1. 由DTFT，理想抽样信号的频谱密度函数为 $\hat{F}(w) = \sum_{n=-\infty}^{\infty} f(nT)\, e^{-jnwT}$

同时，$\hat{F}(w) = \frac{1}{2\pi} F(w) \cdot P(w) = \frac{1}{2\pi} \cdot F(w) * \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(w - nw_0)$

$= \frac{1}{T} \sum_{n=-\infty}^{\infty} F(w - nw_0)$

令 $w=0$，有 $\sum_{n=-\infty}^{\infty} f(nT) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(-nw_0) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F(nw_0)$

故 $T \sum_{n=-\infty}^{\infty} f(nT) = \sum_{n=-\infty}^{\infty} F(nw_0)$. 证毕.

2.

a) $D[x^*(-n)] = X^*(w)$. 故 $D[x(n) * x^*(-n)] = X(w) \cdot X^*(w) = |X(w)|$

b) $D[x(2n)] = \sum_{n=-\infty}^{\infty} x(2n) e^{-jwn}$. 令 $2n=k$，则 k 取偶数. 此时 $x(2n)$ 可写成

$[x(k) + (-1)^k x(k)] \cdot \frac{1}{2}$. 故 $D[x(2n)] = \frac{1}{2} \sum_{k=-\infty}^{\infty} [x(k) + (-1)^k x(k)] \cdot e^{-jw\frac{k}{2}}$

$= \frac{1}{2} X(\frac{w}{2}) + \frac{1}{2} \sum_{k=-\infty}^{\infty} x(k) \cdot e^{-j\frac{w}{2}k} \cdot e^{jk\pi} = \frac{1}{2} [X(\frac{w}{2}) + X(\frac{w}{2} - \pi)]$

故 $D[x(2n+1)] = \frac{1}{2} e^{jw} [X(\frac{w}{2}) + X(\frac{w}{2} - \pi)]$.

c): $D[x(n) - x(n-2)] = D[x(n)] - D[x(n-2)] = X(w) - e^{-jw \cdot 2} \cdot X(w)$

$= (1 - e^{-2jw}) X(w)$

d): $D[x(n) * x(n-1)] = D[x(n)] \cdot D[x(n-1)] = e^{-jw} \cdot X^2(w)$.

3. $D[x(n)] = X(w) = \sum_{n=-\infty}^{+\infty} x(n) \cdot e^{-jnw}$；当且仅当 $n = kL$ 时，$y(n) = x(k)$.

故 $D[y(n)] = Y(w) = \sum_{k=-\infty}^{\infty} y(kL) e^{-jkLw} = \sum_{k=-\infty}^{\infty} x(k) e^{-jkLw} = X(Lw)$. 证毕.