Mask Iterative Hard Thresholding Algorithms for Sparse Image Reconstruction of Objects with Known Contour[†]

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Outline

- Background.
- Signal Model.
- Reconstruction Algorithms for Known Object Contour
  - Mask iterative hard thresholding (mask IHT) and its double overrelaxation (mask DORE) acceleration,
  - Convex Relaxation: Mask fixed-point continuation active set (mask FPC_{AS}) and gradient-projection for sparse reconstruction with debiasing (mask GPSR).
- Simulated X-ray CT Reconstruction Examples.

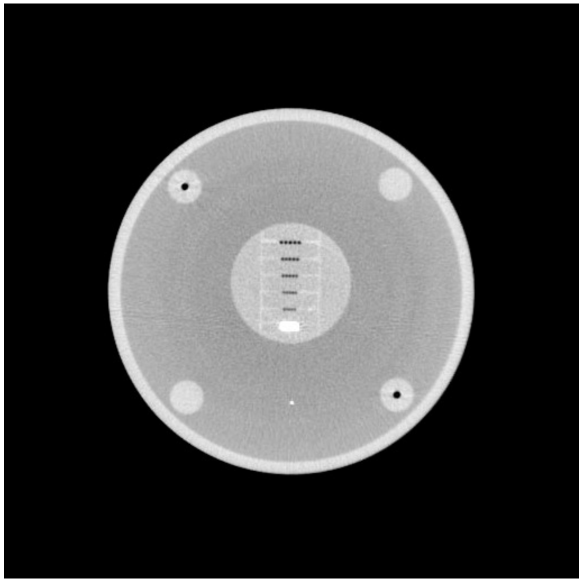
Terminology and Notation

- DSFT and FFT stand for discrete-space and fast Fourier transforms;
- $\| \cdot \|_{\ell_p}$ and "T" are the ℓ_p norm and transpose;
- I_m and $\mathbf{0}_{m \times 1}$ are the identity matrix of size m and $m \times 1$ vector of zeros;
- the thresholding operator $\mathcal{T}_r(\boldsymbol{x})$ keeps the r largest-magnitude elements of a vector $\boldsymbol{x}$ intact and sets the rest to zero, e.g.

$$\mathcal{T}_2([0, 1, -5, 0, 3, 0]^T) = [0, 0, -5, 0, 3, 0]^T;$$

- ρ_H is the spectral norm (i.e. the largest singular value) of a matrix H.

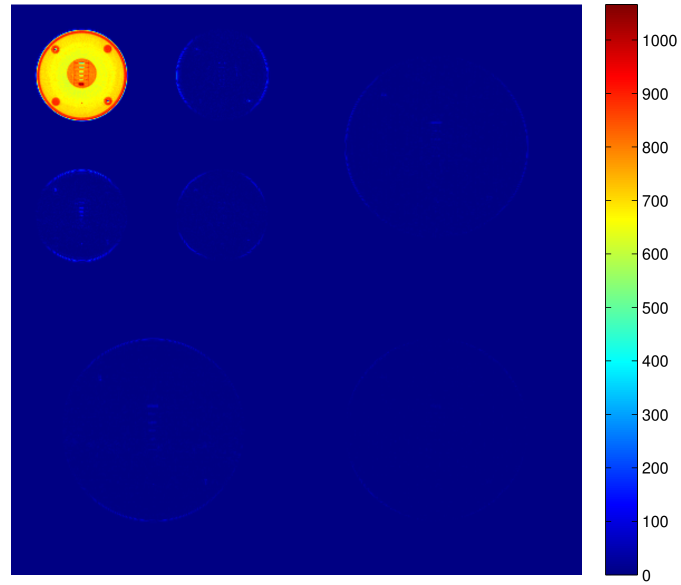
Signal Sparsity



transform

$\longleftrightarrow$

DWT

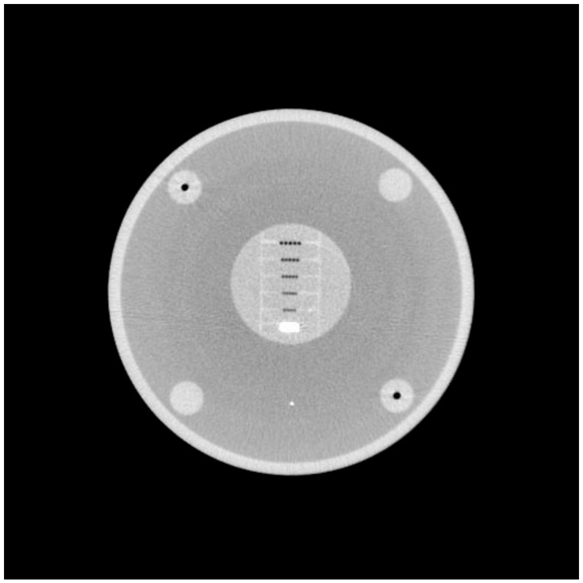


p pixels

significant DWT coeffs $\ll p$

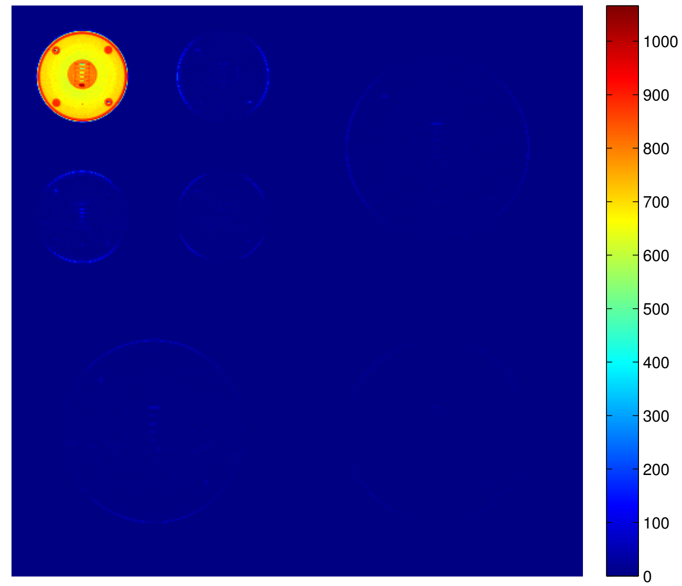
$$\underbrace{\boldsymbol{x}}_{\text{signal}} = \underbrace{\Psi}_{\substack{\text{sparsifying} \\ \text{transform matrix}}} \underbrace{\boldsymbol{s}}_{\substack{\text{sparse signal} \\ \text{transform-coefficient vector}}}$$

