Stochastic Restoration Approaches

- Wiener Filter
- Bayesian Formulation
  - Maximum Likelihood (ML)
  - Maximum *a Posteriori* (MAP)
  - Hierarchical Bayesian

On Stochastic Restoration

- Probability, Random Variables, Random or Stochastic Processes (Random Fields)
- Auto- and cross-correlation, stationary fields

isotropic exponential decay

$R(n_1,n_2) = C \cdot \gamma^{-(|n_1|+|n_2|)}$

complex conjugate

Autocorrelation: $R_{ff}(i,j,k,l) = E[f(i,j)f^*(k,l)]$

WSS: $R_{ff}(i,j,k,l) = R_{ff}(i-k, j-l) = R_{ff}(n_1,n_2)$

- Ergodicity

$$R_{ff}(n_1,n_2) = \lim_{N\to\infty} \frac{1}{(2N+1)^2} \sum_{k_1=-N}^{N} \sum_{k_2=-N}^{N} f(k_1,k_2)f^*(k_1-n_1,k_2-n_2)$$

- Power-Spectrum $\quad P_{ff}(\omega_1,\omega_2) = \mathcal{F}\{R_{ff}(n_1,n_2)\}$

Non-Causal Wiener Restoration Filter

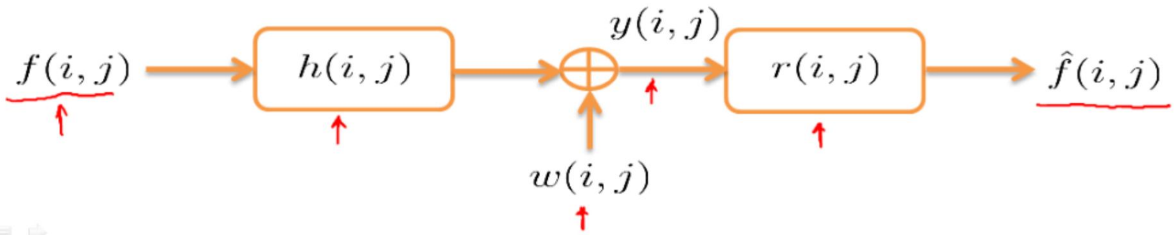
Degradation Model

$$y(i,j) = h(i,j) * *f(i,j) + w(i,j)$$

Objective

$$\hat{f}(i,j) = argmin_{f(i,j)} E\left[|e(i,j)|^2\right] = argmin_{f(i,j)} E\left[|f(i,j) - \hat{f}(i,j)|^2\right]$$

Consider an LSI restoration filter: $\hat{f}(i,j) = r(i,j) * *y(i,j)$



Random Signals and LSI Systems

$f(i,j)$ is WSS with autocorrelation $R_{ff}(i,j)$

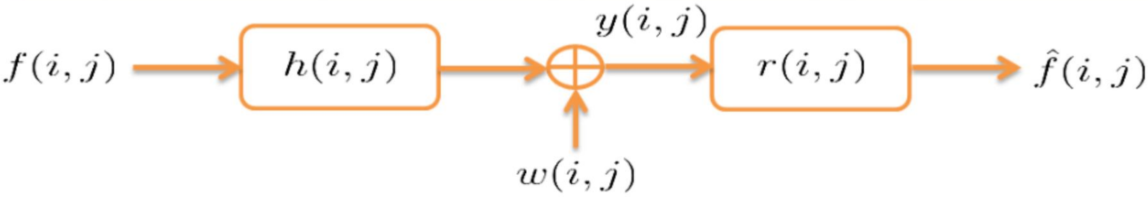
$$R_{yy}(i,j) = R_{ff}(i,j) * *\overset{H(\omega_1,\omega_2)}{h(i,j)} * *\overset{H^*(\omega_1,\omega_2)}{h^*(-i,-j)}$$

$$P_{yy}(\omega_1,\omega_2) = |H(\omega_1,\omega_2)|^2 \cdot P_{ff}(\omega_1,\omega_2)$$

$$R_{fy}(i,j) = R_{ff}(i,j) * *h^*(-i,-j) \longleftrightarrow P_{fy}(\omega_1,\omega_2) = H^*(\omega_1,\omega_2) \cdot P_{ff}(\omega_1,\omega_2)$$

$$R_{yf}(i,j) = R_{ff}(i,j) * *h(i,j) \longleftrightarrow P_{yf}(\omega_1,\omega_2) = H(\omega_1,\omega_2) \cdot P_{ff}(\omega_1,\omega_2)$$

Wiener Restoration Filter



Assumption: $f(i,j)$ and $w(i,j)$ (and therefore $y(i,j)$) are WSS

Solution: **Orthogonality Principle** $\quad \text{error} \perp \text{data}$

$$E[e(i,j)y^*(k,l)] = 0, \quad \forall (i,j),(k,l) \quad \text{or} \quad E[(f(i,j) - \hat{f}(i,j))y^*(k,l)] = 0$$

$$E[f(i,j)\,y^*(k,l)] = E[\hat{f}(i,j)\,y^*(k,l)] = E[y(i,j) ** r(i,j) \cdot y^*(k,l)]$$

$$R_{fy}(i,j) = R_{yy}(i,j) ** r(i,j) \quad \longleftrightarrow \quad R(\omega_1,\omega_2) = \frac{P_{fy}(\omega_1,\omega_2)}{P_{yy}(\omega_1,\omega_2)}$$

Wiener Restoration Filter

Common Assumptions:

