Introduction to Linear Algebra - Week 4 Summary

Generated by LectureMate

February 22, 2025

1 Overview

In week four of Introduction to Linear Algebra, we delve into the concepts of determinants and invertibility. These topics are foundational for understanding matrix diagonalization, eigenvalues, and eigenvectors.

2 Lecture Structure

The week is divided into two parts:

- Two readings
- Two lectures

Today's lecture focuses on:

- Definition of determinants
- Cofactor expansion
- Basic properties of determinants
- Determinants of special matrices

Tomorrow's lecture will cover:

- Using determinants to check matrix invertibility
- Calculating the inverse using the adjugate matrix

3 Cofactor Expansion

The cofactor C_{ij} is defined as the determinant of the matrix obtained by removing the i-th row and j-th column from matrix A. The cofactor is a scalar, specifically a 1×1 matrix.

3.1 Cofactor Properties

The determinant can be expressed using cofactor expansion:

$$\det(A) = \sum_{j=1}^{n} a_{ij} C_{ij}$$

where i is fixed and j varies, or vice versa.

3.2 Cofactor Dimension

The dimension of the cofactor is 1×1, while the reduced matrix has dimensions $(n-1) \times (n-1)$.

4 Determinant Calculation

To calculate the determinant of a 3×3 matrix, we can use cofactor expansion along any row or column. The determinant can be computed as:

$$\det(A) = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

This can be verified by expanding along different rows or columns, yielding the same result.

4.1 Properties of Determinants

1. Swapping two rows or columns changes the sign of the determinant. 2. Multiplying a row or column by a scalar multiplies the determinant by that scalar. 3. Adding a multiple of one row to another does not change the determinant.

5 Special Matrices

For upper triangular matrices, the determinant is simply the product of the diagonal elements:

$$\det(A) = d_1 d_2 d_3$$

This property holds for both upper and lower triangular matrices.

6 Row Echelon Form

To calculate the determinant using row echelon form, we can perform row operations while keeping track of how these operations affect the determinant. The determinant can be calculated as:

$$\det(A) = (-1)^s \det(B)$$

where s is the number of row swaps performed.

7 Example Problem

Given a matrix with one unknown element, we can express the determinant in terms of that unknown. By substituting values, we can analyze the sign of the determinant.

8 Conclusion

The structure of a matrix is crucial in determining its properties, including the determinant. Understanding how to manipulate matrices and apply properties of determinants simplifies calculations and enhances comprehension of linear algebra concepts.