Signal Sparsity



transform

$\longleftrightarrow$



p pixels

DWT

$\#$ significant DWT coeffs $\ll p$

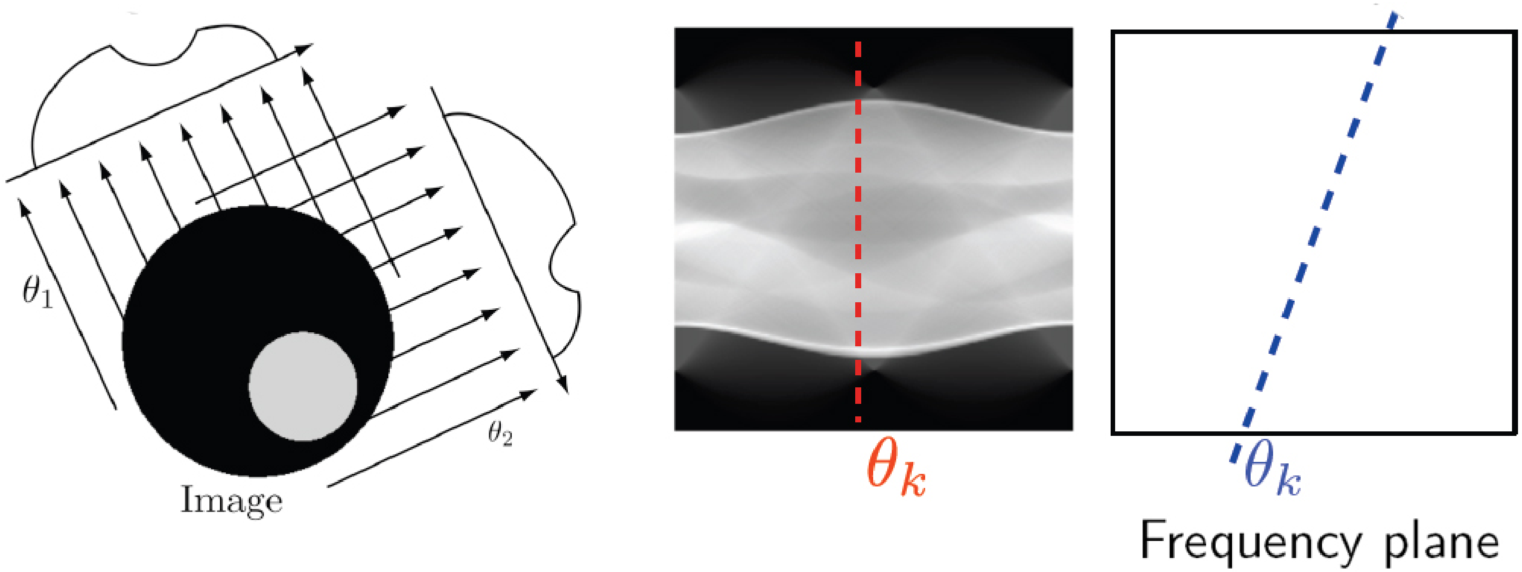
In this example, Ψ is an inverse DWT matrix.

Compressive Sampling (CS)

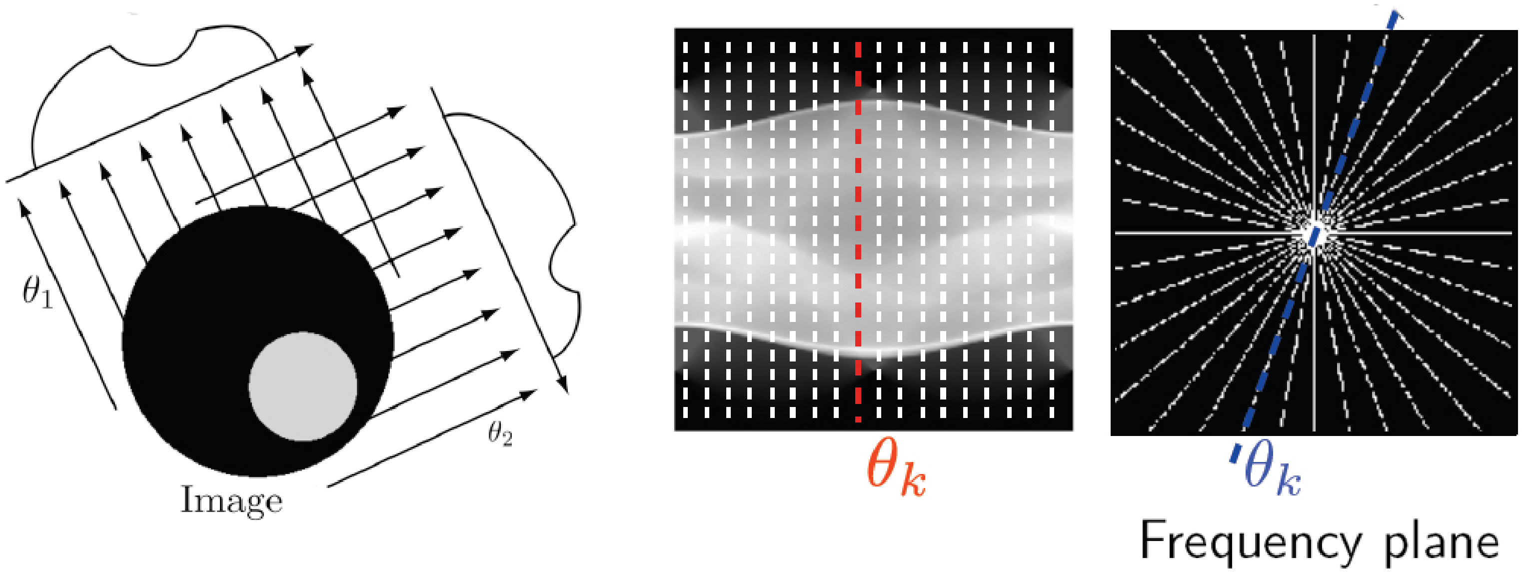
The idea behind compressive sampling is to *sense* the significant signal transform coefficients using a small number of linear measurements:

$$\underbrace{\begin{bmatrix} \boldsymbol{y} \end{bmatrix}_{N\times 1}}_{\text{measurements}} = \underbrace{\begin{bmatrix} \quad & \Phi & \quad \end{bmatrix}_{N\times p}}_{\text{sampling matrix}} \underbrace{\begin{bmatrix} \\ \boldsymbol{x} \\ \\ \end{bmatrix}_{p\times 1}}_{\text{signal vector}} = \Phi\, \Psi\, \boldsymbol{s}.$$

CS Applies Naturally to X-ray Tomography



Fourier slice theorem:
$\mathcal{FT}\{\text{projection at angle } \theta_k\}(\rho) = 2\text{–}\mathcal{FT}\{\text{image}\}(\rho\cos\theta_k, \rho\sin\theta_k)$



Tomographic X-ray projections at discrete angles lead to partial Fourier measurements in the DSFT domain:

$$\underbrace{\boldsymbol{y}}_{\text{2D-DSFT samples}} = \underbrace{\boldsymbol{\Phi}}_{\text{2D-DSFT sampling matrix}} \underbrace{\boldsymbol{x}}_{\text{image}} .$$

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Signal Model

The elements of the $p \times 1$ signal vector $\boldsymbol{x} = [x_1, x_2, \ldots, x_p]^T$ are

$$x_i = \begin{cases} [\Psi\, \boldsymbol{s}]_i, & i \in \mathrm{M} \\ 0, & i \notin \mathrm{M} \end{cases}, \quad i = 1, 2, \ldots, p \tag{1}$$

where $[\Psi\, \boldsymbol{s}]_i$ denotes the ith element of the vector $\Psi\, \boldsymbol{s}$,

- the mask M is the set of $p_{\mathrm{M}} \leqslant p$ indices corresponding to the signal elements inside the contour of the inspected object,

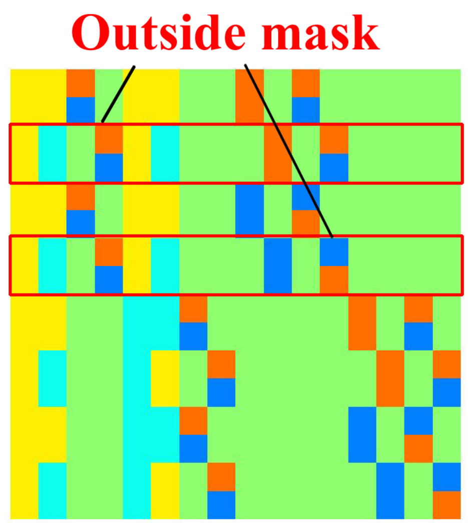
- $\boldsymbol{s}$ is the $p \times 1$ sparse signal transform-coefficient vector, and

