CENG 424 HW 1

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Question 1)

1) $\neg(p \vee r) \wedge (\neg p \Rightarrow (q \vee r)) \wedge \neg q$

$\neg(p \vee r) \wedge (\neg p \Rightarrow (q \vee r)) \wedge \neg q \Leftrightarrow T$ (a) → applied

This is formula name

↳ Denotes I made use of this formula since tableux ends when each formula is applied

(a) → This means I used formula (a) to derive

$\neg(p \vee r) \Leftrightarrow T$ (b) → applied

$\neg p \Rightarrow q \vee r \Leftrightarrow T$ (c)

$\neg q \Leftrightarrow T$ (d) → applied

(b)

$q \vee r \Leftrightarrow \bot$ (e) → applied

(d)

$\boxed{q \Leftrightarrow \bot}$

(e)

$\boxed{\begin{array}{l} q \Leftrightarrow \bot \\ r \Leftrightarrow \bot \end{array}}$

(c) (c)

$\neg p \Leftrightarrow \bot$ (f) applied $q \vee r \Leftrightarrow T$ (g)

(f)

$\boxed{p \Leftrightarrow T}$ $\boxed{q \Leftrightarrow T}$ $\boxed{r \Leftrightarrow T}$

closed closed closed

* At the end, each brach of the semantic tableux is closed. Therefore, the related formula is UNSATISFIABLE.

2) $\neg((p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q)))$

$\neg((p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t \Rightarrow q))) \iff \top$ (a) → applied

$\downarrow$ (a)

$(p \Rightarrow q) \Rightarrow \neg((p \Rightarrow t) \wedge (p \wedge t) \Rightarrow q)) \iff \bot$ (b) → applied

$\downarrow$ (b)

$p \Rightarrow q \iff \top$ (c) → applied

$\neg((p \Rightarrow t) \wedge (p \wedge t) \Rightarrow q)) \iff \bot$ (d) → applied

$\downarrow$ (d)

$(p \Rightarrow t) \wedge ((p \wedge t) \Rightarrow q) \Rightarrow \top$ (e) → applied

$\downarrow$ (e)

$p \Rightarrow t \iff \top$ (f) → applied

$p \wedge t \Rightarrow q \iff \top$ (g) → applied

- (g) $\boxed{p \wedge t \iff \bot}$
- (g) $\boxed{q \iff \top}$
  - $\boxed{p \iff \bot}$
  - $\boxed{q \iff \top}$
    - (f) $\boxed{p \iff \bot}$
    - (f) $\boxed{t \iff \top}$ — open branch

* As it can be seen from the table above, there is an open branch. This means the formula is SATISFIABLE. The open branch gives the interpretation

$I = \{p \leftarrow \sqcup, q \leftarrow \top, t \leftarrow \top\}$, where p can be any boolean value.

Let me try this interpretation I and replace into formula.

$$= \neg((\underbrace{p \Rightarrow \top}_{\top}) \Rightarrow \neg((\underbrace{p \Rightarrow \top}_{\top}) \wedge (\underbrace{p \wedge \top \Rightarrow \top}_{\top})))$$

$$= \neg(\top \Rightarrow \underbrace{\neg(\underbrace{\top \wedge \top}_{\top})}_{\bot})$$

$$= \neg(\top \Rightarrow \bot)$$

$$= \neg(\bot)$$

$\boxed{\top}$ → As we can se I is a model of the formula. Therefore, SATISFIABLE.

Question 2)

1) $\neg(p \vee r) \wedge (\neg p \Rightarrow (q \vee r)) \wedge \neg q$

First, we need to convert the formula into CNF.

$$\neg(p \vee r) \wedge (\neg p \Rightarrow (q \vee r)) \wedge \neg q \Leftrightarrow \neg p \wedge \neg r \wedge (p \vee q \vee r) \wedge \neg q$$

So, the set of clauses S is the following

$$S = \begin{cases} \neg p \rightarrow \text{unit} \\ \neg r \\ \cancel{p} \vee q \vee r \\ \neg q \end{cases}$$