(1) $f(i,j)$ and $w(i,j)$ are uncorrelated

$$E[f(i,j)w^*(i,j)] = E[f(i,j)] \cdot E[w^*(i,j)]$$

(2) Both $f(i,j)$ and $w(i,j)$ are zero mean

v $\rightarrow$ $P_{fy}(\omega_1,\omega_2) = H^*(\omega_1,\omega_2) \cdot P_{ff}(\omega_1,\omega_2)$

v $\rightarrow$ $P_{yy}(\omega_1,\omega_2) = |H(\omega_1,\omega_2)|^2 \cdot P_{ff}(\omega_1,\omega_2) + P_{ww}(\omega_1,\omega_2)$

$$\rightarrow \boxed{R(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2) \cdot P_{ff}(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 \cdot P_{ff}(\omega_1,\omega_2) + P_{ww}(\omega_1,\omega_2)}}$$

Wiener vs CLS Restoration Filters

Wiener

Noise white

$$R(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 + \frac{P_{ww}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)}} = \frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 + \frac{\sigma_{ww}^2}{P_{ff}(\omega_1,\omega_2)}}$$

$P_{ff}(\omega_1,\omega_2) \geq 0, \ \forall(\omega_1,\omega_2)$

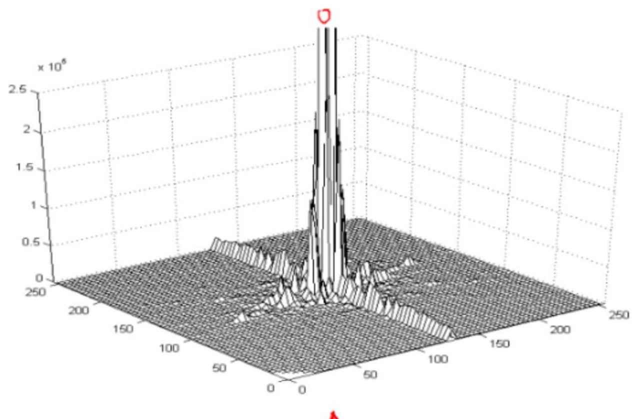
Constrained Least Squares

$$R_{CLS}(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 + \alpha|C(\omega_1,\omega_2)|^2}$$

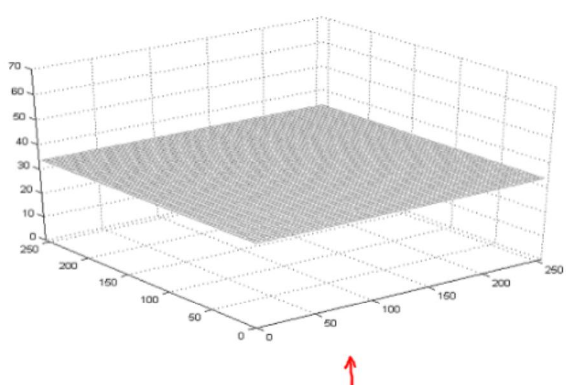
$$\begin{cases} \cdot\ \alpha = \sigma_{ww}^2 \\ \cdot\ |C(\omega_1,\omega_2)|^2 = \frac{1}{P_{ff}(\omega_1,\omega_2)} \end{cases}$$

Example of Power Spectrum of Original Image and Noise

$P_{ff}(\omega_1,\omega_2)$

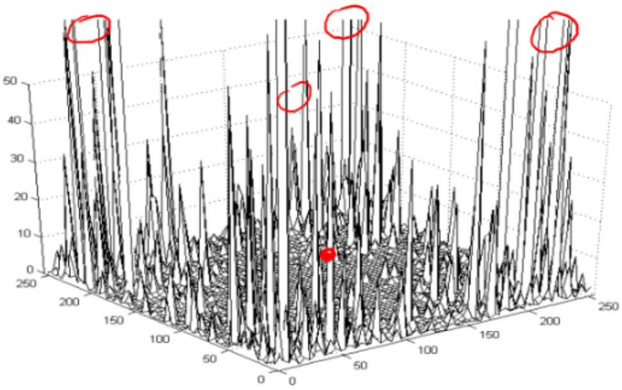


$P_{ww}(\omega_1,\omega_2)$

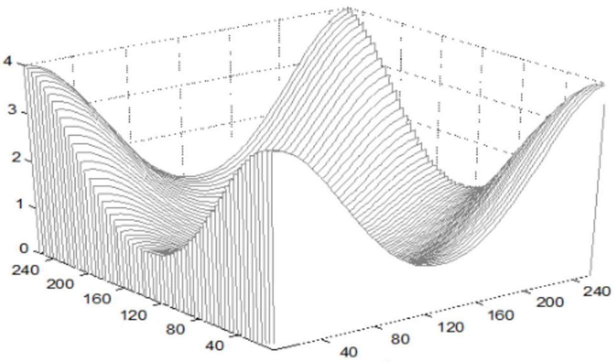


Example of Stabilizing Term

$$\frac{P_{ww}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)} \leftarrow \sigma_{ww}^2$$

