Complex Adaptive Systems and Game Theory: An Unlikely Union

MIRSAD HADZIKADIC,[1] TED CARMICHAEL[1], AND CHARLES CURTIN[2]

[1]*Complex Systems Institute, College of Computing and Informatics, University of North Carolina at Charlotte, Charlotte, North Carolina 28223; and* [2]*MIT-USGS Science Impact Collaborative, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139*

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A Complex Adaptive System is a collection of autonomous, heterogeneous agents, whose behavior is defined with a limited number of rules. A Game Theory is a mathematical construct that assumes a small number of rational players who have a limited number of actions or strategies available to them. The CAS method has the potential to alleviate some of the shortcomings of GT. On the other hand, CAS researchers are always looking for a realistic way to define interactions among agents. GT offers an attractive option for defining the rules of such interactions in a way that is both potentially consistent with observed real-world behavior and subject to mathematical interpretation. This article reports on the results of an effort to build a CAS system that utilizes GT for determining the actions of individual agents. © 2010 Wiley Periodicals, Inc. Complexity 16: 34–42, 2010

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INTRODUCTION

A Complex Adaptive System (CAS) is a collection of autonomous, heterogeneous agents, whose behavior is defined with a limited number of rules. These rules govern the type and number of interactions among agents. The power of the system mainly comes from agents' interactions, not the agents themselves. Each individual interaction generally has only a small or limited direct effect on the outcome of the system. However, the aggregate product of the thousands of these interactions and the accumulated feedbacks among the agents can have a large effect.

A Game Theory (GT) model is a mathematical construct that assumes a small number of rational players who have a limited number of actions or strategies available to them. Each player is trying to select the best next move, given what she believes the other players would want to do to maximize their chances of success. As such, GT is usually used in the strategic contexts such as political crises, military conflicts, advertising strategies, pricing decisions, or investment options.

While it is generally recognized that GT can be frequently useful in explaining the past, anticipating the out-

Corresponding author: T. Carmichael, University of North Carolina at Charlotte, 9201 University City Blvd., Charlotte, North Carolina 28223; e-mail: tedsaid@gmail.com

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TABLE 1

Contrasting Properties of the CAS and GT Paradigms

Property	Complex Adaptive Systems	Game Theory
Number of agents	Thousands	Two, three
Actions-type	Local, tactical	Strategic
Actions-frequency	One per time step, possibly infinite	One in simultaneous games, several in iterative games
Agents' behavior	Any (rational, probabilistic, biased, etc.)	Rational
Implementation	Computer simulation	Payoff table or iterative tree

come of future strategic interactions, and prescribing the actions that are likely lead to a good outcome for a particular participant in the game [1], it has been noted for its failure to systematically account for new findings in behavioral economics regarding the biases in human behavior [2], the effect of randomness on events and outcomes [3], the inability of some of the players to perform the necessary calculations and analysis [1], or the perception that GT is only useful at an isolated, strategic level.

The CAS method has the potential to alleviate some of the shortcomings of GT. For example, CAS agents could use GT rules to determine the outcome of their pairwise interactions. By doing so, thousands of interactions and their cumulative feedbacks could be monitored and aggregated to understand the overall effect on the society. This would demonstrate the utility of GT both at the individual level and at the aggregate, strategic level.

In addition, the agents could be endowed with behavior that reflects the type and distribution of biases and heuristics noticed in the general population. Similarly, random behavior could easily be introduced into the system by having a certain proportion of agents choose their actions randomly. This approach can be extended to other characteristics of the human decision making process, including risk tolerance, degree of independence of opinion, and demographic characteristics.

On the other hand, CAS researchers are always looking for ways to ground the interactions of these agents with sound theories for how these interactions are defined. These interactions happen simultaneously during each time step, only to continue indefinitely as long as the system is running. GT offers an attractive option for defining the rules of such interactions in a way that is both potentially consistent with observed real-world behavior and subject to mathematical interpretation.

Table1 summarizes some of the complementing properties typically found in these two paradigms that could be used to enhance the shortcomings of each system.

