Type I and Type II Sum of Squares and Partial R^2

Type I vs. Type II Sum of Square Terms

Variable	Type I SS	Type II SS
X_1	$SSR(X_1)$	$SSR(X_1 \mid X_2, X_3)$
X_2	$SSR(X_2 \mid X_1)$	$SSR(X_2 \mid X_1, X_3)$
X_3	$\boxed{SSR(X_3 \mid X_1, X_2)}$	$\boxed{SSR(X_3 \mid X_1, X_2)}$

```r
anova(model4)   #type I
```

```
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1 352.27  352.27 57.2768 1.131e-06 ***
x2         1  33.17   33.17  5.3931   0.03373 *  
x3         1  11.55   11.55  1.8773   0.18956    
Residuals 16  98.40    6.15                      
```

```r
library(car)
Anova(model4, type="II")  #type II
```

```
Anova Table (Type II tests)

Response: y
          Sum Sq Df F value Pr(>F)
x1        12.705  1  2.0657 0.1699
x2         7.529  1  1.2242 0.2849
x3        11.546  1  1.8773 0.1896
Residuals 98.405 16               
```

The F tests in the Type II ANOVA table are equivalent to The T tests in the Model summary.

The effect of order of predictors entering the model

Variable	Type I SS	Type II SS
X_3	$SSR(X_3)$	$SSR(X_3 \mid X_1, X_2)$
X_2	$SSR(X_2 \mid X_3)$	$SSR(X_2 \mid X_1, X_3)$
X_1	$SSR(X_1 \mid X_2, X_3)$	$SSR(X_1 \mid X_2, X_3)$

After

```
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x3         1  10.05   10.05  1.6343    0.2193    
x2         1 374.23  374.23 60.8471 7.684e-07 ***
x1         1  12.70   12.70  2.0657    0.1699    
Residuals 16  98.40    6.15                      
```

Sum up to SSR=SSTO-SSE

```
Anova Table (Type II tests)

Response: y
          Sum Sq Df F value Pr(>F)
x3        11.546  1  1.8773 0.1896
x2         7.529  1  1.2242 0.2849
x1        12.705  1  2.0657 0.1699
Residuals 98.405 16
Analysis of Variance Table
```

Before

```
Analysis of Variance Table

Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x1         1 352.27  352.27 57.2768 1.131e-06 ***
x2         1  33.17   33.17  5.3931   0.03373 *  
x3         1  11.55   11.55  1.8773   0.18956    
Residuals 16  98.40    6.15                      
```

Type I SS would change by the order

```
Anova Table (Type II tests)

Response: y
          Sum Sq Df F value Pr(>F)
x1        12.705  1  2.0657 0.1699
x2         7.529  1  1.2242 0.2849
x3        11.546  1  1.8773 0.1896
Residuals 98.405 16
```

Type II SS would not change.

Comments

- Type I SS always sum to SSR for the model with all predictors.
- Type I SS can give different values depending on the order in which variables are specified (e.g., switching X_1 and X_2).
- Type I SS are generally less useful than Type II SS unless you are specifically interested in partitioning variation among an ordered set of predictors.
- Type II SS can be considered a special case of Type I SS.
- When there is no assumption violation and the Type I and II ANOVA tables are similar, the order doesn’t matter in the marginal effect of the predictors given others. We can conclude that the predictors are independent.

Coefficients of partial determination

The relative marginal reduction in the variation in Y associated with some predictor when others are already in the model is

$$R^2_{Y2|1} = 0.232$$

$$R^2_{Y3|12} = 0.105$$

$$R^2_{Y1|2} = 0.031$$

When X2 is added to the model containing X1, the error sum of squares is reduced by 23.2%.
The error sum of squares containing both X1 and X2 is reduced by 10.5% when X3 is added.
Adding X1 to the model containing X2, the error sum of squares is reduced only by 3.1%.

Coefficients of partial determination (example 1)

The relative marginal reduction in the variation in Y associated with some predictor when others are already in the model is

$$R^2_{Y2|1} = 0.232$$

When X2 is added to the model containing X1, the **existing** error sum of squares is reduced by 23.2%. .

