Lecture Summary: Connor Maps

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1 Introduction

Good afternoon everyone. Today, we will explore the concept of Connor maps, which are a method for extracting a formula from a visual representation, such as a truth table or a diagram. This lecture will focus on understanding how to derive Boolean formulas from these representations, particularly from an engineer's perspective.

2 Understanding Universes with Predicates

We begin by considering two predicates, A and B. The number of distinct universes that can be formed with these predicates is determined by the regions they create. Specifically, with two predicates, we can distinguish four regions. Each region can either be inhabited or empty, leading to a total of $2^4 = 16$ different universes.

2.1 Exponential Growth

As we increase the number of predicates, the number of possible universes grows exponentially. For example, with four predicates, the number of distinct universes rises to 65,536. This exponential growth is significant in engineering applications, where understanding the relationships between variables is crucial.

3 Boolean Logic and Formula Extraction

To illustrate the extraction of a formula from a universe, consider the categorical proposition "Every B is A". We can represent this using Boolean logic. The inhabited regions can be described using formulas such as:

- $\neg A \vee B$ (the outside region)
- $A \wedge B$ (the middle region)

However, these formulas can often be simplified to shorter representations.

3.1 Simplification Techniques

We can simplify Boolean formulas by identifying common factors or by negating certain regions. For instance, if we identify that $B \wedge \neg A$ must be false, we can apply De Morgan's laws to derive a simpler formula.

4 Introduction to Connor Maps

Connor maps are a graphical tool that helps in visualizing truth tables and simplifying Boolean expressions. They allow engineers to recognize patterns and derive concise formulas without extensive calculations.

4.1 Pattern Recognition

Humans excel at recognizing patterns, which is a key advantage when using Connor maps. By highlighting true cells in a truth table and attempting to cover them with rectangles, we can derive a Boolean formula that describes the inhabited regions.

5 Constructing Connor Maps

To construct a Connor map:

- Highlight the cells that are true.
- Cover these cells with the largest possible rectangles, where the dimensions of the rectangles are powers of two.

This method ensures that the resulting formula is as short as possible.

5.1 Example of Rectangle Coverage

Consider a truth table with predicates A and B. If we cover an L-shaped region with rectangles, we can derive a formula such as $A \vee \neg B$. The goal is to find the largest rectangles to minimize the number of variables in the final expression.

6 Higher Dimensions and Grey Code

When dealing with four predicates, we can create a four-dimensional truth table. To facilitate the extraction of rectangles, we use a Grey code ordering, which ensures that only one variable changes at a time as we move through the table. This reduces the complexity of identifying rectangles and helps avoid race conditions in circuit design.

6.1 Torus Representation

To visualize the truth table in higher dimensions, we can represent it on a toroidal surface. This allows us to wrap around the edges of the table, making it easier to identify rectangles that span across the boundaries.

7 Conclusion

Connor maps provide a powerful method for simplifying Boolean expressions and extracting formulas from visual representations. While they leverage human pattern recognition, algorithmic methods can also be employed for more complex scenarios. Understanding these concepts is essential for engineers working with logical systems.

7.1 Future Directions

In the next lecture, we will explore symbolic manipulations of formulas, which will complement our understanding of Connor maps and their applications in engineering.