

Homework #1

Due on 13:00, September 16, 2024

Problem 1. Let $\Sigma = \{0, 1\}$.

Please give the state diagram of a DFA recognizing the language

$$L_1 = \{w \mid w \text{ contains at least two 0s and at most one 1}\}.$$

(The problem is from exercise 1.6 in the textbook.)

Problem 2. Let

$$\Sigma_2 = \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}.$$

That is, Σ_2 contains all columns of 0s and 1s of height two. Each string $\mathbf{w} \in \Sigma_2^*$ is written as

$$\mathbf{w} = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \cdots \begin{bmatrix} a_n \\ b_n \end{bmatrix},$$

where $n \geq 0$, a_i and b_i are either 0 or 1 for each $1 \leq i \leq n$. Consider the language

$$L = \{\mathbf{w} \in \Sigma_2^* \mid b_1 b_2 \dots b_n \text{ is the outcome of right shifting } a_1 a_2 \dots a_n \text{ by one bit}\}.$$

For simplicity, we assume the leftmost bit after applying the shift operation is always 0. For example,

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \in L, \quad \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \in L, \quad \text{while} \quad \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} \notin L.$$

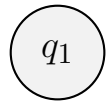
Note that $\epsilon \in L$ since the outcome of right shifting an empty string is still an empty string.

- (a) Construct a DFA for L in less than or equal to 3 states
- (b) Prove that L can not be recognized by a DFA with less than 3 states. Our proof is by contradiction. Assume L can be recognized by a 2-state DFA. Then we must have the following situations (here q_0 is not necessary a start state):



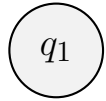
(1)

or



(2)

or



(3)

Explain which cases are not possible. From the remaining case(s), finish the proof.
(This problem is an old exam problem and is considered to be more challenging.
You may see problems with similar flavors in the upcoming exam.)