SENG 331 - Modeling And Simulation with Python

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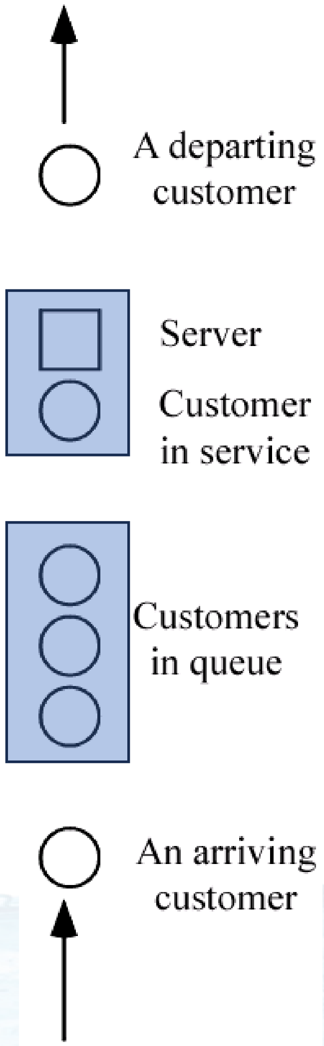
1.3 DISCRETE-EVENT SIMULATION

- *Discrete-event simulation*: Modeling of a system as it evolves over time by a representation where the state variables change instantaneously at separated points in time
  - More precisely, **state** can change at only a ***countable* number** of points in time
  - These points in time are when ***events*** occur
- *Event:* Instantaneous occurrence that may change the state of the system
  - Sometimes get creative about what an "event" is … e.g., end of simulation, make a decision about a system's operation

1.3 Discrete-Event Simulation (cont'd.)

- Example: Single-server queue
  - Estimate expected average delay in queue (line, not service)
  - State variables
    - Status of server (idle, busy) – needed to decide what to do with an arrival
    - Current length of the queue – to know where to store an arrival that must wait in line
    - Time of arrival of each customer now in queue – needed to compute time in queue when service starts
  - Events
    - Arrival of a new customer
    - Service completion (and departure) of a customer
    - Maybe – end-simulation event (a "fake" event) – whether this is an event depends on how simulation terminates (a modeling decision)



1.3.1 Time-Advance Mechanisms

- *Simulation clock*: Variable that keeps the current value of (simulated) time in the model
  - Must decide on, be consistent about, time units
  - Usually no relation between simulated time and (real) time needed to run a model on a computer
- Two approaches for time advance
  - *Next-event time advance* (usually used) … described in detail below
  - *Fixed-increment time advance* (seldom used) … Described in Appendix 1A
    - Generally introduces some amount of modeling error in terms of when events *should* occur vs. *do* occur
    - Forces a tradeoff between model accuracy and computational efficiency

1.3.1 Time-Advance Mechanisms (cont'd.)

- More on next-event time advance
  - Initialize simulation clock to 0
  - Determine times of occurrence of future events – *event list*
  - Clock advances to next (most imminent) event, which is executed
    - Event execution may involve updating event list
  - Continue until stopping rule is satisfied (must be explicitly stated)
  - Clock "jumps" from one event time to the next, and doesn't "exist" for times between successive events … periods of inactivity are ignored

1.3.1 Time-Advance Mechanisms (cont'd.)

- Next-event time advance for the single-server queue

  t_i = time of arrival of i_{th} customer (t_0 = 0)

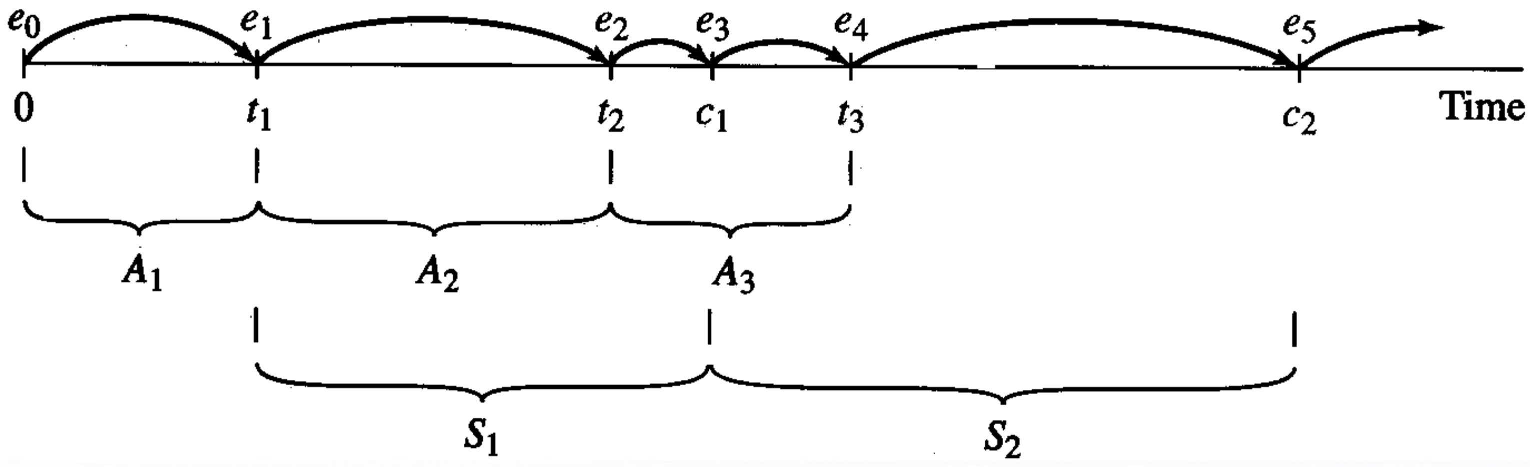
  $A_i = t_i - t_{i-1}$ = interarrival time between (i-1)st and ith customers *(usually assumed to be a random variable from some probability distribution)*

  S_i = service-time requirement of ith customer (another random variable)

  D_i = delay in queue of ith customer

  $C_i = t_i + D_i + S_i$ = time ith customer completes service and departs

  e_j = time of occurrence of the jth event (of any type), j = 1, 2, 3, …



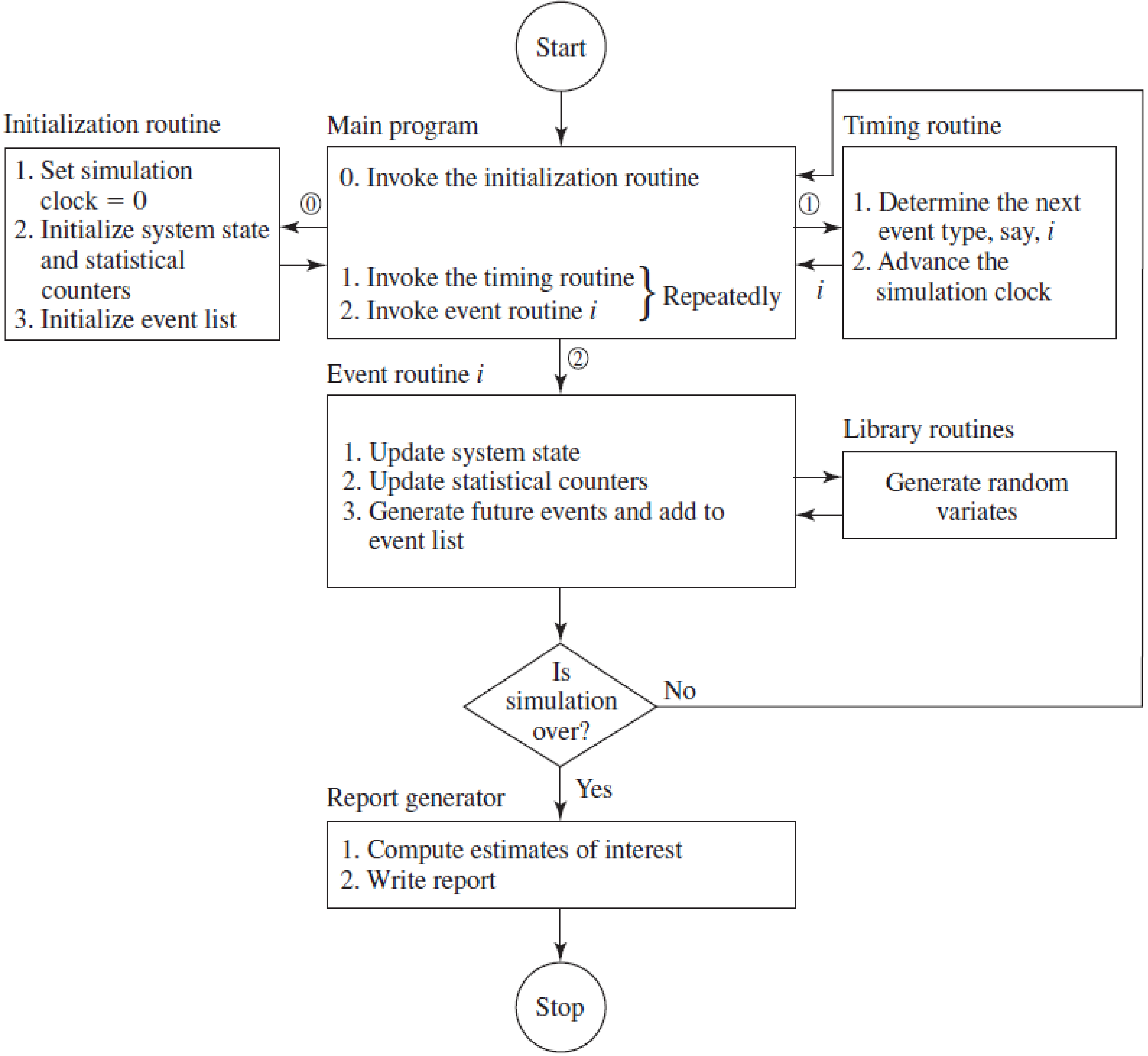
1.3.2 Components and Organization of a Discrete-Event Simulation Model

- Each simulation model must be customized to target system
- But there are several common components, general organization
  - *System state* – variables to describe state
  - *Simulation clock* – current value of simulated time
  - *Event list* – times of future events (as needed)
  - *Statistical counters* – to accumulate quantities for output
  - *Initialization routine* – initialize model at time 0
  - *Timing routine* – determine next event time, type; advance clock
  - *Event routines* – carry out logic for each event type
  - *Library routines* – utility routines to generate random variates, etc.
  - *Report generator* – to summarize, report results at end
  - *Main program* – ties routines together, executes them in right order

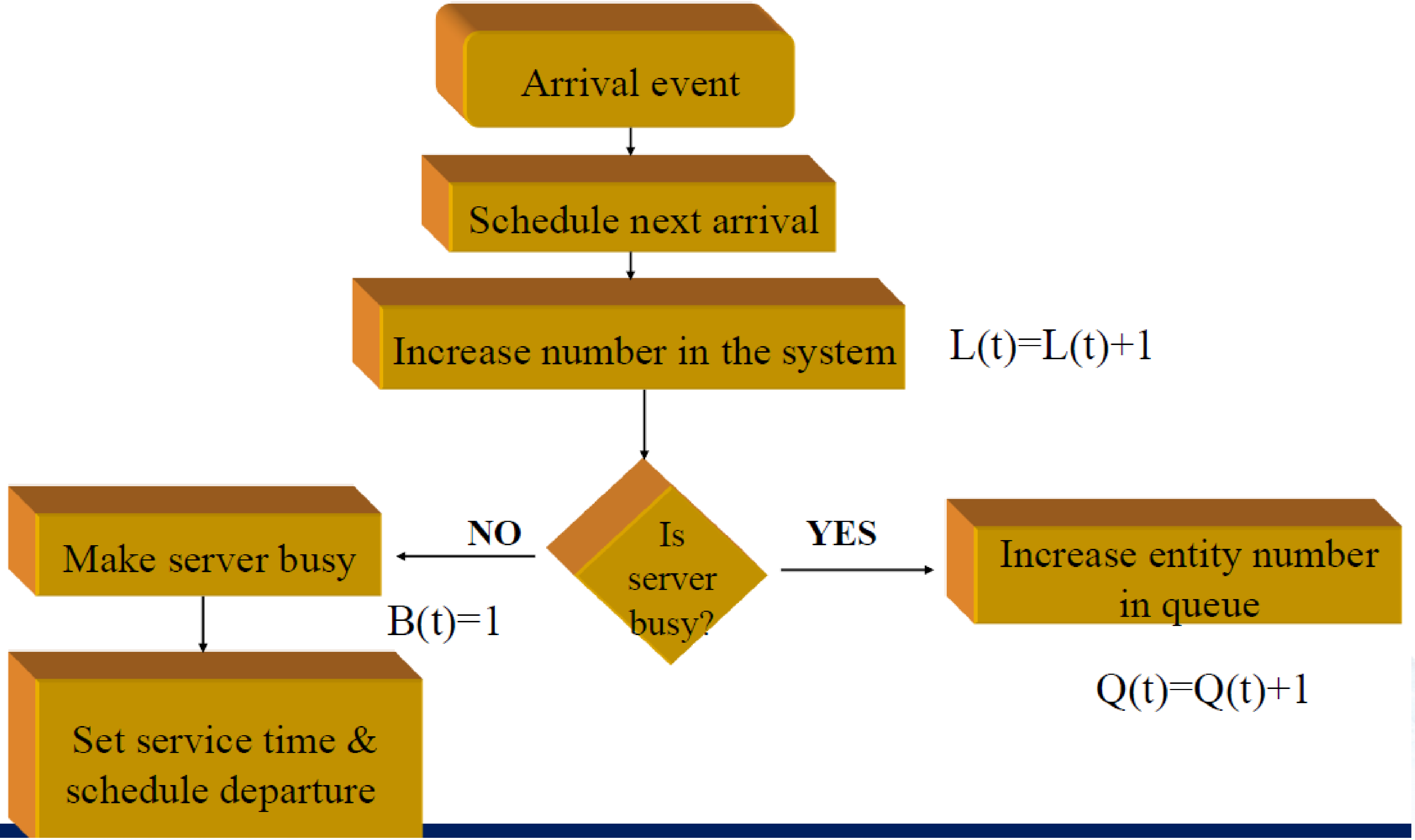
1.3.2 Components and Organization of a Discrete-Event Simulation Model (cont'd.)

