Linear Algebra Lecture Summary

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February 22, 2025

1 Introduction

This week's lecture focused on vectors and planes, building on the previous week's discussion of eigenvalues and eigenvectors. The lecture included a review of fundamental concepts and introduced new ideas related to vector operations.

2 Attendance and Homework

The attendance code was provided at the beginning of the lecture. Homework 303 was released and is due next Wednesday, covering the topics discussed in the previous week.

3 Vectors in R^n

3.1 True or False Question

A question was posed regarding whether two nonzero vectors in R^n must have the same coordinate tips. The consensus was that this statement is false. The key points discussed included:

- Vectors are defined by their length and direction, not by their starting points.
- Two vectors can be equal even if they originate from different points in space.

3.2 Vector Representation

Vectors can be represented abstractly without specific coordinates. For example, vectors can be defined in terms of their direction and magnitude, allowing for operations such as addition using the tip-to-tail method.

4 Vector Operations

4.1 Addition of Vectors

Vectors can be added using the tip-to-tail method. If vectors $\mathbf{u}$ and $\mathbf{v}$ are represented as arrows, the sum $\mathbf{u} + \mathbf{v}$ can be visualized by placing the tail of $\mathbf{v}$ at the tip of $\mathbf{u}$.

4.2 Norm of a Vector

The norm (or length) of a vector $\mathbf{v} = (v_1, v_2, \ldots, v_n)$ is calculated using the Pythagorean theorem:

$$\|\mathbf{v}\| = \sqrt{v_1^2 + v_2^2 + \ldots + v_n^2}$$

4.3 Dot Product

The dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ is defined as:

$$\mathbf{u} \cdot \mathbf{v} = \|\mathbf{u}\| \|\mathbf{v}\| \cos \theta$$

where θ is the angle between the two vectors.

5 Inequalities and Theorems

5.1 Cauchy-Schwarz Inequality

The Cauchy-Schwarz inequality states that:

$$|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \|\mathbf{v}\|$$

5.2 Triangle Inequality

The triangle inequality states that for any vectors $\mathbf{u}$ and $\mathbf{v}$:

$$\|\mathbf{u} + \mathbf{v}\| \leq \|\mathbf{u}\| + \|\mathbf{v}\|$$

A proof of this theorem was conducted during the lecture.

6 Orthogonality

Vectors are orthogonal if their dot product is zero. This occurs when the angle between them is $\frac{\pi}{2}$ radians (90 degrees). The relationship between orthogonality and the dot product was emphasized.

7 Equations of a Straight Line

The lecture concluded with a discussion on the equations of straight lines in various forms. The following forms were identified as representing straight lines:

- Slope-intercept form: $y = mx + c$
- Parametric form: $\mathbf{x} = \mathbf{a} + t\mathbf{b}$

The importance of understanding these forms in higher dimensions was also highlighted.

8 Conclusion

The lecture provided a comprehensive overview of vectors, their properties, and operations. The concepts of orthogonality and the equations of straight lines were also discussed, setting the stage for future topics in linear algebra.