Lecture Summary: Derivatives and Implicit Differentiation

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1 Introduction

In today's lecture, we continued our discussion on derivatives, focusing on applications of derivative laws, derivatives of special functions (including logarithmic functions), and the concept of implicit differentiation.

2 Implicit Differentiation

2.1 Definition

Implicit differentiation is a technique used to differentiate equations that define implicit functions. For example, the equation of a circle $x^2 + y^2 = c$ cannot be easily solved for y in terms of x. Instead, we differentiate both sides with respect to x, treating y as a function of x and applying the chain rule.

2.2 Example: Circle

Given the equation of a circle:

$$x^2 + y^2 = 1$$

Differentiating both sides with respect to x:

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

This leads to:

$$2x + 2y\frac{dy}{dx} = 0$$

Rearranging gives:

$$\frac{dy}{dx} = -\frac{x}{y}$$

This result shows the slope of the tangent to the circle at any point (x, y).

3 Rules of Differentiation

3.1 Product and Quotient Rules

We reviewed the product and quotient rules for differentiation. The product rule states:

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

The quotient rule states:

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

3.2 Composition of Functions

We also discussed the chain rule for differentiating composite functions:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

4 Applications of Implicit Differentiation

Implicit differentiation is particularly useful when dealing with complex algebraic expressions that cannot be easily rearranged into the form $y = f(x)$.

4.1 Example: Logarithmic Functions

For a function defined implicitly, such as $y^2 = e^x$, we can take the natural logarithm of both sides to simplify differentiation:

$$\ln(y^2) = x \implies 2\ln(y) = x$$

Differentiating implicitly gives:

$$\frac{2}{y}\frac{dy}{dx} = 1 \implies \frac{dy}{dx} = \frac{y}{2}$$

5 Conclusion

In conclusion, today's lecture emphasized the importance of implicit differentiation and the application of derivative rules. We practiced several exercises to reinforce these concepts, highlighting the need for familiarity with logarithmic properties and the rules of differentiation.

6 Questions

Students were encouraged to ask questions regarding the material covered, and further exercises were assigned for practice.