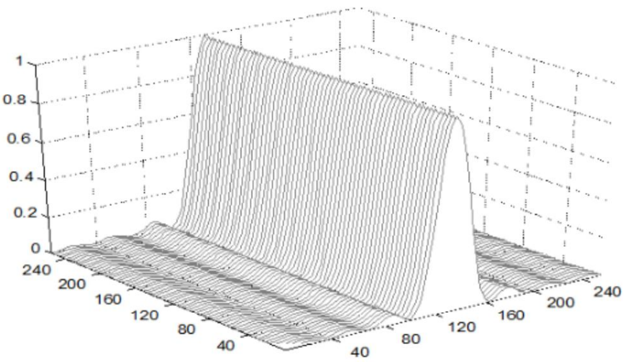


$|C(\omega_1,\omega_2)|^2$



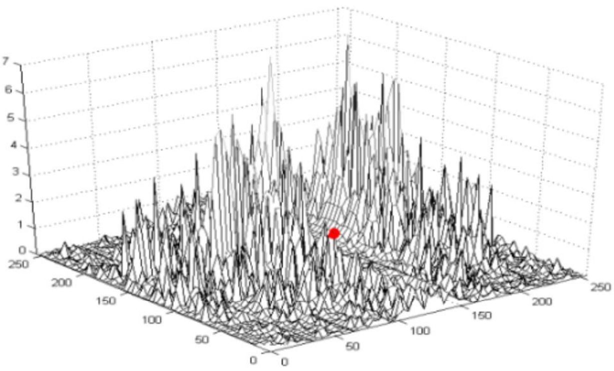
Motion Blur over 8 Pixels

$|H(\omega_1,\omega_2)|^2$

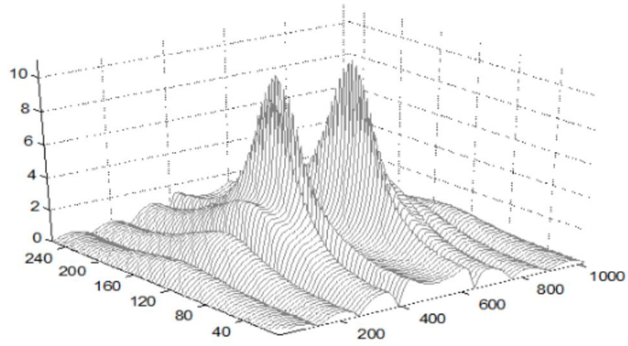


Wiener vs CLS Filter

$R_{WIENER}(\omega_1,\omega_2)$



$R_{CLS}(\omega_1,\omega_2)$

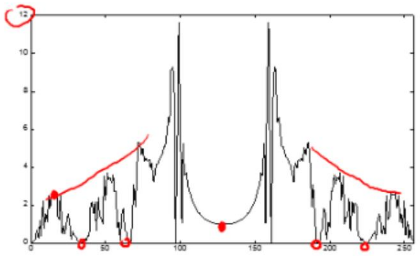


$\alpha = 0.1$

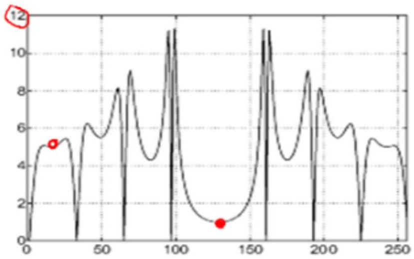
Wiener vs CLS Filter

$w_2=0$

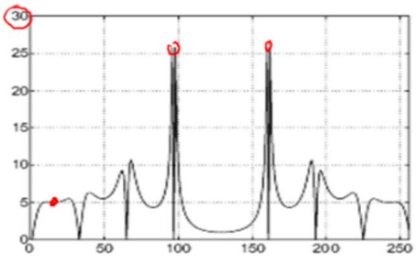
$R_{WIENER}(\omega_1,0)$



$R_{CLS_{iterative}}(\omega_1,0)$



$R_{CLS_{direct}}(\omega_1,0)$



Wiener restoration;
$P_{ff}(\omega_1,\omega_2)$ from original image,
$P_{ww}(\omega_1,\omega_2)$ ideal

Iterative CLS restoration
with C a 2D Laplacian,
α=0.01, k=330

Direct CLS restoration
with C a 2D Laplacian and
α=0.01

Wiener vs CLS Filter

Noisy-blurred image;
1D motion blur over 8 pixels;
BSNR=20dB.

Wiener restoration;
$P_{ff}(\omega_1, \omega_2)$ from original image,
$P_{ww}(\omega_1, \omega_2)$ ideal, ISNR=3.93dB

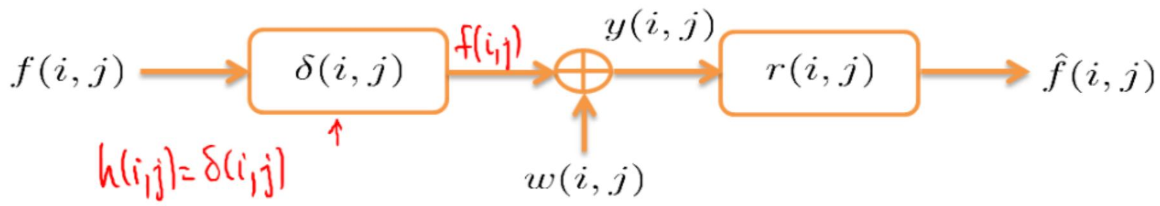
Wiener vs CLS Filter

Wiener restoration;
$P_{ff}(\omega_1, \omega_2)$ from original image,
$P_{ww}(\omega_1, \omega_2)$ ideal, ISNR=3.93dB

Iterative CLS restoration
with C a 2D Laplacian,
α =0.01, k=330, ISNR=-1.01dB

Direct CLS restoration
with C a 2D Laplacian and
α =0.01, ISNR=-1.64dB

Wiener Noise Smoothing Filter



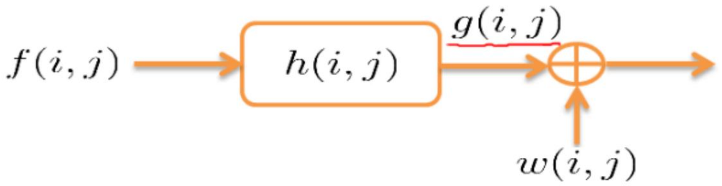
$h(i,j) = \delta(i,j)$

$\longleftrightarrow \quad H(\omega_1,\omega_2) = 1$

$$y(i,j) = f(i,j) + w(i,j)$$

$$R(\omega_1,\omega_2) = \frac{1}{1+\frac{P_{ww}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)}} = \frac{P_{ff}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)+P_{ww}(\omega_1,\omega_2)}$$

