

1> N^2 -squared = N^2 N factorial = $N!$ ascending order 升序

2> The Dictionary Principle: Turning an $O(N)$ problem into an $O(\log N)$ problem.

3> Expression: $! > \&\& = || > * = / = \% > + = -$

4> Linear: n Linearithmic: $n \lg n$

5> Union-find:

| Algorithm | Initialize | Union | Find | Connected |
|-------------|------------|--------------------|----------------|----------------|
| Quick-find | N | N | 1 | 1 |
| Quick-union | N | $\lg N + \sim N +$ | $\lg N \sim N$ | $\lg N \sim N$ |
| Weighted QU | N | $\lg N +$ | $\lg N$ | $\lg N$ |
| QU + PC | N | $1 + \sim N +$ | $1 \sim N$ | $1 \sim N$ |
| WQU + PC | N | $\lg^* N +$ | $\lg^* N$ | $\lg^* N$ |

(+: includes cost of finding roots)

6> Sort:

Selection sort:

```
int N = a.length;
for (int i = 0; i < N; i++) {
    int min = i;
    for (int j = i+1; j < N; j++)
        if (less(a[j], a[min]))
            min = j;
    swap(a, i, min);
}
```

Insertion sort:

```
int N = a.length;
for (int i = 0; i < N; i++)
    for (int j = i; j > 0; j--)
        if (less(a[j], a[j-1]))
            swap(a, j, j-1);
        else break;
```

Shell sort:

```
int N = a.length;
int h = 1;
while (h < N/3) h = 3*h + 1; // 1, 4, 13, 40, ...
while (h >= 1) { // h-sort the array.
    for (int i = h; i < N; i++) {
        for (int j = i; j >= h && less(a[j], a[j-h]); j -= h)
            swap(a, j, j-h);
    }
    h = h/3;
}
```

Merge sort:

```
private static void merge(Comparable[] a,
    Comparable[] aux, int lo, int mid, int hi) {
    for (int k = lo; k <= hi; k++)
        aux[k] = a[k];
    int i = lo, j = mid+1;
    for (int k = lo; k <= hi; k++) {
        if (i > mid) a[k] = aux[j++];
        else if (j > hi) a[k] = aux[i++];
        else if (less(aux[j], aux[i])) a[k] = aux[j++];
        else a[k] = aux[i++];
    }
}
```

```
private static void sort(Comparable[] a, Comparable[] aux, int
    lo, int hi) {
    if (hi <= lo) return;
    int mid = lo + (hi - lo) / 2;
    sort(a, aux, lo, mid);
    sort(a, aux, mid+1, hi);
    merge(a, aux, lo, mid, hi);
}
```

```
private static int partition(Comparable[] a, int lo, int hi) {
    int i = lo, j = hi+1;
    while (true) {
        while (less(a[++i], a[lo])) if (i == hi) break;
        while (less(a[lo], a[--j])) if (j == lo) break;
        if (i >= j) break;
        swap(a, i, j);
    }
    swap(a, lo, j); return j;
}
```

Quick sort:

```
private static void sort(Comparable[] a, int lo, int
    hi) { if (hi <= lo) return; int j = partition(a, lo, hi);
    sort(a, lo, j-1); sort(a, j+1, hi); }
```

| | In-place | Stable | Best | Average | Worst | remarks |
|--------------|----------|--------|---------------|----------------|-------------|--|
| Selection | Y | | $1/2 N^2$ | $1/2 N^2$ | $1/2 N^2$ | N exchanges |
| Insertion | Y | Y | N | $1/4 N^2$ | $1/2 N^2$ | Use for small N or partially ordered |
| Shell | Y | | $N \log_3 N$ | ? | $c N^{3/2}$ | Tight code; subquadratic |
| Merge | | Y | $1/2 N \lg N$ | $N \lg N$ | $N \lg N$ | $N \lg N$ guarantee; stable |
| Timsort | | Y | N | $N \lg N$ | $N \lg N$ | Improve mergesort when preexisting order |
| Quick(3 Way) | Y | | $N \lg N$ | $1.39 N \lg N$ | $1/2 N^2$ | Fastest in practice |

7> Use Linked List to implement Bag, Queue and Stack.

8> Stirling's approximation for $N!$ = $\lg N!$ = $\sim N \lg N$

9> An inversion is a pair of keys that are out of order.

An array is partially sorted if the number of inversions are $\leq CN$.

Number of exchanges equals the number of inversions.

Number of compares = exchanges + $(N - 1)$.

```
private static void sort(Comparable[] a, int lo, int hi) { if (hi <= lo) return; int lt = lo, gt = hi; Comparable v = a[lo]; int i = lo;
while (i <= gt) { int cmp = a[i].compareTo(v); if (cmp < 0) swap(a, lt++, i++); else if (cmp > 0) swap(a, i, gt--); else i++; }
sort(a, lo, lt - 1); sort(a, gt + 1, hi); }
```