

Linear Algebra Lecture Summary: Elementary Matrices

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1 Introduction

This lecture focuses on elementary matrices and their connection to Gaussian elimination and matrix algebra. The session is divided into two main topics: elementary matrices and linear transformations.

2 Elementary Matrices

2.1 Definition

An elementary matrix is a square matrix that can be obtained from the identity matrix by a single elementary row operation. The three types of row operations are:

- Type 1: Swapping two rows.
- Type 2: Multiplying a row by a non-zero scalar.
- Type 3: Adding a multiple of one row to another row.

2.2 Examples of Elementary Matrices

Consider the following matrices:

- Matrix A: Swapping rows 1 and 2.
- Matrix B: Multiplying row 2 by 2.
- Matrix C: Adding 5 times row 1 to row 2.

Only matrices A and C are elementary matrices as they result from a single row operation.

2.3 Connection to Gaussian Elimination

If an elementary row operation is performed on an $m \times n$ matrix A , the result can be expressed as:

$$E \cdot A = A'$$

where E is the corresponding elementary matrix and A' is the resulting matrix.

2.4 Properties of Elementary Matrices

All elementary matrices are invertible. The inverse of an elementary matrix corresponds to the inverse of the row operation it represents. For example, if an elementary matrix E adds 5 times row 1 to row 2, its inverse would subtract 5 times row 1 from row 2.

3 Matrix Operations and Transposes

3.1 Transpose of Matrix Products

The transpose of a product of matrices follows the rule:

$$(A \cdot B)^T = B^T \cdot A^T$$

This rule applies to any number of matrices, and can be proven using induction.

3.2 Elementary Matrices and Transposes

If E is an elementary matrix corresponding to a row operation on A , then:

$$E^T \cdot A^T = A'^T$$

where A' is the result of the row operation.

4 Row Equivalence

Two matrices A and B are said to be row equivalent if one can be transformed into the other through a sequence of elementary row operations. This implies:

$$B = U \cdot A$$

where U is a product of elementary matrices.

5 Conclusion

In summary, elementary matrices serve as the building blocks for all invertible matrices. Every invertible matrix can be expressed as a product of elementary matrices, highlighting their fundamental role in linear algebra.