- More on entities
  - Objects that compose a simulation model
  - Usually include customers, parts, messages, etc. … may include resources like servers
  - Characterized by data values called *attributes*
  - For each entity resident in the model there's a record (row) in a *list*, with the attributes being the columns
- Approaches to modeling
  - *Event-scheduling* – as described above, coded in general-purpose language
  - *Process* – focuses on entities and their "experience," usually requires special-purpose simulation software

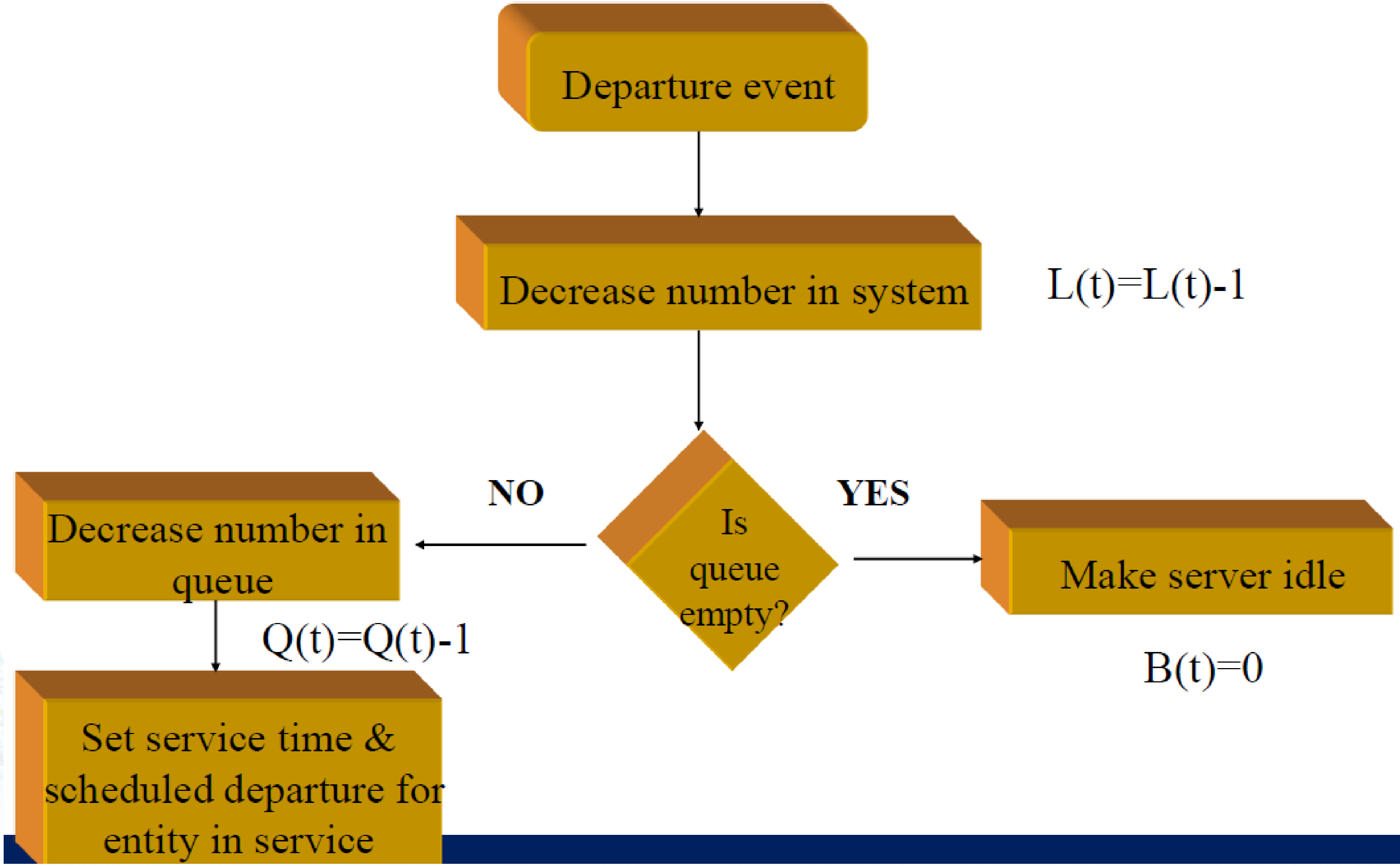
Components and Organization of a Discrete-Event Simulation Model
(cont'd.)



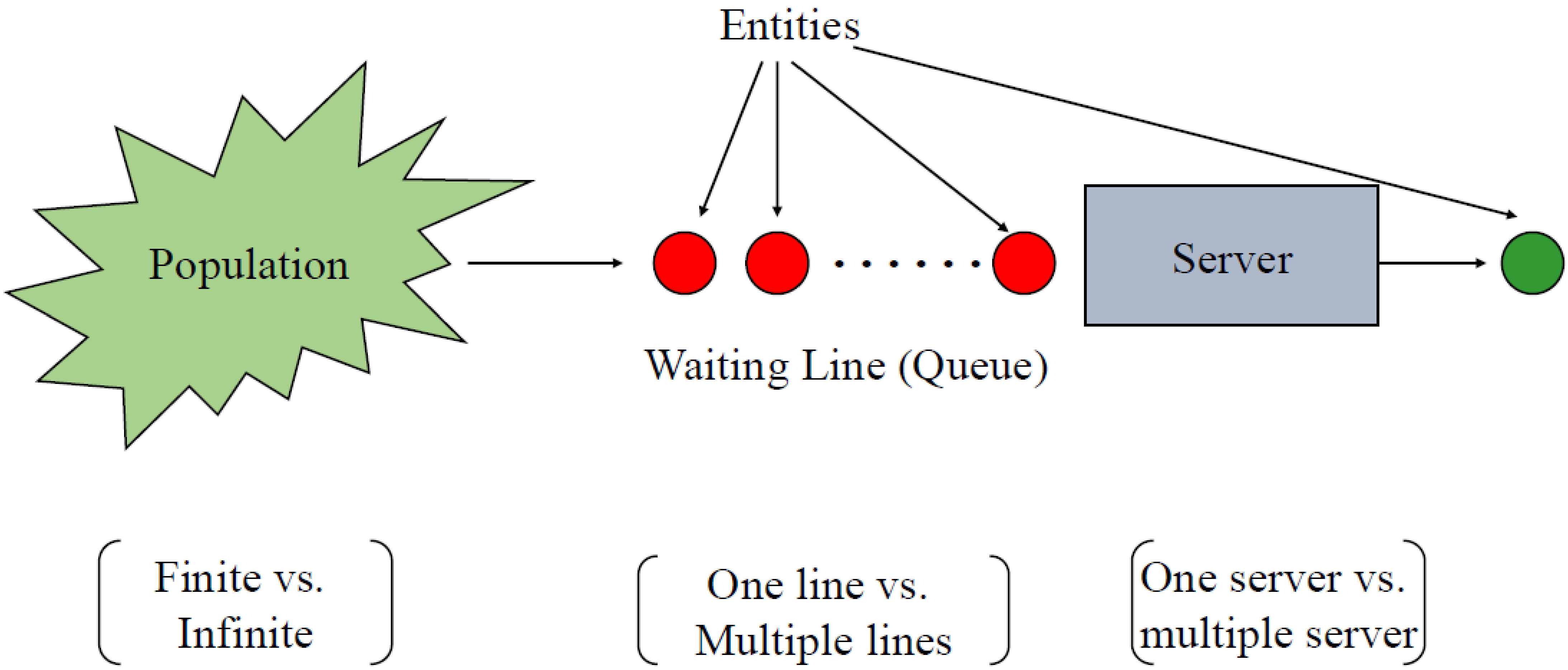
Arrival Event Flow Chart



Departure Event Flow Chart



Example : Queueing Systems



Slides 51 to 75 are adopted from of Bilkent University – IE324 Course Slides

Queueing Systems Characteristics

- Interarrival and Service Times
  - Exponential (M)
  - Deterministic (D)
  - Erlang (E)
  - General (G)
- Queue discipline
  - First Come/In First Served/Out (FCFS/FIFO)
  - Last Come/In First Served/Out (LCFS/LIFO)
  - Earliest Due Date (EDD)
- System Capacity
- Number of Servers

Example Queueing System

- Consider a bank teller queue with one teller (G/G/1) and FIFO queue discipline
  - State Variables: Number of customers in queue
    Busy/Idle Status of Teller (0,1)
  - Activities: Customer Inter Arrival — Customer Service
    Triggers — Triggers
  - Events: Customer Arrival Event — Customer Departure Event
  - Delay: Waiting time in queue

Example – Cont…

- Customer 1 arrives at $t = t_1$ (Note that t_1 *is equal to zero in our case*)
    - If interarrivals are statistical with random variable X_i^A for customer i then the next Customer Arrival Event is scheduled at $t = t_1 + X_2^A$
- Customer 1 enters service at $t = t_2$
    - If service times are statistical with random variable X_i^S for customer i, then the next customer departure event is scheduled at $t = t_2 + X_1^S$

Computation of Statistics

- As a result of a simulation run, we need to obtain statistical information about a performance measure of interest.
  - Average Waiting Time
  - Maximum Waiting Time
  - Average Number of Entities in the System
  - Maximum Number of Entities in the System
  - Server Utilization
  - Average System Time
  - Maximum System Time

- We need to keep track of some more variables to capture the performance measure we wish to investigate.

Computation of Statistics

- P: Total number of customers served so far (# left the system)
- N: Total number of customers passed through the queue

- ΣWQ : Sum of queue times observed so far
- WQ^* : Maximum queue time observed so far

Updated whenever a customer finishes waiting in queue

- ΣTS : Sum of system times observed so far
- TS^*: Max system time observed so far

Updated whenever a customer leaves the system

Computation of Statistics

- $Q(t)$: Function of number of customers waiting in queue (state var.)

  $$Q(t) \in \{0,1,2,\dots\}$$

- $B(t)$: Server busy function (state var.)

  $$B(t) \in \{0,1\}$$

- $L(t) = Q(t) + B(t)$

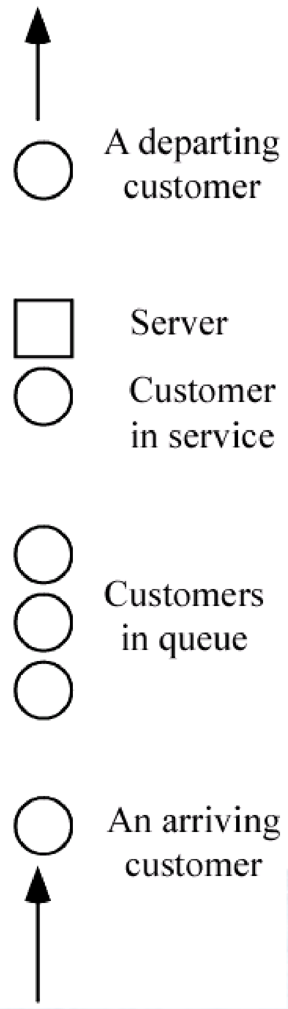
Computation of Statistics

- Using these variables, we can find:
  - Average Waiting Time (In Queue): $\frac{\Sigma WQ}{N}$ (Tally)
  - Average System (Total) Time: $\frac{\Sigma TS}{P}$ (Tally)
  - Average Number of Customers In Queue: $\frac{\int Q(t)}{T_E}$ (Time-persistent)
  - Server Utilization: $\frac{\int B(t)}{T_E}$ (Time-persistent)

- Note that additional variables are needed if further statistical outputs are needed:
  - Maximum idle time, maximum queue length…

1.4.1 Problem Statement

- Recall single-server queueing model
- Assume interarrival times are independent and identically distributed (IID) random variables
- Assume service times are IID, and are independent of interarrival times
- Queue discipline is FIFO
- Start empty and idle at time 0
- First customer arrives after an interarrival time, not at time 0
- Stopping rule: When n_{th} customer has completed delay in queue (i.e., *enters* service) … *n* will be specified as input



1.4.1 Problem Statement (cont'd.)

- Quantities to be estimated
  - *Expected average delay in queue* (excluding service time) of the n customers completing their delays
    - Why "expected?"
  - *Expected average number of customers in queue* (excluding any in service)
    - A *continuous-time average*
    - Area under $Q(t)$ = queue length at time t, divided by $T(n)$ = time simulation ends … see book for justification and details
  - *Expected utilization (proportion of time busy) of the server*
    - Another continuous-time average
    - Area under $B(t)$ = server-busy function (1 if busy, 0 if idle at time t), divided by $T(n)$ … justification and details in book
  - Many others are possible (maxima, minima, time or number in system, proportions, quantiles, variances …)
- Important: *Discrete-time* vs. *continuous-time* statistics

