Calculus Lecture Summary - Week 3

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1 Introduction

Good morning and welcome to week three of our calculus course. Before we dive into today's content, I want to highlight the importance of utilizing available resources such as Math Pals, who are fellow students that can assist you with any questions you may have.

2 Recap of Week 2

To start, let's have a quick recap of what was covered last week. The focus was on limits, specifically:

- Understanding limits from both sides (left and right).
- Checking if limits exist.
- Introduction to continuity of functions.

3 Today's Topics

Today, we will build upon the concepts of limits and continuity and introduce derivatives.

3.1 Understanding Derivatives

The derivative at a point on a curve represents the slope of the tangent line at that point. To visualize this, imagine a family of straight lines that vary as the point moves along the curve.

3.2 Geometric and Analytical Perspectives

The derivative can be understood in multiple ways:

- **Geometric:** The tangent line to the curve.
- **Analytical:** The formal definition of the derivative.
- **Applied:** The derivative as a rate of change.

3.3 Existence of Derivatives

Each derivative is a function in itself, which may or may not have further derivatives. The existence of a derivative at a point is closely tied to the concept of limits.

3.4 Calculating Derivatives

The calculation of the first derivative involves limits. The formal definition is:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

We will explore how this definition applies to various functions, including polynomials.

4 Practical Applications

We will engage in exercises to apply the concept of derivatives as rates of change. For instance, calculating average velocity over time intervals and then finding instantaneous velocity by taking limits as the intervals shrink.

4.1 Example: Average Velocity

To find the average velocity between two points, we calculate:

$$\text{Average Velocity} = \frac{\text{Change in Height}}{\text{Change in Time}}$$

As the time interval approaches zero, this leads us to the instantaneous velocity, which is the derivative.

4.2 Conditions for Derivatives

A function must be continuous at a point for the derivative to exist there. However, continuity alone does not guarantee the existence of a derivative. For example, a function can be continuous but have a corner or vertical tangent at that point.

5 Discussion and Exercises

We will conduct interactive exercises where students can log in and vote on answers to derivative-related questions. This will help reinforce the concepts discussed.

5.1 Example: Modulus Function

The modulus function $|x|$ is continuous everywhere but does not have a derivative at $x = 0$ due to the sharp corner.

6 Conclusion

In conclusion, we have explored the concept of derivatives, their geometric and analytical interpretations, and their practical applications in calculating rates of change. Remember that understanding the underlying principles is crucial for mastering calculus.