This article reports on the results of an effort to build a CAS system that utilizes GT for determining the actions of individual agents. First, we define the concept of CAS in greater detail.

COMPLEX ADAPTIVE SYSTEMS

CAS is a method developed in physics, mathematics, and computational sciences [4–9] to deal with the issue of complexity and complex systems and has been redefined by a growing number of applications in domains as diverse as biology [Volker Frimm, Steve Railsback], political science [Axelrod—evolution of cooperation, etc.], economics [Miller and Holland on economic agents], and others [El Farol on diner model, Epstein and Axtell on Anasazi, Michael Cohen on trash can model]. Complex, dynamical systems are comprised of parts that interact with each other. They are complex because it is impossible to predict their behavior by simply understanding the function of each part, primarily because the function of the overall system depends on the way these parts interact with each other. The diversity of these parts and the richness of their interactions endow a complex system with its capacity to innovate, adapt, and sustain itself. At the same time, these global, emergent properties cannot be studied or readily understood by only inspecting the parts in isolation.

Complex dynamical systems are nonlinear, such that threshold effects allow a small change in the input value to cause disproportionate change in the output variable. They are also adaptive-the system adjusts to both negative and positive changes caused by external or internal factors. They often exhibit emergent properties, i.e., properties not anticipated by the blueprint of the system. One of these properties is the ability of system's parts to self-organize in some manner that is beneficial to this emerging group.

Agent-Based Modeling (ABM) denotes a method for implementing a CAS to provide a computational environment for exploring characteristics of complex systems in a controlled setting. The Santa Fe Institute was one of the pioneers in this field. Since the mid-20th century, there has been a steady effort to apply CAS to areas as diverse as economics, business, political science, government, military, archeology, biology, and ecology [10–33]. Design-

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ing CAS applications is challenging because researchers often do not know what key variables need to be captured to successfully model the system. There has been some evidence that the CAS method itself can be used to identify key system variables [34].

To test the utility of the combination of CAS and GT methodologies, we decided to use the domain of ecology as the testing ground for the system. The following section briefly describes the characteristics of ecological systems.

ECOLOGY

Gunderson and Holling [35] have suggested that "some of the most telling properties of ecological systems emerge from the interactions between slow-moving and fast-moving processes and between processes that have large spatial reach and processes that are relatively localized." This implies that ecological systems are nonlinear, dynamical, and resilient, due to the variability of processes and the diversity of biotic and abiotic agents acting in the systems.

Things get substantially complicated when human social and economic activities are added to the mix. The resulting systems operate in a narrow range between immovable and chaotic states, often called "the edge of chaos" [36]. This area allows systems to constantly search for improvement through adaptation, creativity, and feedback.

Understanding long-term processes is not easy. It requires resources, access, data, and time. This is especially true for processes that have large spatial characteristics. Experiments, while often required to explain phenomena of interest [37, 38], are frequently impossible to conduct either because the observer is not allowed to interfere with the observed world or because the observer cannot physically modify the observed environment.

Levin [39, 40] defines ecosystems as complex adaptive systems: "Ecosystems and the biosphere are complex adaptive systems, in which pattern emerges from, and feeds back to affect, the actions of adaptive individual agents, and in which cooperation and multicellularity can develop and provide the regulation of local environments, and indeed impose regularity at higher levels." CAS tools have also been used in other settings for the study of ecological systems [Neo Martinez (food webs), Herb Gintis (hawk dove evolution), Ben Klemens (ABM hawk dove), Burtsev and Turchin (Nature, 2006, an ABM with hawk/dove characteristics.]

The remaining sections of the article provide as follows: (a) a description of the CAS tool used for experimentation here and a description of the example used throughout the article, (b) the analysis of the experimentation results, and (c) conclusions.

THE CAS TOOL AND THE EXAMPLE

The tool is a computational ABM designed as a general-purpose simulation device for studying threshold effects in various disciplines. The intent is to (a) capture fundamental phenomena that cross disciplinary boundaries and (b) encourage the "export" of interesting findings from one discipline to another. This tool serves as a mechanism for enabling both types of research activity. It is implemented using NetLogo [41].

