Toward solving large combinatorial optimization problem

Group 7 qiqiqiqiqi

Content

01 Problem

- Max cut
- Molecule Simulations

02 DC-QAOA

- Hybrid classical and quantum algorithm
- Implement with Qiskit

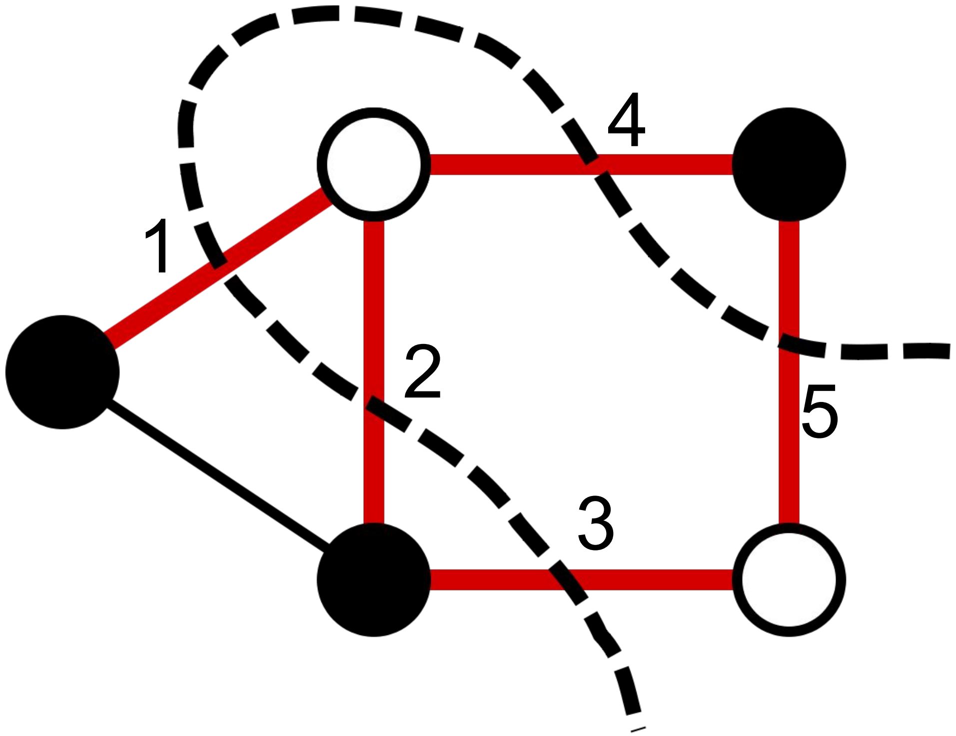
03 Tensor Decomposition

- Learnable parameterized unitary circuit

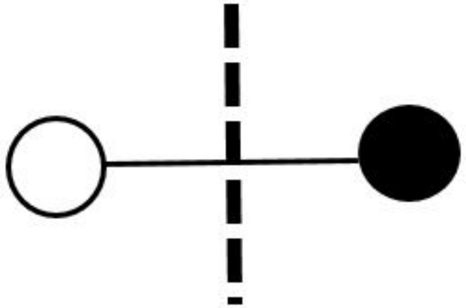
04 Result

- the best result is the combination of the high probability options of each circuit

Problem - Max cut

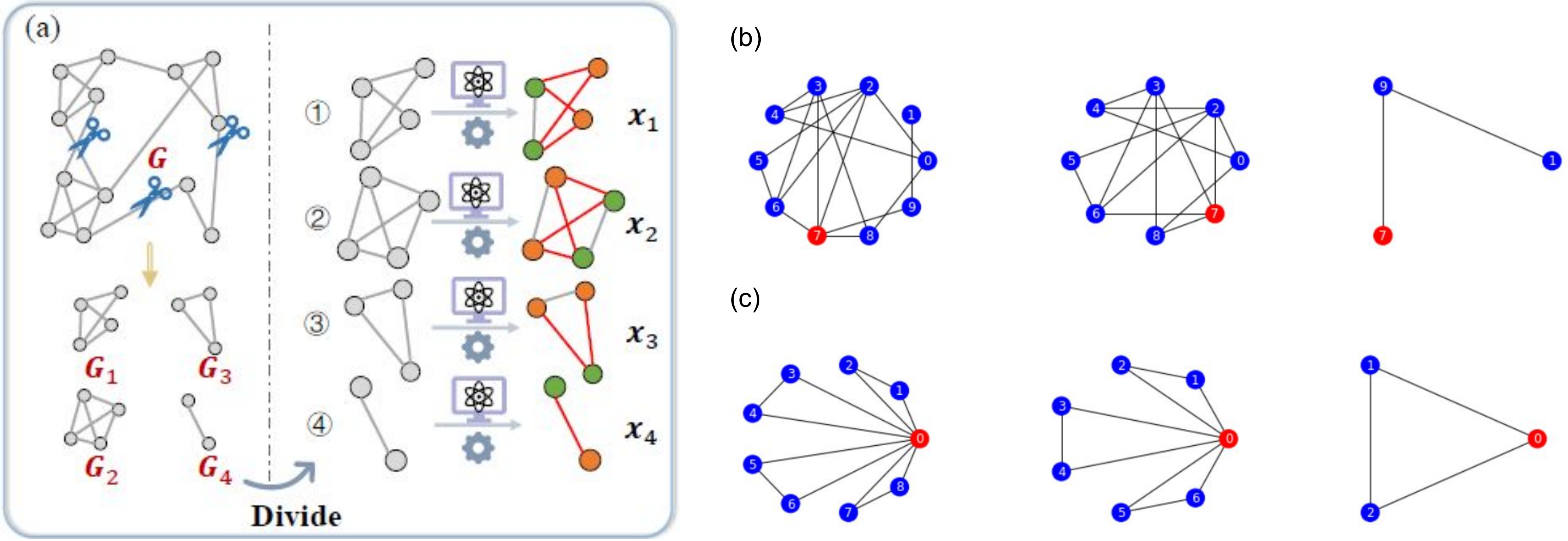


- each node can be assigned to either the "black" or "white" sets (0 or 1)
- partitioning nodes of a graph into two sets, such that the number of edges between the sets is maximum



