Calculus and Its Applications

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1 Introduction

This lecture is the second in the series for the course on Calculus and its Applications. The focus today is on functions, graphs, trigonometric expressions, and logarithms.

2 Administrative Notes

Before diving into the content, students are reminded to contact the course secretary for any changes to workshop groups, as the lecturers do not manage this process.

3 Review of Domain and Range

3.1 Domain

The domain of a function is the set of all possible input values (x-values) for which the function is defined. There is no standard procedure to calculate the domain; it requires understanding when the function is not defined. For example:

- For a square root function, the expression under the square root must be non-negative.
- For a function with a denominator, the denominator must not equal zero.

3.2 Range

The range is the set of all possible output values (y-values). To find the range, one can reverse the function, looking for values of y that correspond to x in the domain. For instance, for the function $y = x^2$:

- Domain: All real numbers.
- Range: $y \geq 0$ (since x^2 is always non-negative).

4 Trigonometric Relationships

The lecture included exercises on trigonometric relationships. A key identity used was:

$$\sin^2 x + \cos^2 x = 1$$

This identity helps in solving equations involving sine and cosine.

4.1 Example Problem

Given a trigonometric equation, students were encouraged to manipulate it using known identities to find solutions. The importance of understanding the signs of sine and cosine in different quadrants was emphasized.

5 Inverse Functions

The concept of inverse functions was discussed, particularly how to verify if two functions are inverses by checking if their compositions yield the identity function.

5.1 Example Problem

For functions $f(x)$ and $g(x)$, if $f(g(x)) = x$ and $g(f(x)) = x$, then f and g are inverses. Graphical representation can also aid in understanding this relationship.

6 Logarithmic Functions

Logarithmic functions were introduced, focusing on their properties and how to manipulate them. The change of base formula was highlighted as essential for solving logarithmic equations.

6.1 Example Problem

To solve an equation involving logarithms, students were encouraged to:

- Use properties of logarithms to combine or separate terms.
- Sketch the graphs of the functions involved to find points of intersection, which represent solutions.

7 Even and Odd Functions

Definitions of even and odd functions were provided:

- A function $f(x)$ is **even** if $f(-x) = f(x)$.
- A function $f(x)$ is **odd** if $f(-x) = -f(x)$.

Examples included $f(x) = x^2$ (even) and $f(x) = x^3$ (odd).

8 Applications of Calculus

The lecture concluded with a discussion on mathematical modeling, specifically in the context of compound interest. The formula for compound interest was presented:

$$A = P(1 + r/n)^{nt}$$

where:

- A is the amount of money accumulated after n years, including interest.
- P is the principal amount (the initial amount of money).
- r is the annual interest rate (decimal).
- n is the number of times that interest is compounded per year.
- t is the number of years the money is invested or borrowed.

The importance of distinguishing between discrete and continuous compounding was emphasized.

9 Conclusion

Students were encouraged to attend workshops to reinforce their understanding of the material covered in the lectures.