Wiener Restoration Filter



$$y(i,j) = g(i,j) + w(i,j)$$

$$P_{gg}(\omega_1,\omega_2) = |H(\omega_1,\omega_2)|^2 \cdot P_{ff}(\omega_1,\omega_2)$$

$|H(\omega_1,\omega_2)|^2$

$$R(\omega_1,\omega_2) = \frac{H^*(\omega_1,\omega_2)\cdot P_{ff}(\omega_1,\omega_2)}{\underbrace{|H(\omega_1,\omega_2)|^2\cdot P_{ff}(\omega_1,\omega_2)}_{P_{gg}(\omega_1,\omega_2)}+P_{ww}(\omega_1,\omega_2)} \cdot \frac{H(\omega_1,\omega_2)}{H(\omega_1,\omega_2)}$$

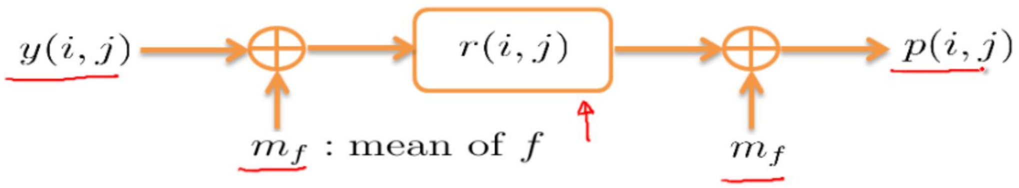
$$= \underbrace{\frac{P_{gg}(\omega_1,\omega_2)}{P_{gg}(\omega_1,\omega_2)+P_{ww}(\omega_1,\omega_2)}}_{\text{Noise Smoothing Wiener}} \cdot \underbrace{\frac{1}{H(\omega_1,\omega_2)}}_{\text{Inverse}}$$

Spatially Adaptive Wiener Noise Smoothing Filter

$$y(i,j) = f(i,j) + w(i,j)$$

$$E[w(i,j)] = 0, \quad P_{ww}(\omega_1,\omega_2) = \sigma_w^2$$

$$R(\omega_1,\omega_2) = \frac{P_{ff}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2) + \sigma_w^2}$$



Other Restoration Filters

$\alpha = 1/2 \qquad 0 \le \alpha \le 1$

Geometric Mean Filter

$$R(\omega_1,\omega_2) = \left(\frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2}\right)^{\alpha} \left(\frac{H^*(\omega_1,\omega_2)}{|H(\omega_1,\omega_2)|^2 + \gamma \frac{P_{ww}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)}}\right)^{1-\alpha}$$

Spatially Adaptive Wiener Noise Smoothing Filter

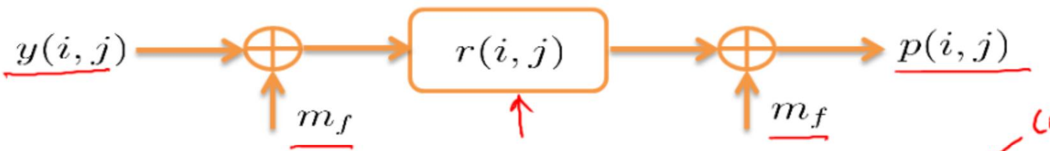
Divide image into stationary regions

For each region use the following model: $f(i,j) = m_f + \sigma_f \cdot v(i,j)$

local mean; st. dev.

$E[v(i,j)]=0,\ \sigma_v^2=1$

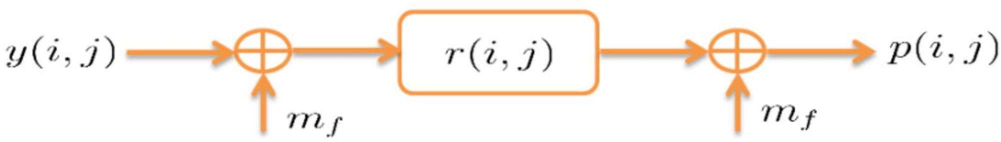
$$R(\omega_1,\omega_2) = \frac{P_{ff}(\omega_1,\omega_2)}{P_{ff}(\omega_1,\omega_2)+\sigma_w^2} = \frac{\sigma_f^2}{\sigma_f^2+\sigma_w^2} \longleftrightarrow r(i,j) = \frac{\sigma_f^2}{\sigma_f^2+\sigma_w^2}\cdot\delta(i,j)$$



(i,j).

$$p(i,j) = (y(i,j)-m_f) ** r(i,j) + m_f = (y(i,j)-m_f)\frac{\sigma_f^2}{\sigma_f^2+\sigma_w^2} + m_f$$

Spatially Adaptive Wiener Noise Smoothing Filter



$$\boxed{p(i,j) = \frac{\sigma_f^2(i,j)}{\sigma_f^2(i,j)+\sigma_w^2}\cdot y(i,j) + \frac{\sigma_w^2}{\sigma_f^2(i,j)+\sigma_w^2}\cdot m_f(i,j)}$$

1) $\sigma_w^2 = 0$ $\quad p(i,j) = y(i,j)$

2) edges: $\sigma_f^2(i,j) \gg \sigma_w^2$

$p(i,j) \approx 1 \cdot y(i,j) + 0 \cdot m_f(i,j) \approx y(i,j)$

3) flat regions: $\sigma_f^2(i,j) \ll \sigma_w^2$

$p(i,j) \approx 0 \cdot y(i,j) + 1 \cdot m_f(i,j) \approx m_f(i,j)$

$m_f(i,j) \simeq m_y(i,j)$

$\sigma_w^2 = ?$

$y = f + w$

$\sigma_y^2 = \sigma_f^2 + \sigma_w^2$

flat region: $\sigma_f^2 \simeq 0$

$\sigma_y^2 \simeq \sigma_w^2$

$\sigma_f^2 = \sigma_y^2 - \sigma_w^2$

Stochastic Restoration Approaches

- Wiener Filter
- Bayesian Formulation
  - Maximum Likelihood (ML)
  - Maximum *a Posteriori* (MAP)
  - Hierarchical Bayesian

