Linear Algebra Revision Lecture Summary

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1 Introduction

The lecture began with an introduction by the instructors, Erica and Kasia. The session was structured to include examples and questions from students, focusing on key concepts in linear algebra.

2 Example 1: Unique Solutions

We considered an $n \times n$ matrix A and a non-zero vector b. The equation $Ax = b$ is given to have a unique solution. We were asked how many solutions the equation $Ax = 0$ has.

2.1 Analysis

To analyze this, we let v be the unique vector such that $Av = b$. We introduced another vector u such that $Au = 0$. Using the linearity of A, we can express:

$$A(u + v) = Au + Av = 0 + b = b$$

This implies that $u + v$ is also a solution to $Ax = b$. If u were non-zero, it would imply multiple solutions to $Ax = b$, contradicting the uniqueness assumption. Thus, we conclude that u must be the zero vector, indicating that the only solution to $Ax = 0$ is the trivial solution.

3 Example 2: Subspaces of R^3

We examined whether certain sets are subspaces of R^3.

3.1 Set Analysis

1. **Set Q**: A plane not passing through the origin. - Conclusion: Not a subspace (does not contain the origin).

2. **Set W**: Defined by $y^2 = z^2$. - Conclusion: Not a subspace (fails closure under addition).

3. **Set U**: A line through the origin. - Conclusion: Is a subspace (contains the origin and is closed under addition and scalar multiplication).

4 Span and Linear Independence

We discussed the concept of span and linear independence through examples involving vectors in R^3.

4.1 Example of Spanning

Given two subspaces U and V, we analyzed the following statements: - U is a subset of V. - V is a subspace of U. - $U \neq V$. - $U = V$.

4.1.1 Analysis of Equality

We concluded that: - If U and V are equal, they must span the same space. - If U is a proper subset of V, then U cannot equal V.

4.1.2 Counterexamples

We provided counterexamples to illustrate that two spans can be equal even if the generating sets are different, as long as they span the same space.

5 Conclusion

The lecture concluded with a summary of key points regarding unique solutions, subspaces, and spans. Students were encouraged to ask questions and engage with the material further.