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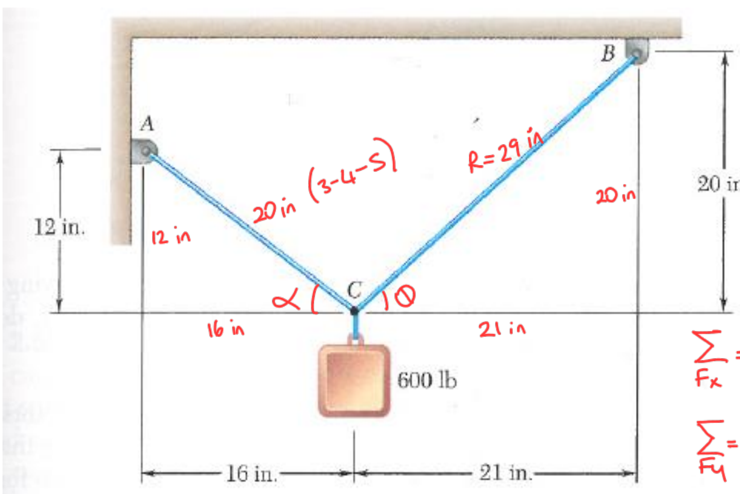
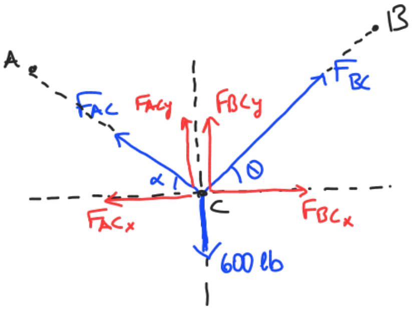
CENG

HOMEWORK 2

Q1)

$$R=\sqrt{20^2+21^2}=\sqrt{400+441}=\sqrt{841}=29$$





$\cos\alpha = 4/5$

$\cos\theta = 21/29$

$\sin\alpha = 3/5$

$\sin\theta = 20/29$

$$\sum_{F_x}=0=F_{ACx}+F_{BCx}=-F_{AC}\cdot\cos\alpha+F_{BC}\cdot\cos\theta=0$$

$$0=-F_{AC}\cdot 4/5+F_{BC}\cdot 21/29 \quad (1)$$

$$\sum_{F_y}=0=F_{ACy}+F_{BCy}-600=F_{AC}\cdot 3/5+F_{BC}\cdot\frac{20}{29}-600=0$$

$$F_{AC}\cdot\frac{3}{5}+F_{BC}\cdot\frac{20}{29}-600=0 \quad (2)$$

I only substitute the variables from formula 1 and 2

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

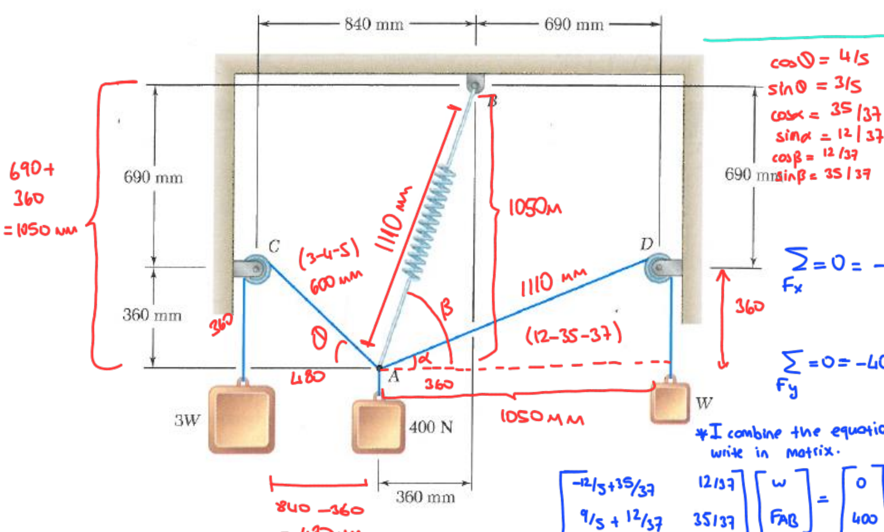
$$(1)\quad -F_{AC}\,\frac{4}{5}+F_{BC}\cdot\frac{21}{29}=0 \Rightarrow \boxed{F_{AC}=F_{BC}\cdot\frac{21}{29}\cdot\frac{5}{4}}$$

$$(2)\quad F_{BC}\cdot\frac{21}{29}\cdot\frac{5}{4}\cdot\frac{3}{5}+F_{BC}\cdot\frac{20}{29}=600 \Rightarrow F_{BC}\left(\frac{21}{29}\cdot\frac{3}{4}+\frac{20}{29}\right)=600$$

$$F_{BC}=486.71\ lb$$

$$F_{AC}=486.71\cdot\frac{21}{29}\cdot\frac{5}{4}\approx\boxed{440.56\ lb}\ (a) \qquad \boxed{F_{BC}=486.71\ lb}\ (b)$$

