

**I. Equivalent way of describing k-promotion.** Let  $D_1 : \text{Inc}^{[q]}(\lambda) \rightarrow \text{Inc}^{[q] \cup \bullet}(\lambda)$  be the map from increasing tableaux labeled with  $1, \dots, q$  to increasing tableaux labeled with  $1, \dots, q, \bullet$  that replaces all 1s with  $\bullet$ s. Let  $\text{Sw}_n$  be the operator that finds all short ribbons containing  $\bullet$  and  $n$ , leaves trivial ribbons unchanged, and switches  $n$  and  $\bullet$  in nontrivial ribbons. Let  $\text{Sw}$  be the operator that finds all instances of  $\bullet$ , determines the minimum integer  $k \leq q$  such that  $k$  labels a box directly right or below an instance of  $\bullet$ , and applies  $\text{Sw}_k$ . Let  $\text{SW}$  be the operator that applies  $\text{Sw}$  until all  $\bullet$ s have no boxes directly below or the right. Let  $\Sigma$  be the operator that cyclically permutes the labels:  $q \rightarrow q-1 \rightarrow \dots \rightarrow 1 \rightarrow q$ .

In [1], k-promotion is described as  $\Sigma \circ \text{Sw}_q \circ \dots \circ \text{Sw}_1 \circ D_1$ . We claim that this is equivalent to  $\Sigma \circ \text{SW} \circ D_1$ .  
COMPLETE

**II. k-promotion commutes with FC.** We can write  $T \in \text{Inc}^{[q] \cup \bullet}(\lambda) \rightarrow \text{Inc}^{[q] \cup \bullet}(\lambda)$  as a set of pairs  $\{(n, (i, j)) : (i, j) \in \lambda\}$  with the appropriate order constraints on  $n$ . Let  $Q(i) = n_i$ , where  $n_i$  is the  $i^{\text{th}}$  element in  $T$ , ordered lowest to highest. Define  $FC((n, (i, j))) = (Q^{-1}(n), (i, j))$ , so that  $FC(T) = \{(Q^{-1}(n), (i, j))\}$ .