Homework #5

Due on **13:00**, December 9, 2024

Problem 1. (30pts)

(Textbook Exercises 3.15) Show that the collection of decidable languages is closed under the operation of:

(a) (15 pts) union
(e) (15 pts) intersection

Problem 2 (30pts)

(Textbook Exercises 4.4) Let $A\varepsilon_{CFG} = \{\langle G\rangle |\ G \text{ is a CFG that generates } \varepsilon\}$. Show that $A\varepsilon_{CFG}$ is decidable.

Problem 3 (40pts)

Let $A = \{\langle M\rangle \mid M \text{ is a DFA that accepts all strings in } (01)^*\}$.

(a) (20 pts) Prove that for a DFA M,

$$M \text{ accepts all strings in } (01)^* \iff \overline{L(M)} \cap (01)^* = \emptyset.$$

You must prove the two directions in a formal way.

(b) (20 pts) Construct a decider H to show that A is decidable.
(Hint: you may use the fact that the language

$$E_{\text{DFA}} = \{\langle M\rangle \mid M \text{ is a DFA with } L(M) = \emptyset\}$$

is decidable.)