Apply unit propogation to $\neg p$ and we get new set of clauses S'

$$S' = \begin{cases} \neg r \rightarrow \text{unit} \\ q \vee \cancel{r} \\ \neg q \end{cases}$$

Now, Apply unit propogation to $\neg r$ to get set of clauses S''

$$S'' = \begin{cases} q \\ \neg q \end{cases}$$

This set means $q \wedge \neg q$ and it always evaluates to $\boxed{\perp}$

This means the formula is $\boxed{\text{unsatisfiable.}}$

2) $\neg(((p \wedge q \wedge \neg w) \Rightarrow r) \Rightarrow \neg(((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w)))) \wedge (q \vee \neg r \vee \neg w)$

* I will make use of definitional clause form transformation in order to convert the formula into <u>CNF.</u> Then I will apply DLL algorithm.

Step 1) $f_1 \Leftrightarrow \neg(((p \wedge q \wedge \neg w) \Rightarrow r) \Rightarrow \neg(((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w)))) \wedge (q \vee \neg r \vee \neg w) \Leftrightarrow T$

This step gives $\left.\boxed{f_1}\right\}$ set of clauses from step (1)

Step 2) $f_1 \Leftrightarrow f_2 \wedge f_3$

where $f_2 \Leftrightarrow \neg(((p \wedge q \wedge \neg w) \Rightarrow r) \Rightarrow \neg(((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w))))$

$f_3 \Leftrightarrow q \vee \neg r \vee \neg w$

$$= f_1 \Leftrightarrow f_2 \wedge f_3$$
$$= (f_1 \vee \neg(f_2 \wedge f_3)) \wedge (\neg f_1 \vee (f_2 \wedge f_3))$$
$$= (f_1 \vee \neg f_2 \vee \neg f_3) \wedge (\neg f_1 \vee f_2) \wedge (\neg f_1 \vee f_3)$$

$$\left.\boxed{\begin{array}{l} f_1 \vee \neg f_2 \vee \neg f_3 \\ \neg f_1 \vee f_2 \\ \neg f_1 \vee f_3 \end{array}}\right\}$$ This is the set of clauses from step (2)

Step 3) $f_2 \iff \neg f_4$

where $f_4 \iff ((p \wedge q \wedge \neg w) \Rightarrow r) \Rightarrow \neg(((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w)))$

$= f_2 \iff \neg f_4$

$= (f_2 \vee f_4) \wedge (\neg f_2 \vee \neg f_4)$

$\boxed{\begin{array}{l} f_2 \vee f_4 \\ \neg f_2 \vee \neg f_4 \end{array}}$ } This is the set of clauses resulting from step (3)

Step 4) $f_3 \iff q \vee \neg r \vee \neg w$

$= (f_3 \vee \neg(q \vee \neg r \vee \neg w)) \wedge (\neg f_3 \vee q \vee \neg r \vee \neg w)$

$= (f_3 \vee (\neg q \wedge r \wedge w)) \wedge (\neg f_3 \vee q \vee \neg r \vee \neg w)$

$= (f_3 \vee \neg q) \wedge (f_3 \vee r) \wedge (f_3 \vee w) \wedge (\neg f_3 \vee q \vee \neg r \vee \neg w)$

$\boxed{\begin{array}{l} f_3 \vee \neg q \\ f_3 \vee r \\ f_3 \vee w \\ \neg f_3 \vee q \vee \neg r \vee \neg w \end{array}}$ } This is the set of clause from step (4)

step 5) $f_4 \iff f_5 \Rightarrow f_6$

where

$f_5 \iff (p \wedge q \wedge \neg w) \Rightarrow r)$

$f_6 \iff \neg(((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w)))$

$= f_4 \iff f_5 \Rightarrow f_6$

$= f_4 \iff \neg f_5 \vee f_6$

$= (f_4 \vee \neg(\neg f_5 \vee f_6)) \wedge (\neg f_4 \vee (\neg f_5 \vee f_6))$