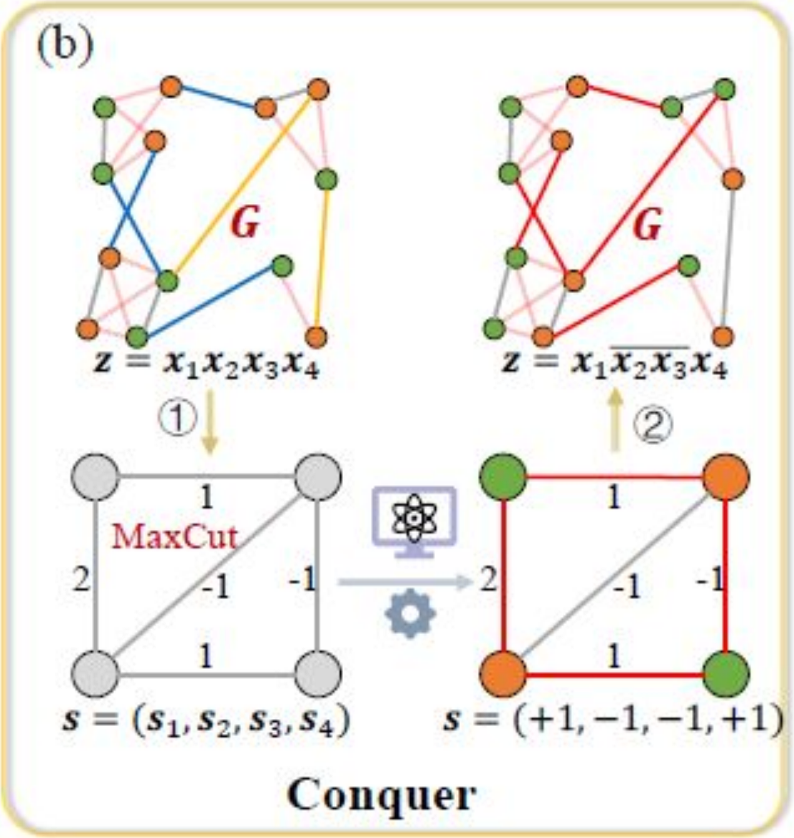
DC-QAOA

Divide and Conquer QAOA



ref: QAOA-in-QAOA: solving large-scale MaxCut problems on small quantum machines

Divide and Conquer QAOA



1x0x000: 100, 0x0x101: 50, …

$\otimes$

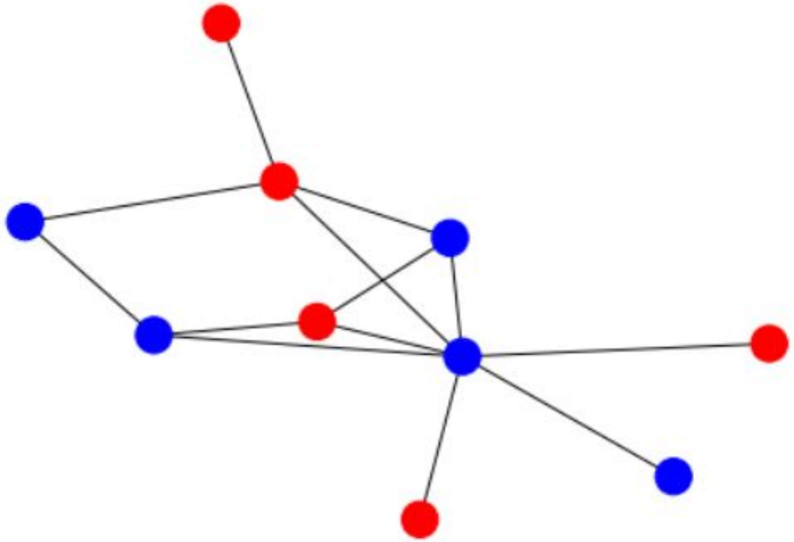
00xxx11: 50, 00xxx01: 48, … top k answer

