^2}$

$[V_{max} = a\omega]$

| $\cos(\omega t + \phi) = \pm 1$

| $(\omega t + \phi) = n\pi$

$\phi = n\pi - \omega t$

Acceleration.

$acc = -a\omega^2 \sin(\omega t + \phi)$.

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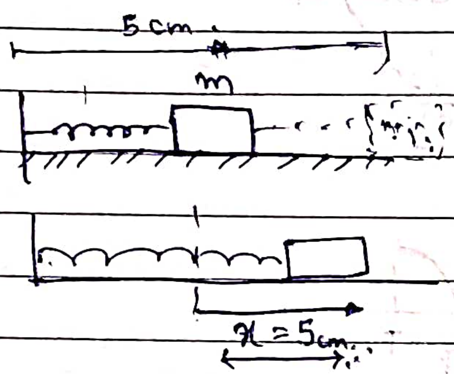
$m = 0.5 kg$
$x = 5cm$.
$\omega = ?$
$V_{max} = ?$
$a_{max} = ?$
$k = 50$

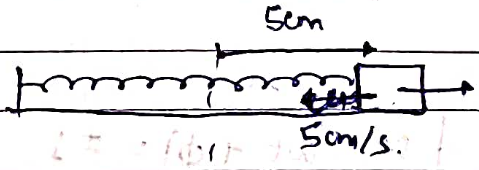


$T = 2\pi\sqrt{\frac{m}{k}}$ $\qquad \omega = \sqrt{\frac{k}{m}} = \frac{\cancel{500}}{0.\cancel{5}} = 10\ rad/sec.$

$V = a\omega = \frac{5}{100} \times 10 = 0.5 m/s$.

$a = -a\omega^2 = -5 m/s^2$.

Q $V_i = 5cm/s$.



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2525

$\frac{x^2}{a^2} + \frac{v^2}{(a\omega)^2} = 0$

$\frac{5cm^2}{a^2} + \frac{(5cm/s)^2}{a^2(10)^2} = 0$

$\frac{25}{10^6} + \frac{25}{10^4} = a^2.$

$\frac{25 + 2500}{10^6}$ $\qquad \frac{25}{10^6} = a^2 - \frac{25}{10^4}$

$V = \cancel{a}\,\omega\sqrt{a^2 - x^2}$

$\frac{25}{10^2} = 10\sqrt{a^2 - \frac{25}{10^4}}$

$\frac{25}{10^3}$

$\frac{5}{10^3} = \sqrt{a^2 - \frac{25}{10^4}}$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

Date ..../..../........

Energy of SHM oscillator

$PE = \frac{m\omega^2 x^2}{2}$

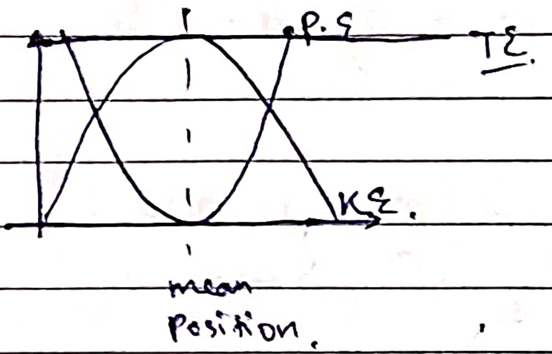
$TE = \frac{m\omega^2 A^2}{2}$

$KE = \Delta$

$PE, \; U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$

$TE = \frac{1}{2}m\omega^2(A^2)$



$\Phi_{mean} \frac{\int f(t)\,dt}{\int dt}$

$\langle KE \rangle = \frac{1}{t}\int (KE)\,dt$

$\frac{1}{t}\int_0^t \frac{1}{2} m\omega^2 A^2 \cos^2(\omega t + \phi)$

$\Rightarrow \frac{m\omega^2 A^2}{2T}\int_0^t \cos^2(\omega t + \phi)\,dt$ $\qquad \langle \cos^2(t) \rangle$

