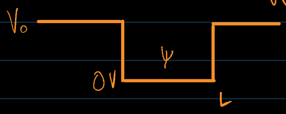
Tuesday, May 6, 2025    11:39 AM

## Drawbacks of Classical $e^-$

- Compton, photo electric, black-body | Electrical conductivity | Wiedmann-Franz $\left(\frac{K}{\sigma T}\right)$ at low temp.
- ferromagnetism, Quantum free $e^-$ theory, $4.5R \neq 3R$

## Sommerfield Model

- $e^-$ move freely in constant potential within boundaries of metal
- discrete energy values, pauli exclusion princpl, ($e^-$ & lattice, $e^-$ & $e^-$) neglect.




$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE\psi}{\hbar^2} = 0 \quad * \longrightarrow \quad E = \frac{n^2 \hbar^2 \pi^2}{2mL^2}$$

## Density of States

- in solid energy level of $e^-$ spread over (energy band) range. Each band has no. of state.
- $g(E)$ = no. of quantum/energy state / Energy / Volume.

$$g(E) = \frac{dN(E)}{dE}\frac{1}{V} \rightarrow g(E)\,dE = dN(E)$$

($dN(E)$: no. of energy state b/w $(E, E+dE)$)

When using fermi → $n = \frac{N}{V} = \int \frac{dN(E)}{V} dE = \int \underbrace{f(E)}_{\text{fermi-dirac}} g(E)\,dE$

($N$: total; $n$: no. of particle per unit volume.)

$E$ for 3D = $E = \frac{\hbar^2 \pi^2}{2mL^2}(n_x^2 + n_y^2 + n_z^2)$  (Give sphere example)

$$dN(E)dE = \frac{1}{8}\left(\left[\frac{4}{3}\pi (n+dn)^3\right] - \left(\frac{4}{3}\pi n^3\right)\right)$$

$$= \frac{\pi}{6}(3\, n^2 dn) = \frac{\pi}{2}(n(ndn))$$

$$E = \frac{n^2 \hbar^2 \pi^2}{2ma^2} = \frac{n^2 h^2}{8ma^2}$$

→ $n\,dn = \left(\frac{8ma^2}{h^2}\right) \times \frac{1}{2}\,dE$

→ $n = \left(\frac{8ma^2 E}{h^2}\right)^{1/2}$

$$dN(E)dE = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} a^3 E^{1/2} dE \Rightarrow \frac{dN(E)dE}{a^3} = \frac{\pi}{2}\left(\frac{8m}{h^2}\right)^{3/2} E^{1/2} dE = g(E)$$

$n = \int f(E)\,g(E)\,dE$ for $T=0$ $f(E) = 1$ → $n = \frac{\pi}{3}\left(\frac{8m}{h^2}\right)^{3/2} E_F^{3/2}$ → $\boxed{E_F = \left(\frac{3}{\pi}\right)^{2/3}\left(\frac{h^2}{8m}\right) n^{2/3}}$

## Bloch Theorem

- In a periodic potential, the wavefunctions of electrons can be represented as product of plane wave & periodic function with same periodicty as crystal lattice.
- assumed: 'a' dis b/w lattice point.
  - Zero at site center
  - max b/w 2 core

$\psi(r) = e^{i\vec{k}\vec{r}} \underbrace{U_k(r)}_{\text{modulating function}} \rightarrow \{\psi(x) = e^{ikx} U(x)\}$ 1-D

Verify it by putting $x = x + Na$.

ik Na

↳ max b/w 2 core

Verify it by putting $x = x + Na$.

