Linear Algebra Lecture Summary

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1 Introduction

This lecture marks the beginning of our linear algebra course. Attendance is crucial, especially on Thursdays. The reading assignments for sections 1.1, 1.2, and 1.3 should be completed prior to this lecture. Quizzes will be released on Monday mornings, and it is recommended to complete the reading quiz before the tutorial on Tuesday.

2 Linear Equations

2.1 Definition

A linear equation is an equation of the form:

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b$$

where a_i and b are constants, and x_i are variables.

2.2 Examples

We discussed several equations to determine which are linear:

- Equation 1: Not linear (quadratic)
- Equation 2: Linear (constant coefficients)
- Equation 3: Not linear (exponential)
- Equation 4: Linear
- Equation 5: Linear (inconsistent)

2.3 Solutions to Linear Equations

The solutions to a linear equation in two variables lie on a straight line. However, in higher dimensions, solutions can lie on a plane or hyperplane. A linear equation in n variables has its solutions lying in an $n-1$ dimensional space.

3 Systems of Linear Equations

3.1 Types of Solutions

A system of two equations in two variables can have:

- Zero solutions (parallel lines)
- One solution (intersecting lines)
- Infinitely many solutions (same line)

3.2 Dimensionality

The number of free parameters in a system of equations determines the dimensionality of the solution space. For example, if we have two equations in three variables, the solution space is one-dimensional (a line).

4 Rank of a System

The rank of a system of equations is the number of meaningful equations. If the rank equals the number of variables, there is a unique solution. If the rank is less than the number of variables, there are infinitely many solutions.

4.1 Theorem 1.2.2

For a system of m equations in n variables:

- If $n = r$, there is a unique solution.
- If $n > r$, there are infinitely many solutions.
- If $n < r$, there are no solutions.

5 Elementary Row Operations

We discussed the three types of elementary row operations:

- Swapping two rows
- Multiplying a row by a non-zero scalar
- Adding a multiple of one row to another row

5.1 Example

Given an augmented matrix, we can perform operations to simplify it. For instance, if we have:

$$\begin{bmatrix} 1 & 1 & 1 & | & 2 \\ 2 & 3 & 1 & | & 5 \\ 3 & 5 & 2 & | & 8 \end{bmatrix}$$

We can perform row operations to reach row echelon form.

6 Conclusion

The lecture concluded with a discussion on the importance of understanding the dimensionality of solutions and the implications of the rank of a system. Students were encouraged to engage with the material and ask questions.