$\frac{m\omega^2 A^2}{2T}\int_0^t \frac{\cos 2\theta + 1}{2}$

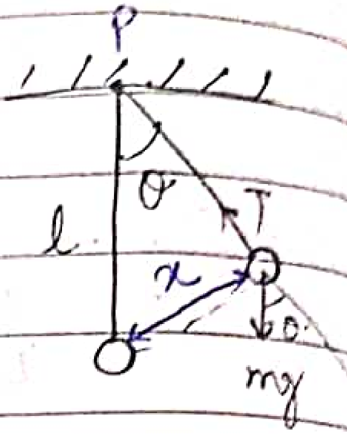
$\frac{m\omega^2 A^2}{4T}\left[\cancel{\int_0^t \cos 2\theta} + \int_0^t dt\right] \Rightarrow \frac{m\omega^2 A^2}{4T} \times T \Rightarrow \frac{1}{4}m\omega^2 A^2.$

$\langle KE \rangle = \frac{1}{4}m\omega^2 A^2.$

Spiral

Simple Pendulum

Forces acting on mass m.



① mg, acting downward

② Tension (T) on string

$mg\cos\theta = T$ (radial component)

$mg\sin\theta =$ restoring force

$$F_t = -ma$$

$$\cancel{m}g\sin\theta = -\cancel{m}\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = -g\sin\theta \quad -①$$

$$\theta = \frac{x}{l} \Rightarrow x = l\theta$$

$$l\frac{d^2\theta}{dt^2} = -g\sin\theta$$

For small ~~os~~ amplitude $\sin\theta \approx \theta$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \cdots$$

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$$\frac{d^2\theta}{dt^2} = \frac{-g}{L}\theta$$

$$\boxed{\frac{d^2\theta}{dt^2} + \frac{g}{L}\theta = 0}$$

eqⁿ of SHM → $\boxed{\frac{d^2x}{dt^2} + \frac{K}{m}x = 0.}$

Time Period

$$T = \frac{2\pi}{\omega}.$$

$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}.}$$

Q. $T = 2\pi\sqrt{\frac{l}{g+a}}$ $\quad T = \frac{\pi}{3}$ s. $\quad l = 1m$

$$\frac{\pi}{3} = 2\pi\sqrt{\frac{l}{g+a}}$$

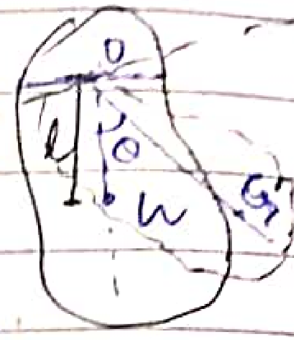
$$\frac{1}{36} = \frac{l}{g+a}$$

$$g + a = 36l$$

$$a = 36 - 10$$

$$= 26\ m/s^2$$

Compound Pendulum.



A rigid body capable of oscillating about a horizontal axis passing through a vertical plane.

Point of suspension →
It is a point about which a body oscillates in a vertical plane passing through Center of gravity needs the axis of Rotation

length of Pendulum → (OG) distance b/w point of suspension and the Centre of Gravitation.

Restoring Torque → $\tau = -mgl\sin\theta$

$$\boxed{\tau = I\alpha} \Rightarrow \tau = I\frac{d^2\theta}{dt^2}$$

$$I\frac{d^2\theta}{dt^2} = -mgl\sin\theta.$$

$$\frac{d^2\theta}{dt^2} = \frac{mgl\sin\theta}{I} = \theta.$$

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For small amplitude, $\sin\theta = \theta$

Time Period →

$$T = \frac{2\pi}{\omega}, \quad \omega = \sqrt{\frac{mgl}{I}}$$

$$\boxed{T = 2\pi\sqrt{\frac{I}{mgl}}}$$

I = MOI of body about Horizontal axis.

I_g → MOI about centre of mass.