we get $\psi(x+Na) = e^{ikNa}\,\psi(x)$

modulating function

## Kronig Penny Model

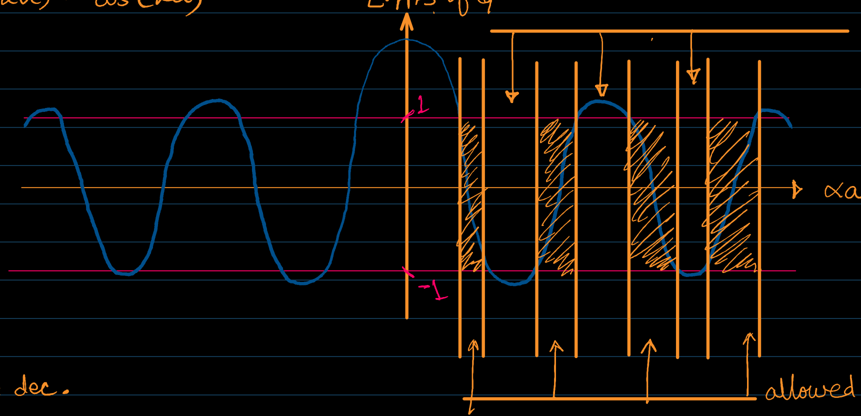
↳ Assumed : 'a' → gap distance , 'b' square potential width.

↳ 0 at core , max b/w ⊕ core.

for gap: $\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$ & for peak: $\frac{\partial^2\psi}{\partial x^2} + \frac{2m(E-V_0)}{\hbar^2}\psi = 0$

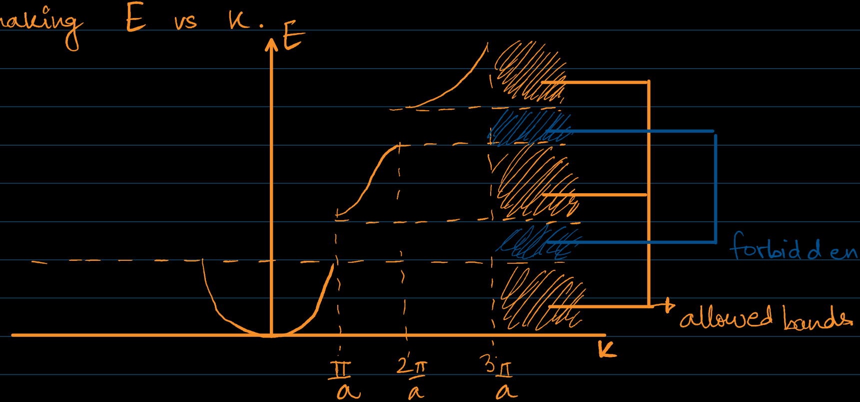
$\alpha^2 = \frac{2mE}{\hbar^2}$ $\beta^2 = \frac{2m}{\hbar^2}(V_0 - E)$

$$P\frac{\sin(\alpha a)}{\alpha a} + \cos(\alpha a) = \cos(ka)$$




- due to RHS cos b/w (-1,1) hence bounded.
- periodicity = $\left(n\frac{\pi}{a}\right)$
- $\alpha a$ inc, forbidden dec.

from $\left(\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0\right)$ When making E vs k.




- $P \rightarrow$ measure of potential barrier strength.

  $P\uparrow \rightarrow$ electron bound more strongly to well.

$P = \infty \rightarrow \frac{\sin\alpha a}{\alpha a} + \frac{\cos\alpha a}{P} = \frac{\cos ka}{P}$

$\sin\alpha a = 0 \rightarrow \alpha = \frac{n\pi}{a} \rightarrow \alpha^2 = \frac{n^2\pi^2}{a^2}$

$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2} \rightarrow E = \underbrace{\frac{n^2\pi^2\hbar^2}{2ma^2}}_{\text{Pot. Well energy}}$

$P = 0 \rightarrow \cos\alpha a = \cos ka \rightarrow \alpha = k = \frac{2\pi}{\lambda}$

$\frac{2mE}{\hbar^2} = k^2 \rightarrow E = \underbrace{\frac{\hbar^2k^2}{2m}}_{\text{free } e^- \text{ energy}} = \frac{1}{2}mv^2.$

- k-values define boundaries of brillioun zone.
- $e^-$ moving have energies from allowed band sep by forb zone.
- Discontinuity at $\left(k = \frac{n\pi}{a}\right) \longrightarrow$ brillioun zone$_1$ = $\left(-\frac{\pi}{a}, \frac{\pi}{a}\right)$, brillioun zone$_2$ = $\left(-\frac{2\pi}{a}, -\frac{\pi}{a}\right)$ & $\left(\frac{\pi}{a}, \frac{2\pi}{a}\right)$
- Each portion a band.

Quantum aspects of Hall's effect.