Model 1, Y~X1

	Df	SS	MS
x1	1	352	352
Residuals	18	143	7.9
Total	19	495	

R^2

$$R^2 = \frac{352}{495} = 71\%$$

Model 3, Y~X1+X2

	Df	SS	MS
x1	1	352	352
x2	1	33	33
Residuals	17	110	6.5
Total	19	495	

$$R^2 = \frac{385}{495} = 78\%$$

Model 4, Y~X1+X2+X3

	Df	SS	MS
x1	1	352	352
x2	1	33	33
x3	1	12	12
Residuals	16	98	6.1
Total	19	495	

$$R^2 = \frac{397}{495} = 80\%$$

$$SSE(X_1) = 143 \qquad SSE(X_2|X_1) = 33$$

$$R^2_{Y2|1} = \frac{SSE(X_2|X_1)}{SSE(X_1)} = \frac{33}{143} = 0.232$$

Coefficients of partial determination (example 2)

The relative marginal reduction in the variation in Y associated with some predictor when others are already in the model is

$R^2_{Y3|1\,2} = 0.105$ When X3 is added to the model containing X1 X2, the **existing** error sum of squares is reduced by 10.5%. .

R^2

Model 1, Y~X1

	Df	SS	MS
x1	1	352	352
Residuals	18	143	7.9
Total	19	495	

$$R^2 = \frac{352}{495} = 71\%$$

Model 3, Y~X1+X2

	Df	SS	MS
x1	1	352	352
x2	1	33	33
Residuals	17	110	6.5
Total	19	495	

$$R^2 = \frac{385}{495} = 78\%$$

Model 4, Y~X1+X2+X3

	Df	SS	MS
x1	1	352	352
x2	1	33	33
x3	1	12	12
Residuals	16	98	6.1
Total	19	495	

$$R^2 = \frac{397}{495} = 80\%$$

$$SSE(X_1, X_2) = 110 \qquad SSE(X_3|X_1X_2) = 12$$

$$R^2_{Y3|12} = \frac{SSE(X_3|X_1X_2)}{SSE(X_1, X_2)} = \frac{12}{110} = 0.105$$

Coefficients of partial determination

A coefficient of partial determination measures the marginal contribution of one X variable when all others Are already included in the model

For example, $Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \epsilon_i$

The relative marginal reduction in the variation in Y associated with X1 when X2 is already in the model is

$$R^2_{Y1|2} = \frac{SSE(X2) - SSE(X1, X2)}{SSE(X2)} = \frac{SSR(X1|X2)}{SSE(X2)}$$

$$R^2_{Y2|1} = \frac{SSE(X1) - SSE(X1, X2)}{SSE(X1)} = \frac{SSR(X2|X1)}{SSE(X1)}$$

Q: Which of following represents the relative marginal reduction in the variation in Y associated with X3 when X1 and X2 are already in the model

A) $R^2_{Y12|3} = \frac{SSR(X1\ X2|X3)}{SSE(X3)}$

B) $R^2_{Y12|3} = \frac{SSR(X3|X1\ X2)}{SSE(X1\ X2)}$

C) $R^2_{Y3|12} = \frac{SSR(X1\ X2|X3)}{SSE(X3)}$

D) $R^2_{Y3|12} = \frac{SSR(X3|X1\ X2)}{SSE(X1\ X2)}$

Type I and Type II Partial coefficient determination R^2

Partial determination can be calculated from the Type I and Type II *SS*:

- Type I Squared Partial Correlation uses Type I *SS*: $R^2 = \frac{SS1}{SS1 + SSE}$
- Type II Squared Partial Correlation uses Type II *SS*: $R^2 = \frac{SS2}{SS2 + SSE}$

Where *SS*1 and *SS*2 are the Type I and Type II sums of squares for a particular predictor variable, and *SSE* is the sum of equared error for the full model.