$= (f_4 \vee (f_5 \wedge \neg f_6)) \wedge (\neg f_4 \vee \neg f_5 \vee f_6)$

$= (f_4 \vee f_5) \wedge (f_4 \vee \neg f_6) \wedge (\neg f_4 \vee \neg f_5 \vee f_6)$

$\boxed{\begin{array}{l} f_4 \vee f_5 \\ f_4 \vee \neg f_6 \\ \neg f_4 \vee \neg f_5 \vee f_6 \end{array}}$ } This is the set of clauses from step (5)

Step 6) $f_5 \iff f_7 \Rightarrow r$

where $f_7 \iff (p \wedge q \wedge \neg w)$

$= f_5 \iff f_7 \Rightarrow r$

$= f_5 \iff \neg f_7 \vee r$

$= (f_5 \vee \neg(\neg f_7 \vee r)) \wedge (\neg f_5 \vee (\neg f_7 \vee r))$

$= (f_5 \vee (f_7 \wedge \neg r)) \wedge (\neg f_5 \vee \neg f_7 \vee r)$

$= (f_5 \vee f_7) \wedge (f_5 \wedge \neg r) \wedge (\neg f_5 \vee \neg f_7 \vee r)$

$\boxed{\begin{array}{l} f_5 \vee f_7 \\ f_5 \vee \neg r \\ \neg f_5 \vee \neg f_7 \vee r \end{array}}$ } This is the set of clauses from step (6)

step 7) $f_6 \iff \neg f_8$

where $f_8 \iff ((p \wedge r) \Rightarrow (q \vee w)) \wedge p \wedge (q \Rightarrow \neg(r \wedge w))$

$= f_6 \iff \neg f_8$

$= (f_6 \vee f_8) \wedge (\neg f_6 \vee \neg f_8)$

$\boxed{\begin{array}{l} f_6 \vee f_8 \\ \neg f_6 \vee \neg f_8 \end{array}}$ } This is the set of clauses from step (7) step 8) $f_7 \Leftrightarrow p \wedge q \wedge \neg w$

$= (f_7 \vee \neg(p \wedge q \wedge \neg w)) \wedge (\neg f_7 \vee (p \wedge q \wedge \neg w))$

$= (f_7 \vee (\neg p \vee \neg q \vee w)) \wedge (\neg f_7 \vee p) \wedge (\neg f_7 \vee q) \wedge (\neg f_7 \vee \neg w)$

$= (f_7 \vee \neg p \vee \neg q \vee w) \wedge (\neg f_7 \vee p) \wedge (\neg f_7 \vee q) \wedge (\neg f_7 \vee \neg w)$

$f_7 \vee \neg p \vee \neg q \vee w$
$\neg f_7 \vee p$
$\neg f_7 \vee q$
$\neg f_7 \vee \neg w$

This is the set of clauses from step (8)

step 9) $f_8 \Leftrightarrow f_9 \wedge p \wedge f_{10}$

where $f_9 \Leftrightarrow (p \wedge r) \Rightarrow (q \vee w)$

$f_{10} \Leftrightarrow (q \Rightarrow \neg(r \wedge w))$

$= f_8 \Leftrightarrow f_9 \wedge p \wedge f_{10}$

$= (f_8 \vee \neg(f_9 \wedge p \wedge f_{10})) \wedge (\neg f_8 \vee (f_9 \wedge p \wedge f_{10}))$

$= (f_8 \vee (\neg f_9 \vee \neg p \vee \neg f_{10})) \wedge (\neg f_8 \vee f_9) \wedge (\neg f_8 \vee p) \wedge (\neg f_8 \vee f_{10})$

$= (f_8 \vee \neg f_9 \vee \neg p \vee \neg f_{10}) \wedge (\neg f_8 \vee f_9) \wedge (\neg f_8 \vee p) \wedge (\neg f_8 \vee f_{10})$

$f_8 \vee \neg f_9 \vee \neg p \vee \neg f_{10}$
$\neg f_8 \vee f_9$
$\neg f_8 \vee p$
$\neg f_8 \vee f_{10}$

