Calculus Lecture Summary - Week 3

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1 Introduction

Good morning and welcome to week three. Before we start, I want to give a shout out to Matt's Pulse, a support system for students, especially for CAP and PPS. This service connects you with fellow students in later years who can assist with any questions you may have.

2 Recap of Week 2

Last week, the topic was limits. A volunteer summarized that we discussed limits from both sides, checking their existence. We also touched on the continuity of functions.

3 Today's Topics

Today, we will build on the concepts of limits and continuity, focusing on derivatives.

3.1 Understanding Derivatives

The derivative at a point on a curve represents the slope of the tangent line at that point. To visualize this, consider a family of straight lines that vary as the point on the curve changes.

3.2 Geometric and Analytical Perspectives

The derivative can be understood in multiple ways:

- Geometrically, as the slope of the tangent line.
- Analytically, through the definition involving limits.
- In applied terms, as a rate of change.

3.3 Existence of Derivatives

Each derivative is a function in itself, which may or may not have further derivatives. The existence of a derivative at a point requires checking limits. The calculation of the first derivative involves taking an increment, typically denoted as h, and examining what happens as h approaches zero.

3.4 First Principles of Derivatives

The formal definition of the derivative is given by:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This limit must exist for the derivative to be defined.

3.5 Example: Derivative of a Polynomial

For a second-degree polynomial, the derivative can be calculated using the first principles. For example, if we have a polynomial $f(x) = ax^2 + bx + c$, the derivative is:

$$f'(x) = 2ax + b$$

4 Interactive Learning

We engaged in an interactive session where students logged in to answer questions related to derivatives and their applications.

4.1 Average Velocity and Instantaneous Velocity

To find average velocity, we calculate the displacement over time. For example, if we have heights at times $t = 1$ and $t = 1.5$, the average velocity is given by:

$$\text{Average Velocity} = \frac{h(1.5) - h(1)}{1.5 - 1}$$

As the time interval shrinks, we approach the instantaneous velocity, which is the derivative.

4.2 Conditions for Derivative Existence

A function must be continuous at a point for the derivative to exist there. However, continuity alone is not sufficient; the function must also not have any corners or vertical tangents at that point.

4.3 Example: Modulus Function

Consider the function $f(x) = |x|$. This function is continuous everywhere but does not have a derivative at $x = 0$ due to the corner point.

5 Conclusion

We concluded the lecture by discussing the importance of understanding derivatives in both theoretical and practical contexts. We also emphasized the need to visualize functions and their derivatives to grasp their behavior better.