Q2)



$\cos\theta = 4/5$

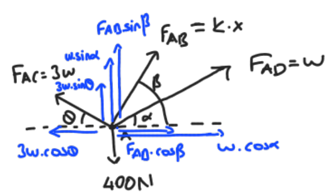
$\sin\theta = 3/5$

$\cos\alpha = 35/37$

$\sin\alpha = 12/37$

$\cos\beta = 12/37$

$\sin\beta = 35/37$



$$\sum_{F_x}=0=-3w\cdot\cos\theta+w\cdot\cos\alpha+F_{AB}\cos\beta=-3w\cdot\frac{4}{5}+w\cdot\frac{35}{37}+F_{AB}\frac{12}{37}=0$$

$$=\boxed{w\left(\frac{-12}{5}+\frac{35}{37}\right)+F_{AB}\cdot\frac{12}{37}=0}\ (1)$$

$$\sum_{F_y}=0=-400+3w\sin\theta+w\cdot\sin\alpha+F_{AB}\cdot\sin\beta=-400+3w\cdot\frac{3}{5}+w\cdot\frac{12}{37}+F_{AB}\cdot\frac{35}{37}=0$$

$$=w\,\frac{9}{5}+w\cdot\frac{12}{37}+F_{AB}\cdot\frac{35}{37}=400$$

$$=\boxed{w\left(\frac{9}{5}+\frac{12}{37}\right)+F_{AB}\cdot\frac{35}{37}=400}\ (2)$$

*I combine the equation 1 and 2 and write in matrix.

$$\begin{bmatrix}-12/5+35/37 & 12/37\\ 9/5+12/37 & 35/37\end{bmatrix}\begin{bmatrix}w\\ F_{AB}\end{bmatrix}=\begin{bmatrix}0\\ 400\end{bmatrix}$$

Applied Cramer's rule

$$|A|=\left(-\frac{12}{5}+\frac{35}{37}\right)\cdot\frac{35}{37}-\frac{12}{37}\left(\frac{9}{5}+\frac{12}{37}\right)\approx -2.06$$

$$w=\frac{-400\cdot 12/37}{|A|}\approx\boxed{62.84\ N = W}\ (a) \qquad F_{AD}=\frac{400(-12/5+35/37)}{|A|}$$

A load of weight 400 N is suspended from a spring and two cords which are attached to the blocks of weight 3W and W as shown. Knowing that the constant of the spring is 800N/m, determine (a) the value of W, (b) the unstretched length of the spring.

$$\boxed{F_{AD}\approx 281.74}$$

$k = 800\ N/m$

$$F_{AD}=k\cdot x \Rightarrow 281.74=800\cdot x$$

$$x=352.17\ mm$$

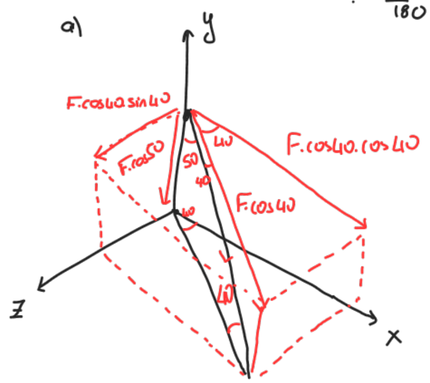
$$1110-352.17=\boxed{757.83\ mm}\ (b)$$

$180 \quad \pi$

$x \quad ?$

$? = \dfrac{x \cdot \pi}{180}$

Q3)