This is the set of clauses from step (9)

step 10) $f_9 \Leftrightarrow f_{11} \Rightarrow f_{12}$

where $f_{11} \Leftrightarrow p \wedge r$

$f_{12} \Leftrightarrow q \vee w$

$= f_9 \Leftrightarrow f_{11} \Rightarrow f_{12}$

$= f_9 \Leftrightarrow \neg f_{11} \vee f_{12}$

$= (f_9 \vee \neg(\neg f_{11} \vee f_{12})) \wedge (\neg f_9 \vee (\neg f_{11} \vee f_{12}))$

$= (f_9 \vee (f_{11} \wedge \neg f_{12})) \wedge (\neg f_9 \vee \neg f_{11} \vee f_{12})$

$= (f_9 \vee f_{11}) \wedge (f_9 \vee \neg f_{12}) \wedge (\neg f_9 \vee \neg f_{11} \vee f_{12})$

$f_9 \vee f_{11}$
$f_9 \vee \neg f_{12}$
$\neg f_9 \vee \neg f_{11} \vee f_{12}$

This is the set of clauses from step (10)

step 11) $f_{10} \Leftrightarrow q \Rightarrow \neg(r \vee w)$

$= f_{10} \Leftrightarrow q \Rightarrow \neg r \vee \neg w$

$= f_{10} \Leftrightarrow \neg q \vee \neg r \vee \neg w$

$= (f_{10} \vee \neg(\neg q \vee \neg r \vee \neg w)) \wedge (\neg f_{10} \vee \neg q \vee \neg r \vee \neg w)$

$= (f_{10} \vee (q \wedge r \wedge w)) \wedge (\neg f_{10} \vee \neg q \vee \neg r \vee \neg w)$

$= (f_{10} \vee q) \wedge (f_{10} \vee r) \wedge (f_{10} \vee w) \wedge (\neg f_{10} \vee \neg q \vee \neg r \vee \neg w)$

$f_{10} \vee q$
$f_{10} \vee r$
$f_{10} \vee w$
$\neg f_{10} \vee \neg q \vee \neg r \vee \neg w$

This is the set of clauses from step (11)

step 12) $f_{11} \Leftrightarrow p \wedge r$

$= (f_{11} \vee \neg(p \wedge r)) \wedge (\neg f_{11} \vee (p \wedge r))$

$= (f_{11} \vee (\neg p \vee \neg r)) \wedge (\neg f_{11} \vee p) \wedge (\neg f_{11} \vee r)$

$= (f_{11} \vee \neg p \vee \neg r) \wedge (\neg f_{11} \vee p) \wedge (\neg f_{11} \vee r)$

$f_{11} \vee \neg p \vee \neg r$
$\neg f_{11} \vee p$
$\neg f_{11} \vee r$

This is the set of clouse from step (12) step 13) $f_{12} \Leftrightarrow q \lor w$

$$= (f_{12} \lor \lnot(q \lor w)) \land (\lnot f_{12} \lor (q \lor w))$$

$$= (f_{12} \lor (\lnot q \land \lnot w)) \land (\lnot f_{12} \lor q \lor w)$$

$$= (f_{12} \lor \lnot q) \land (f_{12} \lor \lnot w) \land (\lnot f_{12} \lor q \lor w)$$

$$\boxed{\begin{array}{l} f_{12} \lor \lnot q \\ f_{12} \lor \lnot w \\ \lnot f_{12} \lor q \lor w \end{array}} \Big\} \text{ This is the set of clauses from step (13)}$$

* My algorithm for converting into CNF has finished. Now, I will write all clauses from each of the steps and apply DLL algorithm.