1.4.2 Intuitive Explanation

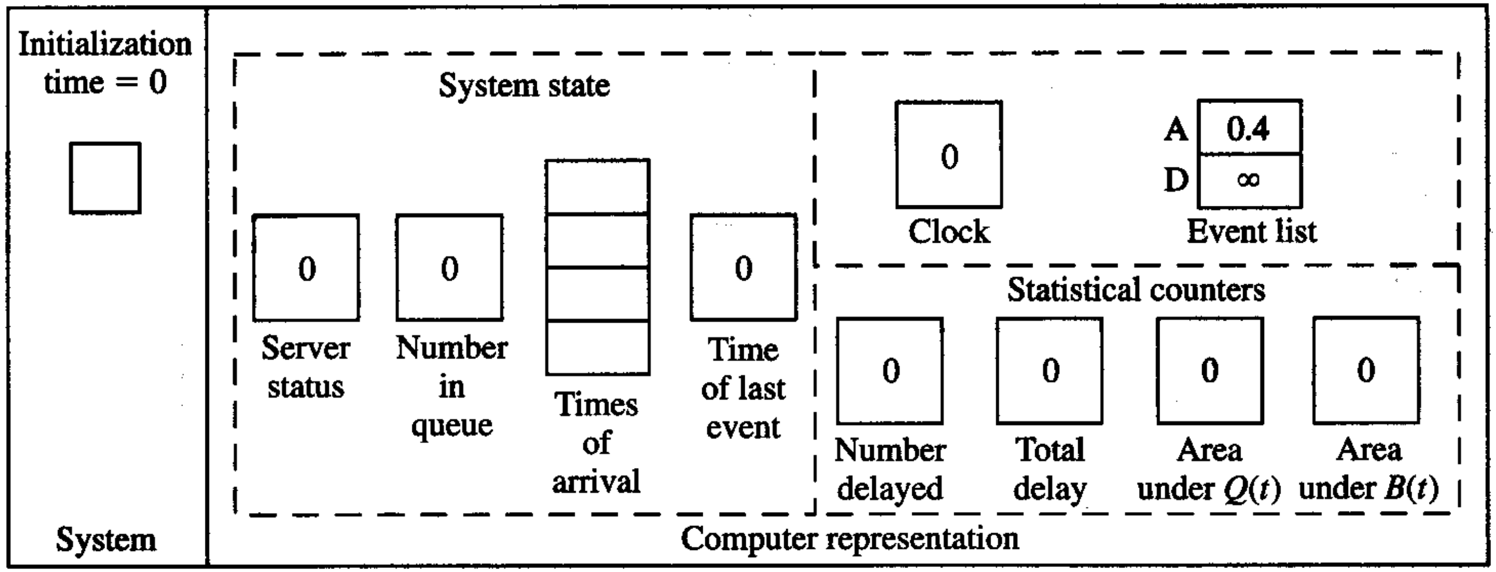
- Given (for now) interarrival times (all times are in minutes):

  0.4, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9, …
- Given service times:

  2.0, 0.7, 0.2, 1.1, 3.7, 0.6, …
- n = 6 delays in queue desired
- "Hand" simulation:
  - Display system, state variables, clock, event list, statistical counters … all *after* execution of each event
  - Use above lists of interarrival, service times to "drive" simulation
  - Stop when number of delays hits n = 6, compute output performance measures

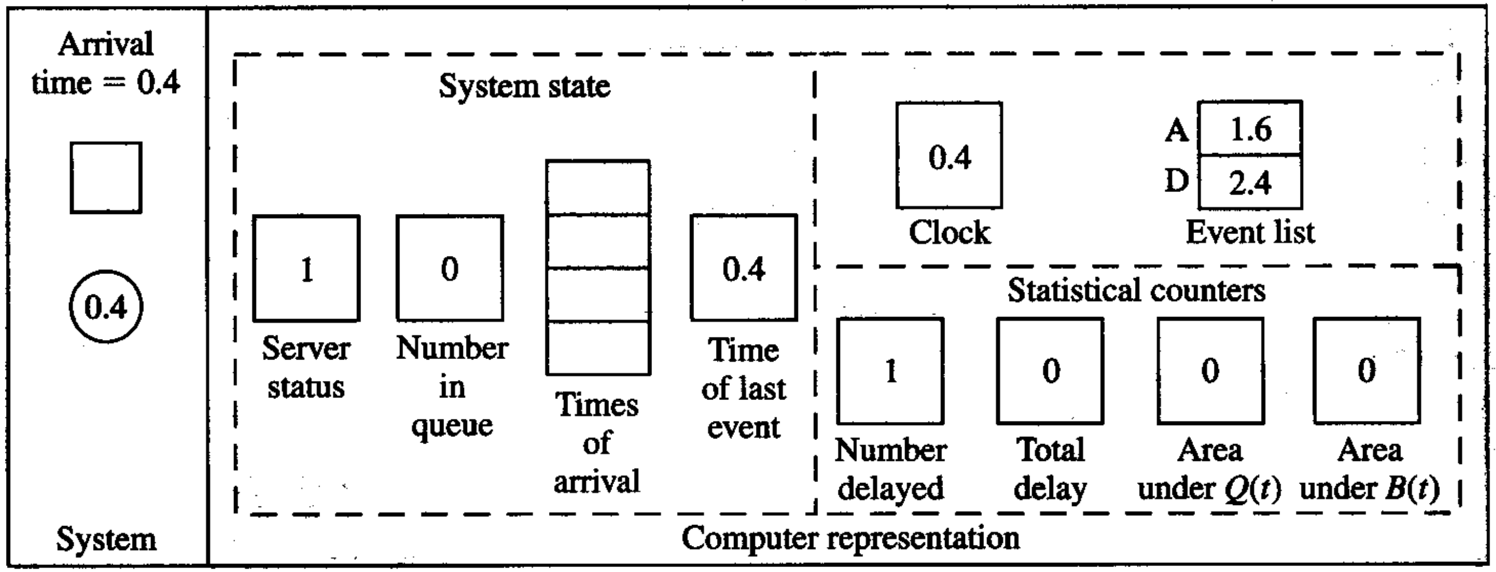
1.4.2 Intuitive Explanation (cont'd)

Status shown is *after* all changes have been made in each case …



Interarrival times: ~~0.4~~, 1.2, 0.5, 1.7, 0.2, 1.6, 0.2, 1.4, 1.9, …
Service times: 2.0, 0.7, 0.2, 1.1, 3.7, 0.6, …

1.4.2 Intuitive Explanation (cont'd)



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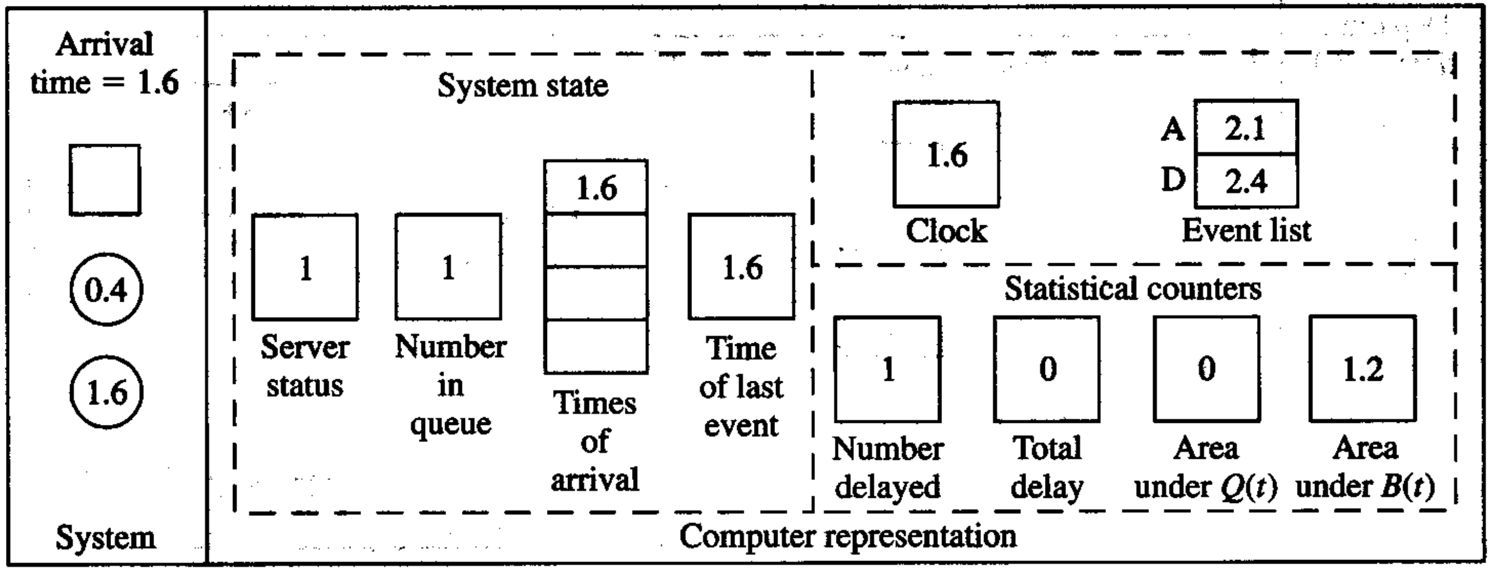

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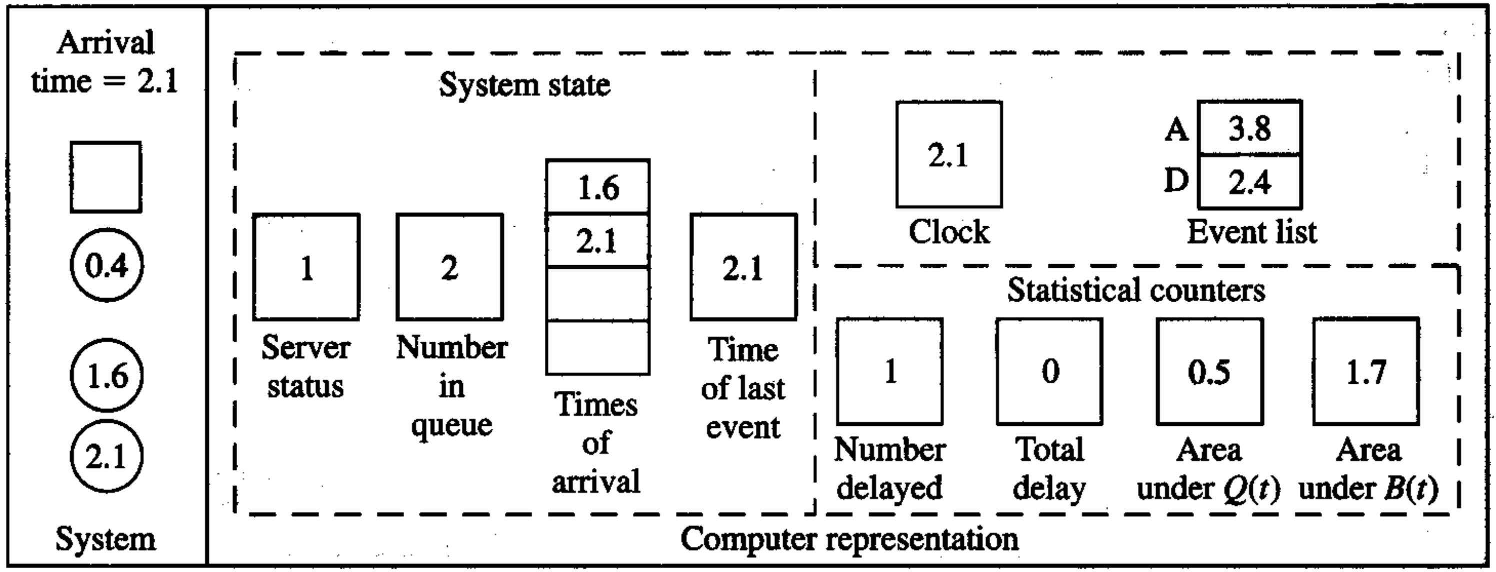

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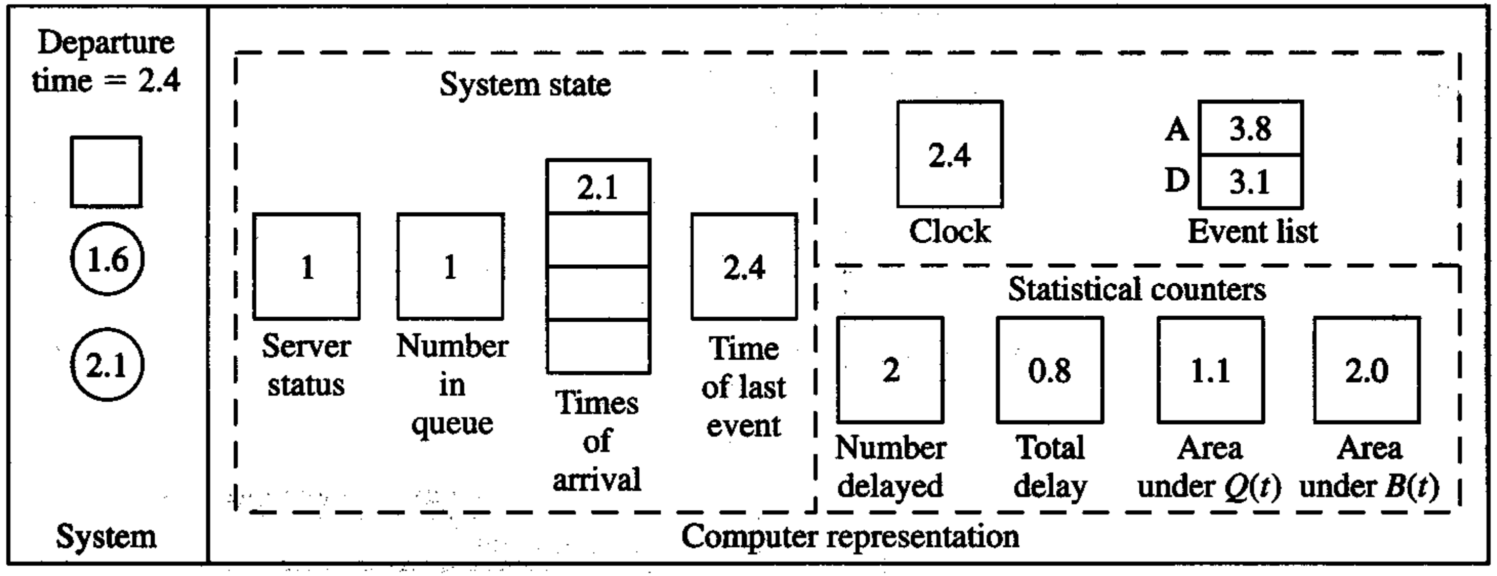