This tool was used to implement a simple but revealing example of an ecological system, the Hawk/Dove GT model as characterized by Gribbin [42]. This example includes two kinds of agent, hawks and doves, and the environment that provides resources. If we think of hawks and doves as birds that compete for the same food sources provided by the environment, then we get an ecological system in its classical interpretation. If, however, we assume that hawks and doves are more or less aggressive political candidates competing for the mindshare of the electorate, then we get a social system with its own dynamics.

For the sake of simplicity, let's assume that hawks and doves are birds, and that they are competing for food. There are certain rules associated with this system. The details of these rules may vary across different formulations of the Hawk/Dove model; here, we use the rules provided in [42].

If birds get to eat, then they score 50 points each. If they run away, however, they score no points. Running away costs them nothing either. Doves do not fight with hawks; they simply run away from the fight. Consequently, they get no points and they lose no points, while hawks get the food and the 50 points. If two doves meet, they make a threat, but they do not fight each other. As a result, the more fearful one flies away and the other gets to eat and collect the 50 points. Unfortunately, because they threatened each other, they both receive a punishment in the amount of 10 points.

Finally, when two hawks meet they always fight. The winner gets to eat and keep 50 points. The loser receives a heavy punishment, 100 points. This is because they often get seriously injured in a fight, and then cannot be effective in collecting food and participating in other fights.

This is actually a game theoretic approach to evolutionary biology [42]. This implementation combines the concepts of CAS and GT to evaluate whether there is a particular evolutionarily stable strategy (ESS) that produces a balance within the given conditions of an ecosystem. Figure 1 depicts the CAS tool interface.

The tool has two kinds of "moving" agents: hawks and doves. The third agent, "patch," remains the same. Patches do not move-they simply provide resources. However, they can influence other patches to provide either more or fewer resources.

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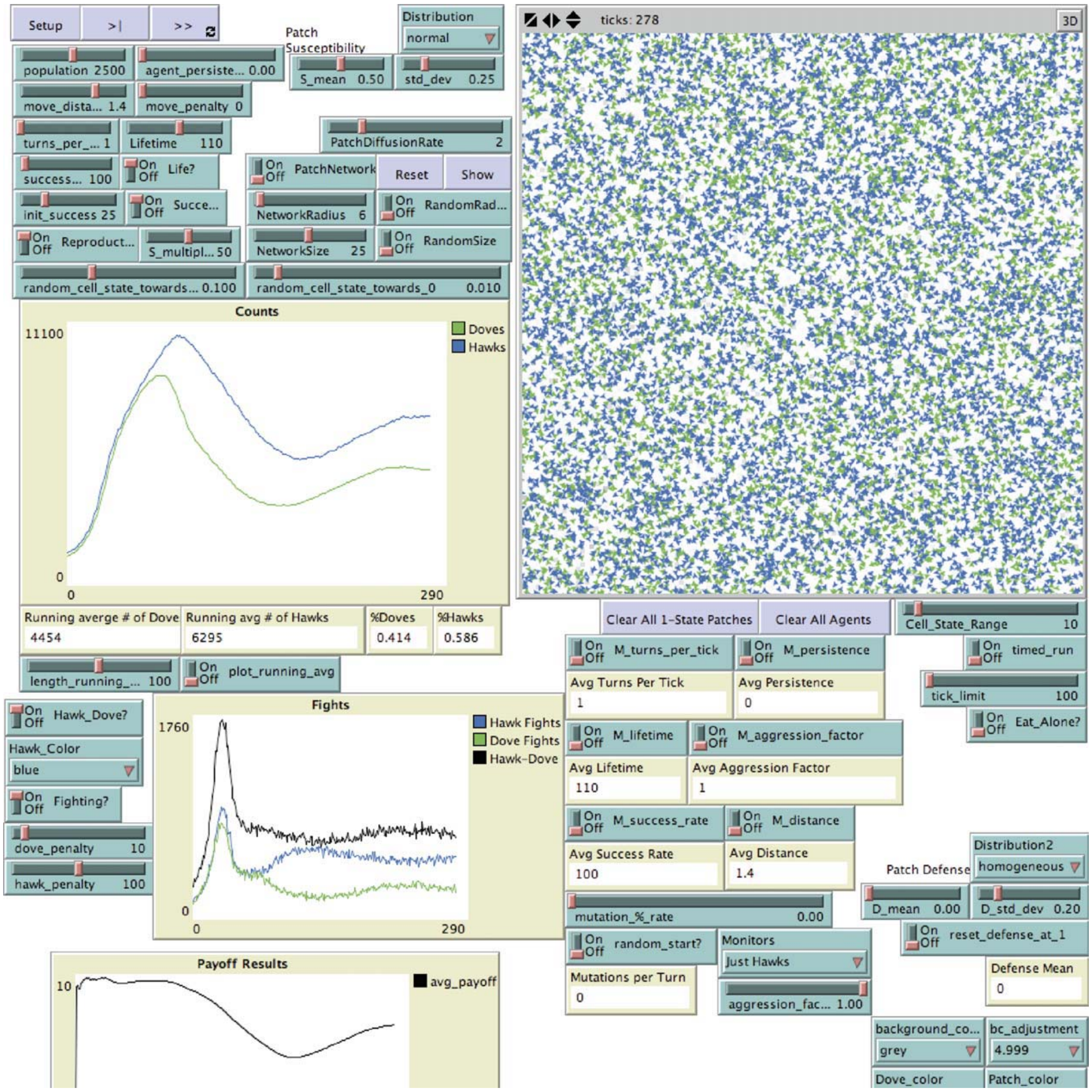
FIGURE 1



