Introduction to Linear Algebra - Week 4 Summary

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1 Overview

In week four of Introduction to Linear Algebra, we delve into the concepts of determinants and invertibility, which are foundational for understanding matrix diagonalization, eigenvalues, and eigenvectors. The week is structured into two parts: two readings and two lectures.

2 Lecture Content

2.1 Determinants

We begin with the definition of determinants, exploring cofactor expansion and basic properties of determinants, particularly for special matrices.

2.1.1 Cofactor Expansion

The cofactor C_{ij} of a matrix A is defined as the determinant of the matrix obtained by removing the i-th row and j-th column from A. The dimension of the cofactor is 1×1, indicating it is a scalar.

- The cofactor is crucial for calculating the determinant using cofactor expansion.
- The determinant can be expressed as:

  $$\det(A) = \sum_{j=1}^{n} a_{ij} C_{ij}$$

  where a_{ij} are the elements of the matrix and C_{ij} are the corresponding cofactors.

2.2 Properties of Determinants

Several properties of determinants were discussed:

- Swapping two rows or columns changes the sign of the determinant.
- Multiplying a row or column by a scalar multiplies the determinant by that scalar.
- Adding a multiple of one row to another does not change the determinant.

2.3 Invertibility and Determinants

We explored how to determine if a matrix is invertible using its determinant:

- A matrix is invertible if and only if its determinant is non-zero.
- The inverse can be calculated using the adjugate matrix.

2.4 Practical Applications

The lecture included practical exercises:

- Calculate the determinant of a 3×3 matrix using cofactor expansion along different rows.
- Use row operations to simplify a matrix to row echelon form and calculate the determinant.
- Analyze the structure of matrices to simplify determinant calculations, particularly for triangular matrices.

3 Conclusion

The key takeaway from this week's lecture is the importance of understanding the structure of matrices when calculating determinants. The properties of determinants allow for simplifications that can significantly reduce computational complexity.