- Ψ is the known orthogonal sparsifying transform matrix satisfying

$$\Psi\, \Psi^T = \Psi^T\, \Psi = I_p.$$

Sparsifying Transform

$$\begin{bmatrix} \\ \boldsymbol{x} \\ \\ \end{bmatrix}_{p\times 1} =$$

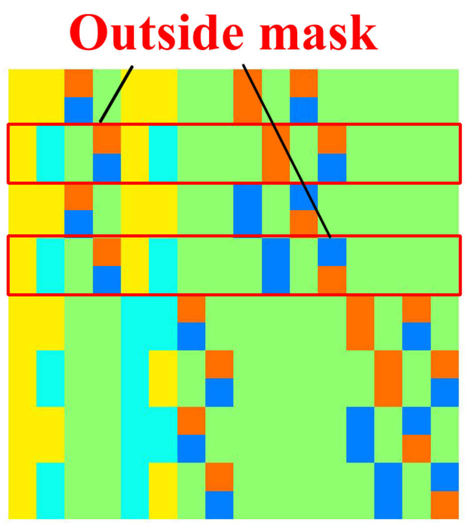


$$\begin{bmatrix} \\ \boldsymbol{s} \\ \\ \end{bmatrix}_{p\times 1}$$

Ψ of size $p \times p$

Sparsifying Transform

$$\begin{bmatrix} \\ \boldsymbol{x} \\ \\ \end{bmatrix}_{p\times 1} =$$



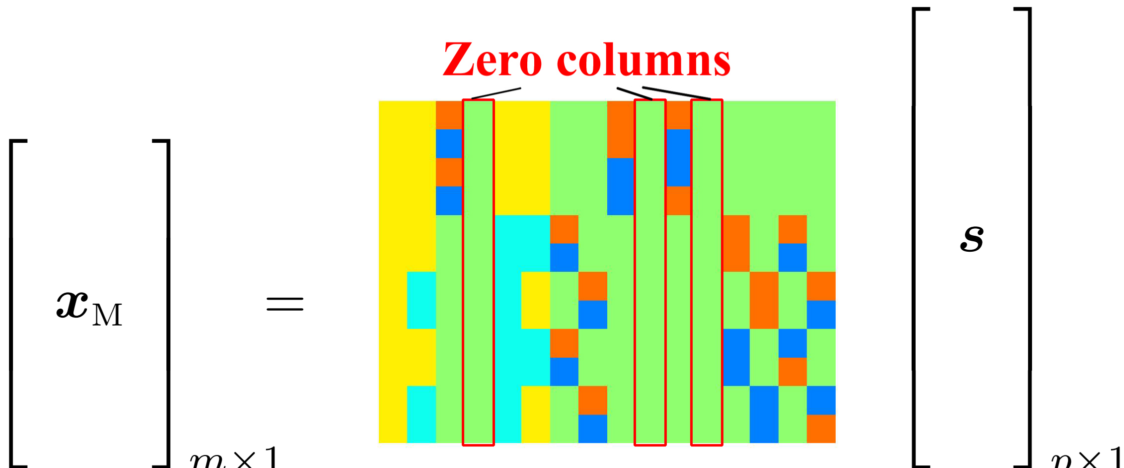
$$\begin{bmatrix} \\ \boldsymbol{s} \\ \\ \end{bmatrix}_{p\times 1}$$

Ψ of size $p \times p$

Remove the rows of Ψ that correspond to the signal indices outside the mask M. Construct the $p_{\mathrm{M}} \times p$ matrix $\Psi_{\mathrm{M},:}$ containing the p_{M} rows of Ψ that correspond to the signal indices within the mask M.

Our Signal Model

The $p_{\mathrm{M}} \times 1$ vector of signal elements *inside the mask* M ($x_i,\ i \in \mathrm{M}$) is



$\Psi_{\mathrm{M},:}$ of size $p_{\mathrm{M}} \times p$

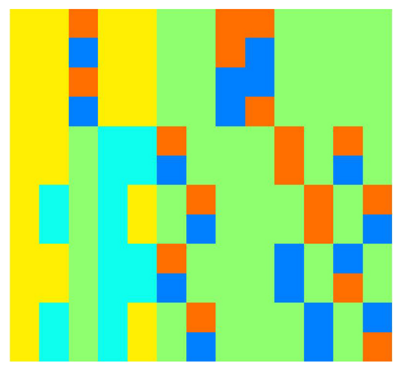
i.e.

$$\boldsymbol{x}_{\mathrm{M}} = \Psi_{\mathrm{M},:}\,\boldsymbol{s}.$$

Our Signal Model (cont.)

Define the set of indices I of nonzero columns of $\varPsi_{\mathrm{M},:}$ containing $p_{\mathrm{I}} \leqslant p$ elements and the corresponding $p_{\mathrm{I}} \times 1$ vector $\boldsymbol{s}_{\mathrm{I}}$ of *identifiable signal transform coefficients* under our signal model. Keep only the identifiable signal transform coefficients:

$$\Big[\ \boldsymbol{x}_{\mathrm{M}}\ \Big]_{m\times 1} =$$

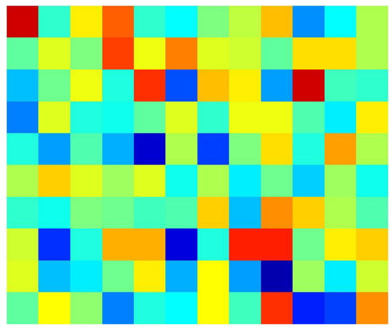


$\varPsi_{\mathrm{M},\mathrm{I}}$ of size $p_{\mathrm{M}} \times p_{\mathrm{I}}$

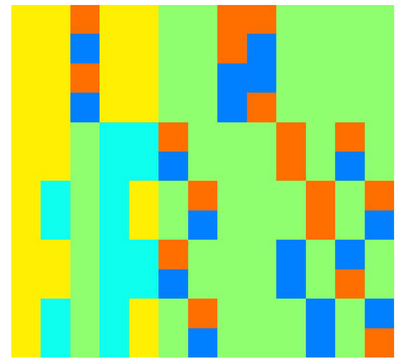
$$\Big[\ \boldsymbol{s}_{\mathrm{I}}\ \Big]_{p_{\mathrm{I}}\times 1}.$$

Our Measurement Model

$$\left[\; \boldsymbol{y} \;\right]_{N\times 1} =$$



$\Phi_{:,\mathrm{M}}$ of size $N \times p_{\mathrm{M}}$



$\Psi_{\mathrm{M},\mathrm{I}}$ of size $p_{\mathrm{M}} \times p_{\mathrm{I}}$

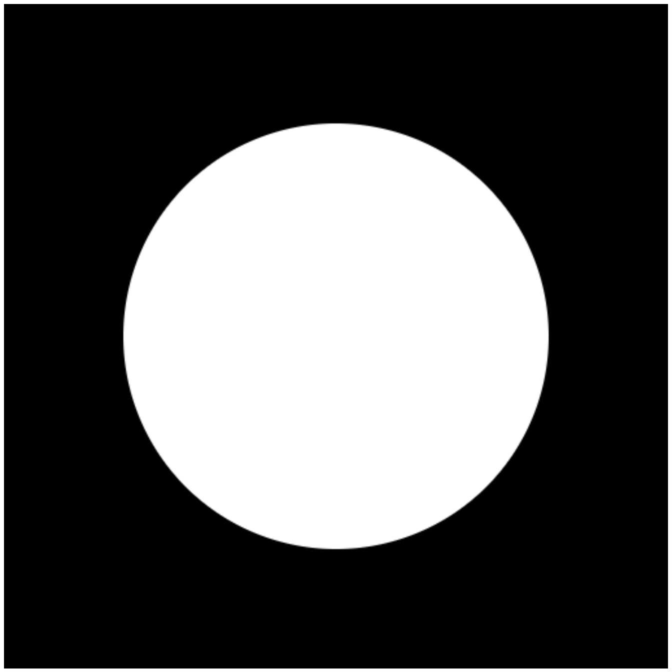
$$\left[\; \boldsymbol{s}_{\mathrm{I}} \;\right]_{p_{\mathrm{I}}\times 1}$$

i.e.