f_1 → unit
$f_1 \lor \lnot f_2 \lor \lnot f_3$
$\cancel{\lnot f_1} \lor f_2$
$\cancel{\lnot f_1} \lor f_3$
$f_2 \lor f_4$
$\lnot f_2 \lor \lnot f_4$
$f_3 \lor \lnot q$
$f_3 \lor w$
$f_3 \lor r$
$\lnot f_3 \lor q \lor \lnot w \lor \lnot r$
$f_4 \lor f_5$
$f_4 \lor \lnot f_6$
$\lnot f_4 \lor \lnot f_5 \lor f_6$
$f_5 \lor f_7$
$f_5 \lor \lnot r$
$\lnot f_5 \lor \lnot f_7 \lor r$
$f_6 \lor f_8$
$\lnot f_6 \lor \lnot f_8$
$f_7 \lor \lnot p \lor \lnot q \lor w$
$\lnot f_7 \lor p$
$\lnot f_7 \lor q$
$\lnot f_7 \lor \lnot w$
$f_8 \lor \lnot f_9 \lor \lnot p \lor \lnot f_{10}$
$\lnot f_8 \lor f_9$
$\lnot f_8 \lor p$
$\lnot f_8 \lor f_{10}$
$f_9 \lor f_{11}$
$f_9 \lor \lnot f_{12}$
$\lnot f_9 \lor \lnot f_{11} \lor f_{12}$
$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\lnot f_{10} \lor \lnot q \lor \lnot r \lor \lnot w$
$f_{11} \lor \lnot p \lor \lnot r$
$\lnot f_{11} \lor p$
$\lnot f_{11} \lor r$
$f_{12} \lor \lnot q$
$f_{12} \lor \lnot w$
$\lnot f_{12} \lor q \lor w$

* Apply unit propagation to f_1 and we get

f_2 → unit
f_3 → unit
$f_2 \lor f_4$
$\cancel{\lnot f_2} \lor \lnot f_4$
$f_3 \lor \lnot q$
$f_3 \lor w$
$f_3 \lor r$
$\cancel{\lnot f_3} \lor q \lor \lnot w \lor \lnot r$
$f_4 \lor f_5$
$f_4 \lor \lnot f_6$
$\lnot f_4 \lor \lnot f_5 \lor f_6$
$f_5 \lor f_7$
$f_5 \lor \lnot r$
$\lnot f_5 \lor \lnot f_7 \lor r$
$f_6 \lor f_8$
$\lnot f_6 \lor \lnot f_8$
$f_7 \lor \lnot p \lor \lnot q \lor w$
$\lnot f_7 \lor p$
$\lnot f_7 \lor q$
$\lnot f_7 \lor \lnot w$
$f_8 \lor \lnot f_9 \lor \lnot p \lor \lnot f_{10}$
$\lnot f_8 \lor f_9$
$\lnot f_8 \lor p$
$\lnot f_8 \lor f_{10}$
$f_9 \lor f_{11}$
$f_9 \lor \lnot f_{12}$
$\lnot f_9 \lor \lnot f_{11} \lor f_{12}$
$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\lnot f_{10} \lor \lnot q \lor \lnot r \lor \lnot w$
$f_{11} \lor \lnot p \lor \lnot r$
$\lnot f_{11} \lor p$
$\lnot f_{11} \lor r$
$f_{12} \lor \lnot q$
$f_{12} \lor \lnot w$
$\lnot f_{12} \lor q \lor w$

* Now apply unit propagation to f_2 and f_3.

$\lnot f_4$ → unit
$q \lor \lnot w \lor \lnot r$
$\cancel{f_4} \lor f_5$
$\cancel{f_4} \lor \lnot f_6$
$\lnot f_4 \lor \lnot f_5 \lor f_6$
$f_5 \lor f_7$
$f_5 \lor \lnot r$
$\lnot f_5 \lor \lnot f_7 \lor r$
$f_6 \lor f_8$
$\lnot f_6 \lor \lnot f_8$
$f_7 \lor \lnot p \lor \lnot q \lor w$
$\lnot f_7 \lor p$
$\lnot f_7 \lor q$
$\lnot f_7 \lor \lnot w$
$f_8 \lor \lnot f_9 \lor \lnot p \lor \lnot f_{10}$
$\lnot f_8 \lor f_9$
$\lnot f_8 \lor p$
$\lnot f_8 \lor f_{10}$
$f_9 \lor f_{11}$
$f_9 \lor \lnot f_{12}$
$\lnot f_9 \lor \lnot f_{11} \lor f_{12}$
$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\lnot f_{10} \lor \lnot q \lor \lnot r \lor \lnot w$
$f_{11} \lor \lnot p \lor \lnot r$
$\lnot f_{11} \lor p$
$\lnot f_{11} \lor r$
$f_{12} \lor \lnot q$
$f_{12} \lor \lnot w$
$\lnot f_{12} \lor q \lor w$