⇩

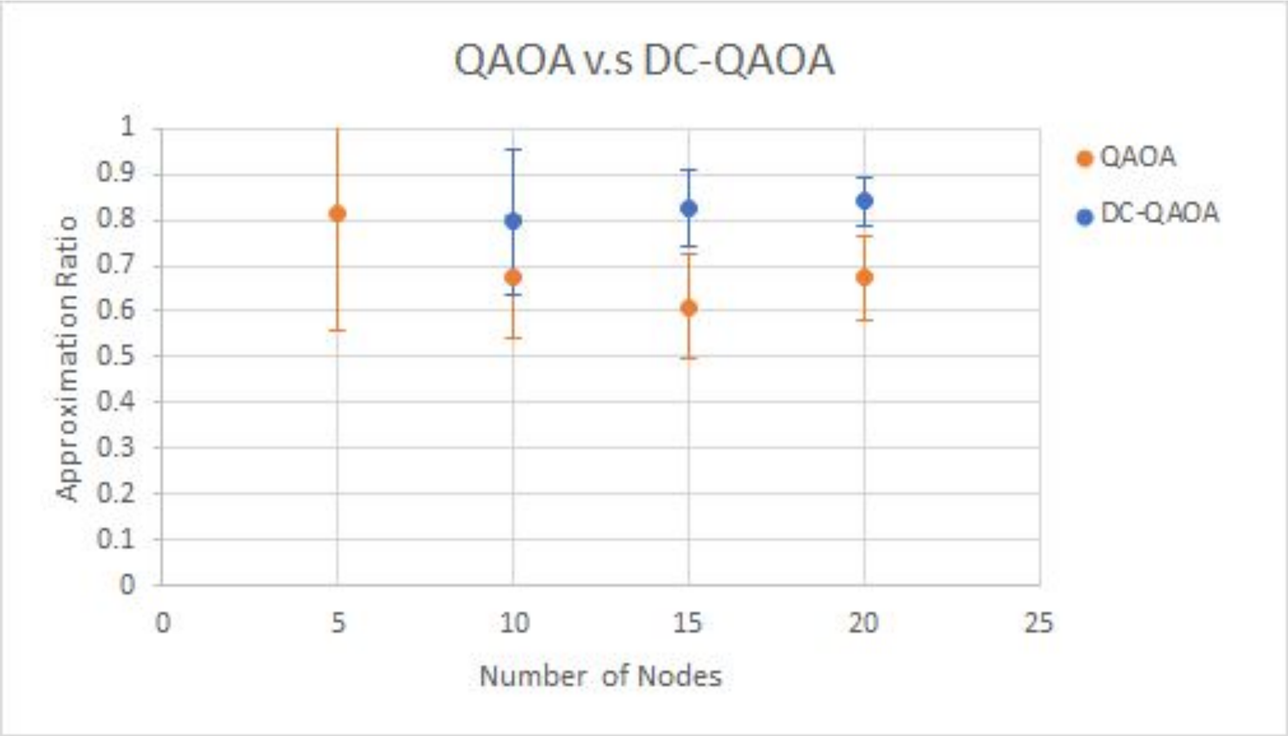
110x000: 50, 000x101: 48, …

ref: QAOA-in-QAOA: solving large-scale MaxCut problems on small quantum machines

Divide and Conquer QAOA



N=10



Tensor Decomposition

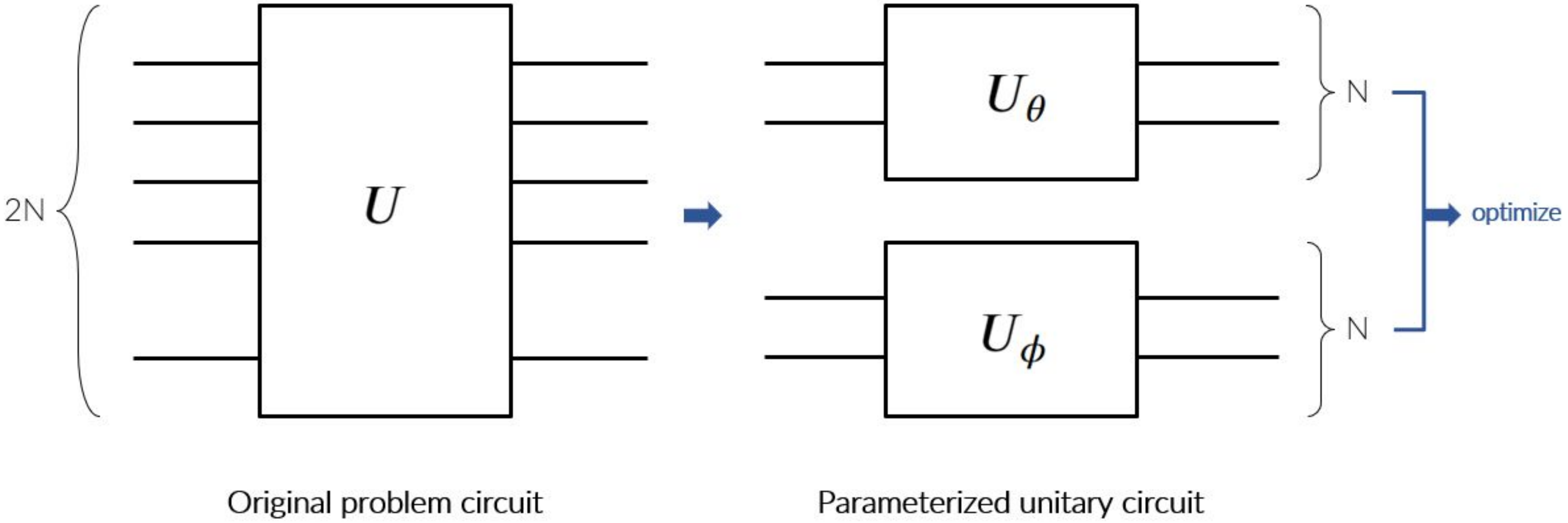
Tensor Decomposition

$$U \approx U_\theta \otimes U_\phi$$

$$U = \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ u_{21} & u_{22} & u_{23} & u_{24} \\ u_{31} & u_{32} & u_{33} & u_{34} \\ u_{41} & u_{42} & u_{43} & u_{44} \end{bmatrix} \quad U_\theta = \begin{bmatrix} u_{\theta 11} & u_{\theta 12} \\ u_{\theta 21} & u_{\theta 22} \end{bmatrix} U_\phi = \begin{bmatrix} u_{\phi 11} & u_{\phi 12} \\ u_{\phi 21} & u_{\phi 22} \end{bmatrix}$$