∥ axis theorem :-

$$I = I_g + ml^2$$

$$I_g = mk^2$$

$$T = 2\pi\sqrt{\frac{mk^2 + ml^2}{mgl}}$$

$$= 2\pi\sqrt{\frac{k^2 + l^2}{gl}}$$

$$T = 2\pi\sqrt{\frac{\frac{k^2}{l^2} + l}{g}}$$

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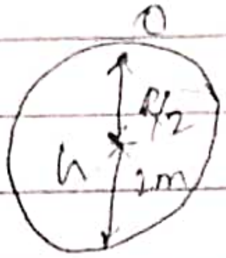
length of Compound pendulum

$\frac{k^2}{l} + l \rightarrow$ equivalent length of Simple Pendulum



G → Centre of gravity.

$L = \frac{K^2}{l} + \underline{l} \rightarrow$ length of Compound Pendulum

Center of Oscillation (O')

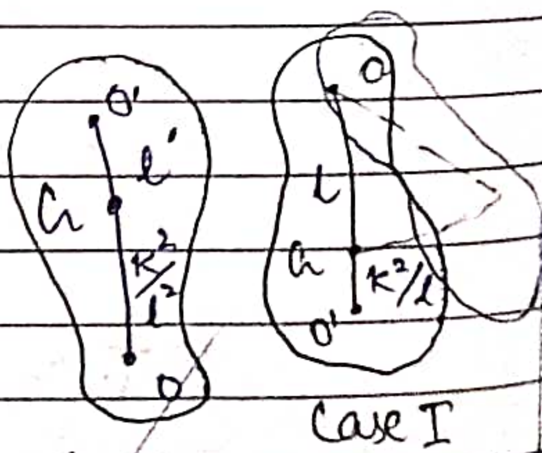
Center of Oscillation is a point below Center of gravity at a distance of $\frac{k^2}{l}$

Interchangeability or Irreversibility of Point of Suspension

Case I $T = 2\pi \sqrt{\frac{k^2}{l} + l / g}$

Case II $T' = 2\pi \sqrt{\frac{k^2}{l'} + l' / g}$

$\frac{k^2}{l'} = l \Rightarrow k^2 = ll'$



Case II

Case I

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$$2\pi\sqrt{\frac{\frac{kl'}{l}+l}{g}} \Rightarrow 2\pi\sqrt{\frac{l'+l}{g}}$$

$L = l' + l$

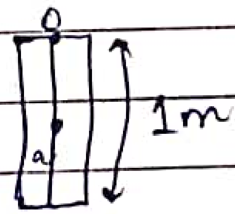
Simple Pendulum.

Q ~~A circular disk of radius = 50cm find the distance of the axis of rotation~~

Q A thin uniform bar of length 1m ab is allowed to oscillate about the horizontal axis passing through it one end.

Calculate (1) equ length of Simple Pendulum

(2) Time Period.



→ $\frac{k^2}{l} + l$.

$I =$

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$$T = 2\pi\sqrt{\frac{k^2+l^2}{lg}}$$

Minimum Time Period.

$$T^2 = 4\pi^2\left(\frac{k^2+l^2}{lg}\right) = \frac{4\pi^2}{g}\left[\frac{k^2}{l}+l\right]$$

Diff. w.r.t l.

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left[\frac{-k^2}{l^2}+1\right]$$

$$\frac{dT}{dl} = 0$$

$$\frac{\cancel{4}^{2}\pi^2}{\cancel{2}Tg}\left[\frac{-k^2}{l^2}+1\right] = 0.$$

$$\frac{k^2}{l^2} = 1$$

$$k^2 = l^2$$

$$k = \pm l$$

$\boxed{k = l}$ $\boxed{k=-l} \rightarrow$ meaningless.

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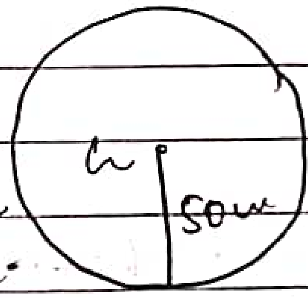
$$\frac{2T\,d^2T}{dl^2} + 2\frac{dT}{dl} = \frac{4\pi^2}{g}\left[\frac{2k^2}{l^3}\right]$$

$$\frac{d^2T}{dl^2} = \frac{4\pi^2}{Tg}\left[\frac{2k^2}{l^3}\right]$$ [+ve so, min. Time Period]

Min^m Time Period.

$$T_{min} = 2\pi\sqrt{\frac{2k}{g}}$$

Q. Radius of Disk = 50 cm
Find the distance of the axis of Rotation from the Center of mass for which the time Period is minimum.



