CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 1

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1. (a) We first multiply both sides by the conjugate, which is $\sqrt{2} - 2\sqrt{3}$. We get the following.

$$\frac{\sqrt{2} + j\sqrt{2}}{2 + 2j\sqrt{3}} = \frac{2\sqrt{2} + 2\sqrt{6} + j(2\sqrt{2} - 2\sqrt{6})}{16} = \frac{\sqrt{2} + \sqrt{6} + j(\sqrt{2} - \sqrt{6})}{8}$$

Re{z} $= \frac{\sqrt{2}+\sqrt{6}}{8}$ and Im{z} $= \frac{\sqrt{2}-\sqrt{6}}{8}$

(b) We convert the numerator and denominator separately into polar form.

$$\text{Calculating magnitude as} \quad r = \sqrt{a^2 + b^2} \quad \text{and phase as} \quad \theta = arctan(\frac{b}{a}).$$

$$\frac{\sqrt{2} + j\sqrt{2}}{2 + 2j\sqrt{3}} = \frac{2e^{\frac{\pi}{4}j}}{4e^{\frac{\pi}{3}j}} = \frac{e^{-\frac{\pi}{12}}}{2}$$

Therefore, the magnitude is $r = \frac{1}{2}$, and the phase is $\theta = -\frac{\pi}{12}$

2. First, we write the formula of the function as a piecewise function formula.

$$x(t) = \begin{cases} 0 & \text{if } -2 \le t \le -1 \\ 1 & \text{if } -1 < t \le 1 \\ 2 - t & \text{if } 1 < t \le 2 \end{cases} \tag{1}$$

Then, we should replace t with $\frac{t}{2} - 2$.

$$x(\frac{t}{2} - 2) = \begin{cases} 0 & \text{if } -2 \le \frac{t}{2} - 2 \le -1 \\ 1 & \text{if } -1 < \frac{t}{2} - 2 \le 1 \\ 2 - (\frac{t}{2} - 2) & \text{if } 1 < \frac{t}{2} - 2 \le 2 \end{cases} \tag{2}$$

We simplify the equation. We get $y(t)$ as the following.

$$x(\frac{t}{2} - 2) = \begin{cases} 0 & \text{if } 0 \le t \le 2 \\ 1 & \text{if } 2 < t \le 6 \\ 4 - \frac{t}{2} & \text{if } 6 < t \le 8 \end{cases} \tag{3}$$

The plot is

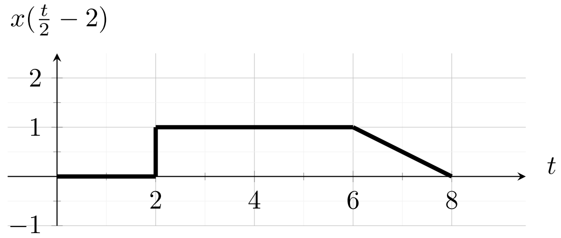

Figure 1: A piecewise function illustrating $x(\frac{t}{2} - 2)$.

3. (a) x[n] = $\delta[n+3] - \delta[n+2] - \delta[n+1] - \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3]$

(b) y[n] = x[2n+2] + x[1-n].

First, we find x[2n+2].
x[2n+2] = $\delta[2n+5] - \delta[2n+4] - \delta[2n+3] - \delta[2n+2] + \delta[2n+1] + 2\delta[2n] + \delta[2n-1]$

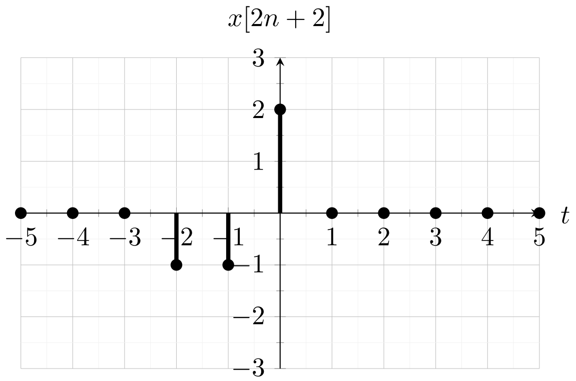

Figure 2: A piecewise function illustrating $x[2n+2]$.