* We apply unit propogation to $\neg f_4$

$q \lor \neg w \lor \neg r$
$f_5 \rightarrow$ unit
$\neg f_6 \rightarrow$ unit
$f_5 \lor f_7$
$f_5 \lor \neg r$
$\cancel{\neg f_5} \lor \neg f_7 \lor r$
$\cancel{f_6} \lor f_8$
$\neg f_6 \lor \neg f_8$

$f_7 \lor \neg p \lor \neg q \lor w$
$\neg f_7 \lor p$
$\neg f_7 \lor q$
$\neg f_7 \lor \neg w$
$f_8 \lor \neg f_9 \lor \neg p \lor \neg f_{10}$
$\neg f_8 \lor f_9$
$\neg f_8 \lor p$
$\neg f_8 \lor f_{10}$

$f_9 \lor f_{11}$
$f_9 \lor \neg f_{12}$
$\neg f_9 \lor \neg f_{11} \lor f_{12}$
$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\neg f_{10} \lor \neg q \lor \neg r \lor \neg w$
$\neg f_{11} \lor p$
$\neg f_{11} \lor r$

$f_{12} \lor \neg q$
$f_{12} \lor \neg w$
$\neg f_{12} \lor q \lor w$
$f_{11} \lor \neg p \lor \neg r$

* Next, we apply unit propogation to f_5 and $\neg f_6$

$q \lor \neg w \lor \neg r$
$\neg f_7 \lor r$
$f_8 \rightarrow$ unit
$f_7 \lor \neg p \lor \neg q \lor w$
$\neg f_7 \lor p$
$\neg f_7 \lor q$
$\neg f_7 \lor \neg w$

$f_8 \lor \neg f_9 \lor \neg p \lor \neg f_{10}$
$\cancel{\neg f_8} \lor f_9$
$\cancel{\neg f_8} \lor p$
$\cancel{\neg f_8} \lor f_{10}$
$f_9 \lor f_{11}$
$f_9 \lor \neg f_{12}$

$\neg f_9 \lor \neg f_{11} \lor f_{12}$
$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\neg f_{10} \lor \neg p \lor \neg r \lor \neg w$
$\neg f_{11} \lor p$

$\neg f_{11} \lor r$
$f_{12} \lor \neg q$
$f_{12} \lor \neg w$
$\neg f_{12} \lor q \lor w$
$f_{11} \lor \neg p \lor \neg r$

* Now, apply unit propogation to f_8

$q \lor \neg w \lor \neg r$
$\neg f_7 \lor r$
$f_7 \lor \cancel{\neg p} \lor \neg q \lor w$
$\neg f_7 \lor p$
$\neg f_7 \lor q$
$\neg f_7 \lor \neg w$

f_9
p
f_{10}
} unit propogation
$f_9 \lor f_{11}$
$f_9 \lor \neg f_{12}$
$\cancel{\neg f_9} \lor \neg f_{11} \lor f_{12}$

$f_{10} \lor q$
$f_{10} \lor r$
$f_{10} \lor w$
$\neg \cancel{f_{10}} \lor \neg q \lor \neg r \lor \neg w$
$\neg f_{11} \lor p$
$\neg f_{11} \lor r$

$f_{12} \lor \neg q$
$f_{12} \lor \neg w$
$\neg f_{12} \lor q \lor w$
$f_{11} \lor \cancel{\neg p} \lor \neg r$

* Next, apply unit propogation to f_9, p and f_{10}. We can also conclude $p \leftarrow \top$ for models since p becomes unit.