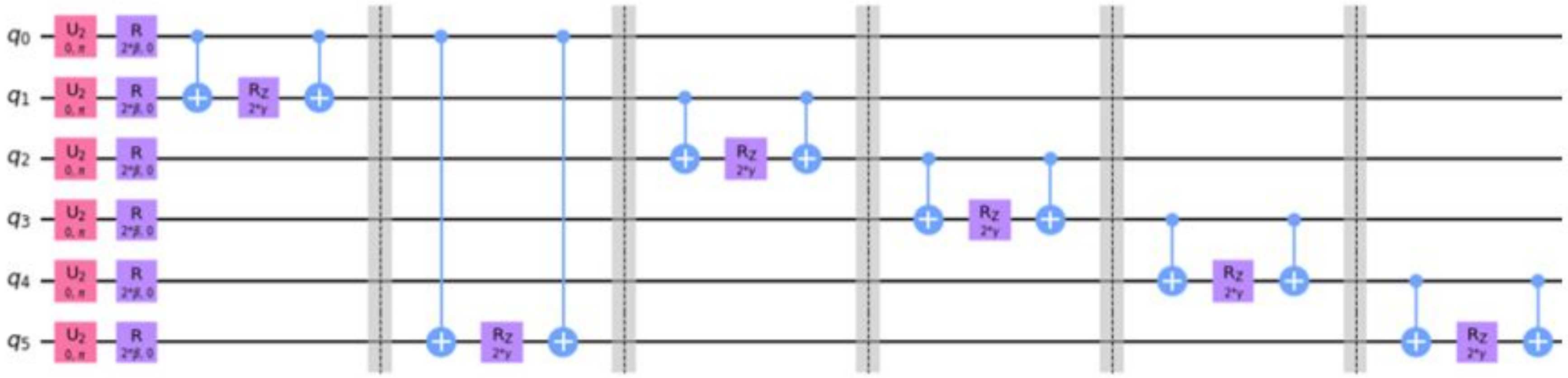
Variational Quantum Singular Value Decomposition, Xin Wang, Zhixin Song, and Youle Wang

