Linear Algebra Lecture Summary

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1 Introduction

This lecture continues from the previous day's discussion on elementary row operations and their applications in solving systems of linear equations. The focus is on understanding row echelon form and reduced row echelon form of matrices.

2 Elementary Row Operations

We began with a review of elementary row operations, which include:

- Swapping two rows.
- Multiplying a row by a non-zero scalar.
- Adding or subtracting a multiple of one row to another row.

The goal is to manipulate matrices to simplify them for solving linear equations.

3 Row Echelon Form

A matrix is in row echelon form if it satisfies the following conditions:

- All non-zero rows are above any rows of all zeros.
- The leading entry of each non-zero row (the first non-zero number from the left) is 1.
- The leading 1 of a row is to the right of the leading 1 of the previous row.

This creates a staircase effect in the matrix.

3.1 Example

Consider the matrix:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

This matrix is in row echelon form.

4 Reduced Row Echelon Form

A matrix is in reduced row echelon form if:

- It is in row echelon form.
- The leading 1 in each row is the only non-zero entry in its column.

This form allows for easy reading of solutions to the system of equations.

4.1 Example

The matrix:

$$\begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

is in reduced row echelon form.

5 Gaussian Elimination

Gaussian elimination is a method used to convert a matrix into row echelon form using the elementary row operations. This process is crucial for solving systems of linear equations.

5.1 Back Substitution

Once a matrix is in row echelon form, back substitution can be used to find the solutions to the system. For example, if we have:

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

we can express the system of equations and solve for the variables.

6 Rank of a Matrix

The rank of a matrix is defined as the number of leading 1s in its row echelon form. It indicates the dimension of the vector space spanned by its rows or columns.

6.1 Example

For the matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

the rank is 2.

7 Free Variables and Solutions

In cases where there are fewer leading 1s than variables, some variables will be free. For example, in the matrix:

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

the variable corresponding to the second column is free.

8 Homogeneous Systems

A system is homogeneous if it can be expressed in the form $Ax = 0$. The solution to a homogeneous system always includes the trivial solution $x = 0$.

9 Conclusion

The lecture concluded with a discussion on the importance of understanding row echelon forms and their applications in solving linear systems. Students were reminded of the upcoming quizzes and encouraged to practice the concepts discussed.