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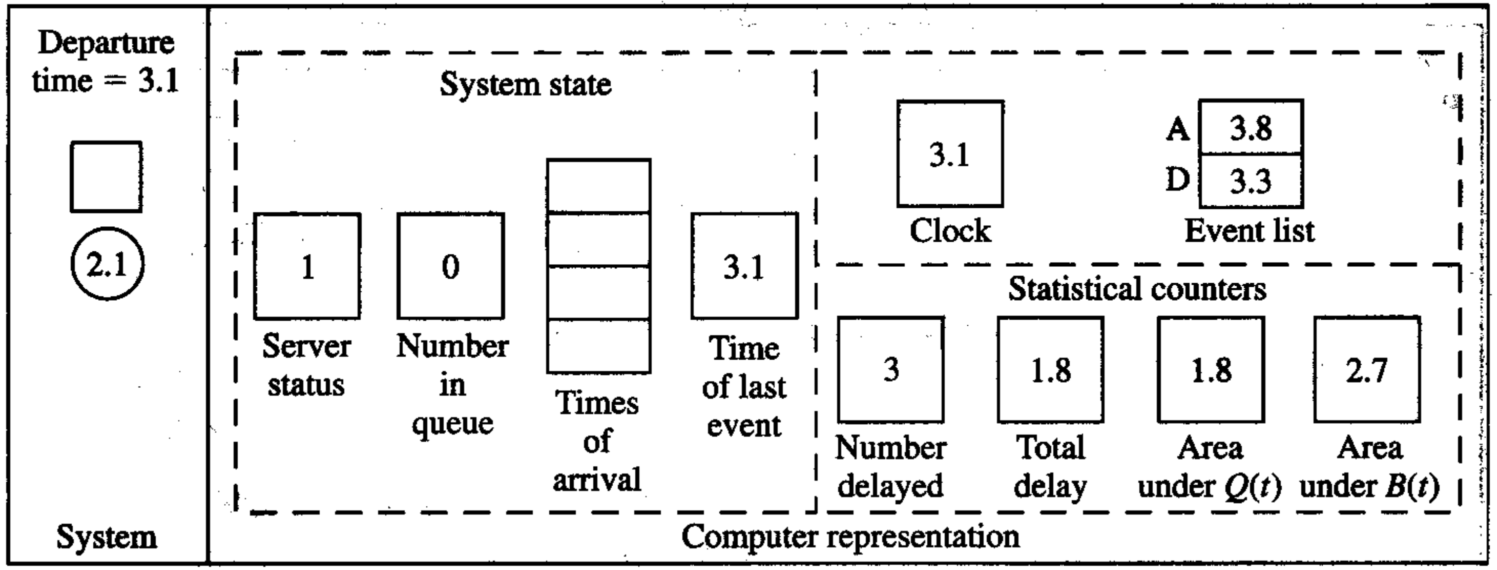

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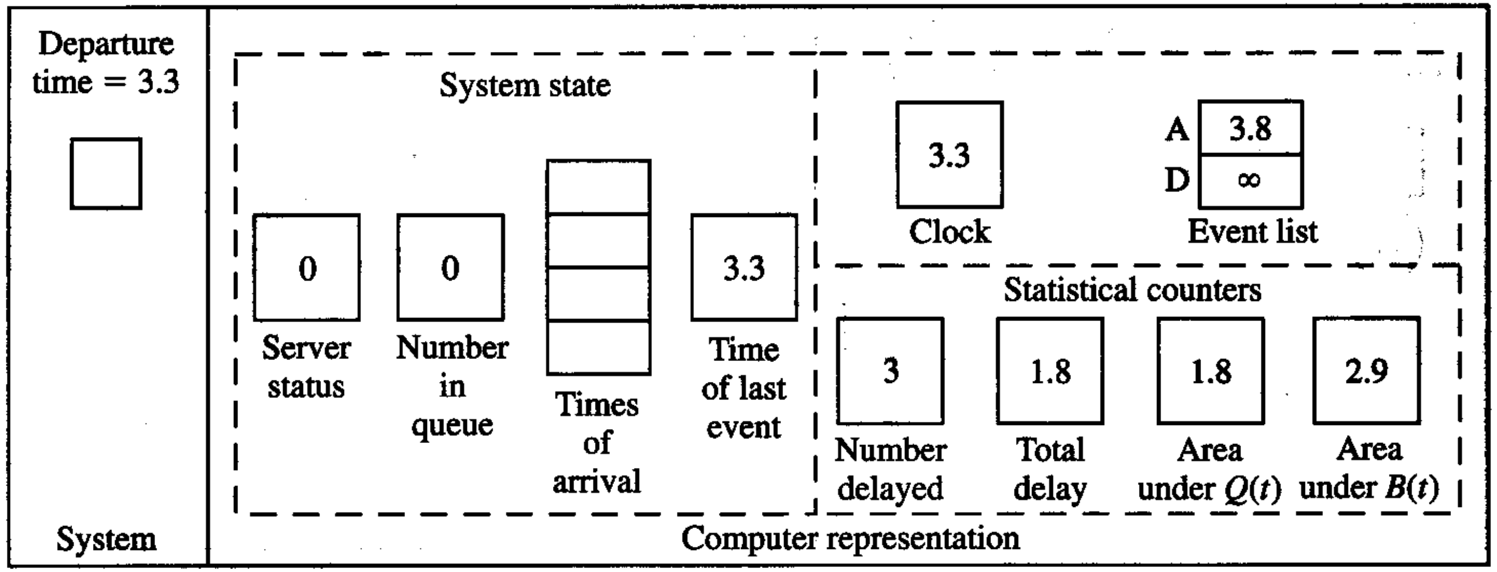

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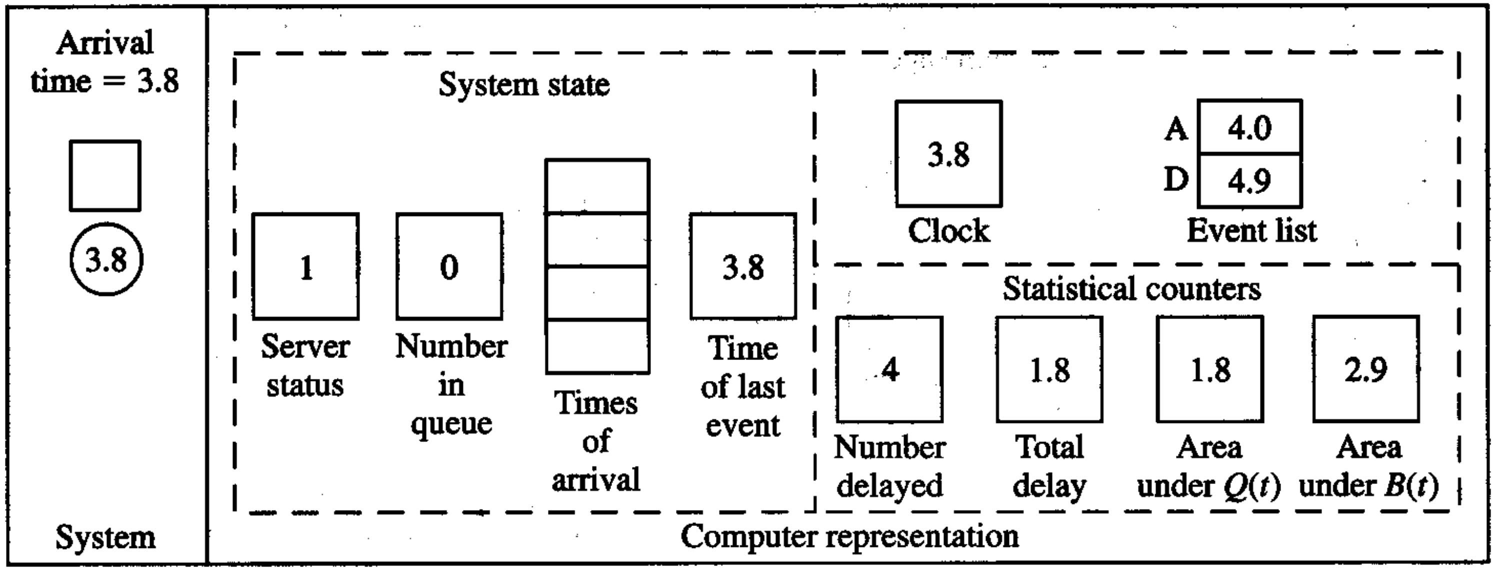

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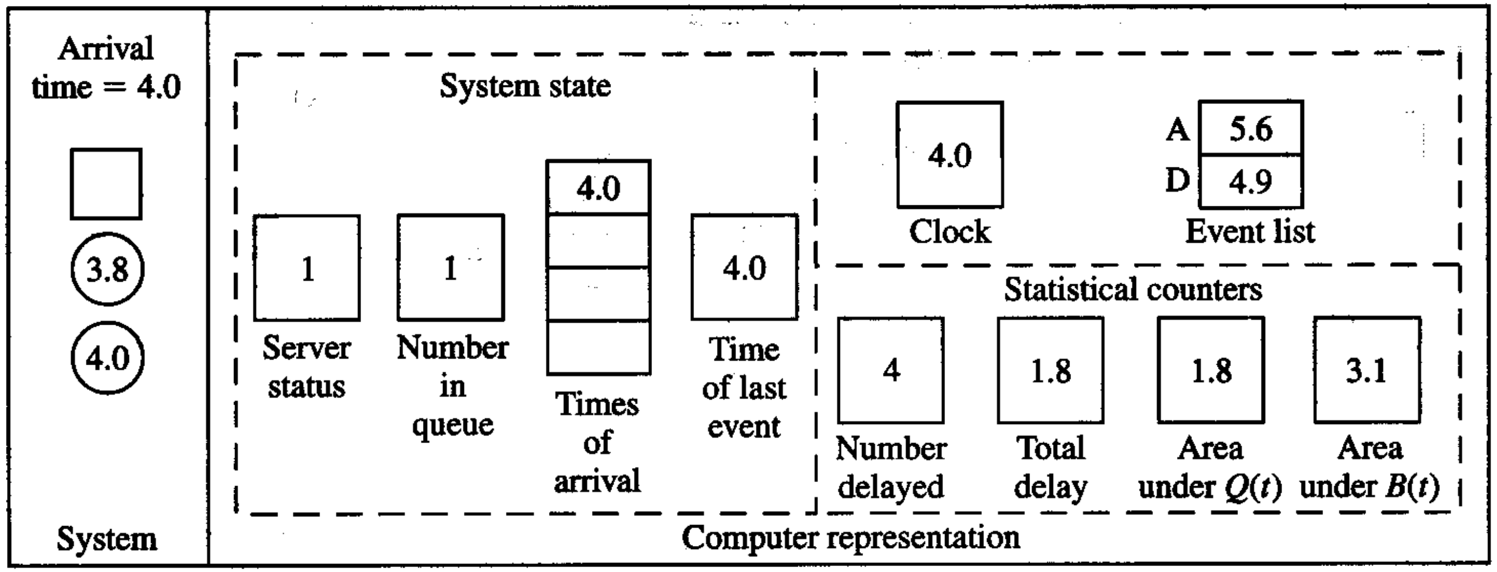

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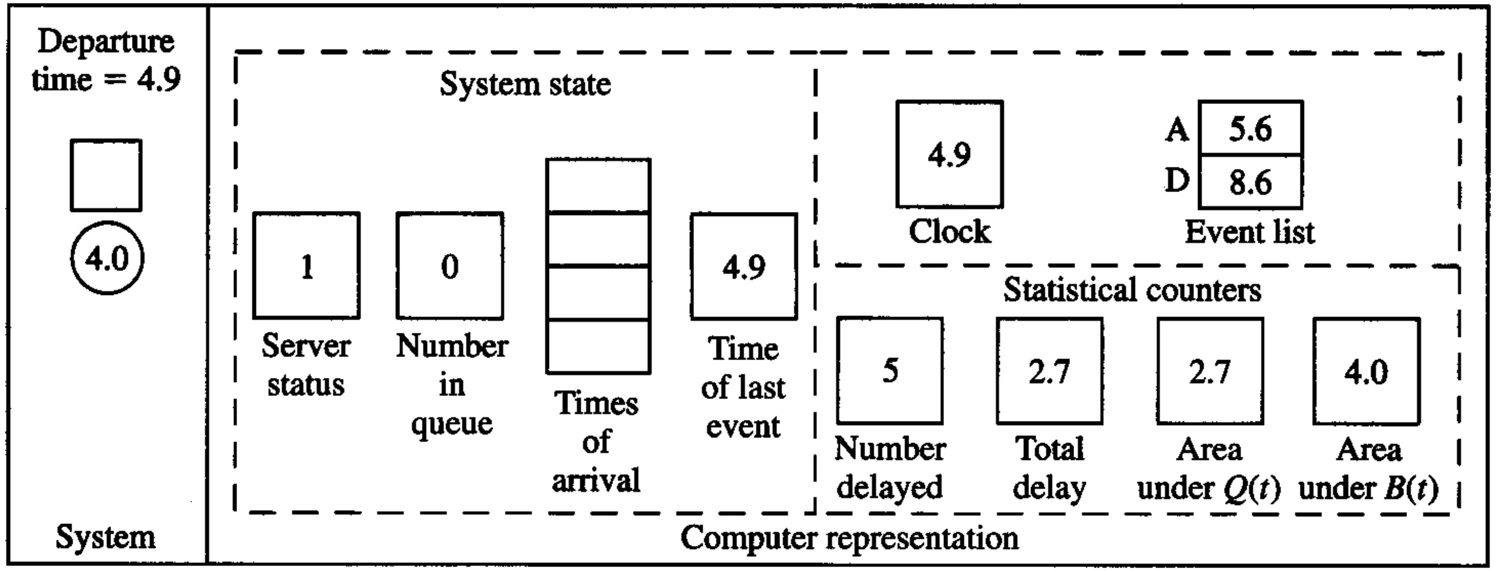