T_{min} ⇒ $l = k$ $\qquad R = 0.5$

$$M.O.I = \frac{1}{2}\cancel{m}r^2 = \cancel{m}k^2$$

$$= \frac{0.25}{2} = k^2$$

$$= \frac{0.5}{\sqrt{2}} = \frac{0.25\sqrt{2}}{}$$

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$$\therefore T_{min} = 2\pi\sqrt{\frac{2K}{g}}$$

$$= 2\pi$$

$$= 2\pi\sqrt{\frac{0.5\times\sqrt{2}}{100}}$$

$$= \frac{2\times3.14}{10}\times\sqrt{7.07}$$

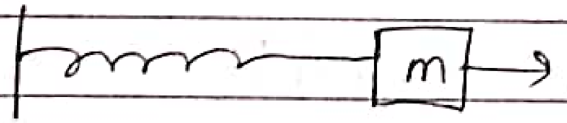
$$= 1.67 \text{ Ans.}$$

Damped Harmonic Motion:-

$$F_d = 6\pi\eta r v$$

$$F_d \propto v$$

→ When the oscillations or the Amplitude die out due to opposing frictional forces.



Force acting on System :-

1) $F_R \cancel{\propto} -Sx \Rightarrow F_R = -Sx$, $S \rightarrow$ Stiffness Const.

2) $F_d \propto v \Rightarrow F_d = -rv$, $r \rightarrow$ damping force

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$$T = 2\pi\sqrt{\frac{k^2+l^2}{lg}}$$

Minimum Time period:

$$T^2 = 4\pi^2\left(\frac{k^2+l^2}{lg}\right) = \frac{4\pi^2}{g}\left[\frac{k^2}{l}+l\right]$$

Diff wrt l.

$$2T\frac{dT}{dl} = \frac{4\pi^2}{g}\left[-\frac{k^2}{l^2}+1\right]$$

$$\frac{dT}{dl} = 0 \Rightarrow k^2 = l^2 \Rightarrow k = \pm l \Rightarrow l = \pm k$$

$\hookrightarrow l = k$ only valid soln

$$\cancel{\frac{d^2T}{dl^2} \Rightarrow \frac{4\pi^2}{}}$$

$$2\frac{dT}{dl} + 2T\frac{d^2T}{dl^2} = \frac{4\pi^2}{g}\left(0 + \frac{2k^2}{l^3}\right)$$

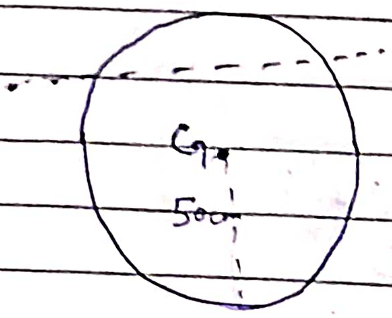
$k = l$, $\frac{dT}{dl} = 0$.

$$2T\frac{d^2T}{dl^2} = \frac{4\pi^2}{g}\cdot\left(\frac{2}{l}\right)$$

$$\frac{d^2T}{dl^2} = \frac{4\pi^2}{2gT}\left(\frac{2}{l}\right) \rightarrow +ve.$$

$$\boxed{T_{min} \Rightarrow 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{2l}{g}}}$$

