Linear Algebra Lecture Summary

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1 Introduction

This lecture continues from the previous day's discussion on elementary row operations and their applications in solving systems of linear equations. The focus is on understanding row echelon form and reduced row echelon form of matrices.

2 Elementary Row Operations

We began with a discussion on what operations can be performed on matrices. The three types of elementary row operations are:

- Swapping two rows.
- Multiplying a row by a non-zero scalar.
- Adding or subtracting a multiple of one row to another row.

2.1 Voting on Operations

Students were asked to vote on which operation to perform next on a given matrix. The majority preferred to multiply a row by a scalar or to add/subtract rows, avoiding illegal operations like squaring a row.

3 Row Echelon Form

A matrix is in row echelon form if it satisfies the following conditions:

- All zero rows are at the bottom of the matrix.
- The leading entry of each non-zero row is 1 (called a leading one).
- The leading one of each row is to the right of the leading one in the previous row.

3.1 Example of Row Echelon Form

For example, the matrix:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

is in row echelon form.

4 Reduced Row Echelon Form

A matrix is in reduced row echelon form if:

- It is in row echelon form.
- Each leading one is the only non-zero entry in its column.

4.1 Example of Reduced Row Echelon Form

For instance, the matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is in reduced row echelon form.

5 Gaussian Elimination

The process of transforming a matrix into row echelon form or reduced row echelon form is known as Gaussian elimination. This method is crucial for solving systems of linear equations.

5.1 Back Substitution

Once a matrix is in reduced row echelon form, back substitution can be used to find the solutions to the system of equations represented by the matrix.

6 Rank of a Matrix

The rank of a matrix is defined as the number of leading ones in its row echelon form. This indicates the number of linearly independent rows in the matrix.

6.1 Example of Rank Calculation

For a matrix in row echelon form:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

the rank is 2, as there are two leading ones.

7 Homogeneous Systems

A system of equations is homogeneous if it can be expressed in the form $Ax = 0$. The solution to a homogeneous system always includes the trivial solution $x = 0$.

8 Consistency of Systems

A system of equations is consistent if it has at least one solution. This can occur if:

- The system has a unique solution.
- The system has infinitely many solutions.

9 Conclusion

The lecture concluded with a reminder about the upcoming quizzes and the importance of understanding these concepts for solving linear systems effectively. Students were encouraged to practice Gaussian elimination and familiarize themselves with the definitions of row echelon form, reduced row echelon form, and rank.