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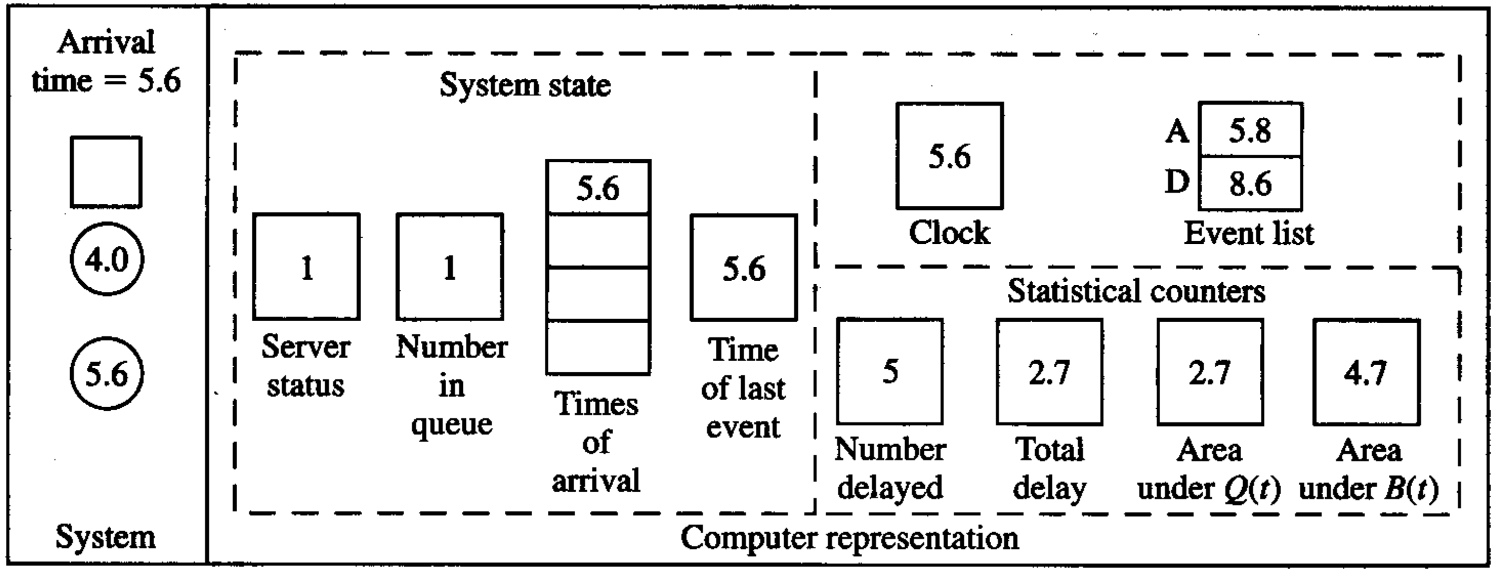

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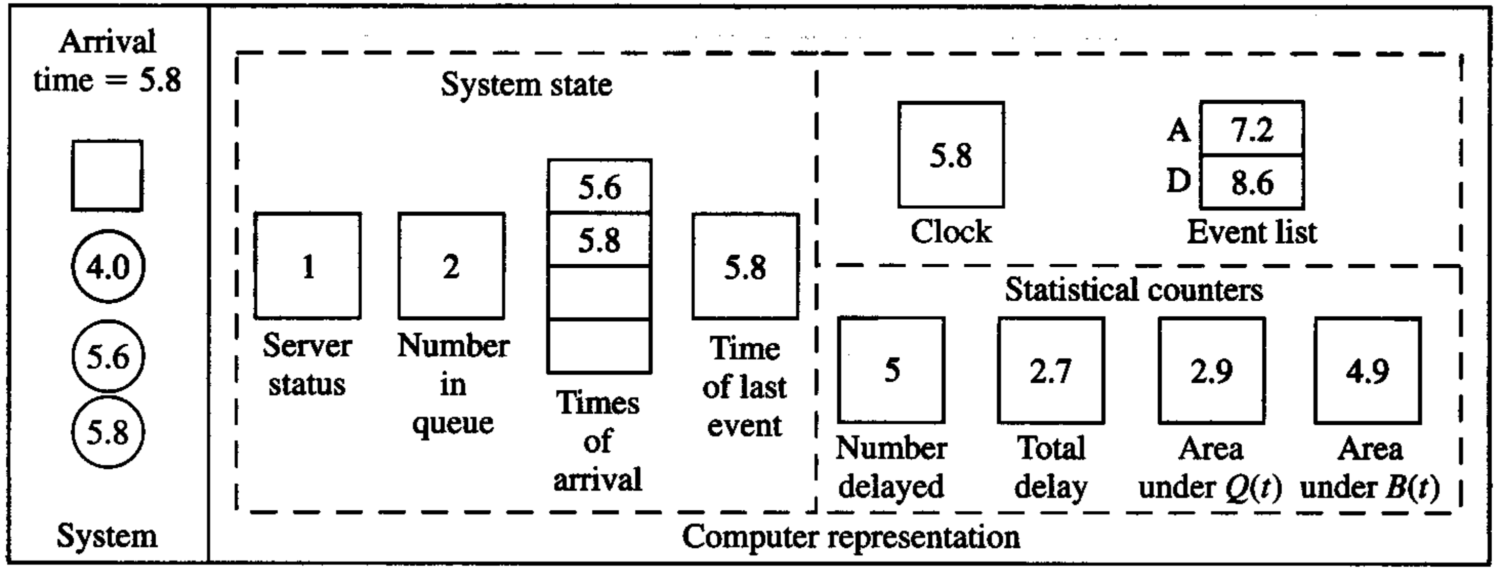

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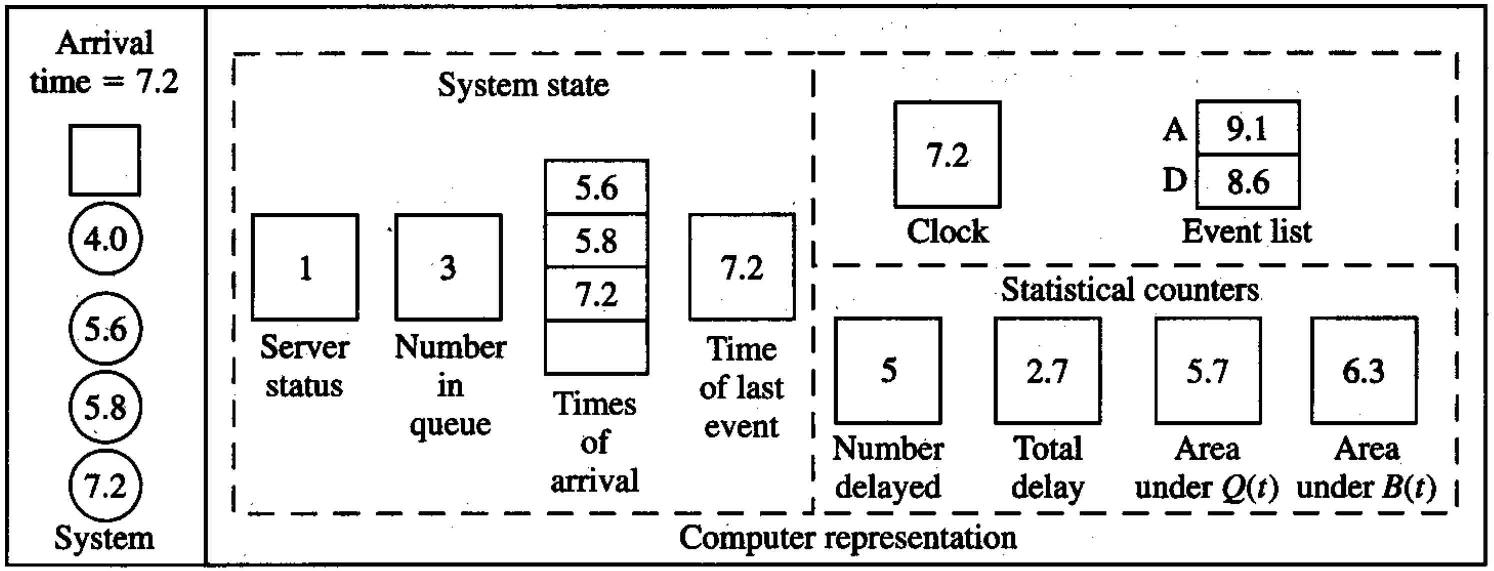

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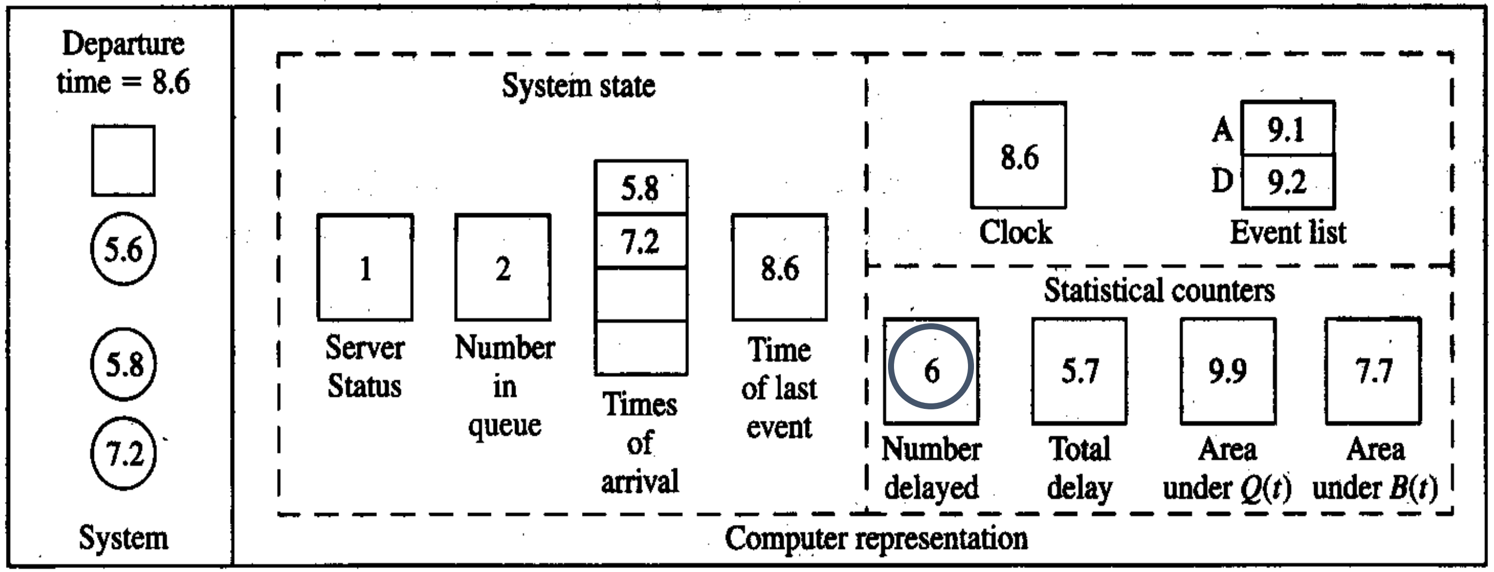

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Service times: ~~2.0~~, ~~0.7~~, ~~0.2~~, ~~1.1~~, ~~3.7~~, ~~0.6~~, …

Final output performance measures:

Average delay in queue = 5.7/6 = 0.95 min./cust.

Time-average number in queue = 9.9/8.6 = 1.15 custs.

Server utilization = 7.7/8.6 = 0.90 (dimensionless)

Computation of Statistics - Reminder

- P: Total number of customers served so far (# left the system)
- N: Total number of customers passed through the queue

- ΣWQ : Sum of queue times observed so far
- WQ^* : Maximum queue time observed so far

Updated whenever a customer finishes waiting in queue

- ΣTS : Sum of system times observed so far
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Updated whenever a customer leaves the system

Computation of Statistics - Reminder

- $Q(t)$: Function of number of customers waiting in queue (state var.)

  $$Q(t) \in \{0,1,2, \dots\}$$

- $B(t)$: Server busy function (state var.)

  $$B(t) \in \{0,1\}$$

- $L(t) = Q(t) + B(t)$

Computation of Statistics - Reminder

- Using these variables, we can find:
  - Average Waiting Time (In Queue): $\frac{\Sigma WQ}{N}$ (Tally)
  - Average System (Total) Time: $\frac{\Sigma TS}{P}$ (Tally)
  - Average Number of Customers In Queue: $\frac{\int Q(t)}{T_E}$ (Time-persistent)
  - Server Utilization: $\frac{\int B(t)}{T_E}$ (Time-persistent)

- Note that additional variables are needed if further statistical outputs are needed:
  - Maximum idle time, maximum queue length…

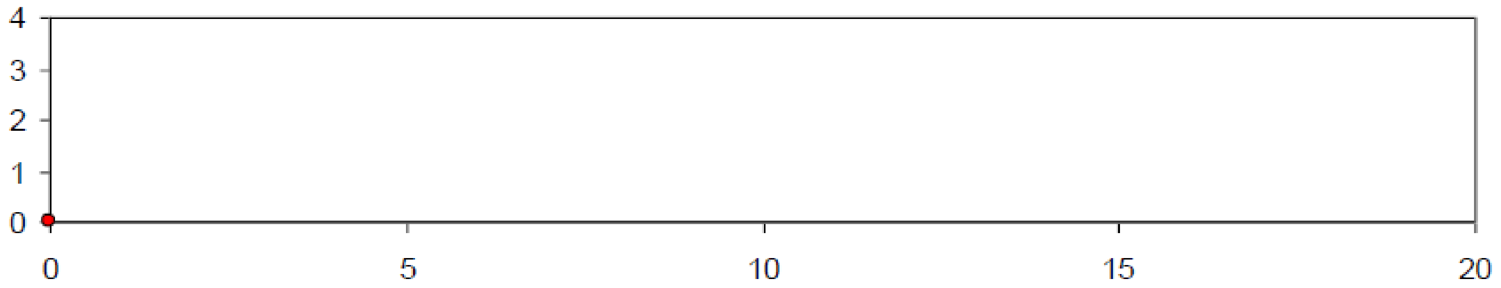
Simulation by Hand

t = 0.00, Initialize

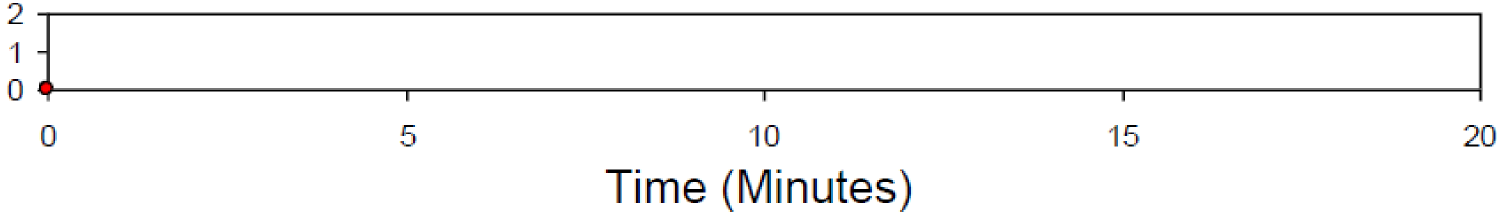
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	0.00	0	0	()	[1, 0.00, Arr] [–, 20.00, End]