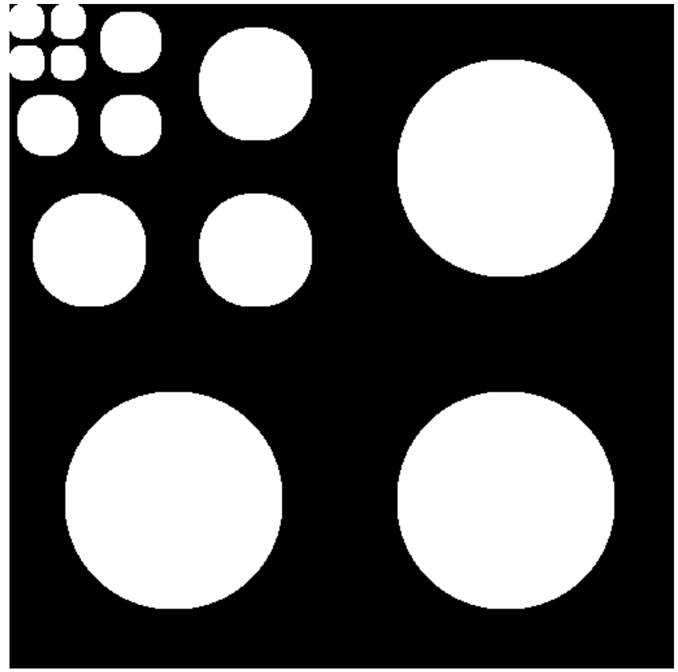
$$\boldsymbol{y} = H\, \boldsymbol{s}_{\mathrm{I}}$$

where $H \triangleq \Phi_{:,\mathrm{M}}\, \Psi_{\mathrm{M},\mathrm{I}}$ is the $N \times p_{\mathrm{I}}$ *sensing matrix*. Here, $\Phi_{:,\mathrm{M}}$ is the *restriction* of the full sampling matrix Φ to the mask index set M consisting of the p_{M} columns of the full sampling matrix Φ that correspond to the signal indices within M.

A Mask and Its $\mathbb{I}$ for Ψ Inverse DWT



(a) A mask



(b) $\mathbb{I}$ for the mask in (a)

Figure 1: The white pixels in (b) correspond to the *identifiable* signal transform coefficients.

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Incorporating Both the Geometric and Signal Sparsity Constraints

We wish to solve the following constrained residual squared error minimization problem:

$$\min_{\boldsymbol{s}} \|\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}\|_{\ell_2}^2 \quad \text{subject to } \|\boldsymbol{s}_{\mathrm{I}}\|_{\ell_0}^2 \leqslant r \tag{2}$$

where $\|\boldsymbol{s}_{\mathrm{I}}\|_{\ell_0}$ counts the number of nonzero elements in the vector $\boldsymbol{s}_{\mathrm{I}}$ and $H = \mathit{\Phi}_{:,\mathrm{M}}\,\mathit{\Psi}_{\mathrm{M},\mathrm{I}}$. We refer to r as the *signal sparsity level* and assume that it is *known*.

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where $\|\boldsymbol{s}_{\mathrm{I}}\|_{\ell_0}$ counts the number of nonzero elements in the vector $\boldsymbol{s}_{\mathrm{I}}$ and $H = \Phi_{:,\mathrm{M}}\,\Psi_{\mathrm{M},\mathrm{I}}$. We refer to r as the *signal sparsity level* and assume that it is *known*.

Finding the exact solution to (2) is intractable in practice.

Tractable Alternatives

Greedy: Develop iterations that *aim at* solving (2) approximately, i.e. decrease the residual squared error in each iteration step while maintaining the specified signal sparsity level.

Convex Relaxation: Substitute ℓ_0 with ℓ_1 in (2) and construct a Lagrange-multiplier formulation:

$$\min_{\boldsymbol{s}}\left(\frac{1}{2}\,\|\boldsymbol{y} - H\,\boldsymbol{s}\|_2^2 + \tau\,\|\boldsymbol{s}\|_1\right) \tag{3}$$

where τ is the regularization parameter that controls the signal sparsity. Note that (3) can be solved in polynomial time.

Mask Iterative Hard Thresholding (Mask IHT)

Assume that the signal estimate $\boldsymbol{s}_{\mathrm{I}}^{(q)}$ is available, where q denotes the iteration index. *Iteration* $(q+1)$ proceeds as follows:

$$\boldsymbol{s}_{\mathrm{I}}^{(q+1)} = \mathcal{T}_r\big(\boldsymbol{s}_{\mathrm{I}}^{(q)} + \mu^{(q)}\, H^T\,(\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q)})\big) \tag{4}$$

where $\mu^{(q)} > 0$ is the appropriately chosen step size. Iterate until $\boldsymbol{s}_{\mathrm{I}}^{(q+1)}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q)}$ do not differ significantly. Upon convergence yielding $\boldsymbol{s}_{\mathrm{I}}^{(+\infty)}$, construct an estimate of the signal vector $\boldsymbol{x}_{\mathrm{M}}$ inside the mask M using $\Psi_{\mathrm{M,I}}\,\boldsymbol{s}_{\mathrm{I}}^{(+\infty)}$.

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For the full mask $\mathrm{M} = \{1, 2, \ldots, m\}$ and constant step size (i.e. $\mu^{(q)} = \mu$ not a function of q), (4) reduces to the standard IHT algorithm in [1].

[1] T. Blumensath and M. E. Davies, "Iterative hard thresholding for compressed sensing," *Appl. Comput. Harmon. Anal.*, vol. 27, no. 3, pp. 265–274, 2009.

DORE Acceleration (Mask DORE)

Assume that two consecutive estimates $\boldsymbol{s}_{\mathrm{I}}^{(q-1)}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q)}$ of identifiable signal coefficients are available from the $(q-1)$-th and q-th iterations, respectively.

Idea: Apply *two consecutive overrelaxation steps* after one mask IHT step to accelerate the convergence of the mask IHT algorithm. These two overrelaxations use the sparse signal estimates $\boldsymbol{s}_{\mathrm{I}}^{(q)}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q-1)}$ from the two most recently completed mask DORE iterations.

Mask DORE Iteration $q+1$

1. Mask IHT step.

$$\widehat{\boldsymbol{s}}_{\mathrm{I}} = \mathcal{T}_r\big(\boldsymbol{s}_{\mathrm{I}}^{(q)} + \mu^{(q)}\, H^T\, (\boldsymbol{y} - H\, \boldsymbol{s}_{\mathrm{I}}^{(q)})\big). \tag{5a}$$

2. First overrelaxation. Minimize the residual squared error $\|\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}\|_{\ell_2}^2$ with respect to $\boldsymbol{s}_{\mathrm{I}}$ lying on the straight line connecting $\widehat{\boldsymbol{s}}_{\mathrm{I}}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q)}$:

$$\bar{\boldsymbol{z}}_{\mathrm{I}} = \widehat{\boldsymbol{s}}_{\mathrm{I}} + \alpha_1\, (\widehat{\boldsymbol{s}}_{\mathrm{I}} - \boldsymbol{s}_{\mathrm{I}}^{(q)}) \tag{5b}$$

which has a *closed-form* solution:

$$\alpha_1 = \frac{(H\,\widehat{\boldsymbol{s}}_{\mathrm{I}} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q)})^T\,(\boldsymbol{y} - H\,\widehat{\boldsymbol{s}}_{\mathrm{I}})}{\|H\,\widehat{\boldsymbol{s}}_{\mathrm{I}} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q)}\|_{\ell_2}^2}.$$

3. Second overrelaxation. Minimize the residual squared error $\|\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}\|_{\ell_2}^2$ with respect to $\boldsymbol{s}_{\mathrm{I}}$ lying on the straight line connecting $\bar{\boldsymbol{z}}_{\mathrm{I}}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q-1)}$:

$$\widetilde{\boldsymbol{z}}_{\mathrm{I}} = \bar{\boldsymbol{z}}_{\mathrm{I}} + \alpha_2\,(\bar{\boldsymbol{z}}_{\mathrm{I}} - \boldsymbol{s}_{\mathrm{I}}^{(q-1)}) \tag{5c}$$

which has a closed-form solution as well:

$$\alpha_2 = \frac{(H\,\bar{\boldsymbol{z}}_{\mathrm{I}} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q-1)})^T\,(\boldsymbol{y} - H\,\bar{\boldsymbol{z}}_{\mathrm{I}})}{\|H\,\bar{\boldsymbol{z}}_{\mathrm{I}} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q-1)}\|_{\ell_2}^2}.$$

4. Thresholding. Threshold $\widetilde{\boldsymbol{z}}_{\mathrm{I}}$ to the sparsity level r: $\widetilde{\boldsymbol{s}}_{\mathrm{I}} = \mathcal{T}_r(\widetilde{\boldsymbol{z}}_{\mathrm{I}})$.

