Problem 1

Let $\Sigma = \{0, 1\}$, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^k 1 \Sigma^*, \text{ and } |u| \ge |v|\}$

Let CFG C: $S \rightarrow 0S0 \mid 1S0 \mid 1S \mid 0S \mid 0A1 \mid 1A1$
$\quad\quad\quad\quad A \rightarrow 0A0 \mid 1A0 \mid 1A \mid 0A \mid 0A1 \mid 1A1 \mid \epsilon$

$C = (\{S, A\}, \{0, 1, \epsilon\}, R : \text{the above rules}, S = S)$

Thus, S must been through A to reach terminal (ϵ), and while

$S \overset{*}{\Rightarrow} A$, there must be at least one 1 on the right side.

Problem 2.

(1) Original grammar
$S \rightarrow XD \mid Y$
$X \rightarrow aXb \mid ab$
$D \rightarrow cD \mid \epsilon$
$Y \rightarrow aBc \mid aYc$
$B \rightarrow bB \mid \epsilon$

(2) remove $D \rightarrow \epsilon$
$S \rightarrow XD \mid Y \mid X$
$X \rightarrow aXb \mid ab$
$D \rightarrow cD \mid c$
$Y \rightarrow aBc \mid aYc$
$B \rightarrow bB \mid \epsilon$

(3) remove $B \rightarrow \epsilon$
$S \rightarrow XD \mid Y \mid X$
$X \rightarrow aXb \mid ab$
$D \rightarrow cD \mid c$
$Y \rightarrow aBc \mid ac \mid aYc$
$B \rightarrow bB \mid b$

Start variable not on the right, skip it.

(4) remove $S \rightarrow X, S \rightarrow Y$
$S \rightarrow XD \mid aBc \mid aYc$
$\quad\quad\quad \mid aXb \mid ab \mid ac$
$X \rightarrow aXb \mid ab$
$D \rightarrow cD \mid c$
$Y \rightarrow aBc \mid aYc \mid ac$
$B \rightarrow bB \mid b$

(5) add $A \rightarrow a$,
$\quad\quad\quad B \rightarrow b$
$\quad\quad\quad C \rightarrow c$
$S \rightarrow XD \mid ABC \mid AYC$
$\quad\quad \mid AXZ \mid AZ \mid AC$
$X \rightarrow AXZ \mid ab AZ$
$D \rightarrow CD \mid c$
$Y \rightarrow ABC \mid AYC \mid AC$
$B \rightarrow ZB \mid b$
$A \rightarrow a$
$Z \rightarrow b$
$C \rightarrow c$

(6) add $AB \rightarrow E \rightarrow AB$
$\quad\quad\quad F \rightarrow AY$
$\quad\quad\quad G \rightarrow AX$
$S \rightarrow XD \mid EC \mid FC$
$\quad\quad \mid GZ \mid AZ \mid AC$
$X \rightarrow GZ \mid AZ$
$D \rightarrow cD \mid c$
$Y \rightarrow EC \mid FC \mid AC$
$B \rightarrow ZB \mid b$
$A \rightarrow a$
$Z \rightarrow b$
$C \rightarrow c$
$E \rightarrow AB$
$F \rightarrow AY$
$G \rightarrow AX$

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