Introduction to Linear Algebra

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February 22, 2025

1 Introduction

This lecture marks the beginning of our course on Linear Algebra. The focus will be on matrix algebra, which includes operations with matrices and vectors. Attendance polls will be conducted regularly, and the first assessment has been announced.

2 Overview of Matrices

2.1 Definition

A matrix is a rectangular array of numbers arranged in rows and columns. The element in the i-th row and j-th column is denoted as a_{ij}.

2.2 Matrix Notation

For an $m \times n$ matrix, the notation is as follows:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

2.3 Matrix Operations

- **Addition:** Two matrices can be added if they have the same dimensions. The sum is obtained by adding corresponding elements.
- **Scalar Multiplication:** Multiplying a matrix by a scalar involves multiplying each element of the matrix by that scalar.
- **Transposition:** The transpose of an $m \times n$ matrix A is an $n \times m$ matrix denoted by A^T, where the rows and columns are swapped.

3 Matrix-Vector Multiplication

3.1 Definition

To multiply an $m \times n$ matrix A by an n-dimensional vector $\mathbf{x}$, we compute:

$$A\mathbf{x} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix}$$

3.2 Properties

- The dimensions must match: If A is $m \times n$, then $\mathbf{x}$ must be $n \times 1$.
- The result is an $m \times 1$ vector.

4 Dot Product

The dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ of the same dimension is defined as:

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \cdots + u_nv_n$$

5 Identity Matrix

The identity matrix I_n is defined as:

$$I_n = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Multiplying any matrix A by the identity matrix yields the original matrix:

$$I_nA = A$$

6 Linear Systems of Equations

A system of linear equations can be represented in matrix form as:

$$A\mathbf{x} = \mathbf{b}$$

where A is the matrix of coefficients, $\mathbf{x}$ is the vector of unknowns, and $\mathbf{b}$ is the vector of constants.

7 Conclusion

In the next lecture, we will explore matrix multiplication and the concept of matrix inverses. Understanding these foundational concepts is crucial for further studies in linear algebra.