MPC Design Decisions and Modeling Assumptions

State Vector Choice

The state vector was chosen as **x = [cart_position, cart_velocity, pole_angle, pole_angular_velocity]^T**, representing the complete observable state of the cart-pole system. This choice includes:

- **Position variables (x, θ)**: Capture the configuration of the system
- **Velocity variables (vx, vθ)**: Enable derivative control and predictive behavior

This 4-dimensional state space is minimal yet complete for the cart-pole problem, allowing the controller to anticipate future behavior based on both positions and rates of change.

Key Modeling Assumptions

1. Linearization Around Upright Equilibrium The nonlinear cart-pole dynamics were linearized around θ = 0 (upright position). This assumption is valid for small-angle deviations typical during stabilization. While initial angles can be ±20°, the controller quickly brings the system into the linear regime where this approximation is accurate.

2. Euler Discretization First-order Euler method was used to discretize the continuous-time dynamics with dt = 0.02s. This simple approach is computationally efficient and sufficiently accurate for the 50 Hz control frequency used in the simulation.

3. Frictionless System The model assumes no friction in the cart motion or pole pivot. This simplification keeps the model linear and matches the ideal pendulum behavior, though real systems would require friction compensation.

4. Point Mass Pole The pole is modeled with its mass concentrated at the center of mass (distance l from pivot), neglecting rotational inertia effects. This is a standard simplification for the cart-pole problem.

5. Receding Horizon Strategy A 30-step prediction horizon (0.6 seconds) was chosen as a balance between:

- **Longer horizons**: Better anticipation but higher computational cost
- **Shorter horizons**: Faster computation but more reactive behavior

The receding horizon approach recomputes the control at each timestep, providing robustness to model uncertainties.

Cost Function Rationale

The cost weights were tuned to prioritize pole stabilization:

- **High pole angle weight (500)**: Makes upright stabilization the primary objective
- **Moderate position weight (50)**: Prevents excessive cart drift while allowing movement for balancing
- **Low control penalty (0.01)**: Permits aggressive control when needed for fast stabilization
- **Terminal cost (2×Q)**: Incentivizes the system to be near equilibrium at the horizon end

This tuning reflects the control objective: stabilize the pole quickly while keeping the cart reasonably centered, using whatever forces are necessary within saturation limits.