Observation Model

$$\mathbf{y} = \mathbf{Hf} + \mathbf{w} \rightarrow w = y - Hf$$

Normal or Gaussian noise

covariance matrix

$$\mathbf{p}(\mathbf{w}) = \mathbf{p}(\mathbf{y}|\mathbf{f}) = \frac{1}{(2\pi)^{N/2}|\mathbf{C}_{\mathbf{ww}}|^{1/2}} \exp\left[-\frac{1}{2}(\mathbf{y}-\mathbf{Hf})^T \mathbf{C}_{\mathbf{ww}}^{-1}(\mathbf{y}-\mathbf{Hf})\right]$$

determinant

White Gaussian noise

$$C_{ww} = \beta^{-1} I \rightarrow C_{ww}^{-1} = \beta I$$

$$|C_{ww}| = \beta^{-N}$$

$$\mathbf{p}(\mathbf{y}|\mathbf{f}) \propto \beta^{N/2} \exp\left[-\frac{1}{2}\beta\|\mathbf{y}-\mathbf{Hf}\|^2\right]$$

proportional

Image Prior Model

Simultaneous Autoregressive (SAR) Image Prior

$$\mathbf{p}(\mathbf{f}) \propto \alpha^{\mathbf{N}/2} \exp\left[-\frac{\mathbf{1}}{\mathbf{2}}\alpha\|\mathbf{Cf}\|^2\right] = \alpha^{\mathbf{N}/2} \exp\left[-\frac{\mathbf{1}}{\mathbf{2}}\alpha\mathbf{f}^{\mathbf{T}}\mathbf{C}^{\mathbf{T}}\mathbf{Cf}\right]$$

$C_{ff}^{-1} = \alpha C^T C$

Joint Distribution

$$\mathbf{p}(\mathbf{f},\mathbf{y}) = \mathbf{p}(\mathbf{y}|\mathbf{f})\mathbf{p}(\mathbf{f})$$
$$\mathbf{p}(\mathbf{y},\mathbf{f}) = \mathbf{p}(\mathbf{f}|\mathbf{y})\mathbf{p}(\mathbf{y})$$

likelihood

Bayes' theorem

prior

Inference based on the posterior: $\mathbf{p}(\mathbf{f}|\mathbf{y}) = \dfrac{\mathbf{p}(\mathbf{y}|\mathbf{f})\mathbf{p}(\mathbf{f})}{\mathbf{p}(\mathbf{y})}$

Maximum likelihood (ML) estimate: $\hat{\mathbf{f}}_{ML} = \arg\max_{\mathbf{f}} \mathbf{p}(\mathbf{y}|\mathbf{f})$

Maximum *a posteriori* (MAP) estimate: $\hat{\mathbf{f}}_{MAP} = \arg\max_{\mathbf{f}} \mathbf{p}(\mathbf{y}|\mathbf{f})\mathbf{p}(\mathbf{f})$

ML Restoration

Gaussian noise: $$\mathbf{p}(\mathbf{y}|\mathbf{f}) \propto \exp\left[-\frac{1}{2}(\mathbf{y}-\mathbf{Hf})^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}(\mathbf{y}-\mathbf{Hf})\right]$$

Maximum Likelihood (ML) estimate:

$$\hat{\mathbf{f}}_{ML} = \arg\max_{\mathbf{f}} \underline{\mathbf{p}(\mathbf{y}|\mathbf{f})} = \arg\max_{\mathbf{f}} \underline{\log \mathbf{p}(\mathbf{y}|\mathbf{f})}$$
$$= \arg\max_{\mathbf{f}} \left[-\frac{1}{2}(\mathbf{y}-\mathbf{Hf})^{\mathbf{T}}\underline{\mathbf{C}_{\mathbf{ww}}^{-1}}(\mathbf{y}-\mathbf{Hf})\right]$$
$$= \arg\min_{\mathbf{f}} \|\mathbf{y}-\mathbf{Hf}\|^2_{\underline{\mathbf{C}_{\mathbf{ww}}^{-1/2}}}$$

generalized inverse

$$\hat{\mathbf{f}}_{ML} = (\mathbf{H}^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}\mathbf{H})^{+}\mathbf{H}^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}\mathbf{y}$$

White Gaussian noise: $\mathbf{C}_{\mathbf{ww}} = \beta^{-1}\mathbf{I} \implies \underline{\hat{\mathbf{f}}_{ML} = (\mathbf{H}^{\mathbf{T}}\mathbf{H})^{+}\mathbf{H}^{\mathbf{T}}\mathbf{y}}$

variance

MAP Restoration

Gaussian noise: $$\mathbf{p}(\mathbf{y}|\mathbf{f}) \propto \exp\left[-\frac{1}{2}(\mathbf{y}-\mathbf{Hf})^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}(\mathbf{y}-\mathbf{Hf})\right]$$ ✓

SAR image prior: $$\mathbf{p}(\mathbf{f}) \propto \exp\left[-\frac{1}{2}\alpha\|\mathbf{Cf}\|^2\right]$$ $\underline{C_{ff}^{-1} = \alpha C^T C}$ ✓

