Calculus and Its Applications: Week 2

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1 Introduction

This lecture, led by Lily, focused on the concepts of limits and continuity in calculus. The session began with some technical difficulties regarding the projector, which were resolved shortly after.

2 Class Structure

The class operates on an interactive model where students are encouraged to discuss questions with their peers before arriving at a collective answer.

3 Limits and Indeterminate Forms

3.1 Initial Problem

The first problem presented was to evaluate the limit:

$$\lim_{t \to 0} \left(\frac{1}{t} - S(t) \right)$$

where $S(t)$ is a function. The naive approach of substituting $t = 0$ leads to an indeterminate form $\infty - \infty$.

3.2 Indeterminate Forms

An indeterminate form occurs when direct substitution in a limit leads to ambiguous results. In this case, both terms approach infinity, making it impossible to determine the limit directly.

3.3 Algebraic Manipulation

To resolve this, the principle of using a function that agrees with the original function except at the problematic point was applied. By manipulating the expression algebraically, it was shown that:

$$\lim_{t \to 0} \left(\frac{1}{t} - S(t) \right) = 1$$

after reducing the expression to a form that allows direct substitution.

4 Further Examples

4.1 Second Problem

The next limit to evaluate was:

$$\lim_{x \to -1} \frac{x^2 + 2x + 1}{x^4 - 1}$$

Again, direct substitution leads to $\frac{0}{0}$, another indeterminate form.

4.2 Factoring

By factoring the numerator and denominator, the expression was simplified, allowing for cancellation of terms:

$$\frac{(x+1)^2}{(x^2-1)(x^2+1)}$$

After cancellation, the limit could be computed, confirming that the limit is indeed 0.

5 Continuity

5.1 Understanding Continuity

The concept of continuity was explored through graphical representations. Students were shown a function with points of discontinuity and asked to analyze the limits from both sides.

5.2 Conditions for Continuity

For a function to be continuous at a point c, the following must hold:

- The limit as x approaches c exists.
- The limit equals the function value at c.

5.3 Example Discussion

A specific example was discussed where the limits from the left and right at $x = 2$ were not equal, leading to the conclusion that the overall limit does not exist at that point.

6 Conclusion

The lecture concluded with a summary of the key concepts of limits and continuity. Students were encouraged to ask questions if they had any uncertainties. The next session was scheduled for Thursday.