Then, we find x[1-n]
x[1-n] = $\delta[4-n] - \delta[3-n] - \delta[2-n] - \delta[1-n] + \delta[-n] + 2\delta[-n-1] + \delta[-n-2]$

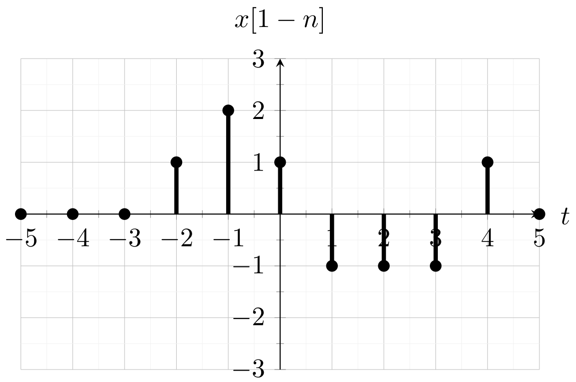

Figure 3: A piecewise function illustrating $x[1-n]$.

Y[n] is the sum of the components that we have found.

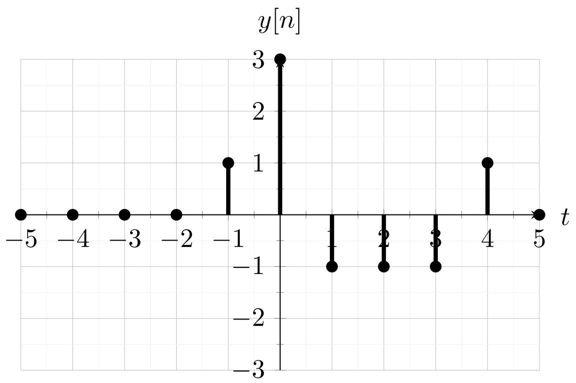

Figure 4: A piecewise function illustrating $y[n]$.

(c) From the figure above, we can formulate the y[n].
y[n] = x[2n+2] + x[1-n] = $\delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4]$

4. (a) Lets say N is the fundamental period. Therefore,

   cos($\frac{5\pi}{2}$n) = cos($\frac{5\pi}{2}$(n+N)) = cos($\frac{5\pi}{2}$n+$\frac{5\pi}{2}$N)

   $\frac{5\pi}{2}N = 2\pi k$ and $k \in \mathbb{N}$

   N = $\frac{4k}{5}$. There exist such N and k that are integers. So the fundamental period is N = 4 (for k = 5). Since we can find a period, this function is periodic.

   (b) Lets say N is the fundamental period. Therefore,

   sin(5n) = sin(5(n+N)) = sin(5n+5N)

   $5N = 2\pi k$ and $k \in \mathbb{N}$

   N = $\frac{2\pi k}{5}$. There do not exist such N and k that are integers. Since we can NOT find a period, this function is NOT periodic.

   (c) Lets say T is the fundamental period. Therefore,

   5sin(4t + $\frac{\pi}{3}$) = 5sin(4(t+T) + $\frac{\pi}{3}$) = 5sin(4t + 4T + $\frac{\pi}{3}$)

   $4T = 2\pi k$ and $k \in \mathbb{N}$ and $T \in \mathbb{R}$

   T = $\frac{\pi k}{2}$. There exist such T and k. So the fundamental period is T = $\frac{\pi}{2}$ (for k = 1). Since we can find a period, this function is periodic.

5.

$$\delta(at) =? \frac{\delta(t)}{|a|}$$

This equation already says a cannot be equal to 0 because of the right side of the equation. We first take the intagral of the both sides.

$$\int_{-\infty}^{\infty} \delta(at)dt =? \int_{-\infty}^{\infty} \frac{\delta(t)}{|a|}dt$$

First, let us consider the case where $a > 0$. This means $|a| = a$. So, our equation becomes $\int_{-\infty}^{\infty} \delta(at)dt =? \int_{-\infty}^{\infty} \frac{\delta(t)}{a}dt$.

Lets use substituting method. Say $u = a * t$. This gives $dt = \frac{du}{a}$. Both of the limits of the integral remain the same, meaning from - ∞ to ∞. So, this gives the following.

$$\int_{-\infty}^{\infty} \frac{1}{a}\delta(u)du =? \int_{-\infty}^{\infty} \frac{\delta(t)}{a}dt$$

.

If we cancel the both sides and change the variables. It can be seen the equation holds for the first case, which is $a > 0$. The first part of the proof is completed. Lets move on with the second part.

Lets now consider the case where $a < 0$. This means $|a| = -a$. So, our equation becomes $\int_{-\infty}^{\infty} \delta(at)dt =? \int_{-\infty}^{\infty} \frac{\delta(t)}{-a}dt$.

