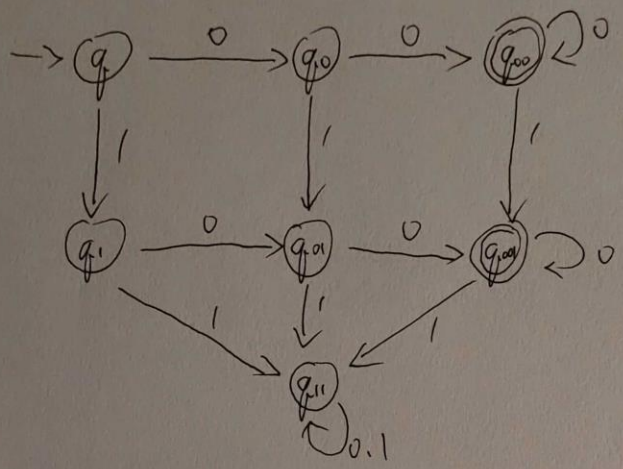
Formal Languages and Automata Theory HW1 r12631055 林東甫

Problem 1.

$\Sigma = \{0, 1\}$, $L_1 = \{w \mid w$ contains at least two 0s and at most one 1$\}$

$L_1(M)$:



$M = (Q, \Sigma, \delta, q_0, F) = (Q = \{q, q_0, q_{00}, q_1, q_{01}, q_{001}, q_{11}\}$

$\Sigma = \{0, 1\}$

$q_0 = q$

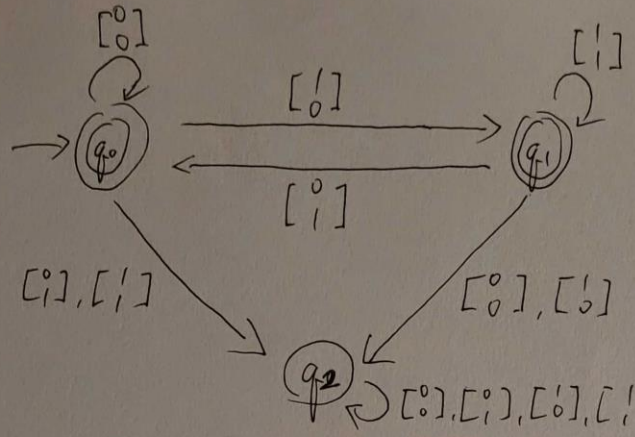
$F = \{q_{00}, q_{001}\}$

The δ function:

	0	1
q	q_0	q_1
q_0	q_{00}	q_{01}
q_{00}	q_{00}	q_{001}
q_1	q_{01}	q_{11}
q_{01}	q_{001}	q_{11}
q_{001}	q_{001}	q_{11}
q_{11}	q_{11}	q_{11}

Problem 2

(a) $L(M)$:



$M = (Q = \{q_0, q_1, q_2\}, \Sigma_2 = \{[0\,0], [1\,1], [1\,0], [0\,1]\}, q_0 = q_0, F = \{q_0, q_1\})$

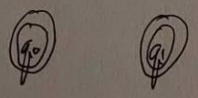
The δ function:

	$[0\,0]$	$[1\,1]$	$[1\,0]$	$[0\,1]$
q_0	q_0	q_2	q_1	q_2
q_1	q_2	q_0	q_2	q_1
q_2	q_2	q_2	q_2	q_2

(b) pf: Assume L can be recognized by a 2-state DFA. ($L(M_3)$)

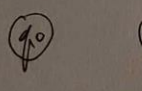
Then we must have the following situations:

Case 1:



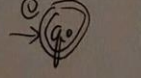
in this case, any w will be accepted by $L(M_3)$ including $[1\,1][0\,0] \notin L$, which is not possible.

Case 2:



in this case, any w will not be accepted by $L(M_3)$ including $[0\,0] \in L$, which is not possible.

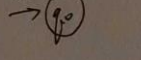
Case 3:



known $[1\,1], [0\,0] \notin L$, $[1\,0], [0\,1] \in L$, i.e.

① if $q_0 = q_0$, $F = \{q_0\}$, then $\begin{cases} \delta(q_0, [1\,0]) = \delta(q_0, [0\,0]) = q_0 \\ \delta(q_0, [0\,1]) = \delta(q_0, [1\,1]) = q_1 \end{cases}$

② if $q_0 = q_0$, $F = \{q_1\}$, then $\begin{cases} \delta(q_0, [1\,0]) = \delta(q_0, [0\,0]) = q_1 \\ \delta(q_1, [1\,1]) = \delta(q_1, [0\,1]) = q_0 \end{cases}$,



Now considering $W_1 = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \notin L$, $W_2 = \begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \notin L$,

in ①, $\delta(q_0, W_1) = q_0$, W_1 is accepted by M_2

in ②, $\delta(q_0, W_2) = q_1$, W_2 is accepted by M_2 } $\rightarrow\leftarrow$

$\therefore$ Case 3 is not possible.

therefore L can <u>not</u> be recognized a 2-state DFA,

$\therefore$ L can <u>not</u> be recognized by a DFA with less than 3 states.

(where 1-state DFA is trivial, considering Case 1 and Case 2)