Radius of disk = 50cm



Axis → one plane ⊥ to plane

$I = \frac{MR^2}{2} \Rightarrow K = \frac{R}{\sqrt{2}}$

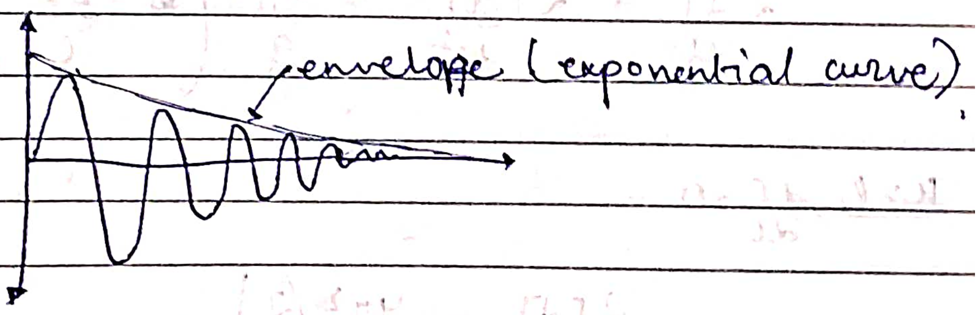
$T_{min} = 2\pi\sqrt{\frac{2 \times R}{g\sqrt{2}}} =$

$$= 2\pi\sqrt{\frac{2 \times 50}{100 \times 10\sqrt{2}}}$$

$$k = l \to \frac{R}{\sqrt{2}} = \frac{50\,cm}{1.4} = 35.35\,cm.$$

$$T = 2\pi\sqrt{\frac{2 \times 0.5}{9.8\sqrt{2}}} = 1.68.$$

Damped Oscillations



The oscillation and amplitud damp out due to opposing frictional forces.

Damping derivation.

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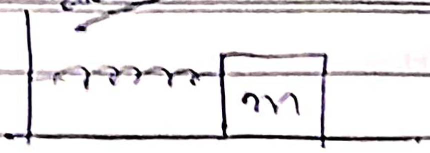

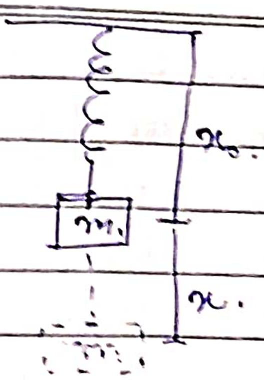

Forces acting in system:

(i) a Restoring force $(F_R) = -Sx = -Kx$.

(ii) Damping force $(F_d) \propto -V$ $= -rV$. $r \rightarrow$ damping coeff.

↳ velocity of object.

Net force $\Rightarrow F = F_R + F_d \Rightarrow -Sx - rV$.

Apply Newton's law of motion

$f = ma$.

$ma = -Sx - rV$.

$$m\frac{d^2x}{dt^2} = -r\frac{dx}{dt} - Sx.$$

$$\frac{d^2x}{dt^2} = \frac{-r}{m}\frac{dx}{dt} - \frac{Sx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{Sx}{m} = 0.$$

$$\boxed{\frac{S}{m} = \omega_0^2 \; ; \; \frac{r}{m} = 2b}$$

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0.$$

damping const

$$\boxed{b = \frac{r}{2m}}$$

natural frequency of oscillator.

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Damped Harmonic Motion.

$$\frac{d^2x}{dt^2} + 2b\frac{dx}{dt} + \omega_0^2 x = 0$$

$$x = A_1 e^{\left(-b+\sqrt{b^2-\omega_0^2}\right)t} + A_2 e^{\left(-b-\sqrt{b^2-\omega_0^2}\right)t}$$

$$2b = \frac{r}{m}.$$

1. $b >> \omega_0$ (Heavily Damped or Overdamping).
2. $b = \omega_0$ (Critical damping).
3. $b < \omega_0$. (lightly damped). (under damped).

1. $b^2 >> \omega_0^2$.

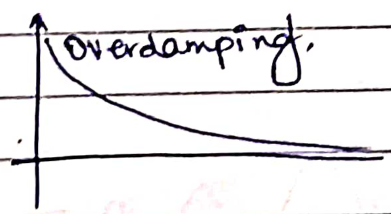
$$\sqrt{b^2-\omega_0^2} = +ve.$$

$$x = e^{-bt}\left[A_1 e^{\left(\sqrt{b^2-\omega_0^2}\right)t} + A_2 e^{\left(-\sqrt{b^2-\omega_0^2}\right)t}\right]$$ exponential fall.

$$e^\theta = \cosh\theta + \sinh\theta.$$

↳ non-periodic func.

use: Dead Beat Galvanometer.