We can use substituting method once again. Say $u = a * t$. This gives $dt = \frac{du}{a}$. However, there is a difference for this case since a is negative (< 0). When $t = -\infty$, then $u = \infty$. Also, when $t = \infty$, $u = -\infty$ (This is caused by the equation $u = a * t$). This means the lower and upper limits of the integral swaps. This gives the following equation. (**Note:** The limits of the integral has changed.)

$$\int_{\infty}^{-\infty} \frac{1}{a}\delta(u)du =? \int_{-\infty}^{\infty} \frac{\delta(t)}{-a}dt$$

If we swap the limits of the integral since the order is reversed, we multiply it by -1.

$$-\int_{-\infty}^{\infty} \frac{1}{a}\delta(u)du =? \int_{-\infty}^{\infty} \frac{\delta(t)}{-a}dt$$

Cancelling $-1/a$'s from both sides shows the equation holds again as well. Therefore, the proof is completed for both $a < 0$ and $a > 0$.

6. (a) y[n] = y_2[n] = y_1[n-2] = 4x_1[n-2] + 2x_1[n-3] = 4x[n-2] + 2x[n-3]
   y[n] = 4x[n-2] + 2x[n-3] is the overall formula for the given system.

   (b) The system becomes as the following.
   First system: $y_1[n] = x_1[n-2]$
   Second system: $y_2[n] = 4y_1[n] + 2y_1[n-1]$

   y[n] = y_2[n] = 4y_1[n] + 2y_1[n-1] = 4x_1[n-2] + 2x_1[n-3] = 4x[n-2] + 2x[n-3]

   y[n] = 4x[n-2] + 2x[n-3] is the overall formula for the given system. This is the same as the part A. Thus, the system is COMMUTATIVE.

   (c) Assume,
   $y_a[n] = H\{x_a[n]\} = 4x_a[n-2] + 2x_a[n-3]$

   $y_b[n] = H\{x_b[n]\} = 4x_b[n-2] + 2x_b[n-3]$

   We should check whether the following formula holds.
   $\alpha * y_a[n] + \beta * y_b[n] = H\{\alpha * x_a[n] + \beta * x_b[n]\}$

   The lefthand of the equations is:
   $\alpha * y_a[n] + \beta * y_b[n] = 4 * \alpha * x_a[n-2] + 2 * \alpha * x_a[n-3] + 4 * \beta * x_b[n-2] + 2 * \beta * x_b[n-3]$

   The righthand of the equations is:
   $H\{\alpha * x_a[n] + \beta * x_b[n]\} = 4(\alpha * x_a[n-2] + \beta * x_b[n-2]) + 2(\alpha * x_a[n-3] + \beta * x_b[n-3])$

   From the above equations, we see that the formula holds. Righthand and lefthands are equal to each other. Thus, LINEAR.

   (d) A system is time-invariant if shifting input in time causes an identical shift in the output as well, $y[n] = H\{x[n]\}$ implies $y[n + n_0] = H\{x[n + n_0]\}$

   $y[n + n_0] = H\{x[n + n_0]\} = 4x[n - 2 + n_0] + 2x[n - 3 + n_0]$
   We can see the amount of shifting is the same for both sides of the equations. Therefore, this is TIME-INVARIANT.

7.

```
from sympy import *

n = symbols('n', integer=True)
x_1 = IndexedBase('x_1')
x_2 = IndexedBase('x_2')
a = symbols('a')
b = symbols('b')

def partA():
    def H(x):
        return n*x

    y_1 = H(x_1[n])
    y_2 = H(x_2[n])

    leftside = a*y_1 + b*y_2
    rightside = H(a*x_1[n] + b*x_2[n])

    print("Leftside is: ", leftside)
    print("Rightside is: ", rightside)

    if(leftside.equals(rightside)):
        print("The given system is a Linear system")
    else:
        print("The given system is a Non-Linear system")

def partB():
    def H(x):
        return x**2

    y_1 = H(x_1[n])
    y_2 = H(x_2[n])

    leftside = a * y_1 + b * y_2
    rightside = H(a * x_1[n] + b * x_2[n])

    print("Leftside is: ", leftside)
    print("Rightside is: ", rightside)

    if (leftside.equals(rightside)):
        print("The given system is a Linear system")
    else:
        print("The given system is a Non-Linear system")
```

(a)

```
------------ PART A -------------
Leftside is:  a*n*x_1[n] + b*n*x_2[n]
Rightside is:  n*(a*x_1[n] + b*x_2[n])
The given system is a Linear system
```

(b)

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------------ PART B -------------
Leftside is:  a*x_1[n]**2 + b*x_2[n]**2
Rightside is:  (a*x_1[n] + b*x_2[n])**2
The given system is a Non-Linear system
```

Our code implies part A is Linear, whereas part B is Non-Linear.