Tensor Decomposition



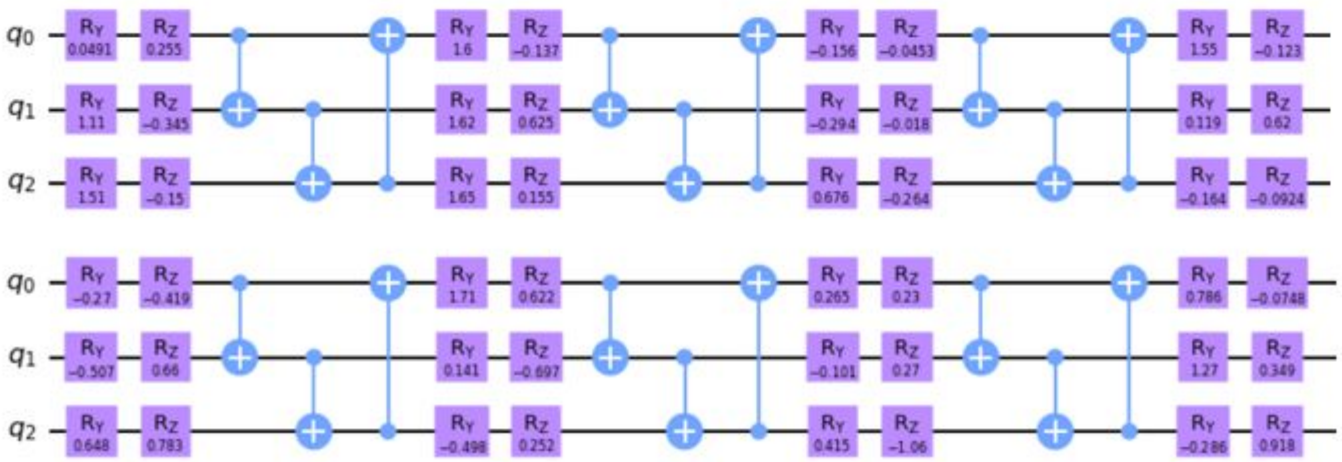
Tensor Decomposition

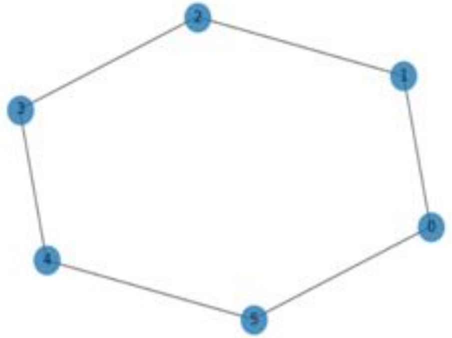
$U(\beta, \gamma)$



U_θ

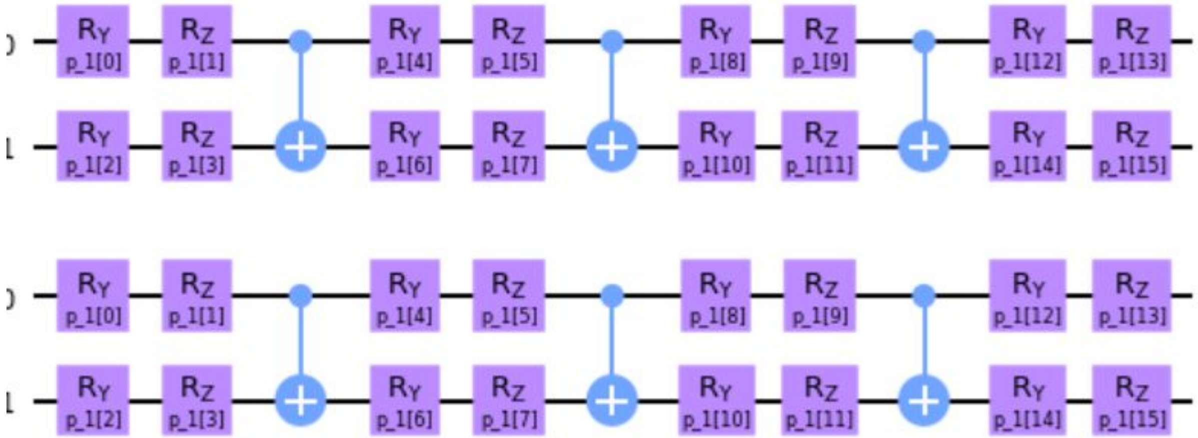
U_ϕ

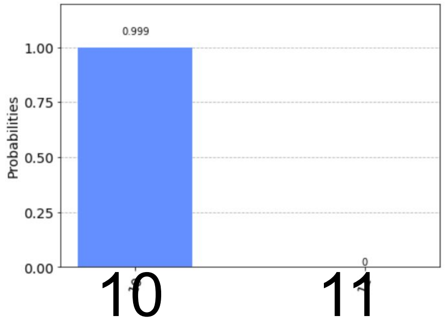


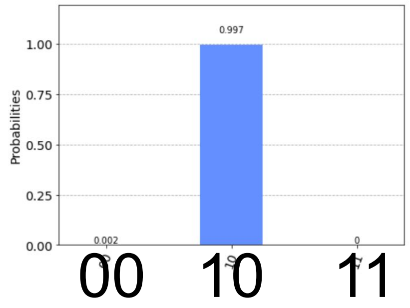


qubits=4,6