N: 0 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00	Area under $Q(t)$ 0.00	Area under $B(t)$ 0.00

$Q(t)$ graph



$B(t)$ graph



Interarrival times	1.73, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	2.90, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

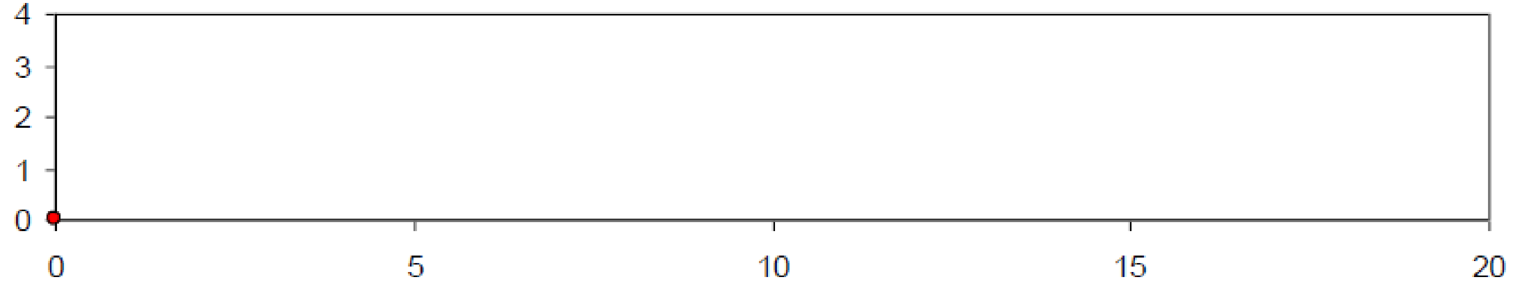
Simulation by Hand

t = 0.00, Arrival of Part 1

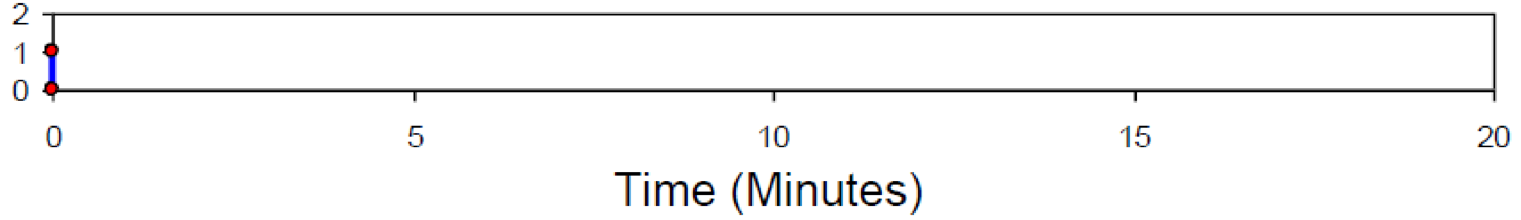
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
1	0.00	1	0	() 0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]

N: 1 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00	Area under $Q(t)$ 0.00	Area under $B(t)$ 0.00

$Q(t)$ graph



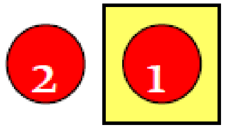
$B(t)$ graph



Interarrival times	~~1.73~~, 1.35, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

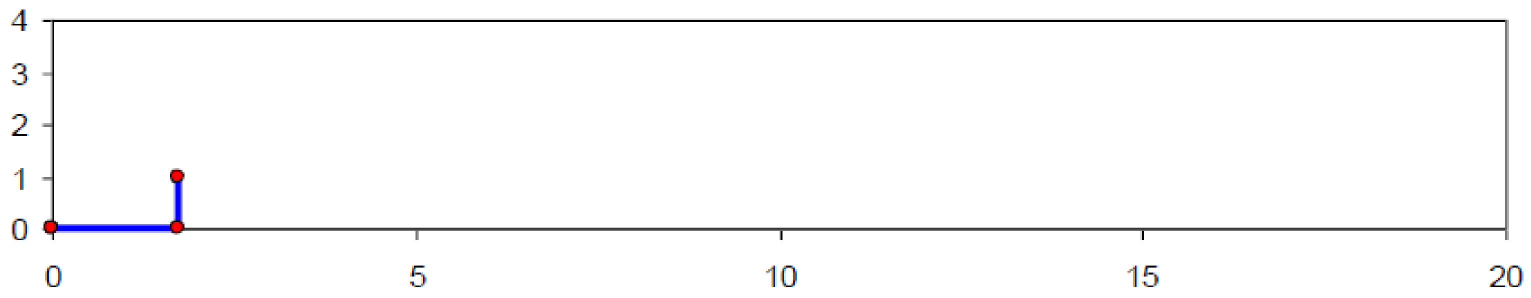
Simulation by Hand

t = 1.73, Arrival of Part 2

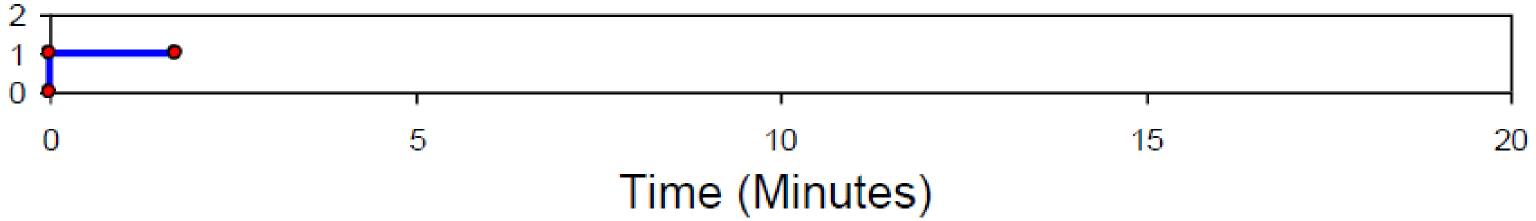
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	1.73	1	1	(1.73) 0.00	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00, End]

N: 1 P: 0	Σ WQ: 0.00 Σ TS: 0.00 WQ*: 0.00 TS*: 0.00	Area under $Q(t)$ 0.00	Area under $B(t)$ 1.73

$Q(t)$ graph



$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, 1.76, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

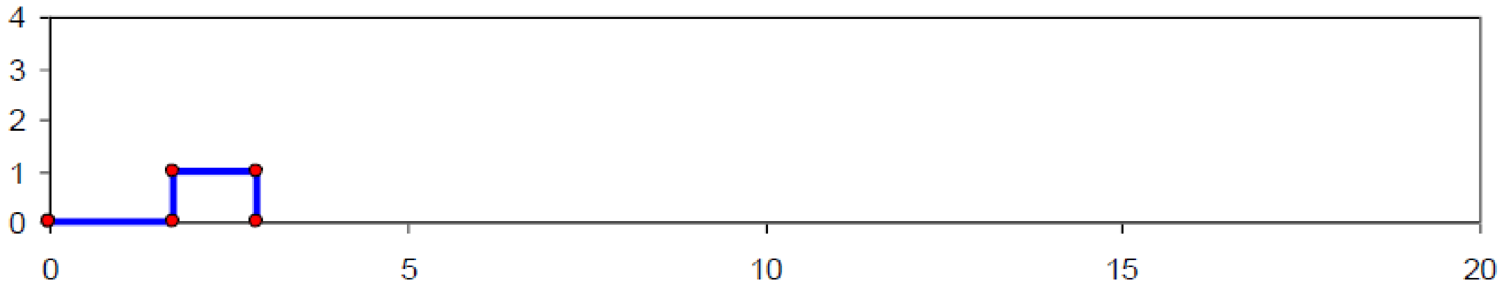
Simulation by Hand

t = 2.90, Departure of Part 1

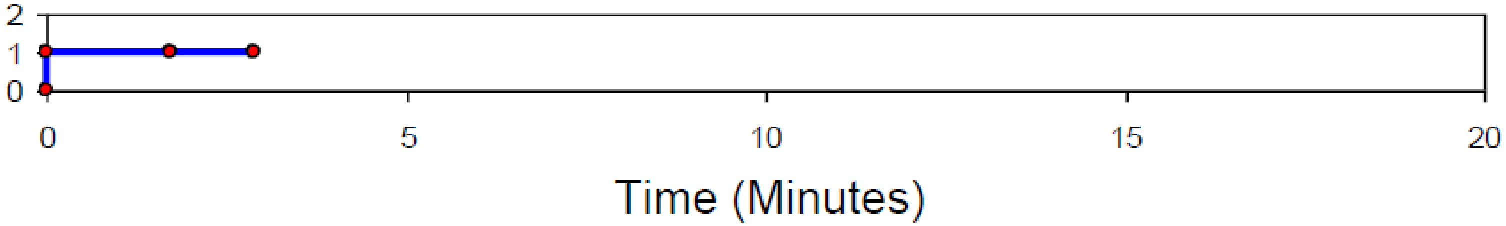
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
2	2.90	1	0	() 1.73	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 1.17	Area under $B(t)$ 2.90

$Q(t)$ graph



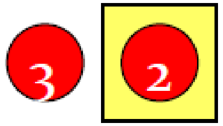
$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, 0.71, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

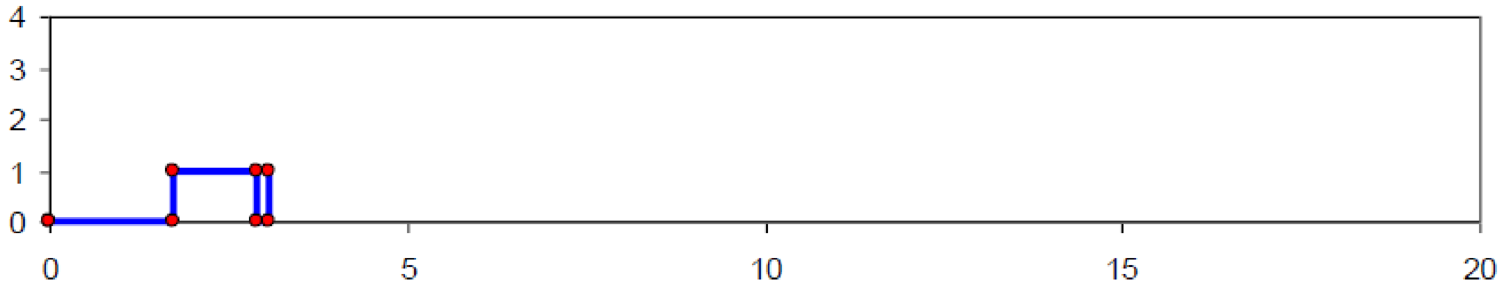
Simulation by Hand

t = 3.08, Arrival of Part 3

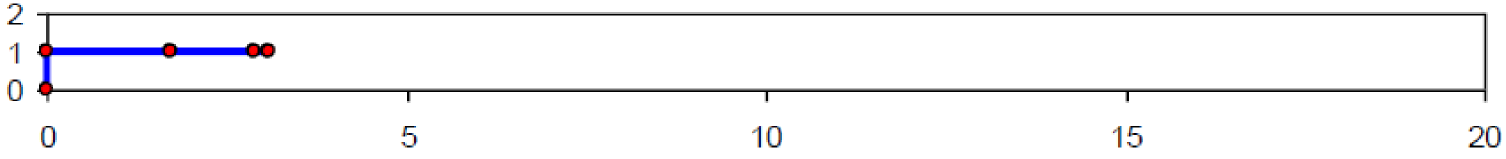
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	3.08	1	1	(3.08) 1.73	[4, 3.79,Arr] [2, 4.66,Dep] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 1.17	Area under $B(t)$ 3.08

$Q(t)$ graph



$B(t)$ graph

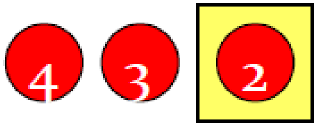


Time (Minutes)

Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, 0.62, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand

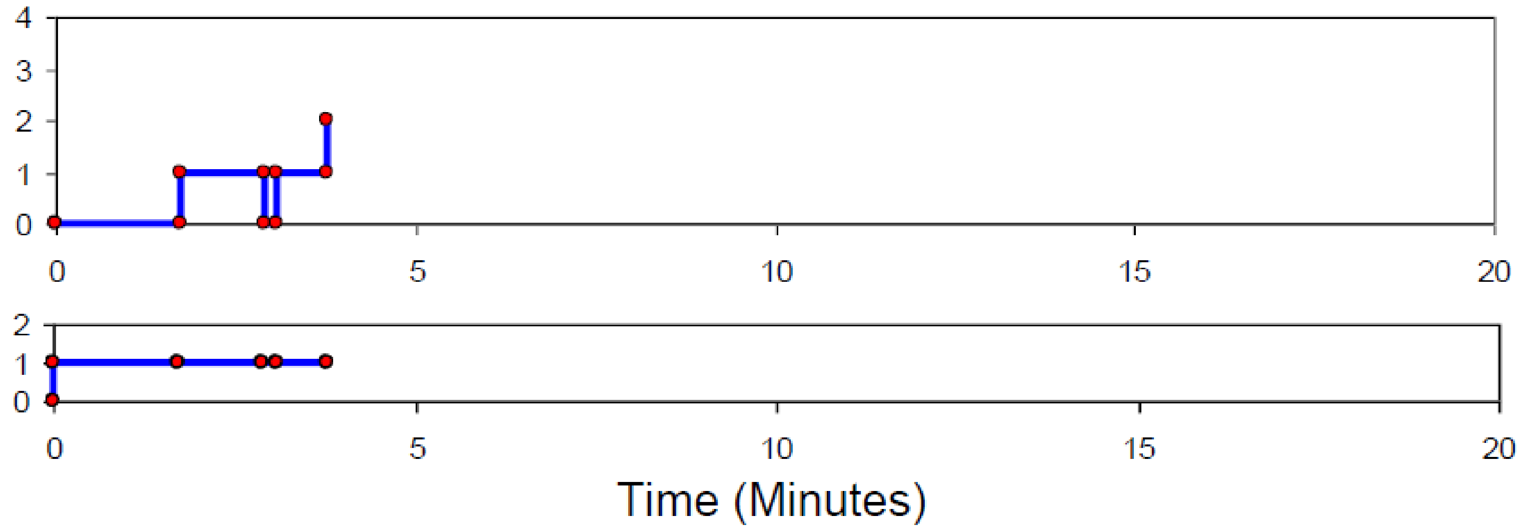
t = 3.79, Arrival of Part 4

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	3.79	1	2	(3.79, 3.08) 1.73	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 1.88	Area under $B(t)$ 3.79

