Homework 4

Due on 13:00, October 28, 2024

Problem 1. Let x and y be two string such that

$$x = x_1 x_2 \ldots x_n \text{ and } y = y_1 y_2 \ldots y_m, \text{ where } x_i, y_i \in \{0, 1\}.$$

We define the *Kronecker product* of x and y, denoted as $x \otimes y$, to be the string

$$(x_1 \cdot y) \circ (x_2 \cdot y) \circ \ldots \circ (x_n \cdot y).$$

In the above definition, $\circ$ denotes concatenation and $x_i \cdot y$ is defined as multiplying each character in y by x_i. When either x or y is ϵ, we define $x \otimes y = \epsilon$. For example, we have:

$$01 \otimes 101 = (0 \cdot 101) \circ (1 \cdot 101) = 000101.$$

(a) In this problem, you are required to design a two-tape Turing machine that calculates the *Kronecker product*. The machine starts with the tape content:

$$\begin{aligned} &\text{tape1 :} x\#y \sqcup \ldots \\ &\text{tape2 :} \sqcup \ldots \end{aligned}$$

and it should end with the tape content:

$$\begin{aligned} &\text{tape1 :} x\#y \sqcup \ldots \\ &\text{tape2 :} x \otimes y \sqcup \ldots \end{aligned}$$

Following the textbook, we define the two-tape TM's tape to be only infinite on the right side. Also, the tape heads are allowed to move right, move left or stay. The machine can assume the input string is properly in the form $x\#y$ where $x, y \in \{0, 1\}^*$ (i.e., the machine does not have to check the format), and the heads are initialized at the start of the tapes as usual.

You only have to give the diagram. For specifying the transitions, please follow the notation used in lecture slides "chap3_multitapeTM1.pdf". Moreover, you can simplify the diagram by using notations like

$$\begin{cases} \{0, 1, \#\} \to 1, R \\ \sqcup \to S \end{cases}$$

if the first tape reads any character in $\{0, 1, \#\}$, writes 1 and goes right, while the second tape reads $\sqcup$ and stays.

The state q_{reject} and the edges to q_{reject} can be ignored. The machine should accept after $x \otimes y$ is constructed. The machine should use

$$\Sigma = \{0, 1, \#\} \text{ and } \Gamma = \{0, 1, \#, \sqcup\}$$

and the number of states should be less than or equal to 7(including q_{accept} and q_{reject}. (Hint: Make good use of the second tape to reduce the number of states.)

(b) Simulate the machine on the string

10#01

and show the configuration at each step. Multiple steps that involve only head movements (no state or tape change) can be condensed into one step.

Problem 2. Explain why the following is not a description of a legitimate Turing machine.

M_{bad} = "On input $\langle p \rangle$, a polynomial over $x_1, \ldots, x_k$:

1. Try all possible settings of $x_1, \ldots, x_k$ to integer values.
2. Evaluate p on all of these settings.
3. If any of these settings evaluates to 0, *accept*; otherwise, *reject*."