5. Decision. If $\|\boldsymbol{y} - H\,\widetilde{\boldsymbol{s}}_{\mathrm{I}}\|_{\ell_2}^2 < \|\boldsymbol{y} - H\,\widehat{\boldsymbol{s}}_{\mathrm{I}}\|_{\ell_2}^2$, assign $\boldsymbol{s}_{\mathrm{I}}^{(q+1)} = \widetilde{\boldsymbol{s}}_{\mathrm{I}}$; otherwise, assign $\boldsymbol{s}_{\mathrm{I}}^{(q+1)} = \widehat{\boldsymbol{s}}_{\mathrm{I}}$ and complete *Iteration* $q+1$.

Iterate until $\boldsymbol{s}_{\mathrm{I}}^{(q+1)}$ and $\boldsymbol{s}_{\mathrm{I}}^{(q)}$ do not differ significantly. Upon convergence yielding $\boldsymbol{s}_{\mathrm{I}}^{(+\infty)}$, construct an estimate of the signal vector $\boldsymbol{x}_{\mathrm{M}}$ inside the mask M using $\Psi_{\mathrm{M,I}}\,\boldsymbol{s}_{\mathrm{I}}^{(+\infty)}$.

Our Adaptive Step Size Selection: Basic Idea

In *Iteration* $q+1$ of our mask DORE IHT scheme, approximately find the largest step size $\mu^{(q)}$ that satisfies

$$\|\boldsymbol{y} - H\,\widehat{\boldsymbol{s}}_{\mathrm{I}}\|_{\ell_2}^2 \leqslant \|\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q)}\|_{\ell_2}^2 \tag{6}$$

where $\widehat{\boldsymbol{s}}_{\mathrm{I}}$ is computed using (5a):

$$\widehat{\boldsymbol{s}}_{\mathrm{I}} = \mathcal{T}_r\big(\boldsymbol{s}_{\mathrm{I}}^{(q)} + \mu^{(q)}\,H^T\,(\boldsymbol{y} - H\,\boldsymbol{s}_{\mathrm{I}}^{(q)})\big).$$

Note: $\mu^{(q)}$ exists and is lower bounded by $1/\rho_H^2$.

Convex Relaxation

We solve the convex problem in (3)

$$\min_{\boldsymbol{s}}\left(\frac{1}{2}\left\|\boldsymbol{y}-H\,\boldsymbol{s}_{\mathrm{I}}\right\|_{\ell_2}^2+\tau\left\|\boldsymbol{s}_{\mathrm{I}}\right\|_{\ell_1}\right)$$

using the fixed-point continuation active set (mask $\mathrm{FPC}_{\mathrm{AS}}$) method and gradient-projection for sparse reconstruction with debiasing[1] method (mask GPSR) in [2] and [3].

---

[1] Here, debiasing is performed if possible.

[2] Z. Wen, W. Yin, D. Goldfarb, and Y. Zhang, "A fast algorithm for sparse reconstruction based on shrinkage, subspace optimization, and continuation," *SIAM J. Sci. Comput.*, vol. 32, no. 4, pp. 1832–1857, 2010.

[3] M. A. T. Figueiredo, R. D. Nowak, and S. J. Wright, "Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems," *IEEE J. Select. Areas Signal Processing*, vol. 1, no. 4, pp. 586–597, 2007.

Full Mask

Full mask $\mathrm{M} = \{1, 2, \ldots, p\}$ means *unknown* object contour; then

- mask FPC_{AS} and mask GPSR reduce to the FPC_{AS} and GPSR methods for compressive sampling in e.g. [2] and [3];

- mask DORE reduces to a double overrelaxation iterative hard thresholding method with an adaptive step size, which we refer to as DORE.

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Initialization and Performance Metric

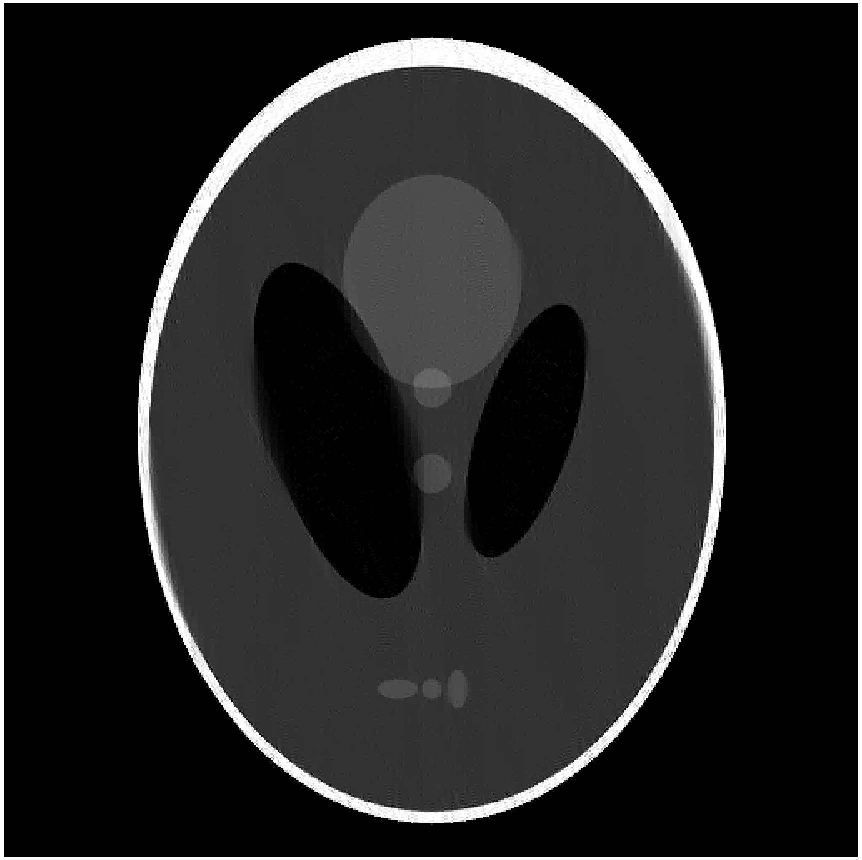
We initialize all (mask and traditional) sparse signal reconstruction methods using the standard *filtered backprojection* (FBP) reconstruction, which ignores both the signal sparsity and geometric object contour information.

Our performance metric is the peak signal-to-noise ratio (PSNR) of a reconstructed image $\widehat{\boldsymbol{x}} = [\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_p]^T$ inside the mask M:

