Calculus and Its Applications

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1 Introduction

Welcome back to our calculus course. This lecture continues our exploration of limits and continuity of functions. We will discuss the Squeeze Theorem, continuity definitions, and practical examples.

2 Squeeze Theorem

2.1 Understanding the Theorem

Whenever faced with an inequality involving three functions and asked about the limit, one should immediately think of the Squeeze Theorem. However, before applying it, it is crucial to verify that all assumptions of the theorem are satisfied.

2.2 Example Problem

Consider the functions:

$$f(x) = -\frac{1}{3}x^3 + x^2 - \frac{7}{3}$$

$$h(x) = \cos\left(\frac{\pi}{2}x\right)$$

We want to find the limit as x approaches 2.

2.3 Checking Assumptions

1. **Defined Functions**: Both functions $f(x)$ and $h(x)$ are defined for all x not equal to 2, satisfying the first assumption. 2. **Open Interval**: We consider an open interval around 2, such as $(1.5, 2.5)$, where the inequalities hold.

Since both conditions are satisfied, we can apply the Squeeze Theorem:

$$\lim_{x\to 2} g(x) = \lim_{x\to 2} f(x) = \lim_{x\to 2} h(x)$$

Calculating these limits gives:

$$\lim_{x\to 2} f(x) = -1 \quad \text{and} \quad \lim_{x\to 2} h(x) = -1$$

Thus, $\lim_{x\to 2} g(x) = -1$.

3 Continuity of Functions

3.1 Definition of Continuity

A function is continuous over a closed interval if it is continuous at every point in the open interval and continuous from the right at the left endpoint and from the left at the right endpoint.

3.2 Example Problem

Consider the function:

$$f(x) = \frac{x^2 - 4x + 3}{x^2 - 1}$$

We need to determine if it is continuous over the closed interval $[0, 3]$.

3.3 Identifying Discontinuities

The function is not defined at $x = 1$, which is within the interval $[0, 3]$. Therefore, it cannot be continuous over this interval.

4 Piecewise Functions

4.1 Example Problem

Consider a piecewise function defined as:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$$

We need to check continuity at $x = 1$.

4.2 Checking Continuity

1. **Value at 1**: $f(1) = \sqrt{1} = 1$. 2. **Left Limit**: $\lim_{x \to 1^-} f(x) = 1^2 = 1$. 3. **Right Limit**: $\lim_{x \to 1^+} f(x) = \sqrt{1} = 1$.

Since the left limit, right limit, and function value at 1 are all equal, $f(x)$ is continuous at $x = 1$.

5 Conclusion

In this lecture, we explored the Squeeze Theorem, continuity definitions, and practical examples. Understanding these concepts is crucial for further studies in calculus and analysis.

Thank you for your participation, and I look forward to our next session.