1. $f(t) = e^{-at} \cdot u(t) = \begin{cases} e^{-at}, & t>0 \\ 0, & t\le 0. \end{cases} \quad (a>0)$

Ft: $\int_{-\infty}^{\infty} e^{-at} \cdot u(t) \cdot e^{-j\omega t} dt = \int_{0}^{\infty} e^{-(a+j\omega)t} dt$

$= \frac{1}{a+j\omega}$. $\therefore F(\omega) = \frac{1}{a+j\omega}$. $(a>0)$.

2. $F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt = \int_{0}^{\tau} t \cdot e^{-j\omega t} dt + \int_{\tau}^{2\tau} \tau \cdot e^{-j\omega t} dt.$

$= \left(-\frac{t}{j\omega} + \frac{1}{\omega^2}\right) e^{-j\omega t} \Big|_{0}^{\tau} + \frac{\tau}{-j\omega} e^{-j\omega t} \Big|_{\tau}^{2\tau}$

$= -\frac{\tau}{j\omega} e^{-2j\omega\tau} + \frac{1}{\omega^2} e^{-j\omega\tau} - \frac{1}{\omega^2}.$

3.

(1) $F(\omega) = \int_{-\infty}^{\infty} e^{\frac{-t^2}{20}} \cdot e^{-j\omega t} dt = \sqrt{20} \int_{-\infty}^{\infty} e^{-m^2 - 2\sqrt{5} j\omega m} dm.$

$= \sqrt{20} \int_{-\infty}^{\infty} e^{-(m+\sqrt{5} j\omega)^2 - 5\omega^2} dm$

$= 2\sqrt{5\pi} \cdot e^{-5\omega^2}.$

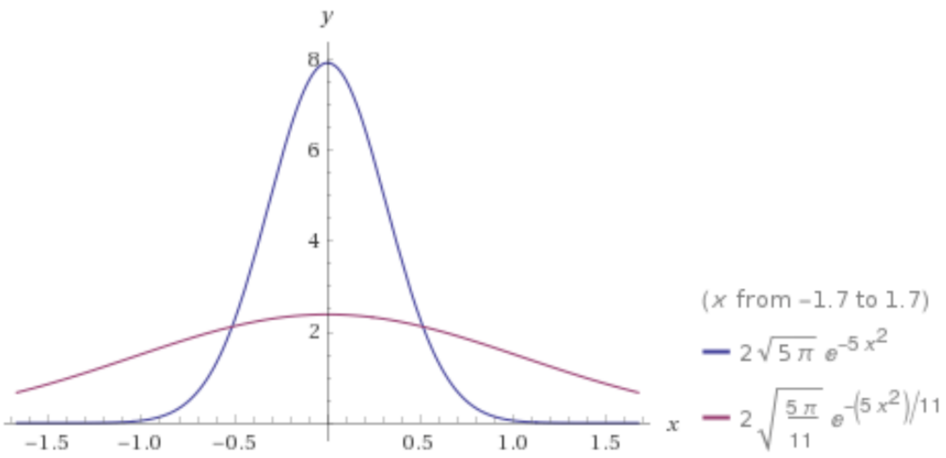
(2) $F(\omega) = \int_{-\infty}^{\infty} e^{\frac{-11}{20}t^2} \cdot e^{-j\omega t} dt = \sqrt{\frac{20}{11}} \int_{-\infty}^{\infty} e^{-m^2 - 2\sqrt{\frac{5}{11}} j\omega m} dm$

$= \sqrt{\frac{20}{11}} \int_{-\infty}^{\infty} e^{-(m+\sqrt{\frac{5}{11}} j\omega)^2 - \frac{5}{11}\omega^2} dm$

$= 2\sqrt{\frac{5\pi}{11}} \cdot e^{-\frac{5}{11}\omega^2}.$

plot	$2\sqrt{5\pi}\ e^{-5x^2}$
	$2\sqrt{5\times\frac{\pi}{11}}\ e^{-5/11\,x^2}$

Plots:



后一个函数是前一个函数被两边朝向远离原点方向的力拉伸后的结果。