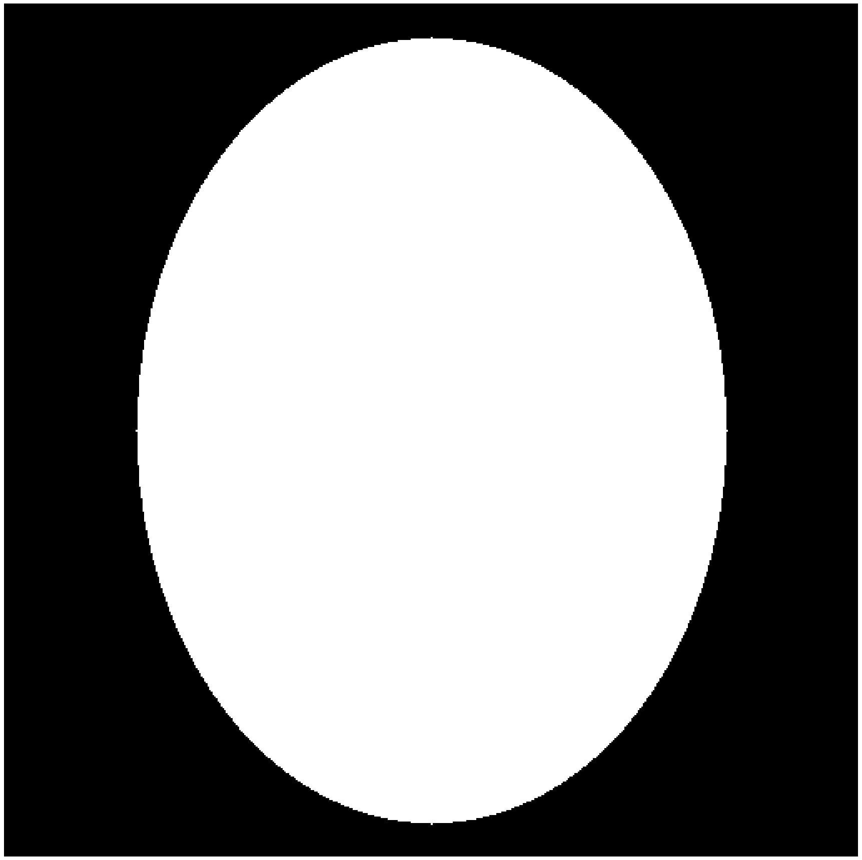
$$\text{PSNR (dB)} = 10 \log_{10} \left\{ \frac{[(\max_{i \in \mathrm{M}} x_i) - (\min_{i \in \mathrm{M}} x_i)]^2}{\sum_{i \in \mathrm{M}} (\widehat{x}_i - x_i)^2 / p_{\mathrm{M}}} \right\}$$

where $\boldsymbol{x}$ is the true image.

Shepp-Logan Phantom Reconstruction



(a) Sampled Shepp-Logan phantom of size 512^2.



(b) Mask.

Shepp-Logan Phantom Reconstruction (cont.)

- We take limited-angle projections with 1° spacing between projections. Each projection of an analog Shepp-Logan phantom is computed using its analytical sinogram and then sampled by a receiver array containing 511 elements.

- We compute FFT of each projection, yielding $N = 512$ frequency-domain measurements.

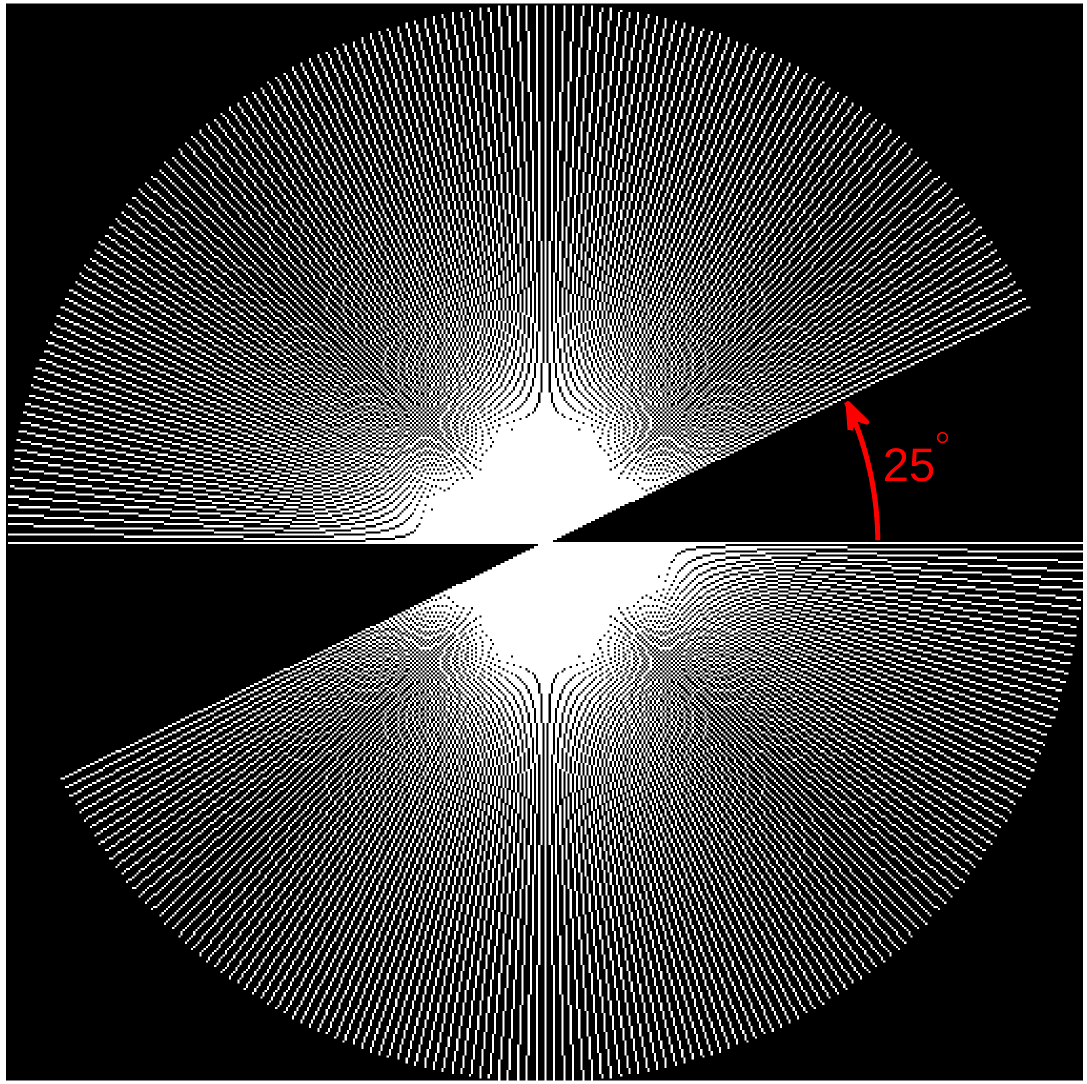


Figure 2: Limited-angle sampling pattern in the frequency domain.

Shepp-Logan Phantom Reconstruction (cont.)

- The $N \times p$ *sampling matrix* $\mathit{\Phi}$ is constructed using *2D-DSFT matrix* efficiently implemented by nonuniform fast Fourier transform (NUFFT) [4].

- We select the inverse Haar (Daubechies-2) DWT matrix to be the orthogonal sparsifying transform matrix $\mathit{\Psi}$; the true signal vector $\boldsymbol{s}$ consists of the Haar wavelet transform coefficients of the phantom and is sparse:

$$\|\boldsymbol{s}\|_{\ell_0} = 7866 \approx 0.00382\, p.$$

[4] J. Fessler and B. Sutton, "Nonuniform fast Fourier transforms using min-max interpolation," *IEEE Trans. Signal Process.*, vol. 51, no. 2, pp. 560–574, Feb. 2003.