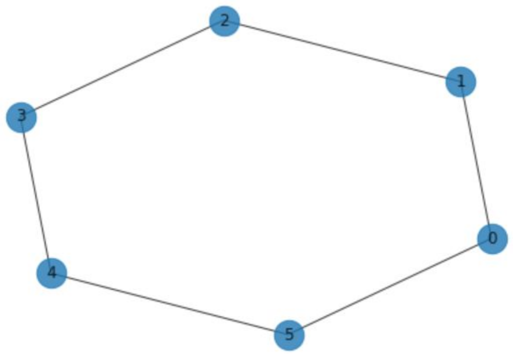
4 qubits

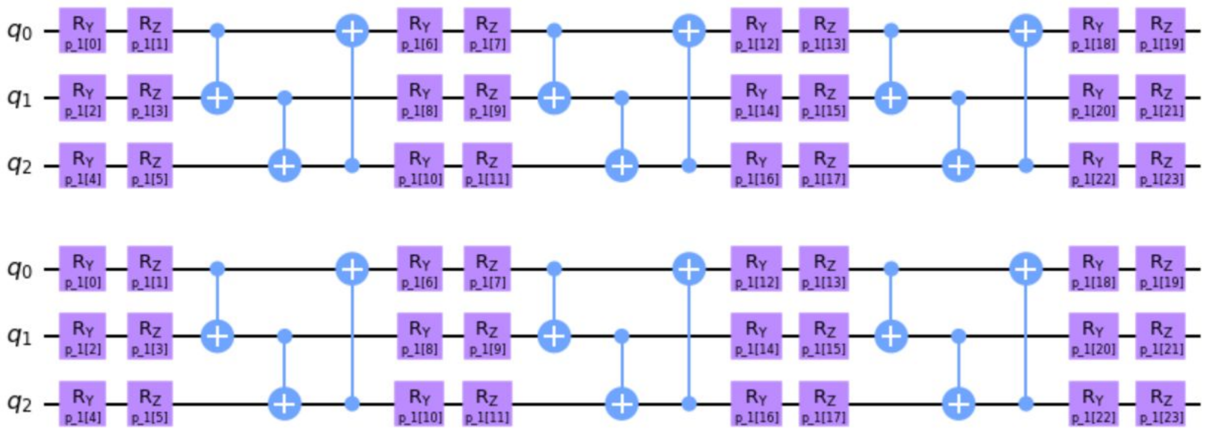


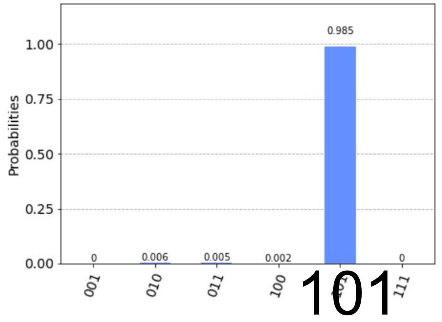


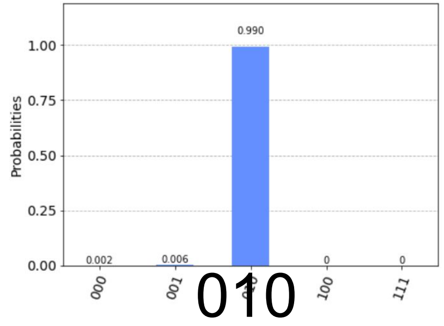


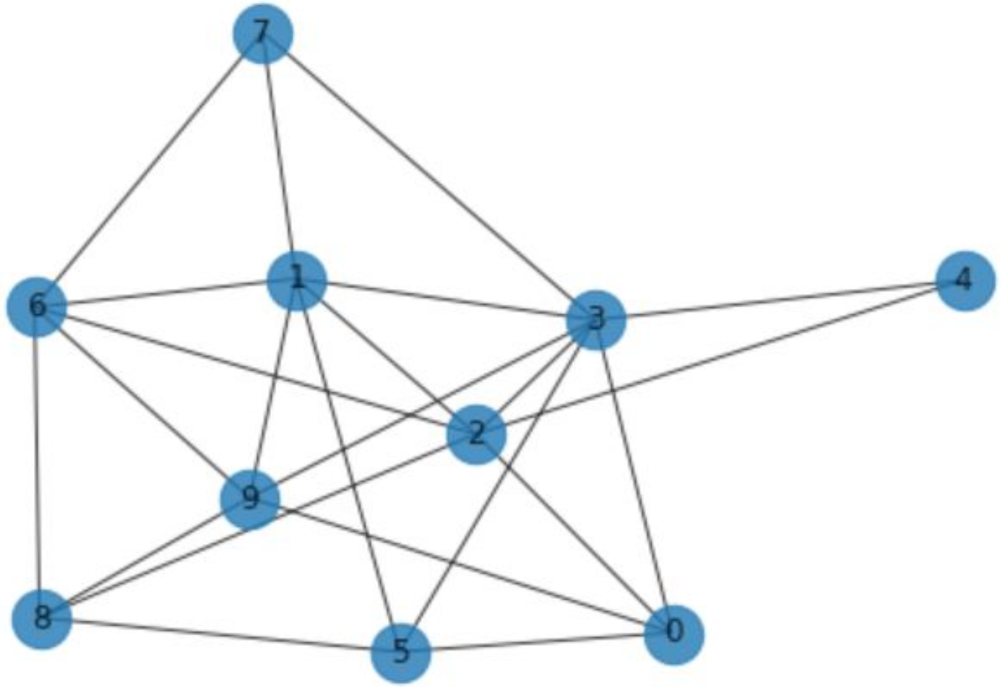
6 qubits











qubits=10

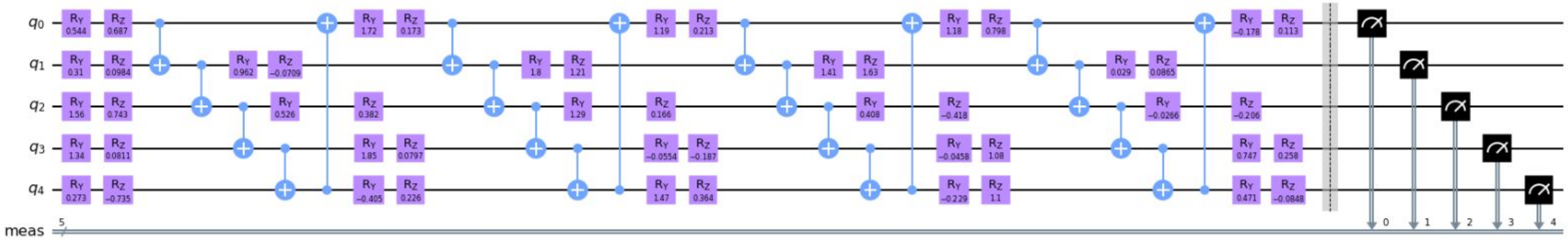
Best 4 solutions:

