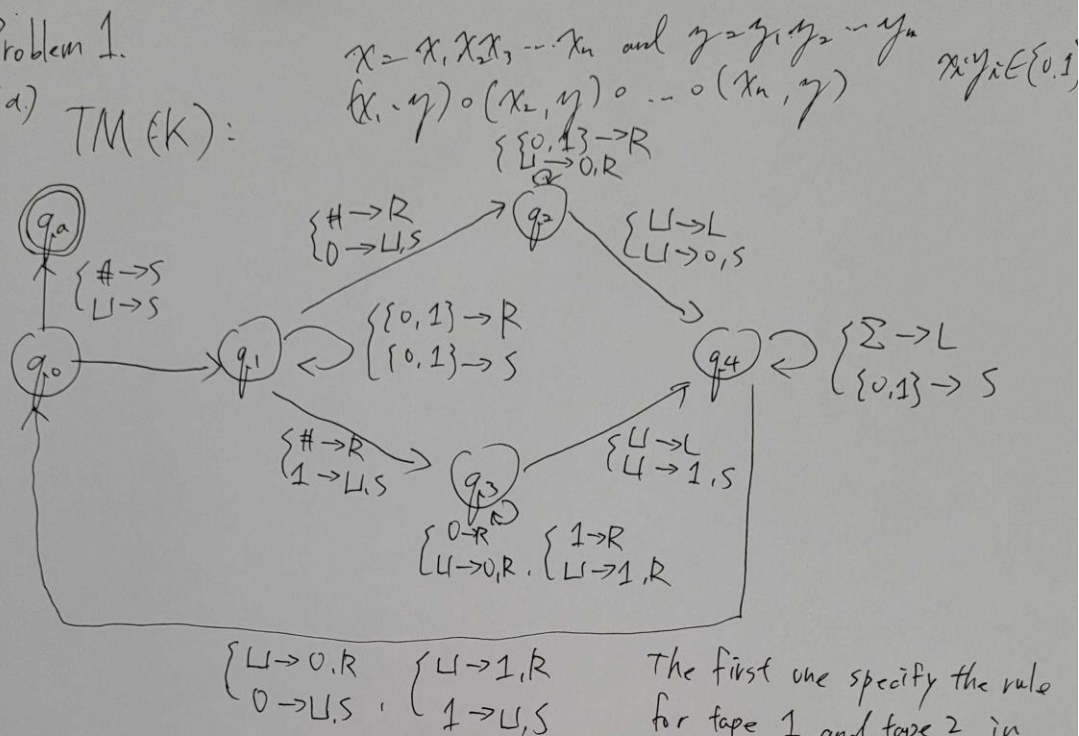


Problem 1.

(a) TM (K):



At q_0 , we copy the current character in x to the second tape and mark its position with L .

At q_1 , we move the head of first tape to the $\#$ in the middle.

Then, according to the character we copied earlier, we transition to either q_2 or q_3 , where we can scan through y and calculate $x_i \cdot y$ on the second tape. When y ends, the machine goes to q_4 while storing x_i onto second tape again.

At q_4 , the machine restore x_i from the ~~second~~ head of the first tape back to the position of x_i which was marked with L .

Then, we restore x_i from the second tape and move to the next character of x . If there are more characters of x , the machine repeats the steps above. Otherwise, it accept.

(b)

$$\begin{array}{c} q_0 01 \# 10 \sqcup \\ q_0 \end{array} \Rightarrow \begin{array}{c} \sqcup q_1 1 \# 10 \sqcup \\ q_1 0 \sqcup \end{array} \Rightarrow \begin{array}{c} \sqcup 1 q_1 \# 10 \sqcup \\ q_1 0 \sqcup \end{array}$$

$$\Rightarrow \begin{array}{c} \sqcup 1 \# q_2 10 \sqcup \\ q_2 \sqcup \end{array} \Rightarrow \begin{array}{c} \sqcup 1 \# 1 q_2 0 \sqcup \\ 0 q_2 \sqcup \end{array} \Rightarrow \begin{array}{c} \sqcup 1 \# 10 q_2 \sqcup \\ 00 q_2 \sqcup \end{array}$$

$$\Rightarrow \begin{array}{c} \sqcup 1 \# 1 q_4 0 \sqcup \\ 00 q_4 0 \sqcup \end{array} \Rightarrow \dots \Rightarrow \begin{array}{c} q_4 \sqcup 1 \# 10 \sqcup \\ 00 q_4 0 \sqcup \end{array}$$

$$\Rightarrow \begin{array}{c} 0 q_0 1 \# 10 \sqcup \\ 00 q_0 \sqcup \end{array} \Rightarrow \begin{array}{c} 0 \sqcup q_1 \# 10 \sqcup \\ 00 q_1 1 \sqcup \end{array} \Rightarrow \begin{array}{c} 0 \sqcup \# q_3 10 \sqcup \\ 00 q_3 \sqcup \end{array}$$

$$\Rightarrow \begin{array}{c} 0 \sqcup \# 1 q_3 0 \sqcup \\ 00 1 q_3 \sqcup \end{array} \Rightarrow \begin{array}{c} 0 \sqcup \# 10 q_3 \sqcup \\ 00 10 q_3 \sqcup \end{array} \Rightarrow \begin{array}{c} 0 \sqcup \# 1 q_4 0 \sqcup \\ 00 10 q_4 1 \sqcup \end{array}$$

$$\Rightarrow \dots \Rightarrow \begin{array}{c} 0 q_4 \sqcup \# 10 \sqcup \\ 00 10 q_4 1 \sqcup \end{array} \Rightarrow \begin{array}{c} 0 1 q_0 \# 10 \sqcup \\ 00 10 q_0 \sqcup \end{array} \Rightarrow \text{accept.}$$

Problem 2.

1. The statement "try all possible settings of $x_1, \dots, x_k \dots$ " implies that the TM ~~might~~ need to explore an infinite number of possible integer values for each variable.

Since TM is designed for computation the proceed in a finite and discrete manner. It operate on a tape with finite set of symbols or states.

2. "Evaluate p_i on all of these settings." suggests a form of non-deterministic action where the TM is simultaneously checking all possible combinations of integers for the variables.

A standard TM is deterministic in each step, given a specific state and symbol on the tape, there should be a unique transition to every next state.

3. In this statement, the TM should accept if any of settings evaluates to 0. But in a standard TM, the acceptance or rejection of an input is determined by reaching a specific state of accept or reject. It's not based on arbitrary conditions as evaluation of a polynomial. TM use a finite set of states and transitions to decide whether to accept or reject an input, ~~and~~ different from mathematical expressions.