A snapshot of the part of the CAS tool's interface.

The various parameters of the tool provide the modeler with multiple dimensions in which to evaluate the system. The most relevant parameters include as follows:

- Initial population size (hawks and doves)
- Speed and lifetime of hawks and doves
- Initial and accumulated resources (successes)

- Conditions for spawning new hawks and doves (*success_rate*)
- Rate of increase or reduction of resources
- Aggression factor for hawks
- Rate of mutation for assorted factors.

The following section explains and analyzes the results obtained during various runs of the simulation tool.

ANALYSIS

The CAS tool is versatile and it allows modelers to evaluate the system under an enormous number of conditions. The project presented here involves a small subset of these conditions that are relevant to reducing the perceived shortcomings of GT. The results are organized around common themes presented in the following subsections.

Evolutionarily Stable Strategy

Gribbin [42] postulated that based on the game theory framework neither a society with a large majority of hawks nor a society with a large majority of doves would become a stable solution in this ecological system, due to the given payoff structure. As he put it: "Neither extreme is stable, and from either end there is an evolution to the middle. The point of balance is actually reached, for this particular choice of numbers, when there are five doves for every seven hawks. In that situation, it turns out that each individual gets 6.25 points per conflict, on average."

In summary, for the given set of conditions, there is a stable solution that is not necessarily the best solution. Namely, the best solution would include only doves, where the average payoff for each conflict would be $(50 - 10) = 40$ points for the winning dove, and (-10) points for the losing dove. The average then is $(40 - 10)/2 = 15$, which is much greater than the 6.25 that Gribbin predicted.

Hawks and doves move randomly in the environment. They compete for food, according to the GT rules, every time they arrive at the food source at the same time. Runs are stopped when the equilibrium is reached. The scenario depicted by Gribbin expected 41.7% doves and 58.3% hawks (5 to 7 ratio) with the payoff of 6.25. However, in formulating the Hawk and Dove example as a CAS, both hawks and doves have a limited lifetime, and spawn new agents as a positive function of collected resources. Also, if any agent's resources drop below zero, that agent is removed from the system.

