Lecture Summary on Derivatives and Implicit Differentiation

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1 Introduction

In today's lecture, we continued our discussion on derivatives, focusing on applications of derivative laws, derivatives of special functions, and implicit differentiation.

2 Implicit Differentiation

2.1 Definition

Implicit differentiation is a technique used to differentiate equations that define implicit functions. For example, consider the equation of a circle given by:

$$x^2 + y^2 = c$$

To differentiate this equation, we apply the differentiation rules to both x and y, treating y as a function of x and multiplying by $\frac{dy}{dx}$ when differentiating y.

2.2 Example

To find the slope of the tangent to the circle at a specific point, we can differentiate the equation implicitly:

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(c)$$

This leads to:

$$2x + 2y\frac{dy}{dx} = 0$$

Rearranging gives:

$$\frac{dy}{dx} = -\frac{x}{y}$$

This result shows how implicit differentiation can yield the same result as explicit differentiation, even when the function cannot be easily rearranged.

3 Rules of Differentiation

3.1 Product and Quotient Rules

We also reviewed the product and quotient rules for differentiation. If u and v are functions of x, then:

- Product Rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
- Quotient Rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

3.2 Composition of Functions

We also discussed the chain rule for differentiating composite functions. If f and g are functions, then:

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

4 Applications of Implicit Differentiation

Implicit differentiation is particularly useful when dealing with complex algebraic expressions that cannot be easily rearranged into the form $y = f(x)$.

4.1 Example with Logarithms

Consider an equation where logarithmic functions are involved. To differentiate an equation like:

$$y^2 = e^{x^2}$$

we can take the natural logarithm of both sides to simplify the differentiation process:

$$\ln(y^2) = \ln(e^{x^2}) \implies 2\ln(y) = x^2$$

Differentiating both sides implicitly leads to:

$$\frac{2}{y}\frac{dy}{dx} = 2x$$

Thus, we find:

$$\frac{dy}{dx} = \frac{xy}{1}$$

5 Conclusion

In conclusion, today's lecture emphasized the importance of implicit differentiation and the rules of differentiation. We practiced several exercises to solidify our understanding of these concepts. It is crucial to remember the rules of logarithms, product and quotient rules, and the chain rule when solving differentiation problems.

6 Questions

At the end of the lecture, students were encouraged to ask questions to clarify any doubts regarding the material covered.