Lecture Summary on Eigenvalues and Eigenvectors

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1 Introduction

In this lecture, we focused on finding eigenvalues and eigenvectors of matrices. We began with a review of the definitions and moved on to practical calculations.

2 Finding Eigenvalues

To find the eigenvalues of a matrix A, we solve the characteristic equation given by:

$$\det(\lambda I - A) = 0$$

where λ is the eigenvalue and I is the identity matrix. We explored a specific matrix and determined that its eigenvalues were 1, 2, and 5.

2.1 Example Calculation

Given a matrix A, we calculated the determinant:

$$\det(\lambda I - A) = (\lambda - 1)(\lambda - 2)(\lambda - 5) = 0$$

This yielded the eigenvalues 1, 2, and 5.

3 Finding Eigenvectors

For each eigenvalue, we find the corresponding eigenvector by solving the equation:

$$(A - \lambda I)\mathbf{v} = 0$$

where $\mathbf{v}$ is the eigenvector.

3.1 Example Calculation for $\lambda = 1$

For $\lambda = 1$, we set up the matrix:

$$1I - A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$$

We performed row reduction to find the eigenvector. The general solution was:

$$\mathbf{v} = t \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

indicating that any scalar multiple of this vector is also an eigenvector.

3.2 Example Calculation for $\lambda = 2$

For $\lambda = 2$, we similarly found:

$$2I - A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$

The row reduction led to the eigenvector:

$$\mathbf{v} = t \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$$

4 Multiplicity of Eigenvalues

We discussed the concept of algebraic and geometric multiplicity. The algebraic multiplicity refers to the number of times an eigenvalue appears as a root of the characteristic polynomial, while geometric multiplicity refers to the number of linearly independent eigenvectors associated with that eigenvalue.

4.1 Example of Double Eigenvalue

For a matrix with a double eigenvalue, we found that it could have fewer eigenvectors than its algebraic multiplicity. This situation can lead to a matrix being non-diagonalizable.

5 Diagonalization of Matrices

A matrix A is diagonalizable if there exists an invertible matrix P such that:

$$P^{-1}AP = D$$

where D is a diagonal matrix containing the eigenvalues of A. We noted that if the algebraic multiplicity equals the geometric multiplicity for all eigenvalues, the matrix is diagonalizable.

5.1 Example of Diagonalization

For a matrix with distinct eigenvalues, we constructed the matrix P from the eigenvectors and the diagonal matrix D from the eigenvalues.

6 Conclusion

In summary, we learned how to find eigenvalues and eigenvectors, the significance of multiplicities, and the conditions under which a matrix can be diagonalized. Understanding these concepts is crucial for applications in various fields, including systems of differential equations and stability analysis.