These complications, while more realistic, introduce subtle but important effects on the average payoffs for each species. When a hawk or dove reaches the end of their life, they are removed from the system and their currently held resources are lost, even though no contest for resources took place. This effect creates a negative pressure on the overall payoff that is not accounted for in the pure GT rules. Conversely, there can also exist an extra positive pressure on average payoff. For example, when an agent loses more points (due to conflict) than it has in reserve, then the average payoff of the system is reduced by the existing reserve, not the full penalty as described by Gribbin. Thus, the theoretical penalty is in some cases more than the effective penalty of losing a conflict. These additional complications, though more realistic than traditional GT rules, need to be carefully explored when combining GT with CAS.

Because of the positive and negative pressures on average payoffs, simulation runs in these experiments with relatively short lifetimes for agents and a low *success_rate* will not equal the ratio or average payoff as calculated by Gribbin. However, as both of these variables are scaled up, the cumulative effect of these positive and negative pressures on average payoff is diminished. Subsequent experiments in increasing the threshold *success_rate* and the lifetime of the agents confirm that the average ratio tends to stochastically approach Gribbin's ratio of 5 to 7 (41.6% doves) and that the average payoff increases toward 6.25 as the percentage of doves approaches this ratio.

SENSITIVITY

The evolutionarily stable strategy (ESS) produced in the previous experiment is stable under the given conditions. What would happen if some or all of the conditions were changed, as they often do in real life? Since the number of possible combinations of parameter values in the CAS tool is exceedingly large, it is helpful to demonstrate the range of effects that a small subset of these parameters can cause.

Starting Conditions

The results presented in Table 2 are obtained with the *success_rate* parameter scaled up along with lifetime, and the simulation run-time similarly scaled to fully measure the effects. What happens if the *success_rate* parameter is adjusted independently? Figure 1 illustrates the simulation run with *success_rate* incremented slowly over time, beginning at 150, and increased by 10 each 2000 time steps. (Lifetime is held constant at 200 steps for these runs.)

As expected, the total population for both hawks and doves decreases over time, due to the increasing threshold for generating new. At each change in *success_rate*, the system adjusts but stabilizes at a new population levels. (Figure 3 below illustrates this stability by incrementing *success_rate* from 160 to 190, and then allowing the system to run for an extended period of time.) Also, the average payoff increases with each change due to the larger proportion of doves in the population. However, when *success_rate* reaches 200, the population levels are

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TABLE 2

The Results of the Baseline Game Theory Simulation

Success_Rate	Lifetime	% doves	Average Payoff
500	1000	33.75	1.74
600	1200	33.87	1.94
700	1400	34.62	2.35
800	1600	35.05	2.52
900	1800	35.77	2.79
1000	2000	36.15	3.22
1200	2400	37.14	3.78
1400	2800	37.38	3.99
1600	3200	37.90	4.34
2000	4000	38.71	4.64
3000	6000	40.48	5.76
4000	8000	41.12	5.99

noticeably less stable (more volatile) and at 210 both populations die out completely.

Parameters do not necessarily influence the system outcome in the same way during the initial and the mature stages of the population growth. For example, the system is more sensitive to fluctuations in the initial value of *success_rate* than to its changes after the system achieves stability. Figures 2 and 3 show that incrementing the *success_rate* repeatedly does not lead to population collapse until this parameter reaches ~200. Multiple experiments were run (20 iterations per level) using initial *success_rate* settings of 160, 170, 180, 190, and 200. At the 160 level, the population stabilized as before. For all other runs, the population collapsed in every case, generally between ~900 and ~1500 simulation time steps.

ADDITIONAL EXPERIMENTS

The following two subsections bring more realism to the example considered thus far. In the interest of brevity, only changes to a baseline ESS case are considered.