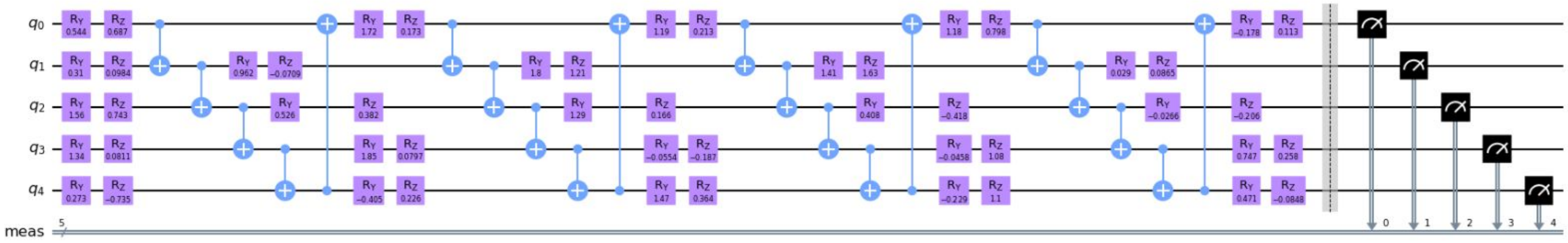
0 0 1 1 0 | 1 0 1 0 1

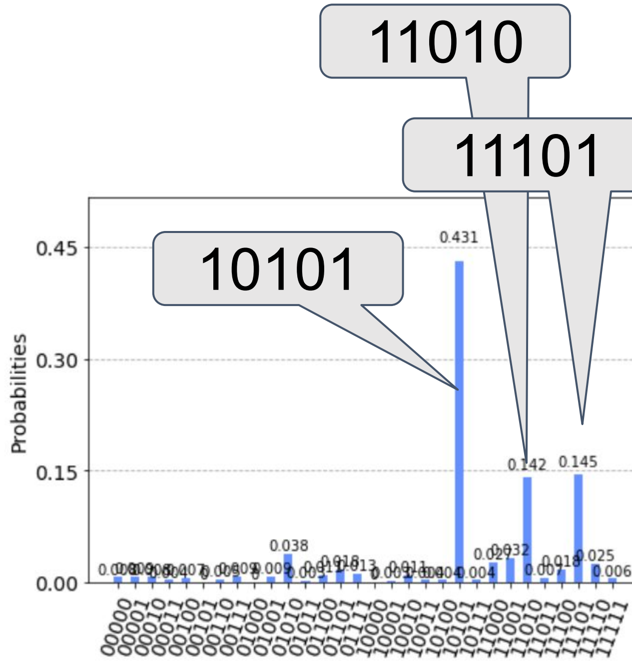
0 0 0 1 1 | 1 0 1 0 1

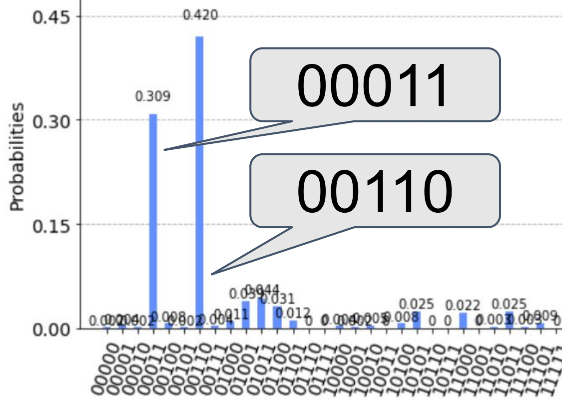
0 0 1 1 0 | 1 1 1 0 1

0 0 1 1 0 | 1 1 0 1 0

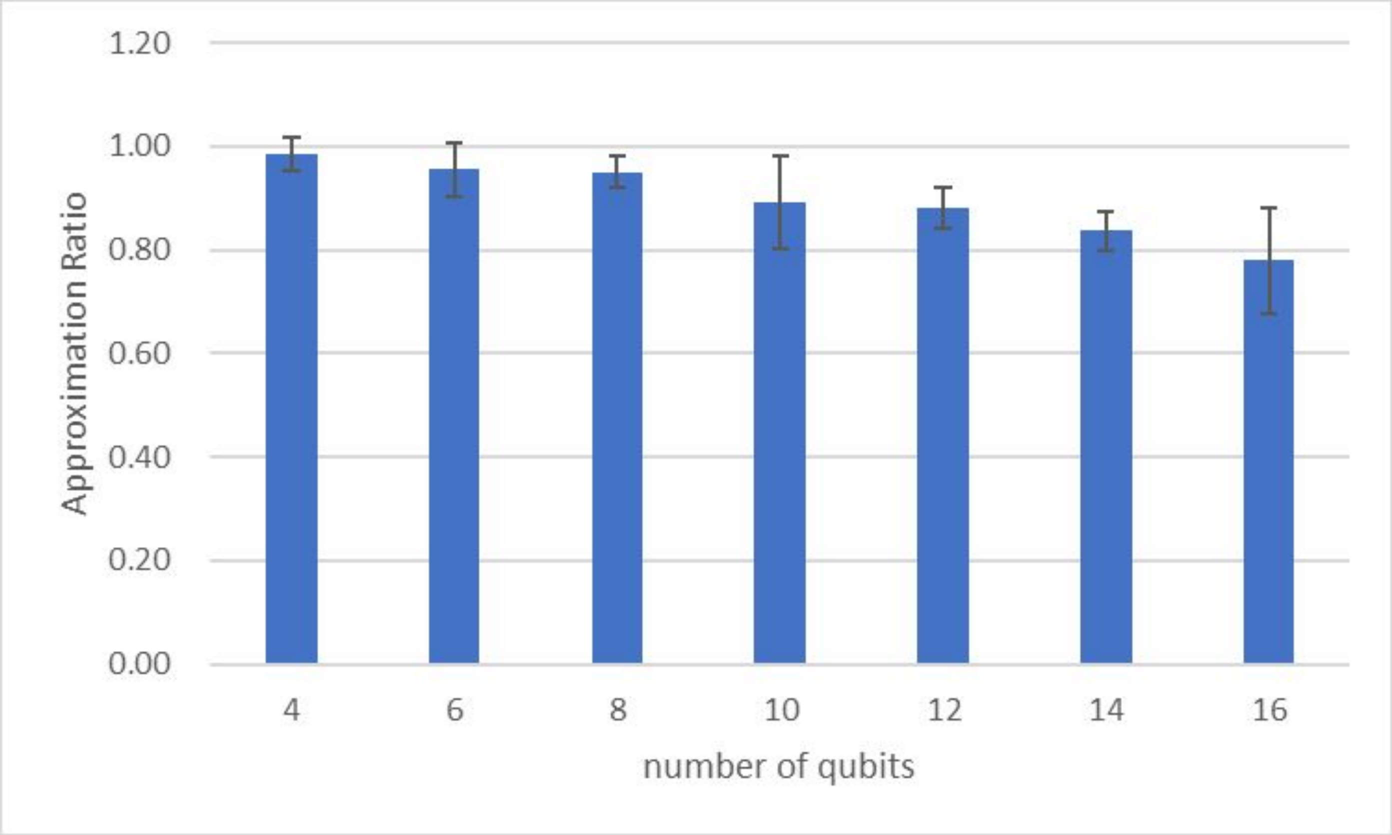








The relationship between the number of qubits and the approximation ratio



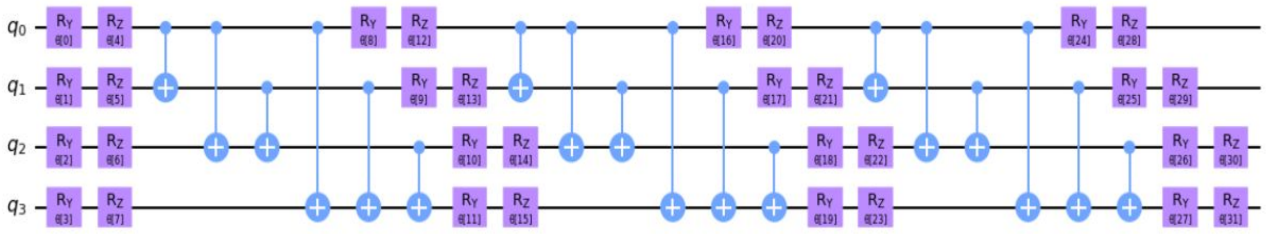
- the graph was drawn randomly.
- each bar shows the ratio of the edges found by our method to the edges calculate from SDP
- The more the qubits are, the more the error is.

