Calculus and Its Applications: Week 2

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1 Introduction

In this lecture, we explored the concepts of limits and continuity in calculus. The session began with some technical difficulties regarding the projector, which delayed the start of the discussion. However, the class was engaged in introducing themselves and discussing the structure of the course.

2 Limits

2.1 Indeterminate Forms

The lecturer presented a limit problem:

$$\lim_{t \to 0} \left(\frac{1}{t} - S(t) \right)$$

The initial approach of substituting $t = 0$ led to an indeterminate form of $\infty - \infty$. This situation is known as an indeterminate form, which requires further analysis.

2.2 Algebraic Manipulation

To resolve the indeterminate form, the lecturer emphasized the importance of finding a function that agrees with the original function everywhere except at the problematic point. By manipulating the expression algebraically, we can simplify the limit:

$$\lim_{t \to 0} \frac{1 - tS(t)}{t}$$

This manipulation allows us to cancel terms and compute the limit by direct substitution, leading to a result of 1.

3 Continuity

3.1 Definition of Continuity

The lecturer introduced the concept of continuity, emphasizing that for a function to be continuous at a point, the following conditions must be met:

- The limit of the function as x approaches the point must exist.
- The limit must equal the value of the function at that point.

3.2 Example Problem

An example was presented to illustrate continuity:

$$\lim_{x \to -1} \frac{x^2 + 2x + 1}{x^4 - 1}$$

Upon factoring and simplifying, the limit was found to be 0, confirming that the function is continuous at that point.

4 Graphical Interpretation

The lecturer displayed a graph depicting a function with discontinuities. The class discussed the implications of colored and uncolored circles on the graph, which indicate whether the function is defined at specific points.

4.1 Left and Right Limits

The limits as x approaches a point from the left and right were analyzed. The class concluded that if these two limits are not equal, the overall limit does not exist, leading to discontinuity.

5 Conclusion

The lecture concluded with a summary of the key concepts of limits and continuity. The importance of understanding indeterminate forms and the conditions for continuity were emphasized. The class was encouraged to ask questions and engage with the material further.