Homework 3

Due on 13:00, October 14th, 2024

Problem 1 (40 pts)

Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words,

$$B = \{uv \mid u \in \Sigma^*,\, v \in \Sigma^* 1 \Sigma^*, \text{ and } |u| \geq |v|\}.$$

Give a CFG that generates B.

Problem 2 (60 pts)

Consider the context-free grammar (V, Σ, R, S) with:

$$V = \{S, B, D, X, Y\} \quad \text{and} \quad \Sigma = \{a, b, c\},$$

where the rule set R contains the following rules:

$$\begin{aligned} S &\to XD \mid Y \\ X &\to aXb \mid ab \\ D &\to cD \mid \epsilon \\ Y &\to aBc \mid aYc \\ B &\to bB \mid \epsilon \end{aligned}$$

Please convert the grammar (1) to CNF by the procedure in Theorem 2.9 of the textbook (in our slides chap2 CNF2.pdf). In removing ? $\to \varepsilon$ rules, consider the order as

$$D, B.$$

In removing rules of a single variable on the right, consider the order as

$$S, X, Y.$$