Applications of Derivatives

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1 Introduction

In this lecture, we continued our discussion on the applications of derivatives, focusing on optimization problems, limits, and their practical applications. The importance of mid-semester feedback was also emphasized.

2 Mid-Semester Feedback

The instructor reminded students that the mid-semester feedback is open until February 14th. Students were encouraged to submit their feedback to improve the course experience.

3 Optimization Problems

3.1 Maxima and Minima

We explored optimization problems, specifically finding maxima and minima of functions. The following steps were outlined:

- Identify the function to be optimized.
- Determine the constraints of the problem.
- Use the first derivative test to find critical points.
- Evaluate the function at critical points and endpoints of the interval.

3.2 Example 1

The first example involved finding the maximum revenue given by the function:

$$R(x) = 900 - 60x^{1/3}$$

with constraints that x (the number of units sold) must be non-negative and the revenue must also be non-negative.

The critical points were found by setting the derivative equal to zero:

$$R'(x) = -20x^{-2/3} = 0 \implies x = 1000$$

The maximum revenue was calculated to be:

$$R(1000) = 3000$$

3.3 Example 2

The second example required finding two positive numbers whose product is 750, for which the sum of one and ten times the other is minimized. The function to minimize was:

$$S(x) = x + 10\left(\frac{750}{x}\right)$$

The critical point was found using the first derivative test, and it was determined that this problem did not have constraints on x except positivity, making it an unbounded optimization problem.

The second derivative test confirmed that the critical point was a local minimum, and since the second derivative was positive everywhere, it was also an absolute minimum.

4 Limits

4.1 Indeterminate Forms

We discussed limits, particularly those that result in indeterminate forms. An example limit was:

$$\lim_{x \to \frac{\pi}{2}^-} \tan(x)\cos(x)$$

This limit was transformed using logarithmic properties and L'Hôpital's rule to evaluate it correctly.

4.2 Example Limit Calculation

The limit was evaluated as follows:

$$\lim_{x \to \frac{\pi}{2}^-} \tan(x)\cos(x) = e^0 = 1$$

The importance of justifying computations with theoretical results was emphasized throughout the discussion.

5 Conclusion

The lecture concluded with a review of the methods used to solve optimization problems and evaluate limits. Students were encouraged to practice these techniques and ensure they understand the theoretical foundations behind their computations.