T4 写基本关联矩阵

(1) 去掉 V_5

$$B_5 = \begin{array}{c}\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix} \\ \begin{array}{cccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10}. \end{array}\end{array}$$

$$|B_5 \cdot B_5^T| = \begin{vmatrix} -1 & 0 & -1 & 3 \\ 0 & -1 & 4 & -1 \\ -2 & 4 & -1 & 0 \\ 4 & -2 & 0 & -1 \end{vmatrix} = 101\text{个}$$

(2) 去掉 e_3:

$$B_5' = \begin{array}{c}\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 1 & -1 & -1 \end{pmatrix} \\ \begin{array}{ccccccccc} e_1 & e_2 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 & e_{10} \end{array}\end{array}.$$

$$|B_5' \cdot B_5'^T| = \begin{vmatrix} -1 & 0 & -1 & 2 \\ 0 & -1 & 4 & -1 \\ -2 & 4 & -1 & 0 \\ 4 & -2 & 0 & -1 \end{vmatrix} = 57\text{个}$$

$$\therefore n(e_3) = 101 - 57 = 44\text{个}$$

(3) 去掉 e_8:

$$B_5'' = \begin{array}{c}\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{pmatrix} \\ \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_9 & e_{10}. \end{array}\end{array}$$

$$|B_5'' \cdot B_5''^T| = \begin{vmatrix} -1 & 0 & -1 & 3 \\ 0 & -1 & 4 & -1 \\ -2 & 4 & -1 & 0 \\ 3 & -2 & 0 & -1 \end{vmatrix} = 60\text{个}$$

$\therefore$ 不含 e_8 的有60个

T_5 写 $B_1 = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -1 & -1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$.

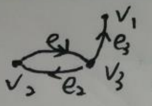
(1). $\overrightarrow{B_1} \cdot B_1^T = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -2 \\ 0 & -1 & 3 & -1 \\ -2 & 0 & 0 & 3 \end{vmatrix} = 27-3=24$个

(2) 划掉 e_3, $B_1' = \begin{pmatrix} -1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -1 & 0 & 1 & -1 & 0 & 0 \end{pmatrix}$.

$\overrightarrow{B_1'} \cdot B_1'^T = \begin{vmatrix} 1 & 0 & 0 & 0 \\ -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 \\ -2 & 0 & 0 & 2 \end{vmatrix} = 18-2=16$个.

(3). 划掉 e_6. $B_1'' = \begin{pmatrix} -1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & -1 & -1 & -1 & 1 & 1 & 0 & 0 \end{pmatrix}$

$\overrightarrow{B_1''} \cdot B_1''^T = 18-3=15$个　　$\therefore n(e_6)=24-15=9$个

T9 考虑图 

，考察其对于 v_1 的基本关联矩阵：$B_1 = \begin{array}{c} \begin{pmatrix}1&-1&0\\-1&1&1\end{pmatrix} \\ \;e_1\;\; e_2\;\; e_3 \end{array}$

$\overline{B_1} = \begin{pmatrix}0&-1&0\\-1&0&0\end{pmatrix}$. $\left|\overline{B_1}\cdot\overline{B_1}^T\right| = \left|\begin{pmatrix}0&-1&0\\-1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&-1\\-1&0\\0&0\end{pmatrix}\right| = \begin{vmatrix}1&0\\0&1\end{vmatrix} = 1$

$\left|\overline{B_1}\cdot B_1^T\right| = \left|\begin{pmatrix}0&-1&0\\-1&0&0\end{pmatrix}\cdot\begin{pmatrix}1&-1\\-1&1\\0&1\end{pmatrix}\right| = \begin{vmatrix}1&-1\\-1&1\end{vmatrix} = 0$. 但由图，以 v_1 为根的树数目明显为0. 故例子成立.

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T_{11}：基本回路矩阵：$\begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$.

高斯消元有 $C_f = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 & 1 \end{pmatrix}$.

$$S_{f_{11}} = -C_{f_{12}}^T = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 1 \\ 0 & -1 & 1 & -1 \\ 0 & -1 & 0 & -1 \end{pmatrix}$$

$$\therefore \quad S_f = \begin{pmatrix} 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

T13.

考虑去除任一结点 V 的所有边：由题意，这一定恰好只产生了两个连通支，即余下部分和结点 V。

则若少去掉一条边，该图重新连通。

故每一个点其相关的所有边构成割集，且"1"数目为偶

则 $d(V_i)$ 为偶数．由任意性知存在Euler回路