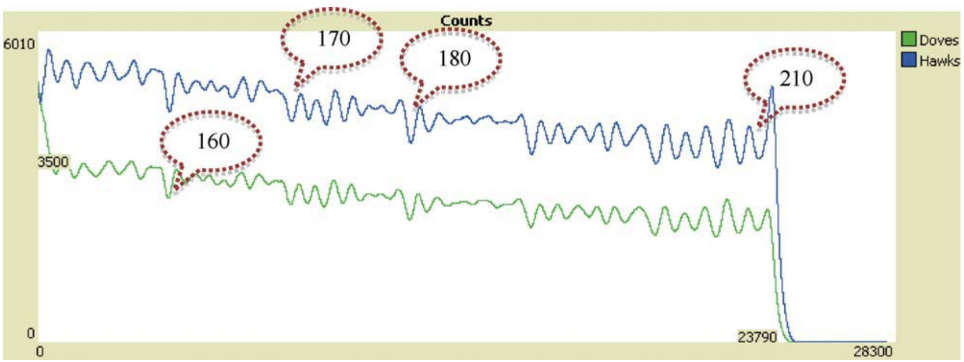
Eating Alone

To evaluate the predictions of game theory for the hawks-and-doves example, the previous experiments were conducted under the assumption that every attempt to find food leads to a conflict between agents. However, in reality, it is possible for doves and hawks to find food when there is no one else around. In such cases, they consume the food without negative consequences. Compared with the ESS version, this situation ends in a stable population comprising 65% hawks and 35% doves, with the lower average payoff of 2.9.

Effects of Penalties

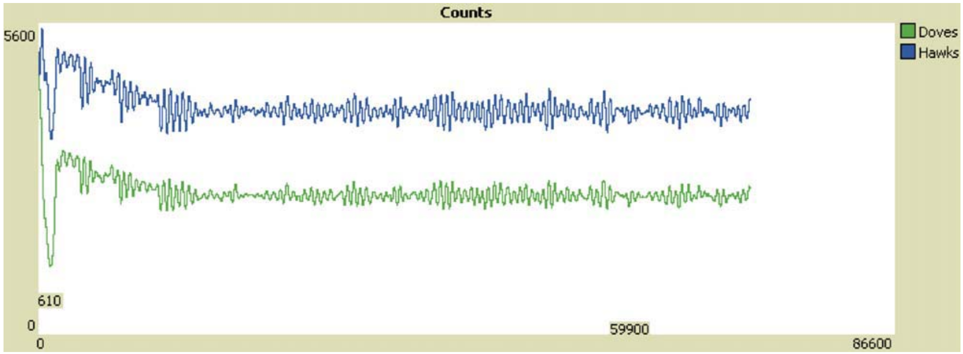
The ESS version imposes no penalty on the movement of hawks and doves, i.e., they do not waste energy unless they are punished for losing a conflict. To evaluate the effect of penalties on the system, a penalty of one point per move is assessed. Introducing even such a minimal penalty changes the situation drastically, as the whole population vanishes rather quickly, within 300 time steps (ticks). However, if both hawks and doves are allowed to eat food when alone, then the resulting population includes almost half the number of doves present under the ESS strategy, with a much lower average payoff of −5.84 per conflict, reflecting the larger presence of hawks in the population.

FIGURE 2



Effects for adjusting success_rate while the simulation is in progress.

FIGURE 3



After initialized, success_rate is incremented to 190, then run for an additional 60,000 ticks, illustrating population stability at this level.

