Calculus and Its Applications

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1 Introduction

Good afternoon everyone, and welcome to the course on Calculus and Its Applications. My name is Chris Sanguine, the course organizer, and I will be discussing the course structure, assessment methods, and the role of artificial intelligence in our learning process.

2 Course Overview

2.1 Course Structure

This course consists of two lectures per week and one 1.5-hour workshop. The course is designed to provide a comprehensive understanding of calculus and its applications.

2.2 Assessment

The assessment for this course is divided into:

- 60% written exam
- 40% coursework

All relevant dates and deadlines can be found on the course's online learning platform.

2.3 Expectations

Success in this course requires consistency in attendance and participation. Students are expected to dedicate 10 to 12 hours per week to this module, which includes lectures, workshops, and independent study.

3 Artificial Intelligence in Assessment

We will also discuss the implications of artificial intelligence in our assessments. While technology can assist in solving problems, it is crucial to understand the underlying principles of calculus.

3.1 Historical Context

The debate over the use of technology in education is not new. Historical figures like William Uhtred argued for the importance of understanding the fundamentals before relying on instruments.

4 Calculus Fundamentals

4.1 Integration and Differentiation

We will revisit some fundamental concepts of calculus, such as the integral of the function $\frac{1}{1+x^2}$, which is the arctangent function. The derivative of the arctangent function is given by:

$$\frac{d}{dx}\arctan(x) = \frac{1}{1+x^2}$$

4.2 Chain Rule Example

Consider the function $y = \arctan\left(\frac{3x+1}{3-x}\right)$. To differentiate this, we apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{1+\left(\frac{3x+1}{3-x}\right)^2} \cdot \frac{d}{dx}\left(\frac{3x+1}{3-x}\right)$$

This leads to a complex expression that simplifies back to $\frac{1}{1+x^2}$.

4.3 Geometric Interpretation

Understanding the geometric interpretation of functions is essential. We will sketch the arctangent function and analyze its properties, including discontinuities and tangents.

5 Functions and Graphs

5.1 Definition of a Function

A function is defined as a relation that assigns exactly one output for each input. We will explore the concepts of domain and range, and how to calculate them.

5.2 Domain and Range

To find the domain of a function, we must identify the values of x for which the function is defined. For example, for the function $f(x) = \sqrt{x+2}$, the domain is:

$$x + 2 \geq 0 \implies x \geq -2$$

The range is determined by the outputs of the function, which in this case is $[-1, \infty)$.

5.3 Y-Intercept

The y-intercept of a function is found by evaluating the function at $x = 0$. For example, if $f(x) = x^2 - 1$, then the y-intercept is:

$$f(0) = 0^2 - 1 = -1$$

5.4 Composition of Functions

When composing two functions, we must ensure that the range of the first function overlaps with the domain of the second. For example, if $f(x)$ and $g(x)$ are two functions, the composition $(f \circ g)(x)$ is defined only if the output of $g(x)$ is within the domain of f.

6 Conclusion

In conclusion, this course will cover a wide range of topics in calculus, emphasizing the importance of understanding the fundamentals while also utilizing technology effectively. Please ensure you attend the workshops and engage with the material actively.