a) $F_x = F.\cos 40.\cos 40$

$F_x = 4.2.\cos 40.\cos 40$

$\boxed{F_x \approx 2.46 \ kN}$

$F_y = -F.\cos 50$

$F_y = -4.2.\cos 50$

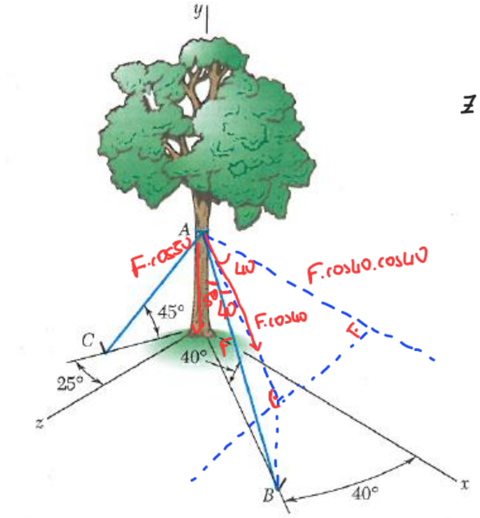
$\boxed{F_y \approx -2.7 \ kN}$

$F_z = F.\cos 40.\sin 40$

$F_z = 4.2.\cos 40.\sin 40$

$\boxed{F_z \approx 2.07 \ kN}$

$$\boxed{F = \left\{ 2.46\hat{i} - 2.7\hat{j} + 2.07\hat{k} \right\} kN}$$



b) $F = \sqrt{(2.46)^2 + (2.7)^2 + (2.07)^2} = 4.2$

$\theta_x = \arccos\left(\dfrac{2.4}{4.2}\right) = \boxed{55.15° = \theta_x}$

$\theta_y = \arccos\left(\dfrac{-2.7}{4.2}\right) = \boxed{130° = \theta_y}$

$\theta_z = \arccos\left(\dfrac{2.07}{4.2}\right) = \boxed{60° = \theta_z}$

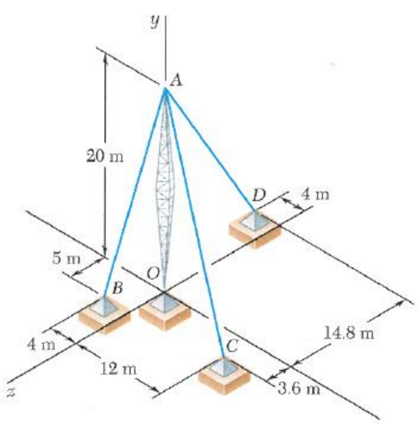
To stabilize a tree partially uprooted in a storm, cables AB and AC are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that the tension in cable AB is 4.2 kN, determine (a) the components of the force exerted by this cable on the tree, (b) the angles θ_x, θ_y and θ_z that the force forms with axes at A which are parallel to the coordinate axes.

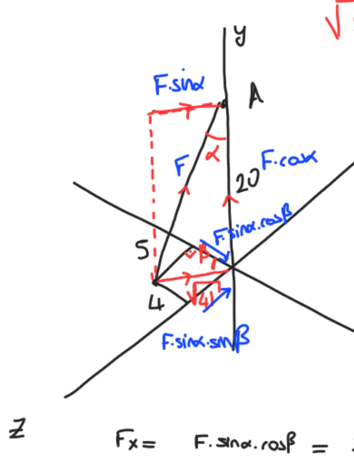
Q4)

$x \quad \pi$

$? \quad 180$

$? = \dfrac{x \cdot 180}{\pi}$





$\sqrt{5^2 + 4^2} = \sqrt{41}$

$\tan\alpha = \dfrac{\sqrt{41}}{20}$ $\boxed{\alpha \approx 17.75}$

$\sin\beta = \dfrac{5}{\sqrt{41}}$

$\cos\beta = \dfrac{4}{\sqrt{41}}$

$\sin\alpha = \sin\left[\tan^{-1}\left(\dfrac{\sqrt{41}}{20}\right)\right] \approx 0.3$

$\cos\alpha = \cos\left[\tan^{-1}\left(\dfrac{\sqrt{41}}{20}\right)\right] \approx 0.95$

$F_x = F.\sin\alpha.\cos\beta = 2100.\sin\alpha.\cos\beta \approx \boxed{400 \ N}$ F_x

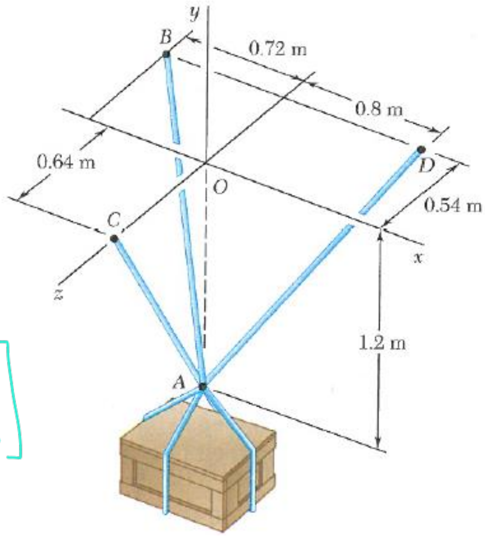
$F_y = F.\cos\alpha = 2100.\cos\alpha = \boxed{2000 \ N}$ F_y

$F_z = -F.\sin\alpha.\sin\beta = -2100.\sin\alpha.\sin\beta = \boxed{-500 \ N}$ F_z

$$\boxed{F = \left\{ 400\hat{i} + 2000\hat{j} - 500\hat{k} \right\} N}$$

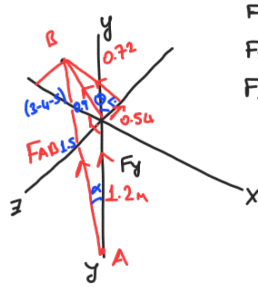
A transmission tower is held by three guy wires anchored by bolts at B,C and D. If the tension in wire AB is 2100 N, determine the components of the force exerted by the wire on the bolt at B.