$q \lor \neg w \lor \neg r$
$\neg f_7 \lor r$
$f_7 \lor \neg q \lor w$
$\neg f_7 \lor q$
$\neg f_7 \lor \neg w$

$\neg f_{11} \lor f_{12}$
$\neg q \lor \neg r \lor \neg w$
$\neg f_{11} \lor \neg r$
$f_{12} \lor \neg q$

$f_{12} \lor \neg w$
$\neg f_{12} \lor q \lor w$
$f_{11} \lor \neg r$

* Now, we cannot apply unit propogation. Therefore, I will apply splitting.

$q = \bot$

$\neg w \lor \neg r$
$\neg f_7 \lor r$
$\neg f_7 \rightarrow$ unit
$\neg f_7 \lor \neg w$
$\neg f_{11} \lor f_{12}$
$\neg f_{11} \lor \neg r$
$f_{12} \lor \neg w$
$\neg f_{12} \lor w$
$f_{11} \lor \neg r$

Left Branch

$q = \top$

$\neg f_7 \lor r$
$f_7 \lor w$
$\neg f_7 \lor \neg w$
$\neg f_{11} \lor f_{12}$
$\neg r \lor \neg w$
$\neg f_{11} \lor \neg r$
$f_{12} \rightarrow$ unit
$f_{12} \lor \neg w$
$f_{11} \lor \neg r$

Right Branch

* Now, apply unit propagation to left branch with $\neg f_7$ and to right branch with f_{12}

Left branch

$\times\ \neg w \vee \neg r$

$\neg f_{11} \vee f_{12}$

$\times\ \neg f_{11} \vee \neg r$

$f_{12} \vee \neg w$

$\neg f_{12} \vee w$

$\times\ f_{11} \vee \neg r$

Right branch

$\neg f_7 \vee r$

$f_7 \vee w$

$\neg f_7 \vee \neg w$

$\neg r \vee \neg w$

$\neg f_{11} \vee \neg r$

$f_{11} \vee \neg r$

* For left branch, $\neg r$ is pure. So, we can eliminate clauses with $\neg r$. This also means $\neg r$ evaluates to T and $\boxed{r \leftrightarrow \bot}$

Left branch becomes

$\times\ \neg f_{11} \vee f_{12}$

$f_{12} \vee \neg w$

$\neg f_{12} \vee w$

Now, $\neg f_{11}$ becomes pure. So, we can eliminate clauses with $\neg f_{11}$.

Final set of clauses for left branch becomes

$$\boxed{\begin{array}{l} f_{12} \vee \neg w \\ \neg f_{12} \vee w \end{array}}$$

By choosing $w \leftarrow T$ and $f_{12} \leftarrow T$, we can evaluate this to T (TRUE). Therefore, the formula is SATISFIABLE.

Also, we do not need to worry about right branch since left branch satisfies and it is enough for us to determine satifiability.

Also, the optimizations give the Interpretation I such that

$I = \{p \leftarrow T, q \leftarrow \bot, r \leftarrow \bot, w \leftarrow T\}$. So, I will also check if I is a model.

Lets replace Interpretation above into the formula.

$$\neg(\underbrace{(\underbrace{(\top\wedge\bot\wedge\neg\top)}_{\bot}\Rightarrow\bot)}_{\top}\Rightarrow\neg(\underbrace{(\underbrace{(\top\wedge\bot)}_{\bot}\Rightarrow\underbrace{(\bot\vee\top)}_{\top})}_{\top}\wedge\top\wedge\underbrace{(\bot\Rightarrow\neg\overbrace{(\bot\wedge\top)}^{\bot})}_{\top})))\wedge\underbrace{(\bot\vee\overbrace{\neg\bot}^{\top}\vee\overbrace{\neg\top}^{\bot})}_{\top}$$

$$\neg(\top \Rightarrow \underbrace{\neg(\underbrace{\top\wedge\top\wedge\top}_{\top}))}_{\bot})\wedge\top$$

$$\underbrace{\neg(\top\Rightarrow\bot)}$$

$$\underbrace{\neg(\bot)}$$

$$\top \quad \checkmark$$

$$I \models \text{formula}$$

* Interpretation I results in $\top$ (True). Therefore, we can conclude I is indeed a model. Hence, SATISFIABLE.