Maximum *a posteriori* (MAP) estimate:

$$\hat{\mathbf{f}}_{MAP} = \arg\max_{\mathbf{f}} \underline{\log \mathbf{p}(\mathbf{y}|\mathbf{f})\mathbf{p}(\mathbf{f})} = \arg\min_{\mathbf{f}} \left[\|\mathbf{y}-\mathbf{Hf}\|^2_{\underline{\mathbf{C}_{\mathbf{ww}}^{-1/2}}} + \alpha\|\mathbf{Cf}\|^2\right]$$

$$\boxed{\hat{\mathbf{f}}_{MAP} = (\mathbf{H}^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}\mathbf{H} + \alpha\mathbf{C}^{\mathbf{T}}\mathbf{C})^{+}\mathbf{H}^{\mathbf{T}}\mathbf{C}_{\mathbf{ww}}^{-1}\mathbf{y}}$$

White Gaussian noise: $\mathbf{C}_{\mathbf{ww}} = \beta^{-1}\mathbf{I} \implies \boxed{\hat{\mathbf{f}}_{MAP} = (\mathbf{H}^{\mathbf{T}}\mathbf{H} + \frac{\alpha}{\beta}\mathbf{C}^{\mathbf{T}}\mathbf{C})^{+}\mathbf{H}^{\mathbf{T}}\mathbf{y}}$

Noise variance

λ

Comments

- The ML and MAP optimization may not result in "nice" closed form solutions; it clearly depend on the form of the noise model and the image prior
- The image and noise covariances (or parameters α and β in the simpler case) were assumed to be known
- The blur function was also assumed to be known
- One way to address the last two points is through the hierarchical Bayesian paradigm

Hierarchical Bayesian Paradigm

Form joint distribution likelihood

$$\mathbf{p}(\mathbf{f}, \mathbf{h}, \boldsymbol{\Omega}, \mathbf{y}) = \underbrace{\mathbf{p}(\mathbf{y}|\mathbf{f}, \mathbf{h}, \boldsymbol{\Omega})}_{\beta}\underbrace{\mathbf{p}(\mathbf{f}|\boldsymbol{\Omega})}_{\alpha}\underbrace{\mathbf{p}(\mathbf{h}|\boldsymbol{\Omega})}_{\gamma}\mathbf{p}(\boldsymbol{\Omega})$$

$\Omega = \{\alpha, \beta, \gamma\}$

Inference on $\mathbf{f}, \mathbf{h},$ and Ω is based on

$$\mathbf{p}(\mathbf{f}, \mathbf{h}, \boldsymbol{\Omega}|\mathbf{y}) = \frac{\mathbf{p}(\mathbf{y}|\mathbf{f}, \mathbf{h}, \boldsymbol{\Omega})\mathbf{p}(\mathbf{f}|\boldsymbol{\Omega})\mathbf{p}(\mathbf{h}|\boldsymbol{\Omega})\mathbf{p}(\boldsymbol{\Omega})}{\mathbf{p}(\mathbf{y})}$$

Ω	Hyperparameters
$\mathbf{p}(\mathbf{y}\|\mathbf{f}, \mathbf{h}, \boldsymbol{\Omega})$	Observation model
$\mathbf{p}(\mathbf{f}\|\boldsymbol{\Omega})$	Image prior
$\mathbf{p}(\mathbf{h}\|\boldsymbol{\Omega})$	Blur prior
$\mathbf{p}(\boldsymbol{\Omega})$	Hyperprior

Bayesian Inference Methods

Maximum Likelihood (ML) estimate

$$\{\hat{\mathbf{f}}, \hat{\mathbf{h}}, \hat{\Omega}\}_{ML} = \arg\max_{\mathbf{f},\mathbf{h},\mathbf{\Omega}} \mathbf{p}(\mathbf{y}|\mathbf{f},\mathbf{h},\mathbf{\Omega})$$

Maximum *a posteriori* (MAP) estimate

$$\{\hat{\mathbf{f}}, \hat{\mathbf{h}}, \hat{\Omega}\}_{MAP} = \arg\max_{\mathbf{f},\mathbf{h},\mathbf{\Omega}} \mathbf{p}(\mathbf{y}|\mathbf{f},\mathbf{h},\mathbf{\Omega})\mathbf{p}(\mathbf{f}|\mathbf{\Omega})\mathbf{p}(\mathbf{h}|\mathbf{\Omega})\mathbf{p}(\mathbf{\Omega})$$

Variational approximation to the posterior

Prior Models

Total Variation (TV) Image Prior

$$\mathbf{p}(\mathbf{f}|\alpha) \propto \frac{\mathbf{1}}{\mathbf{Z}(\alpha)} \exp\left[-\alpha \mathbf{TV}(\mathbf{f})\right]$$

$$\mathbf{TV}(\mathbf{f}) = \sum_{\mathbf{i}} \sqrt{(\mathbf{\Delta}_{\mathbf{i}}^{\mathbf{h}}(\mathbf{f}))^{\mathbf{2}} + (\mathbf{\Delta}_{\mathbf{i}}^{\mathbf{v}}(\mathbf{f}))^{\mathbf{2}}}$$

SAR Blur Prior

$$\mathbf{p}(\mathbf{h}|\gamma) \propto \gamma^{\mathbf{N}/\mathbf{2}} \exp\left[-\frac{\mathbf{1}}{\mathbf{2}}\gamma\|\mathbf{Ch}\|^{\mathbf{2}}\right]$$

Blind Restoration

Degraded
Gaussian blur, var=9
BSNR=40 dB

SAR image model
SAR blur model
ISNR=1.29dB

TV1 image model
SAR blur model
ISNR=1.99dB

TV2 image model
SAR blur model
ISNR=1.95dB

Babacan, S. D., R. Molina, and A. K. Katsaggelos, "Variational Bayesian Blind Deconvolution Using a Total Variation Prior", *IEEE Transactions on Image Processing*, vol. 18, issue 1, pp. 12 - 26, Jan. 2009.