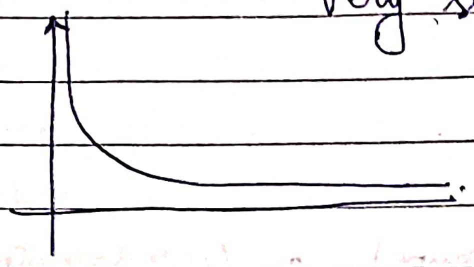


② $b^2 = \omega_0^2$

$$\sqrt{b^2 - \omega_0^2} = 0.$$

$$x = e^{-bt}(A_1 + A_2).$$

• Critical damping :→ takes less time to reach equilibrium compared to overdam
→ Very steep.



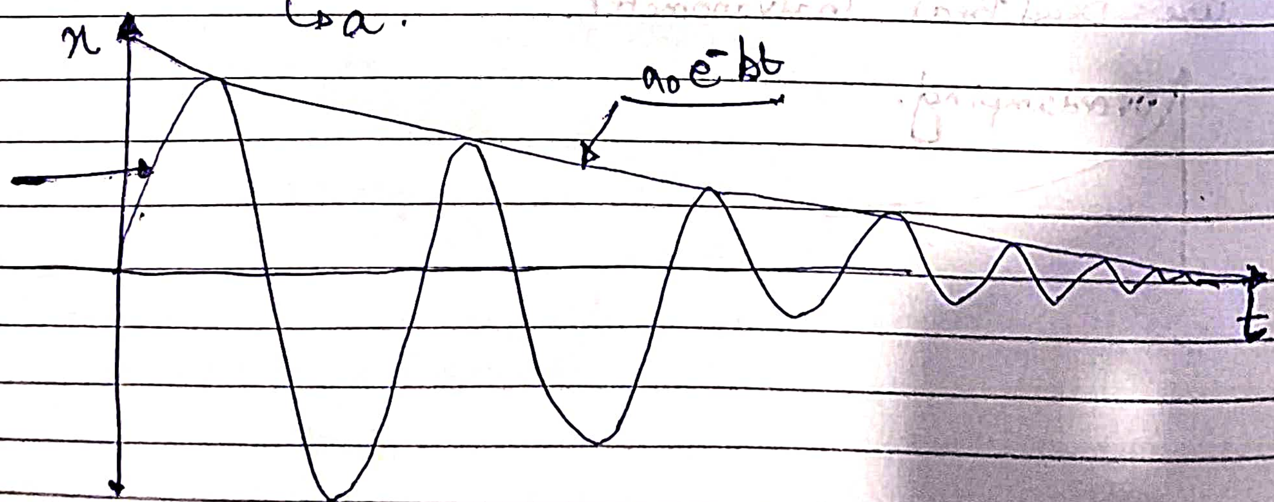
③ $b^2 < \omega_0^2$. (lightly damped or under damped).

$$\sqrt{b^2 - \omega_0^2} = i\underbrace{\sqrt{\omega_0^2 - b^2}}_{= \omega'}$$

$$x = e^{-bt}\left[A_1 e^{i\omega' t} + A_2 e^{-i\omega' t}\right]$$

$$x = a_0 e^{-bt} \sin\left[\sqrt{\omega_0^2 - b^2}\, t + \phi\right]$$

~~Let~~ $x = \underbrace{a_0 e^{-bt}}_{\to a} \sin(\omega' t + \phi)$



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$$\omega' = \sqrt{\omega_0^2 - b^2} \quad , \quad b = \frac{r}{2m}.$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}$$

$r = 0 \rightarrow$ damping is zero.

$$\omega' = \sqrt{\frac{k}{m}} = \omega_0.$$

$$T' = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{r^2}{4m^2}}}$$

(1) Logarithmic decay/Decrement (λ).

It measures the rate at which amplitude decays.

$$a = a_0 e^{-bt}$$

$$a = a_0 \quad \rightarrow t = 0.$$

$$a = a_0 e^{-bT} \quad t = T$$