Introduction to Linear Algebra

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1 Introduction

This lecture marks the beginning of our course on Linear Algebra, presented by Cordelia. The course will alternate between Cordelia and Erik, with Erik taking over in the following weeks. Attendance polls will be conducted every Thursday, and students are encouraged to participate using the provided QR codes.

2 Course Announcements

- The first hand-in assessment has been released and is due next Wednesday at 5:00 PM. This assessment will cover material from last week and this week's lectures.
- Students are reminded to check the course platform for announcements and updates.

3 Lecture Overview

Today's lecture focuses on matrix algebra, which involves calculations with matrices and vectors. Key topics include:

- Basic calculation rules for matrices
- Scalar multiplication
- Transposition of matrices
- Matrix-vector multiplication

4 Matrices

A matrix is defined as a rectangular array of numbers. The elements of a matrix are denoted by a_{ij}, where i indicates the row and j indicates the column. The notation for a matrix with m rows and n columns is referred to as an $m \times n$ matrix.

4.1 Matrix Elements

The elements of a matrix can be defined by a specific rule. For example, if we define the elements by the rule $b_{ij} = i - j$, we can construct the matrix based on this rule.

4.2 Matrix Addition

Matrices can only be added if they have the same dimensions. For example, if we have matrices A and B, they can be added if both are $m \times n$ matrices.

4.3 Scalar Multiplication

Scalar multiplication involves multiplying every element of a matrix by a scalar k. For example, if we have a matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

then multiplying by a scalar k gives:

$$kA = \begin{pmatrix} k \cdot 1 & k \cdot 2 \\ k \cdot 3 & k \cdot 4 \end{pmatrix}$$

4.4 Transposition

The transpose of a matrix A, denoted A^T, is obtained by swapping its rows and columns. For an $m \times n$ matrix, the transpose will be an $n \times m$ matrix.

5 Matrix-Vector Multiplication

Matrix-vector multiplication is defined as follows: if A is an $m \times n$ matrix and x is a vector with n components, the product Ax is given by:

$$Ax = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

This results in a new vector with m components.

5.1 Example of Matrix-Vector Multiplication

Consider the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

and the vector

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

The product Ax is calculated as:

$$Ax = \begin{pmatrix} 1 \cdot x_1 + 2 \cdot x_2 \\ 3 \cdot x_1 + 4 \cdot x_2 \end{pmatrix}$$

6 Properties of Matrix-Vector Multiplication

If $Ax = 0$ and $x \neq 0$, it does not imply that $A = 0$. This is a crucial distinction in linear algebra, as it introduces the concept of linear dependence.

7 Dot Product

The dot product of two vectors u and v of the same dimension is defined as:

$$u \cdot v = u_1 v_1 + u_2 v_2 + \cdots + u_n v_n$$

This operation is fundamental in various applications, including projections and determining angles between vectors.

8 Identity Matrix

The identity matrix I_n is defined as a square matrix with ones on the diagonal and zeros elsewhere. For example, the 3×3 identity matrix is:

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Multiplying any matrix A by the identity matrix will yield the original matrix:

$$AI_n = A$$

9 Conclusion

In this lecture, we covered the basics of matrix algebra, including definitions, operations, and properties. The next lecture will delve into matrix multiplication and further explore these concepts.