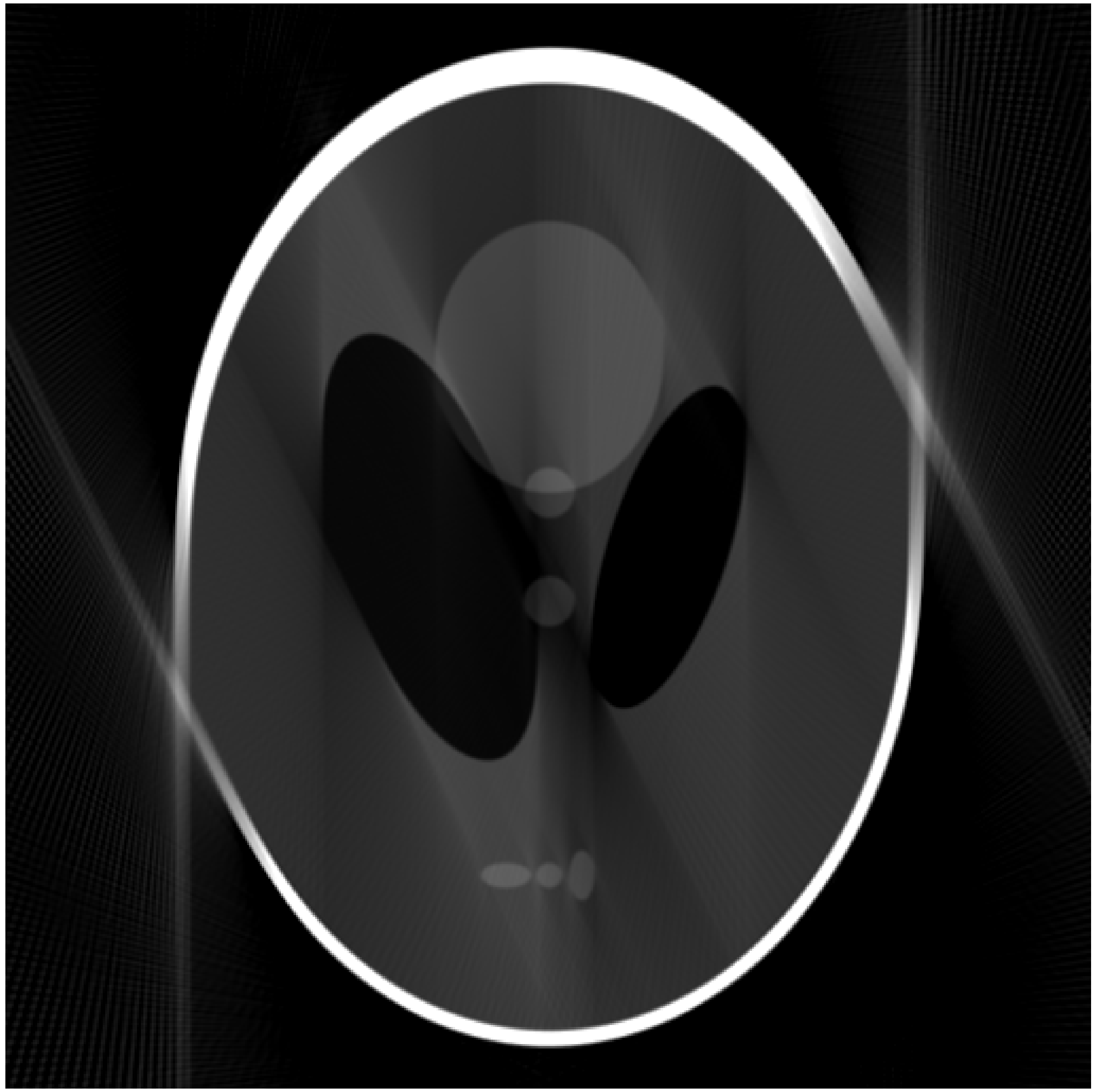
Methods Compared

- the mask DORE and DORE schemes with signal sparsity level r set to $r = 7000$ and $r = 8000$, respectively, where r are tuned for good PSNR performance;

- the mask GPSR ($a = -5$) and GPSR ($a = -5$) schemes, with the regularization parameter

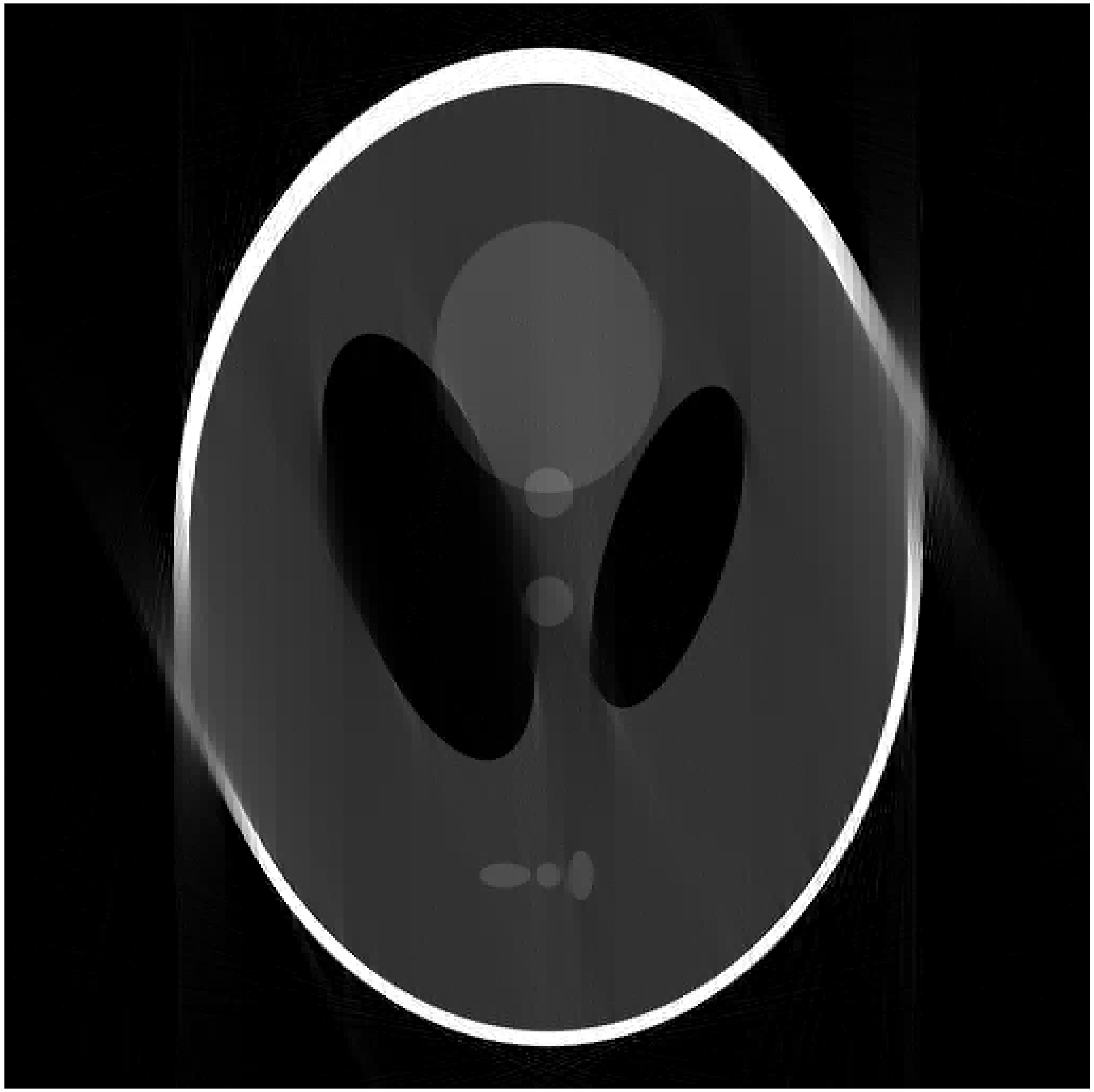
$$\tau = 10^a \, \|H^T \boldsymbol{y}\|_{\ell_\infty}$$

where a are tuned for good PSNR performance.



FBP reconstruction (PSNR $= 19.9$ dB).

