（科目：离散） 数 学 作 业 纸

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T1 (3) $(\forall x)(P(x)\vee q)\to(\exists x)(P(x)\wedge q) = \neg((\forall x)(P(x)\vee q))\vee((\exists x)(P(x)\wedge q))$

$= ((\exists x)\neg(P(x)\vee q))\vee((\exists x)P(x)\wedge q) = ((\exists x)(\neg P(x)\wedge\neg q))\vee((\exists x)P(x)\wedge q)$

$= ((\exists x)\neg P(x)\wedge\neg q)\vee((\exists x)P(x)\wedge q)$

(4). $(\forall y)(\exists x)((P(x)\to q)\vee S(y)) = (\exists x)(P(x)\to q)\vee(\forall y)S(y)$

$= (\exists x)(\neg P(x)\vee q)\vee(\forall y)S(y) = ((\exists x)\neg P(x)\vee q)\vee(\forall y)S(y)$

$= (\neg(\forall x)P(x)\vee q)\vee(\forall y)S(y) = ((\forall x)P(x)\to q)\vee(\forall y)S(y)$.

(5). $(\forall x)P(x)\to q = \neg(\forall x)P(x)\vee q = (\exists x)\neg P(x)\vee q = (\exists x)(\neg P(x)\vee q) = (\exists x)(P(x)\to q)$

(6). $(\exists x)(P(x)\to Q(x)) = (\exists x)(\neg P(x)\vee Q(x)) = (\exists x)\neg P(x)\vee(\exists x)Q(x)$

$= \neg(\forall x)P(x)\vee(\exists x)Q(x) = (\forall x)P(x)\to(\exists x)Q(x)$.

(7). $(\exists x)P(x) \to (\forall x)Q(x) = \neg(\exists x)P(x) \vee (\forall x)Q(x) = ((\forall x)\neg P(x)) \vee (\forall x)Q(x)$

$\Rightarrow (\forall x)(\neg P(x) \vee Q(x)) = (\forall x)(P(x) \to Q(x))$

(8). $(\exists x)P(x) \wedge (\forall x)Q(x) = (\exists x)P(x) \wedge (\forall y)Q(y) = (\exists x)(P(x) \wedge (\forall y)Q(y))$

$\Rightarrow (\exists x)(P(x) \wedge Q(x))$

(9). $((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)R(x)) \vee ((\forall x)P(x) \wedge (\forall x)Q(x) \wedge (\exists x)S(x))$

$= ((\forall x)P(x) \wedge (\forall x)Q(x)) \wedge ((\exists x)R(x) \vee (\exists x)S(x))$

$= (\forall x)(P(x) \wedge Q(x)) \wedge (\exists x)(R(x) \vee S(x))$

T2

(1) 不是普遍有效. 可在 $\{1,2\}$ 域上分析. 若 $P(1)=T$, $P(2)=Q(1)=Q(2)=F$

则 $(\exists x)(P(x)\leftrightarrow Q(x))$ 为 T, $((\exists x)P(x)\leftrightarrow(\exists x)(Q(x))$ 为 F.

(2) 不是普遍有效

(3) 普遍有效

(4) 不是普遍有效.

(5) 普遍有效.

$((\exists x)P(x)\rightarrow(\exists x)Q(x))\rightarrow(\exists x)(P(x)\rightarrow Q(x))$

$=(\neg(\exists x)P(x)\vee(\exists x)Q(x))\rightarrow(\exists x)(\neg P(x)\vee Q(x))$

$=\neg(\neg(\exists x)P(x)\vee(\exists x)Q(x))\vee(\exists x)\neg P(x)\vee(\exists x)Q(x)$

$=((\exists x)P(x)\wedge\neg(\exists x)Q(x))\vee(\exists x)\neg P(x)\vee(\exists x)Q(x)$

$=(((\exists x)P(x)\vee(\exists x)\neg P(x))\wedge(\neg(\exists x)Q(x)\vee(\exists x)\neg P(x)))\vee(\exists x)Q(x)$

$=\underline{\neg(\exists x)Q(x)}\vee(\exists x)\neg P(x)\vee\underline{(\exists x)Q(x)}=T\vee(\exists x)\neg P(x)=T.$

(6) 不是普遍有效 (7) 不是普遍有效 (8) 不是普遍有效.