The partial correlation $r = \sqrt{R^2}$

Type I Coefficients of Partial Determination

- The order matters! Suppose the variables enter the model in the order of X3, X2, X1

$$R^2_{Y\ 3} = \frac{SSR(X3)}{SST} = \frac{10.05}{495} = 0.02$$

$$R^2_{Y\ 2|3} = \frac{SSR(X2|X3)}{SSE(X3)}$$

$$= \frac{SSR(X2|X3)}{SSR(X2|X3) + SSE(X3\ X2)} = \frac{374.23}{374.23 + 12.7 + 98.4} = 0.77$$

$$R^2_{Y\ 1|3\ 2} = \frac{SSR(X1|X3\ X2)}{SSE(X3\ X2)}$$

$$= \frac{SSR(X1|X3\ X2)}{SSR(X1|X3\ X2) + SSE(X3\ X2\ X1)} = \frac{12.7}{12.7 + 98.4} = 0.114$$

```
Response: y
          Df Sum Sq Mean Sq F value    Pr(>F)    
x3         1  10.05   10.05  1.6343    0.2193    
x2         1 374.23  374.23 60.8471 7.684e-07 ***
x1         1  12.70   12.70  2.0657    0.1699    
Residuals 16  98.40    6.15                      
```

- Type II coefficients of partial determination can be denoted and computed the same way as type I.
  - ➢ $R^2_{Y\ 2|3}$ is also the type II coefficient of partial determination of X2 in the MLR with just X2 and X3 predictors.
  - ➢ $R^2_{Y\ 1|3,2}$ is also the type II coefficient of partial determination of X1 in the MLR with just X1, X2 and X3 predictors.

Type II Coefficients of Partial Determination

- Measures the marginal contribution of one X variable when all other variables are already included in the model.

$$R^2_{Y\ 1|2,3} = \frac{SSR(X1|X2{,}X3)}{SSR(X1|X2{,}X3)+SSE}$$

$$R^2_{Y\ 2|1,3} = \frac{SSR(X2|X1\ X3)}{SSR(X2|X1{,}X3)+SSE}$$

$$R^2_{Y\ 3|1,2} = \frac{SSR(X3|X1\ X2)}{SSR(X3|X1{,}X2)+SSE}$$

- Type II coefficient of partial determination of a predictor (X_i) is its Type I coefficient when it entering the model last.
    - ❖ The order of other predictors entering the model doesn't matter.

Compute the Type II Coefficients of Partial Determination of X1 and X3 from the type I ANOVA table

$$R^2_{Y\ 1|2,3} = \frac{SSR(X1|X2,X3)}{SSE(X2,X3)} = \quad 0.114$$

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
x3	1	10.05	10.05	1.6343	0.2193	
x2	1	374.23	374.23	60.8471	7.684e-07	***
x1	1	12.70	12.70	2.0657	0.1699	
Residuals	16	98.40	6.15			

$$R^2_{Y\ 3|1,2} = \frac{SSR(X3|X1\ X2)}{SSE(X1,X2)} = 0.105$$

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)	
x1	1	352.27	352.27	57.2768	1.131e-06	***
x2	1	33.17	33.17	5.3931	0.03373	*
x3	1	11.55	11.55	1.8773	0.18956	
Residuals	16	98.40	6.15			

Partial Coefficient of Determination R^2, connection between type I and type II

Variable	Type I (order 1,2,3)	Type I (order 3,2,1)	Type I (order 1,3,2)	Type II
X1	0.711	0.114	0.711	0.114
X2	0.231	0.771	0.071	0.071
X3	0.105	0.02	0.26	0.105

Type II R^2 is the same as type I for a predictor when it is the last one entering the model.

Partial Correlation Coefficient r, (in population, ρ) and Coefficients of Determination (R^2)

- Correlation coefficient measures the linear association between two (continuous) variables.
- *Partial correlation* measures the strength and direction of a linear association between two continuous variables while controlling one or more other continuous variables. $r_{Y\,3} = \pm\sqrt{R^2_{Y\,3}}$ $r_{Y\,2|\,3} = \pm\sqrt{R^2_{Y\,2|3}}$ $r_{Y\,1|2,3} = \pm\sqrt{R^2_{Y\,1|2,3}}$
- In MLR, $R^2 = SSR/SST$ is the proportion of variation explained by the linear model, while the *coefficient of partial determination* for X_k measures the marginal increase in SSR that results from including X_k in the model.