

# Linear Algebra Lecture Summary

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## 1 Introduction

Welcome to the first lecture of linear algebra. Attendance is important, and quizzes will be released on Monday mornings. The focus of this lecture is on linear equations and systems of linear equations.

## 2 Lecture Overview

### 2.1 Reading Assignments

Students are expected to have completed readings from sections 1.1, 1.2, and 1.3. The upcoming homework will cover material from these sections and will include questions focusing on proof skills.

### 2.2 Online Resources

Students should log on to the online assessment platform, referred to as "stack," before Monday to avoid technical issues.

## 3 Linear Equations

### 3.1 Definition

A linear equation is an equation of the form:

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

where  $a_i$  and  $b$  are constants, and  $x_i$  are variables.

### 3.2 Identifying Linear Equations

A poll was conducted to identify which equations are linear. The results indicated that:

- Equation 1 is not linear (quadratic).
- Equation 2 is linear (constant coefficients).
- Equation 3 is not linear (exponential).

- Equation 4 is linear.
- Equation 5 is linear (inconsistent but still linear).

### 3.3 Solutions to Linear Equations

The solutions to a linear equation in two variables lie on a straight line. However, in higher dimensions, solutions can lie on a plane or hyperplane.

## 4 Systems of Linear Equations

### 4.1 Types of Solutions

A system of two equations in two variables can have:

- No solutions (parallel lines).
- One solution (intersecting lines).
- Infinitely many solutions (same line).

### 4.2 Dimensionality of Solutions

The dimension of the solution space is determined by the number of free parameters. For example, a linear equation in  $n$  variables has solutions lying in an  $n - 1$  dimensional space.

## 5 Elementary Row Operations

### 5.1 Operations Overview

Elementary row operations include:

- Swapping two rows.
- Multiplying a row by a non-zero scalar.
- Adding a multiple of one row to another row.

### 5.2 Augmented Matrices

Students practiced finding the augmented matrix for a system of equations and identifying valid row operations. It is important to note that combining operations can lead to confusion, and clarity is essential.

## 6 Conclusion

The lecture concluded with a discussion on the importance of understanding the dimensionality of solutions and the implications of the rank of a system of equations. Students were encouraged to ask questions and engage with the material.