CENG 424 LOGIC FOR CS HW3

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Q1)

1.1) I will first state my constants.

Object constants = {Me, Nothing}

Relation constants:

knows(x,y) = x knows y is true

Relational Formula: $\boxed{\underbrace{\forall x. \left(knows(I,x) \Rightarrow (x \Leftrightarrow nothing) \right)}_{\text{This is also equal to "Only thing I know is nothing"}}}$

1.2) I will first state my constants

Object constants: {period}

Relation constants:

punctuation(x): x is a punctuation mark

abbreviation(x): x is abbreviated

period(x): x is a period.

Function constants

sentence(x)=y : x belongs to the sentence y

$$\boxed{\forall x. \left((x \Leftrightarrow period) \Rightarrow \left(punctuation(x) \wedge \left(\underbrace{end(sentence(x))}_{\text{the sentence which x belongs to has ended}} \vee \underbrace{\exists y. abbreviation(y)}_{\text{there exists an abbreviation of something}} \right) \right) \right)}$$

* We can translate it as "If something is period, then it is a punctuation mark. Additionally, there is something abbreviated or sentence has ended".

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Q2)

1) $\forall x.\ (P(x) \lor Q(x))$ — Premise
2) $\exists x.\ \neg Q(x)$ — Premise
3) $\forall x.\ (R(x) \Rightarrow \neg P(x))$ — Premise
4) $P(c) \lor Q(c)$ — UI 1
5) $\neg Q(c)$ — EI 2
6) $P(c)$ — Assumption
7) $P(c)$ — copy of 6
8) $Q(c)$ — Assumption
9) $\perp$ — $\perp$, 8-5
10) $P(C)$ — $\perp$ lemma
11) $P(c)$ — $\lor$elimination 4, 6-7, 8-10
12) $R(c) \Rightarrow \neg P(c)$ — UI 3
13) $\neg R(c)$ — MT 12
14) $\boxed{\exists y.\ \neg R(y)}$ — EG 13

$\varphi = \exists y.\ \neg R(y)$

Here, I made use of $\lor$ elimination from Natural Deduction

We here reached the conclusion (φ). Therefore, this is the end of proof.

Q3)

I will first write the set of all premises, Then make use of Natural deduction

p
r
s
t
u
v
w
x
y
z

1) $p(a, b, 0.6)$ — premise
2) $p(a, c, 0.4)$ — premise
3) $p(b, d, 0.3)$ — premise
4) $p(b, e, 0.5)$ — premise
5) $p(b, f, 0.2)$ — premise
6) $p(c, g, 0.9)$ — premise
7) $p(c, h, 0.1)$ — premise
8) $p(e, i, 0.9)$ — premise
9) $p(e, j, 0.1)$ — premise

Δ, premises from the given three in breadth first manner.

10) $\forall v \forall w \forall x \forall y \forall z\ (p(v, w, y) \land p(w, x, z) \Rightarrow p(v, x, y * z))$ — premise
11) $\forall w \forall x \forall y \forall z\ (p(b, w, y) \land p(w, x, z) \Rightarrow p(b, x, y * z))$ — UI 10
12) $\forall x \forall y \forall z\ (p(b, e, y) \land p(e, x, z) \Rightarrow p(b, x, y * z))$ — UI 11
13) $\forall y \forall z\ (p(b, e, y) \land p(e, j, z) \Rightarrow p(b, j, y * z))$ — UI 12
14) $\forall z\ (p(b, e, 0.5) \land p(e, j, z) \Rightarrow p(b, j, 0.5 * z))$ — UI 13
15) $p(b, e, 0.5) \land p(e, j, 0.1) \Rightarrow p(b, j, 0.5 * 0.1)$ — UI 14
16) $p(b, e, 0.5) \land p(e, j, 0.1)$ — $\land$ introduction, 4-9

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17)	$p(b, j, 0.5 * 0.1)$	MP 15-16
18)	$\forall w \forall x \forall y \forall z (p(a, w, y) \wedge p(w, x, z) \Rightarrow p(a, x, y * z))$	UI 10
19)	$\forall x \forall y \forall z (p(a, b, y) \wedge p(b, x, z) \Rightarrow p(a, x, y * z))$	UI 18
20)	$\forall y \forall z (p(a, b, y) \wedge p(b, j, z) \Rightarrow p(a, j, y * z))$	UI 19
21)	$\forall z (p(a, b, 0.6) \wedge p(b, j, z) \Rightarrow p(a, j, 0.6 * z))$	UI 20
22)	$p(a, b, 0.6) \wedge p(b, j, 0.5 * 0.1) \Rightarrow p(a, j, 0.6 * 0.5 * 0.1)$	UI 21
23)	$p(a, b, 0.6) \wedge p(b, j, 0.5 * 0.1)$	$\wedge$ introduction, 1-17
24)	$\boxed{p(a, j, 0.6 * 0.5 * 0.1)}$	MP 22-23

We have found the probability from a to j.

Hence, the result is equal to $\underline{0.6 \times 0.5 \times 0.1} = \boxed{0.03}$

* The result is a byproduct of the proof $\frac{3}{10}$ as it can be seen above.