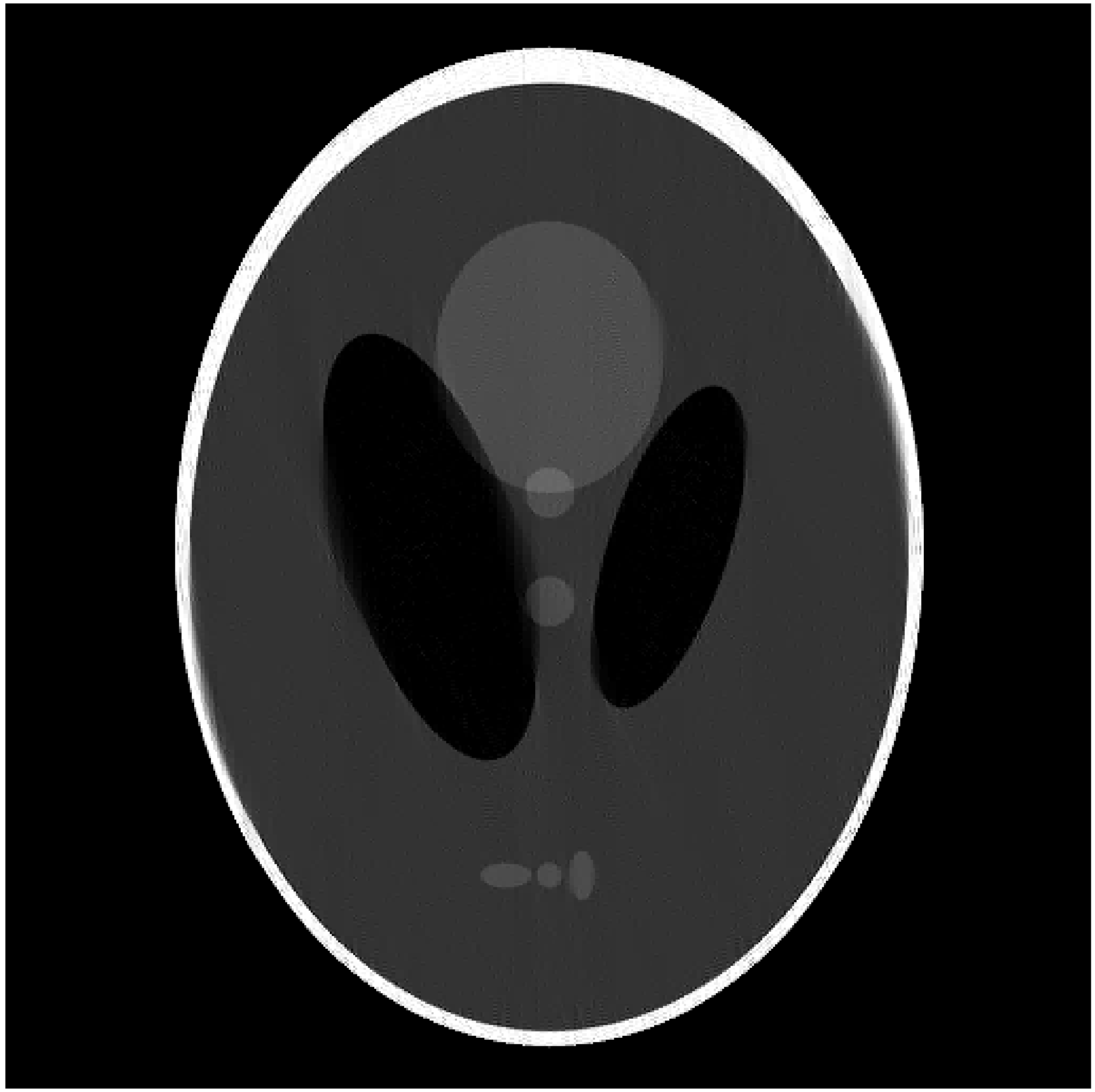
DORE reconstruction (PSNR $= 22.7$ dB).



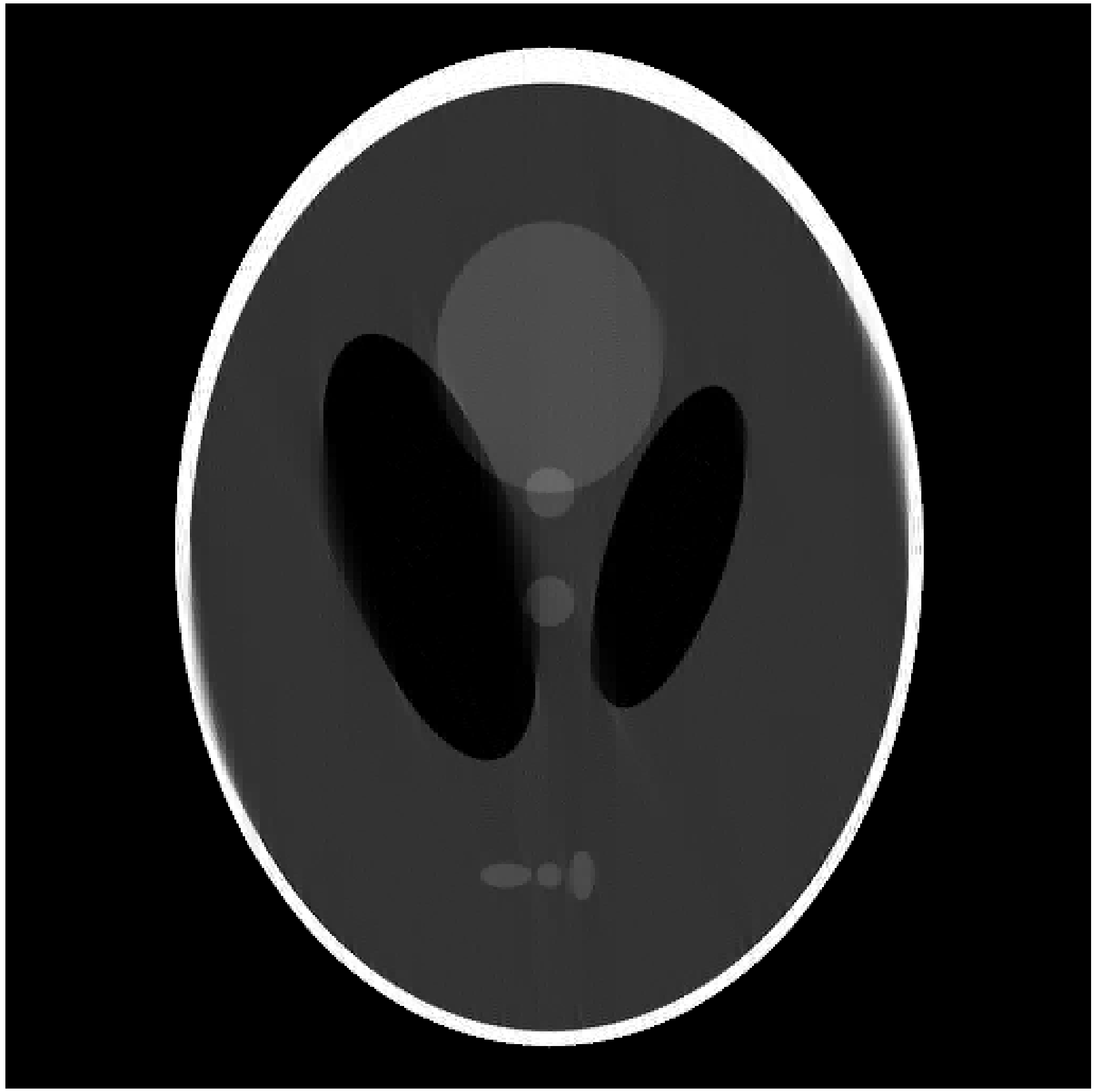
GPSR reconstruction (PSNR $= 22.9$ dB).

$\mathsf{FPC_{AS}}$ reconstruction (PSNR $= 22.5$ dB).

Mask DORE reconstruction (PSNR $= 25.8$ dB).



Mask GPSR reconstruction (PSNR $= 25.3$ dB).



Mask $\mathsf{FPC}_{\mathsf{AS}}$ reconstruction (PSNR $= 26.4$ dB).

References

[1] T. Blumensath and M. E. Davies, "Iterative hard thresholding for compressed sensing," *Appl. Comput. Harmon. Anal.*, vol. 27, no. 3, pp. 265–274, 2009.

[2] Z. Wen, W. Yin, D. Goldfarb, and Y. Zhang, "A fast algorithm for sparse reconstruction based on shrinkage, subspace optimization, and continuation," *SIAM J. Sci. Comput.*, vol. 32, no. 4, pp. 1832–1857, 2010.

[3] M. A. T. Figueiredo, R. D. Nowak, and S. J. Wright, "Gradient projection for sparse reconstruction: Application to compressed sensing and other inverse problems," *IEEE J. Select. Areas Signal Processing*, vol. 1, no. 4, pp. 586–597, 2007.

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