Known Blur BSNR = 40dB

Degraded
Gaussian blur, var=9
BSNR=40 dB

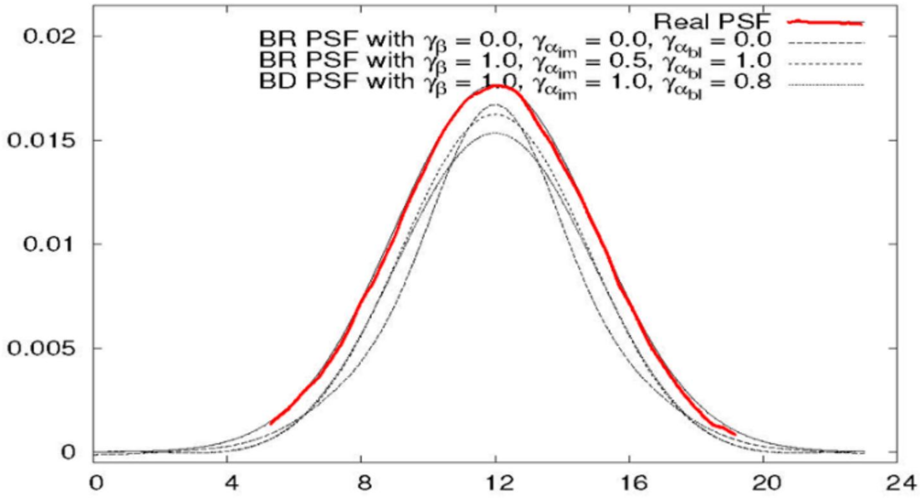
SAR image model
SAR blur model
ISNR=3.90dB

TV1 image model
SAR blur model
ISNR=4.84dB

TV2 image model
SAR blur model
ISNR=4.64dB

Estimated Blur

1D Slice through origin of the original and estimated PSFs



Blind Spatially Varying Restoration

Chen, Z., D. S. Babacan, R. Molina, and A. K. Katsaggelos, "Variational Bayesian Methods For Multimedia Problems", IEEE Transactions on Multimedia, June 2014.

Other Recovery Problems

- Image Super-Resolution
- Video Super-Resolution
- Pansharpening
- The Dual Exposure Problem

Basic Premise for Super-Resolution

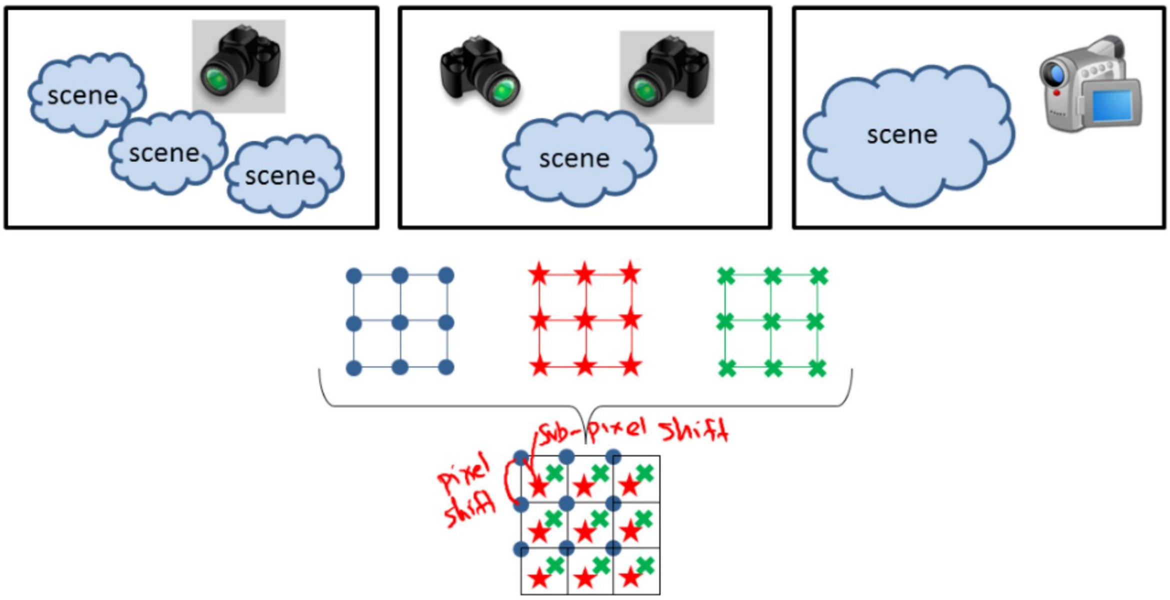
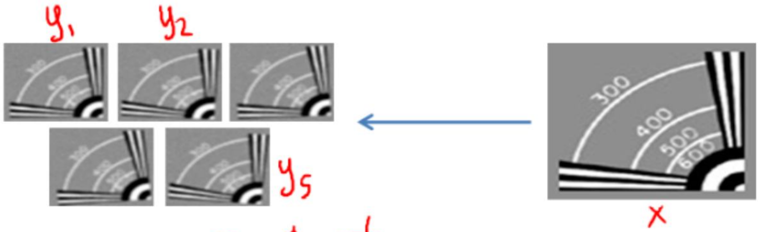


Image Super-Resolution



$$\mathbf{y}_k = \mathbf{A}\,\mathbf{H}_k\,\mathbf{C}(\mathbf{s}_k)\,\mathbf{x} + \mathbf{n}_k = \mathbf{B}_k(\mathbf{s}_k)\mathbf{x} + \mathbf{n}_k$$