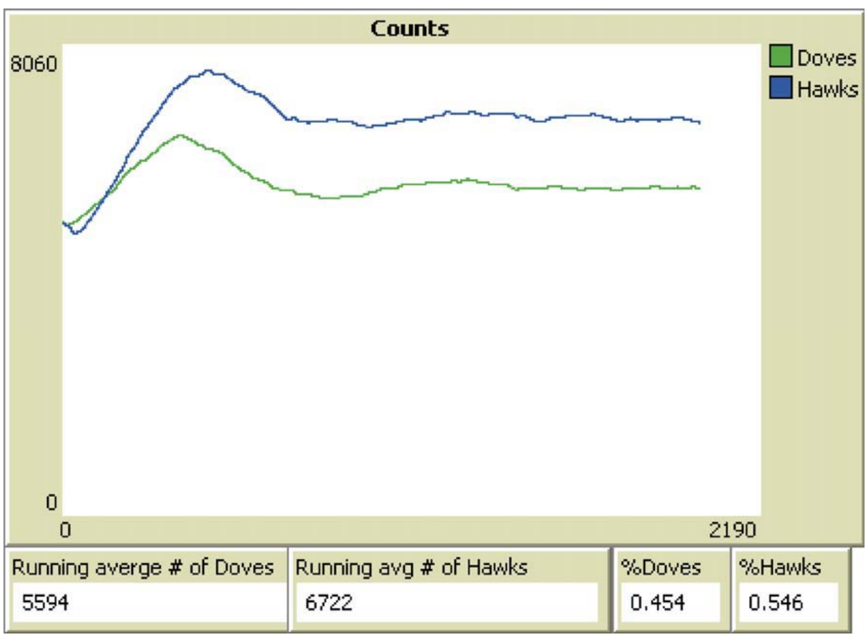
RISK

While it is clear that the hawks are programmed with an inherently more risky strategy than the doves are, it is unclear exactly how that added risk manifests itself among the hawk population. For example, while the hawks do incur a higher penalty during conflict with other hawks, they also benefit from that aggression when confronting a dove by winning the food without penalty. Furthermore, it is not definitive whether this higher-risk strategy is advantageous for the hawks or not.

Risk Advantages versus Drawbacks

To explore this issue in more depth, we experimented with a few adjustments to the model. The *success_rate*

FIGURE 4



Relative number of hawks and doves after 2000 simulation time steps.

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parameter was set to 240, in conjunction with increasing the lifetime parameter to 350. We also added a "spawn_penalty," such that whenever a hawk or dove exceeds the required number of points of *success_rate* to spawn, the spawn_penalty is deducted from that number, and the two resultant agents—the original and the new—split the remaining points as their new current value.

As with previous examples, it seems to be that the hawk strategy is more advantageous than that dove strategy, as the hawks stabilize with a higher average population. In the sample run indicated below, the hawks have about 55% of the population and the doves ~45% (Figure 4).

Upon closer examination, however, this strategy is decidedly less beneficial to the individual hawks. Their average current age at the end of 2000 simulation ticks is only about 90 ticks, whereas the doves current age averages ~175 ticks.

Exploring Risk with Mutation

Another way to gauge the relative risk associated with each type of agent is to allow the *success_rate* for individual hawks and doves to mutate over time. For each agent type, a lower *success_rate* represents higher risk due to the spawn penalty. That is, since the resultant number of current successes is lower after spawning a new agent and deducting the penalty, it is more likely that these agents will die out due to subsequent confrontation penalties. In contrast, a higher *success_rate* would give the agents a larger cushion after spawning a new agent.

Since the hawks incur more risk with their strategy—as illustrated by their average age being lower than that of the doves—then they have less room to adopt a risky (i.e., lower) success_rate. Therefore over time, we expect to see the hawks mutate toward a higher *success_rate* than that of the doves.

A sample simulation run of 60,000 ticks shows this to be the case, with the hawks increasing their *success_rate* from the initial 240 to ~275, and the doves decreasing theirs to ~130. Interestingly, the doves' percentage of the population drops to a low of about 38% around 15,000 ticks. At this point, the total number of agents is ~10,400, which is about 1800 below their previously stable level. However, as the *success_rate* for each group continues to adjust toward an optimal level, both hawks and doves increase their populations, until the overall total levels out to just over 16,000. At this point, the doves constitute ~42% of all the agents. The average age for the doves has decreased to about 165 ticks, whereas the hawks' average age has increased to ~135 ticks.

CONCLUSIONS

The research presented in this article demonstrates that a CAS environment can be a functional test-bed for evaluating the effect of GT strategies in a well-defined population or environment, thus expanding its utility beyond the limited number of strategic players. This research also demonstrates that the predictions of GT do not match the reality if either the conditions in the environment or the behavior of the agents changes over time. Furthermore, constructing a traditional GT in terms of a typical CAS, such that agents have a limited lifetime and can generate replacement agents as a positive function of GT rewards, can introduce additional complications that affect the "pure" GT theoretical results. Thus, the integration between the CAS and the GT both extends the utility of the CAS method and the applicability of the GT to a wider range of real-world situations.

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