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, 14.28, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand

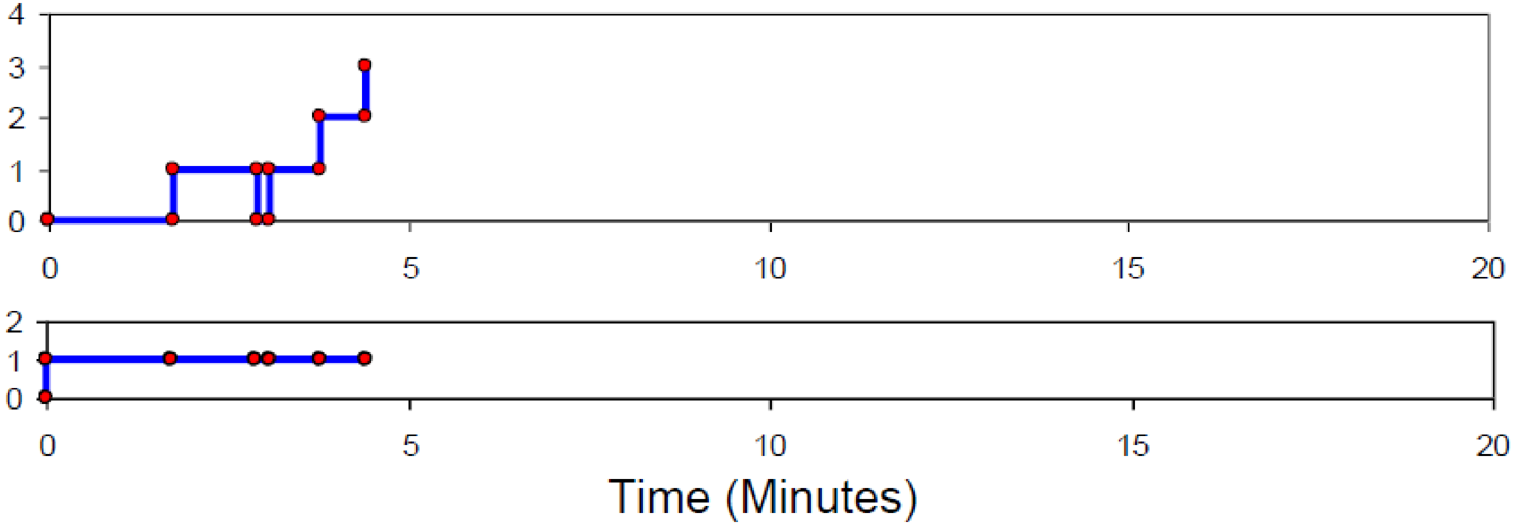
t = 4.41, Arrival of Part 5

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4 3 2	4.41	1	3	(4.41, 3.79, 3.08) 1.73	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 2 P: 1	Σ WQ: 1.17 Σ TS: 2.90 WQ*: 1.17 TS*: 2.90	Area under $Q(t)$ 3.12	Area under $B(t)$ 4.41

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, 3.39, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

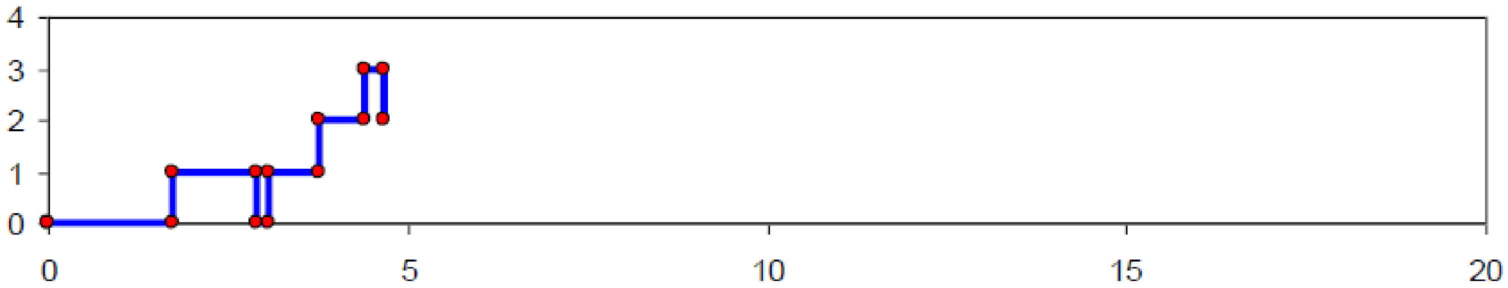
Simulation by Hand

t = 4.66, Departure of Part 2

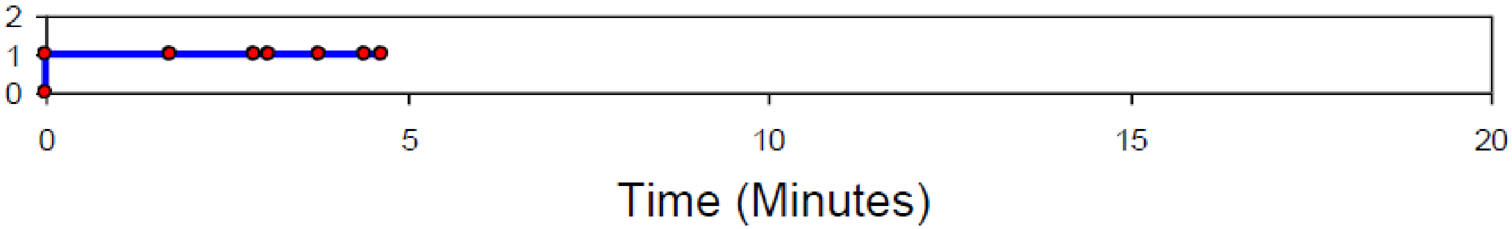
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4 3	4.66	1	2	(4.41, 3.79) 3.08	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 3 P: 2	Σ WQ: 2.75 Σ TS: 5.83 WQ*: 1.58 TS*: 2.93	Area under $Q(t)$ 3.87	Area under $B(t)$ 4.66

$Q(t)$ graph



$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, 4.52, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

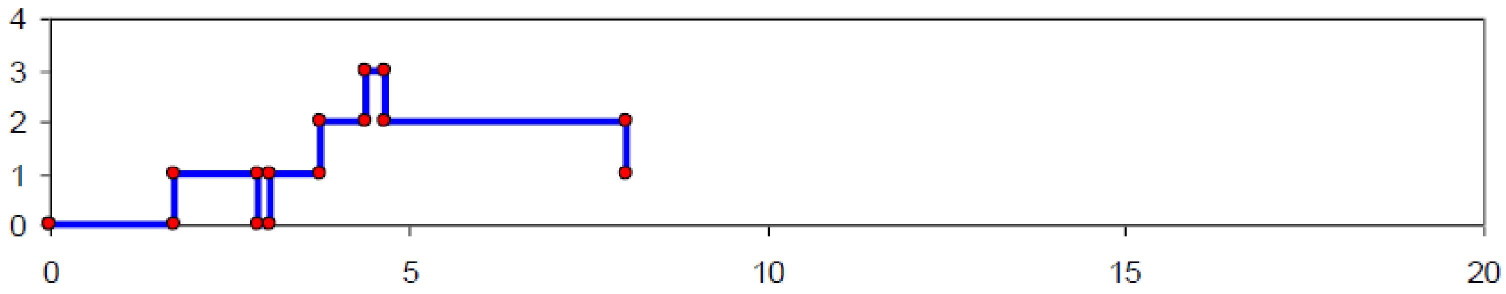
Simulation by Hand

t = 8.05, Departure of Part 3

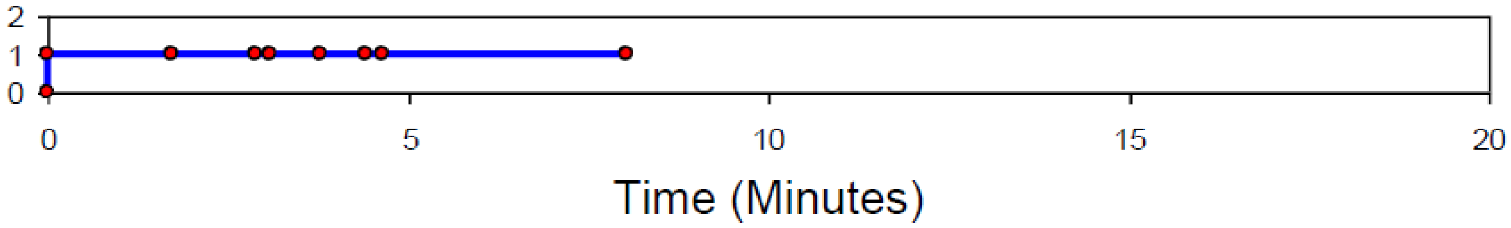
System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
5 4	8.05	1	1	(4.41) 3.79	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]

N: 4 P: 3	Σ WQ: 7.01 Σ TS: 10.80 WQ*: 4.26 TS*: 4.97	Area under $Q(t)$ 10.65	Area under $B(t)$ 8.05

$Q(t)$ graph



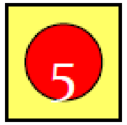
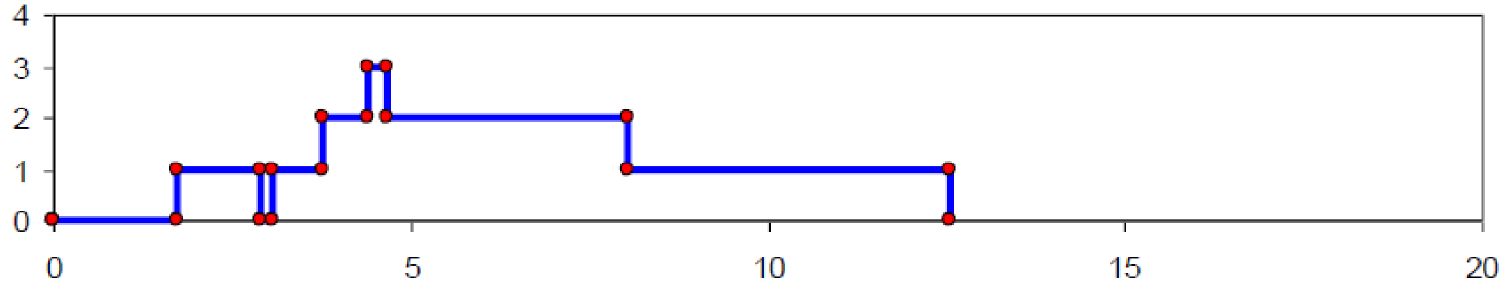
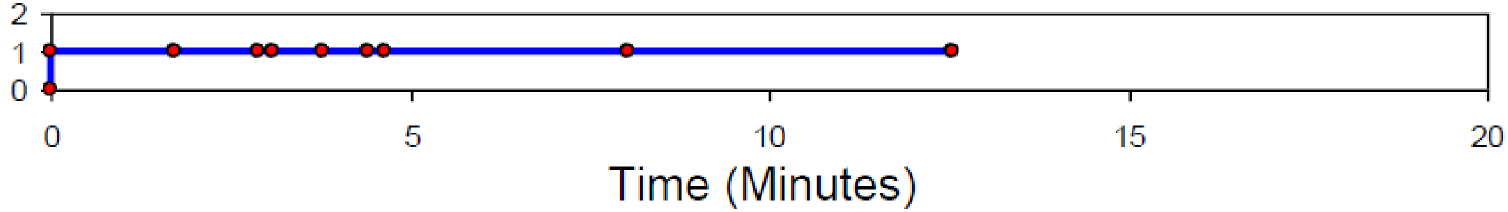
$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, 4.46, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand

t = 12.57, Departure of Part 4

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	12.57	1	0	() 4.41	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
N: 5 P: 4	Σ WQ: 15.17 Σ TS: 19.58 WQ*: 8.16 TS*: 8.78			Area under $Q(t)$ 15.17	Area under $B(t)$ 12.57
$Q(t)$ graph					
$B(t)$ graph					
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...				
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...				

