Linear Algebra: Subspaces and Spanning Sets

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1 Introduction

This lecture covers the concepts of subspaces and spanning sets in vector spaces. The discussion is divided into two main topics: subspaces and spanning sets, followed by linear independence and dimension in the next lecture.

2 Announcements

- An event was advertised, details were not provided.
- An attendance poll was conducted.
- Mid-semester feedback poll results are being reviewed, and feedback will be provided soon.

3 Reading Material

The reading for this week includes:

- Chapter 5
- One page from Chapter 6 covering axioms for vector addition and scalar multiplication.

4 Vector Spaces

Vector spaces can be defined abstractly, not just in terms of R^n. The properties that define a vector space can apply to various mathematical objects, such as:

- Functions
- Polynomials
- Matrices

5 Subspaces

A subspace is a set of vectors that satisfies the following axioms:

- S1: The zero vector is in the subspace.
- S2: If x and y are in the subspace, then $x + y$ is also in the subspace.
- S3: If x is in the subspace and c is a scalar, then cx is also in the subspace.

5.1 Example: Null Space

Consider the set of all vectors x in R^n such that $Ax = 0$. This set is a subspace, known as the null space, because it satisfies all three axioms.

5.2 Example: Non-Subspace

For the set of vectors x such that $Ax = b$ where $b \neq 0$, this is not a subspace because the zero vector is not included.

6 Spanning Sets

The span of a set of vectors is the set of all linear combinations of those vectors. If U is a set of vectors, then the span of U, denoted $\mathrm{span}(U)$, is a subspace of R^n.

6.1 Properties of Spanning Sets

- The span of a set of vectors is the smallest subspace containing those vectors.
- If a vector can be expressed as a linear combination of other vectors in the set, it does not add new information to the span.

6.2 Example: Geometric Interpretation

Given vectors $u = (1, 1)$ and $v = (1, -1)$, the span of these vectors is the entire R^2. Any vector (x, y) in R^2 can be expressed as a linear combination of u and v.

7 Conclusion

The concepts of subspaces and spanning sets are fundamental in linear algebra. Understanding these concepts allows for the exploration of more complex topics such as linear independence and dimension, which will be discussed in the next lecture.