Linear Algebra: Eigenvalues and Eigenspaces

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1 Introduction

In today's lecture, we discussed several important concepts related to eigenvalues and eigenspaces, including diagonalizability, multiplicity, and their implications. We also revisited some statements from the previous lecture.

2 Review of Previous Statements

2.1 Statement 3: Diagonalizability and Similarity

We proved that if a matrix A is diagonalizable, then it is similar to its transpose A^T. The proof involves the definition of diagonalizability, which states that A is similar to a diagonal matrix D. We showed that the diagonal matrix of the transpose can be expressed as $P^{-1}A^TP = D^T$, where $D^T = D$ since transposing does not affect the diagonal elements.

2.2 Statement 4: Similarity and Non-zero Eigenvalues

We also established that if A is diagonalizable and B is similar to A, then B has non-zero eigenvalues. The proof follows a similar structure to the previous statement, utilizing the properties of similarity and diagonalization.

2.3 Statement 5: Powers of Similar Matrices

The final statement discussed was that if A is similar to B, then all powers of A are similar to the corresponding powers of B. We approached this by dividing the proof into cases based on the value of n (positive, negative, or zero). For $n = 0$, both matrices are the identity. For $n > 0$, we showed that A^2 is similar to B^2 using the similarity relation. For negative integers, we used the property that $A^{-n} = (A^n)^{-1}$.

3 Eigenvalues and Eigenspaces

3.1 Definitions

The multiplicity of an eigenvalue λ of a matrix A is defined as the algebraic multiplicity of λ as a root of the characteristic polynomial. The characteristic polynomial is obtained by solving $\det(A - \lambda I) = 0$.

3.2 Properties of Eigenvalues

We discussed the importance of the characteristic polynomial being completely factored. If the polynomial does not factor completely, the matrix cannot be diagonalized. We also noted that if the characteristic polynomial has distinct roots, the corresponding eigenvectors are linearly independent.

4 Exercises and Applications

4.1 Matching Exercise

We conducted a matching exercise involving logical implications related to eigenvalues and diagonalizability. Students were tasked with matching hypotheses with their corresponding conclusions.

4.2 True or False Statements

We also worked through several true or false statements regarding eigenvalues and diagonalizability. Key points included:

- If the eigenvectors of A span R^n, then the eigenvectors of A^{-1} also span R^n.
- The sum of the eigenvalue multiplicities of an $n \times n$ matrix equals n only if the characteristic polynomial factors completely.

5 Conclusion

In conclusion, today's lecture reinforced the connections between eigenvalues, eigenspaces, and diagonalizability. Understanding these concepts is crucial for further studies in linear algebra and its applications.