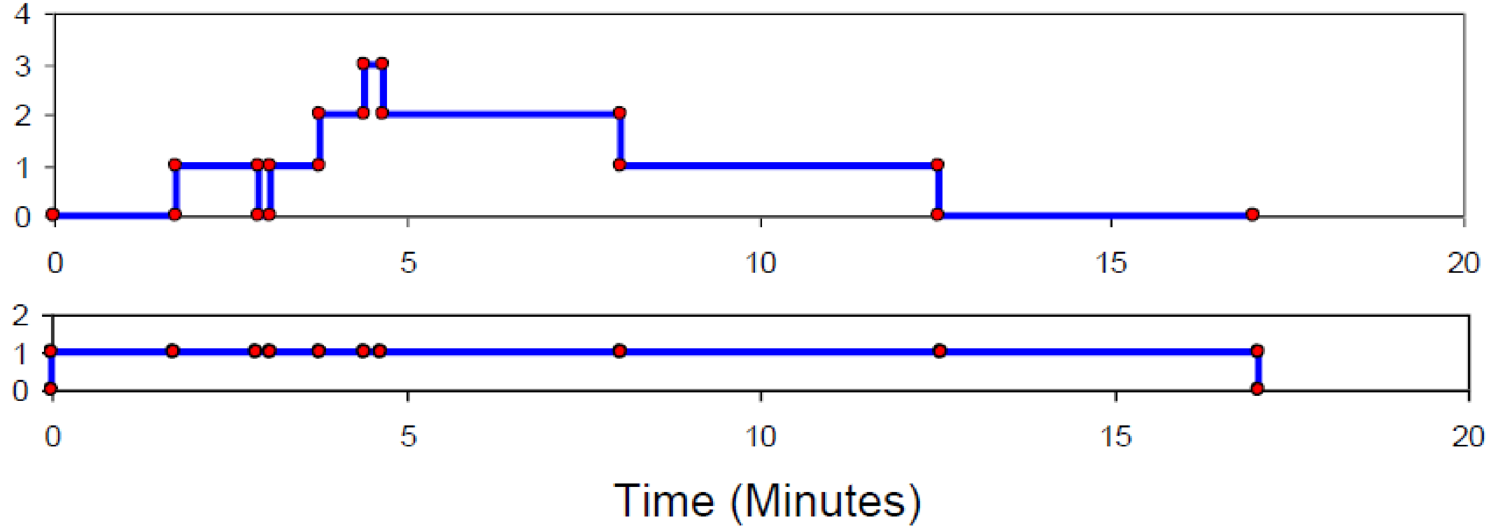
Simulation by Hand

t = 17.03, Departure of Part 5

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
	17.03	0	0	()	[6, 18.69, Arr] [–, 20.00, End]
N: 5 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62		Area under $Q(t)$ 15.17		Area under $B(t)$ 17.03

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, 0.70, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, 4.36, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand

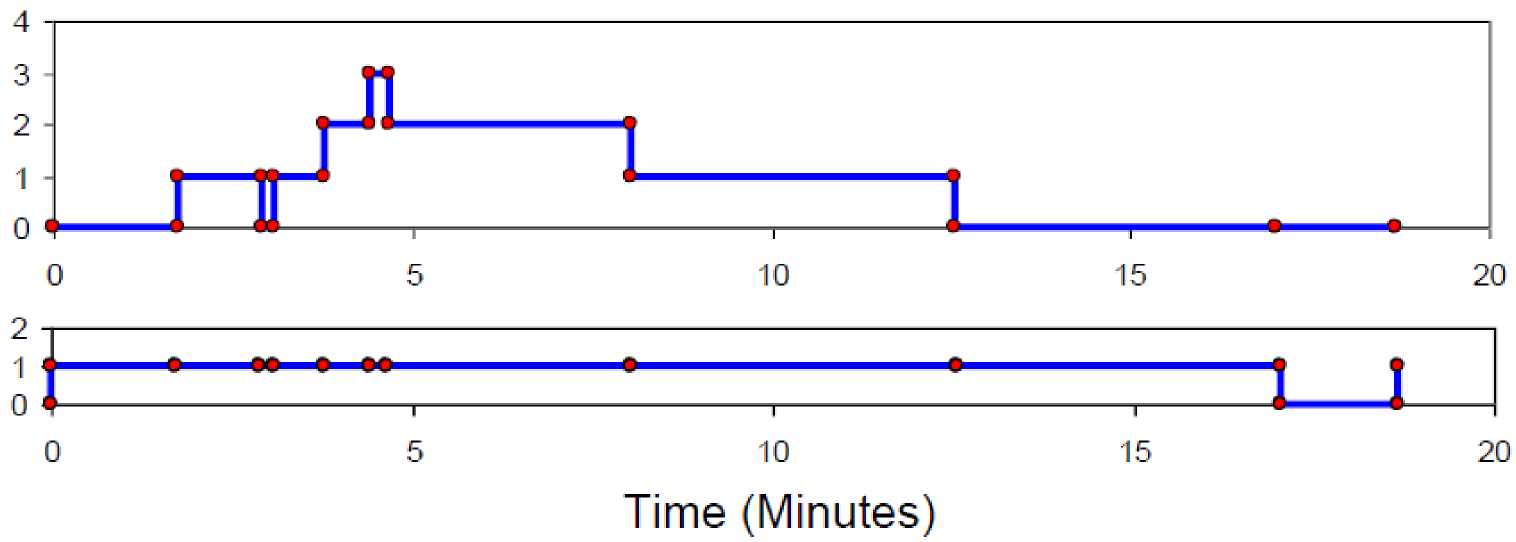
t = 18.69, Arrival of Part 6

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
6	18.69	1	0	() 18.69	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]

N: 6 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62	Area under $Q(t)$ 15.17	Area under $B(t)$ 17.03

$Q(t)$ graph

$B(t)$ graph



Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, 15.52, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

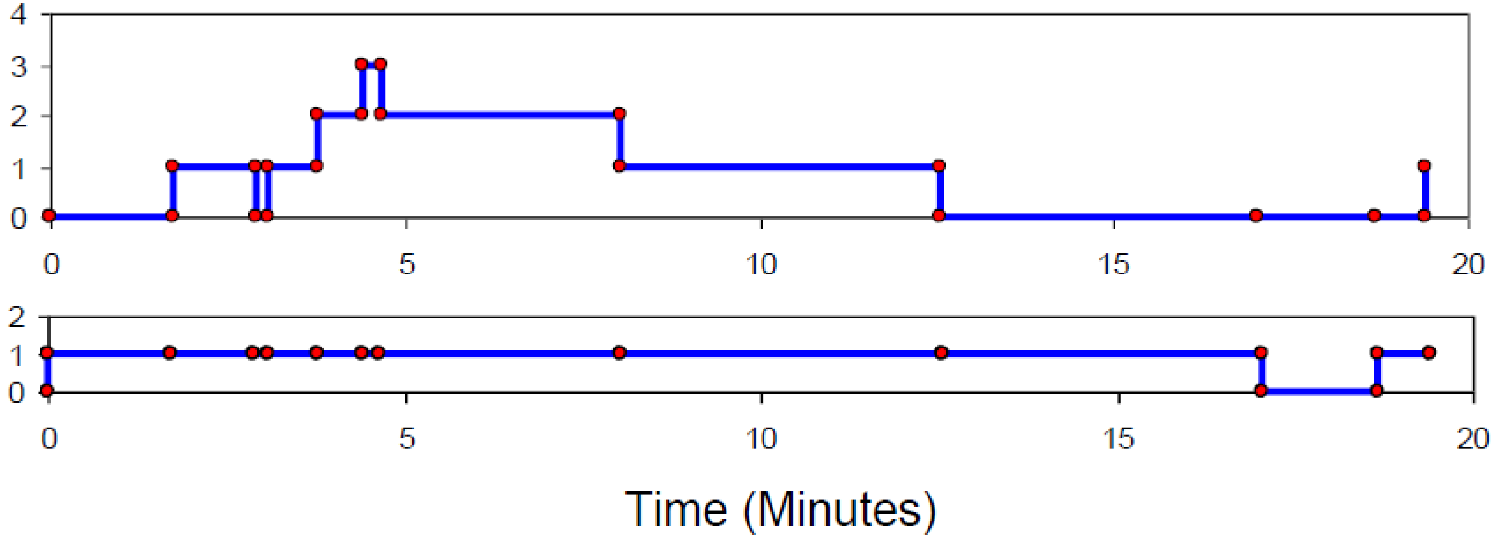
Simulation by Hand

t = 19.39, Arrival of Part 7

System	Clock	$B(t)$	$Q(t)$	Arrival times (Q) S	Event calendar
7 6	19.39	1	1	(19.39) 18.69	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
N: 6 P: 5	Σ WQ: 15.17 Σ TS: 32.20 WQ*: 8.16 TS*: 12.62		Area under $Q(t)$ 15.17		Area under $B(t)$ 17.73

$Q(t)$ graph

$B(t)$ graph

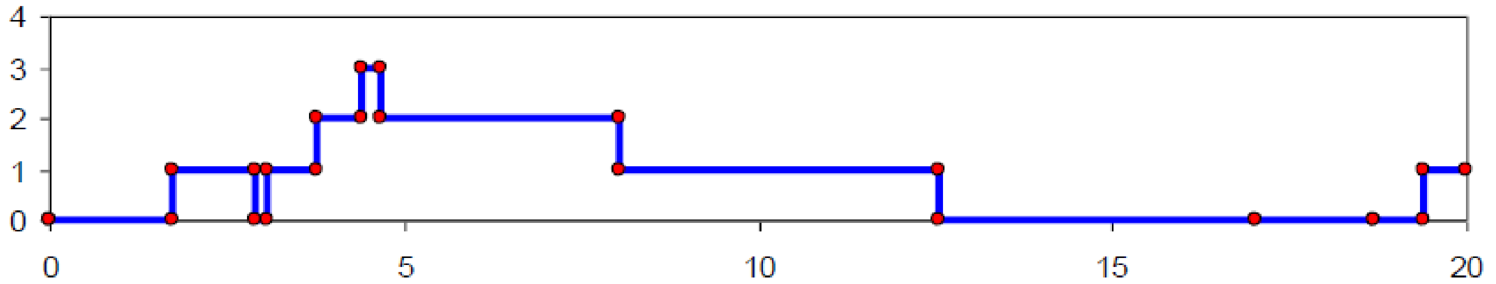


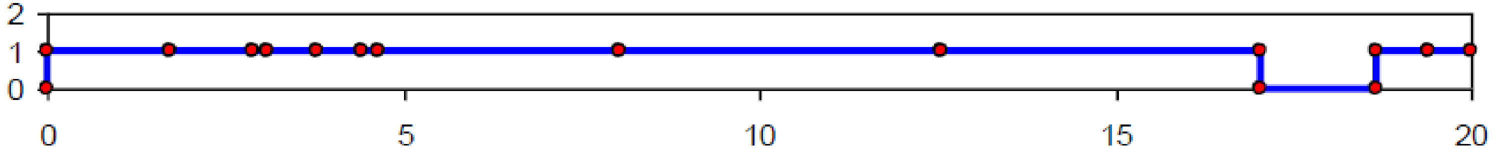
Interarrival times	~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...
Service times	~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...

Simulation by Hand

t = 20.00, The End

<table>
<tr><td>System
7 6</td><td>Clock
20.00</td><td>$B(t)$
1</td><td>$Q(t)$
1</td><td colspan="2">Arrival times (Q) S
(19.39) 18.69</td><td>Event calendar
[6, 23.05, Dep]
[8, 34.91, Arr]</td></tr>
<tr><td>N: 6
P: 5</td><td colspan="3">Σ WQ: 15.17
Σ TS: 32.20
WQ*: 8.16
TS*: 12.62</td><td colspan="2">Area under $Q(t)$
15.78</td><td>Area under $B(t)$
18.34</td></tr>
<tr><td>$Q(t)$ graph

$B(t)$ graph</td><td colspan="6">


</td></tr>
<tr><td>Interarrival times</td><td colspan="6">~~1.73~~, ~~1.35~~, ~~0.71~~, ~~0.62~~, ~~14.28~~, ~~0.70~~, ~~15.52~~, 3.15, 1.76, 1.00, ...</td></tr>
<tr><td>Service times</td><td colspan="6">~~2.90~~, ~~1.76~~, ~~3.39~~, ~~4.52~~, ~~4.46~~, ~~4.36~~, 2.07, 3.36, 2.37, 5.38, ...</td></tr>
</table>

Just-Finished Event			Variables		Attributes		Statistical Accumulators									Event Calendar
Entity No.	Time t	Event Type	$Q(t)$	$B(t)$	Arrival Times: (In Queue)	In Service	P	N	ΣWQ	WQ^*	ΣTS	TS^*	$\int Q$	Q^*	$\int B$	[Entity No., Time, Type]
–	0.00	Init	0	0	( )	–	0	0	0.00	0.00	0.00	0.00	0.00	0	0.00	[1, 0.00, Arr] [–, 20.00, End]
1	0.00	Arr	0	1	( )	0.00	0	1	0.00	0.00	0.00	0.00	0.00	0	0.00	[2, 1.73, Arr] [1, 2.90, Dep] [–, 20.00, End]
2	1.73	Arr	1	1	(1.73)	0.00	0	1	0.00	0.00	0.00	0.00	0.00	1	1.73	[1, 2.90, Dep] [3, 3.08, Arr] [–, 20.00 End]
1	2.90	Dep	0	1	( )	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	2.90	[3, 3.08, Arr] [2, 4.66, Dep] [–, 20.00, End]
3	3.08	Arr	1	1	(3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.17	1	3.08	[4, 3.79, Arr] [2, 4.66, Dep] [–, 20.00, End]
4	3.79	Arr	2	1	(3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	1.88	2	3.79	[5, 4.41, Arr] [2, 4.66, Dep] [–, 20.00, End]
5	4.41	Arr	3	1	(4.41, 3.79, 3.08)	1.73	1	2	1.17	1.17	2.90	2.90	3.12	3	4.41	[2, 4.66, Dep] [6, 18.69, Arr] [–, 20.00, End]
2	4.66	Dep	2	1	(4.41, 3.79)	3.08	2	3	2.75	1.58	5.83	2.93	3.87	3	4.66	[3, 8.05, Dep] [6, 18.69, Arr] [–, 20.00, End]
3	8.05	Dep	1	1	(4.41)	3.79	3	4	7.01	4.26	10.80	4.97	10.65	3	8.05	[4, 12.57, Dep] [6, 18.69, Arr] [–, 20.00, End]
4	12.57	Dep	0	1	( )	4.41	4	5	15.17	8.16	19.58	8.78	15.17	3	12.57	[5, 17.03, Dep] [6, 18.69, Arr] [–, 20.00, End]
5	17.03	Dep	0	0	( )	–	5	5	15.17	8.16	32.20	12.62	15.17	3	17.03	[6, 18.69, Arr] [–, 20.00 End]
6	18.69	Arr	0	1	( )	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.03	[7, 19.39, Arr] [–, 20.00, End] [6, 23.05, Dep]
7	19.39	Arr	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.17	3	17.73	[–, 20.00, End] [6, 23.05, Dep] [8, 34.91, Arr]
–	20.00	End	1	1	(19.39)	18.69	5	6	15.17	8.16	32.20	12.62	15.78	3	18.34	[6, 23.05, Dep] [8, 34.91, Arr]