LR observation

Downsampling

PSF

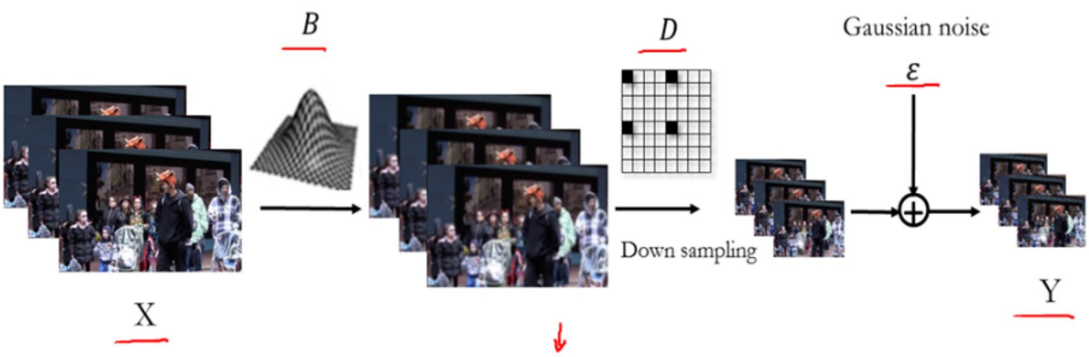
Motion Warping

HR image

Noise

?

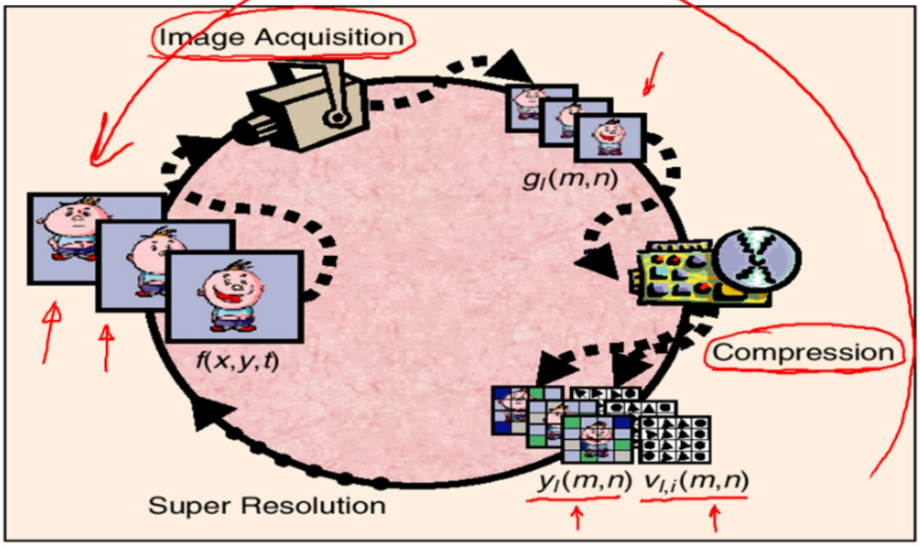
Video Super-Resolution



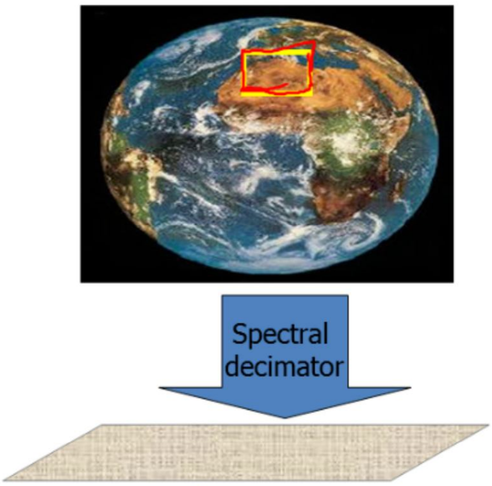
$$Y_k = DBX_k + \varepsilon_k$$

$$Y_i = DBC(d_{i,k})X_k + n_{i,k}$$

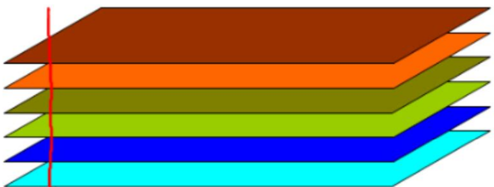
SR of Compressed Video



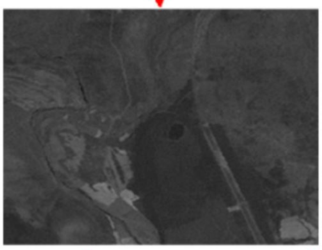
Pansharpening Problem

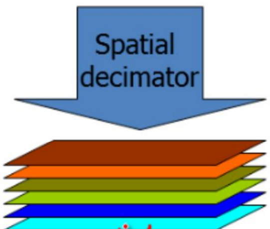


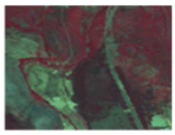
With an ideal sensor we would have high resolution multispectral images



y_b , $b=1,..,B$







$$x = \sum_{b=1}^{B} \lambda_b y_b + v$$

panchromatic x

$$Y_b = DHy_b + n_b, \quad b = 1, \cdots, B$$

The Dual Exposure Problem

Acquisition Model

Long Exposure Image

Camera Shake Blur

$$\mathbf{y}_1 = \mathbf{H}\mathbf{x} + \mathbf{n}_1$$

Short Exposure Image

Geometric Registration

$$\mathbf{y}_2 = \lambda_1 \mathbf{C}\mathbf{x} + \lambda_2 \mathbf{1} + \mathbf{n}_2$$

Photometric Registration

Preprocessing

$$\mathbf{y}_2 = \mathbf{x} + \mathbf{n}_2$$