2024 Formal Languages and Automata Theory HW5 r12631055 林東甫

Problem 1.

(a) Union

Let L_1, L_2 are two decidable languages accepted by Turing Machines M_1, M_2. Then $L_1 \cup L_2$ can be designed as follows:

Given an input w, simulate M_1 on w. If M_1 accepts then accepts, else simulate M_2 on w. If M_2 accepts then accept else reject. If L_1, L_2 are decidable, then $L1 \cup L2$ is decidable.

(e) Intersection

On the given input as (a), run the TMs of both L_1 and L_2.

Accept if and only if ~~L_1~~ M_1 and ~~L_2~~ M_2 both accept else reject.

P~~roblem~~ 2.

~~To show $A_{\varepsilon CFG}$ is decidable, we can construct a Turing Machine (TM) and show TM M decides $A_{\varepsilon CFG}$. Let M be $\forall$ G:~~

~~If G generates ε then M accepts else rejects.~~

Problem 2

The language $A_{\varepsilon CFG} = \{\langle G\rangle \mid G$ is a CFG that generates $\varepsilon\}$.

As a CFG that generates ε, i.e. S can derive ε by a sequence of rules in G, where S is the start symbol of G. for example:

if $S \to A$, $A \to \varepsilon$, then S can derive ε i.e. G generates ε.

Procedure

① ~~mark q_0~~ Run through all the production rules in G. that start with S.

② repeat the process: $A \to X_1 \cdots X_n$. If it reach other terminal, then ends, If it derive ε, then accepts.

③ check all the production rules in G from S, If there are no ε, rejects.

Problem 3.

(a) Prove that for a DFA M:

M accepts all string in $(01)^*$ $\Longleftrightarrow$ $\overline{L(M)} \cap (01)^* = \emptyset$

pf. $\Rightarrow$: Assume for contradiction that $\overline{L(M)} \cap (01)^* \neq \emptyset$.

$\exists x \in \overline{L(M)} \cap (01)^*$ then $x \notin L(M)$ and $x \in (01)^*$.

That is x is a string in $(01)^*$ rejected by M. $\rightarrow\leftarrow$

$\Leftarrow$: Assume for contradiction that M does not accepts all the string in $(01)^*$. $\exists x \in (01)^*$ rejected by M, then $x \in \overline{L(M)} \cap (01)^*$. $\rightarrow\leftarrow$

(b) Since regular languages are closed under complement and intersection we can construct a DFA that recognizes $\overline{L(M)} \cap (01)^*$.

Also, there exists a decider H that decides E_{DFA}. Therefore, we design the following TM recognizing A.

On input $\langle M \rangle$ where M is a DFA:

1. Construct DFA B that recognizes $\overline{L(M)} \cap (01)^*$
2. Run decider H on input $\langle B \rangle$.
3. If H accepts, accept. If H rejects, reject.