

Homework 6

Due on 13:00, December 16, 2024

Problem 1. Assume f and g are functions $f, g : \mathbb{N} \rightarrow \mathbb{R}^+$. Prove or disprove the following statements by using the following definitions.

Definition 1. We say $f(n) = O(g(n))$ if there exists $c > 0$ and $n_0 \in \mathbb{N}$ such that for every integer $n \geq n_0$,

$$f(n) \leq c g(n).$$

Definition 2. We say $f(n) = o(g(n))$ if for each $c > 0$, there exists $n_0 \in \mathbb{N}$ such that for every integer $n \geq n_0$,

$$f(n) \leq c g(n).$$

(a) Let $f(n) = e^{2n}$ and $g(n) = e^n$. $f(n) = O(g(n))$

(b) Let $f(n) = n!$ and $g(n) = n^n$. $f(n) = o(g(n))$

For proving the statements, you need to show the existence of n_0 for one c or all c 's, depending on the definition of big-O or small-o. For disproving the statements, you must prove the opposite of the definition by also showing details.