Applications of Integrals and Riemann Sums

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1 Introduction

In today's lecture, we continued our exploration of integrals, focusing on Riemann sums and their applications. We discussed the historical context of integrals, particularly their use in calculating areas under curves.

2 Riemann Sums

2.1 Definition

A Riemann sum is defined for a function f on a closed interval $[a, b]$ with a regular partition P. The width of each subinterval is denoted as Δx. The Riemann sum is given by:

$$S = \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

where x_i^* is a point in the interval $[x_{i-1}, x_i]$.

2.2 Regular Partition

A regular partition divides the interval $[a, b]$ into n segments of equal length:

$$\Delta x = \frac{b-a}{n}$$

2.3 Limit of Riemann Sums

If f is continuous and non-negative on $[a, b]$, the area under the curve can be found as the limit of the Riemann sums as n approaches infinity:

$$\text{Area} = \lim_{n\to\infty} S$$

3 Applications of Integrals

3.1 Velocity and Acceleration

We discussed a problem involving acceleration $a(t)$ and its relationship to velocity $v(t)$. The key points included: - The relationship $a(t) = \frac{dv}{dt}$ - To find $v(t)$, we integrate $a(t)$,

which introduces an arbitrary constant. - Two conditions are needed to uniquely determine the function.

3.2 Average Velocity

The average velocity over a time interval can be calculated by integrating the velocity function and dividing by the time interval. This was illustrated with examples where students had to determine the correct interpretation of average velocity versus total distance.

3.3 Volume of Water in a Tank

We analyzed a scenario involving the volume of water in a tank, emphasizing the importance of understanding the rate of change. The integral was used to determine the total volume of water that has flowed out over a specified time.

4 Understanding Riemann Sums

4.1 Right and Left Hand Sums

We explored the difference between right-hand and left-hand Riemann sums: - Right-hand sums start from the right endpoint of each subinterval. - Left-hand sums start from the left endpoint.

Both methods converge to the same integral under the right conditions, particularly when the function is continuous.

4.2 Example Problems

We worked through several example problems, emphasizing the importance of understanding the components of Riemann sums, such as Δx and the choice of x_i^*.

5 Conclusion

The lecture concluded with a reminder of the importance of Riemann sums in approximating areas under curves and their applications in real-world scenarios. Students were encouraged to practice problems involving Riemann sums and integrals to solidify their understanding.