Q5)



$$\begin{bmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{bmatrix}$$

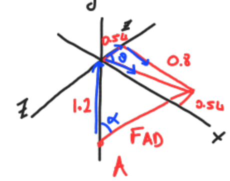
I will draw each force in a different diagram



$F_y = F_{AB} \cdot \cos\alpha = F_{AB} \cdot 4/5 = F_{AB} \cdot 0.8 = F_y$

$F_x = -F_{AB} \sin\alpha \sin\theta = -F_{AB} \cdot \frac{3}{5} \cdot \frac{4}{5} = -F_{AB} \cdot 0.48 = F_x$

$F_z = -F_{AB} \cdot \sin\alpha \cos\theta = -F_{AB} \cdot \frac{3}{5} \cdot \frac{3}{5} = -F_{AB} \cdot 0.36 = F_z$



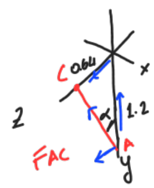
$\theta = \arctan(0.8/0.54) \approx 55.98°$

$\alpha = \arctan\left(\frac{\sqrt{0.8^2 + 0.54^2}}{1.2}\right) \approx 38.81°$

$F_x = F_{AD} \cdot \sin\alpha \cdot \sin\theta = F_{AD} \cdot \sin 38.81 \cdot \sin 55.98° = F_{AD} \cdot 0.51 = F_x$

$F_y = F_{AD} \cdot \cos\alpha = F_{AD} \cdot \cos 38.81 = F_{AD} \cdot 0.78 = F_y$

$F_z = -F_{AD} \cdot \sin\alpha \cdot \cos\theta = -F_{AD} \sin 38.81° \cdot \cos 55.98° = -F_{AD} \cdot 0.35 = F_z$



$\alpha = \arctan\left(\frac{0.64}{1.2}\right) \approx 28.07°$

$F_x = 0$

$F_y = F_{AC} \cdot \cos\alpha = F_{AC} \cdot \cos 28.07° = F_{AC} \cdot 0.89 = F_y$

$F_z = F_{AC} \cdot \sin\alpha = F_{AC} \cdot \sin 28.07° = F_{AC} \cdot 0.46 = F_z$

A 750-kg crate is supported by three cables as shown. Determine the tension in each cable.

* Above, I have computed each component of the forces.

$\sum x = 0 = -F_{AB} \cdot 0.48 + F_{AD} \cdot 0.51 = 0$

$\sum y = 0 = F_{AB} \cdot 0.8 + F_{AD} \cdot 0.78 + F_{AC} \cdot 0.89 - \underbrace{750g}_{\text{gravity}} \Rightarrow F_{AB} \cdot 0.8 + F_{AD} \cdot 0.78 + F_{AC} \cdot 0.89 = 750g$

$\sum z = 0 = -F_{AB} \cdot 0.36 - F_{AD} \cdot 0.35 + F_{AC} \cdot 0.46 = 0$

$$\underbrace{\begin{bmatrix} -0.48 & 0.51 & 0 \\ 0.8 & 0.78 & 0.89 \\ -0.36 & -0.35 & 0.46 \end{bmatrix}}_{A} \begin{bmatrix} F_{AB} \\ F_{AD} \\ F_{AC} \end{bmatrix} = \begin{bmatrix} 0 \\ 750g \\ 0 \end{bmatrix}$$

$|A| \approx -0.67$

* I implemented matrix determinant computing code in pyton to get the determinants.

I will apply Cramer Rule

$$F_{AB} = \frac{\begin{vmatrix} 0 & 0.51 & 0 \\ 750g & 0.78 & 0.89 \\ 0 & -0.35 & 0.46 \end{vmatrix}}{|A|} \approx \boxed{261.51g}$$

$$F_{AD} = \frac{\begin{vmatrix} -0.48 & 0 & 0 \\ 0.8 & 750g & 0.89 \\ -0.36 & 0 & 0.46 \end{vmatrix}}{|A|} \approx \boxed{246.12g}$$

$$F_{AC} = \frac{\begin{vmatrix} -0.48 & 0.51 & 0 \\ 0.8 & 0.78 & 750g \\ -0.36 & -0.35 & 0 \end{vmatrix}}{|A|} \approx \boxed{391.93g}$$

I choose $g \approx 9.8$.

$$\begin{aligned} F_{AB} &\approx 261.51 \times 9.8 \approx \boxed{2562.8\ N} \\ F_{AD} &\approx 246.12 \times 9.8 \approx \boxed{2412\ N} \\ F_{AC} &\approx 391.93 \times 9.8 \approx \boxed{3840.91\ N} \end{aligned}$$