Tensor Decomposition - Quantum Chemistry

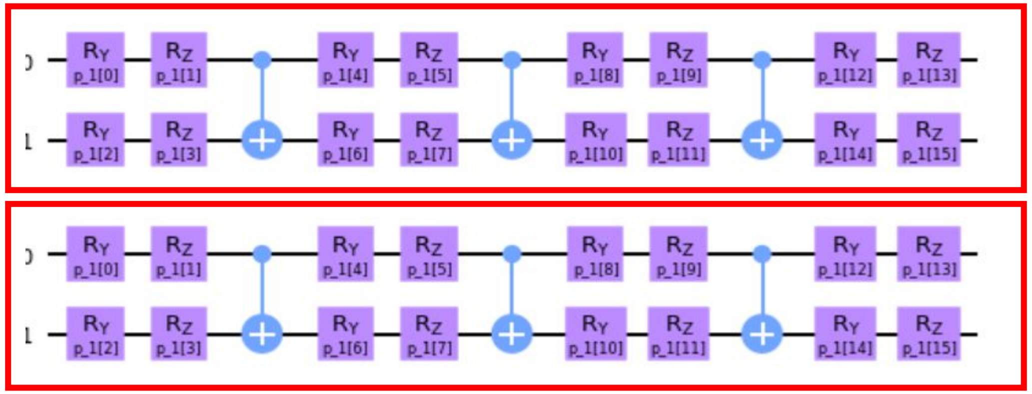
H_2 molecule VQE

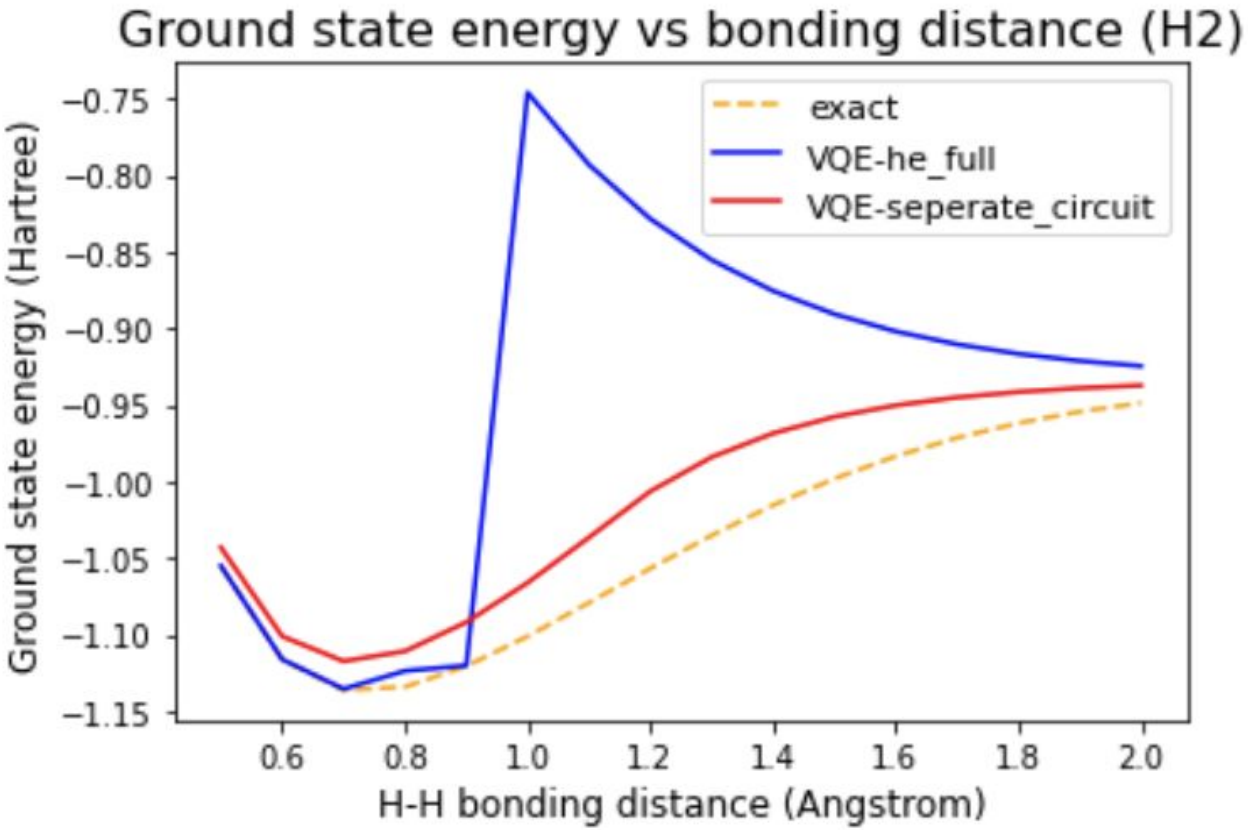
4 qubits (depth=3)

fully entangled hardware-efficient circuit



tensor decomposition circuits





how we get exact result:
scipy.sparse.linalg.eigsh(hamiltonian.to_matrix(sparse=True))

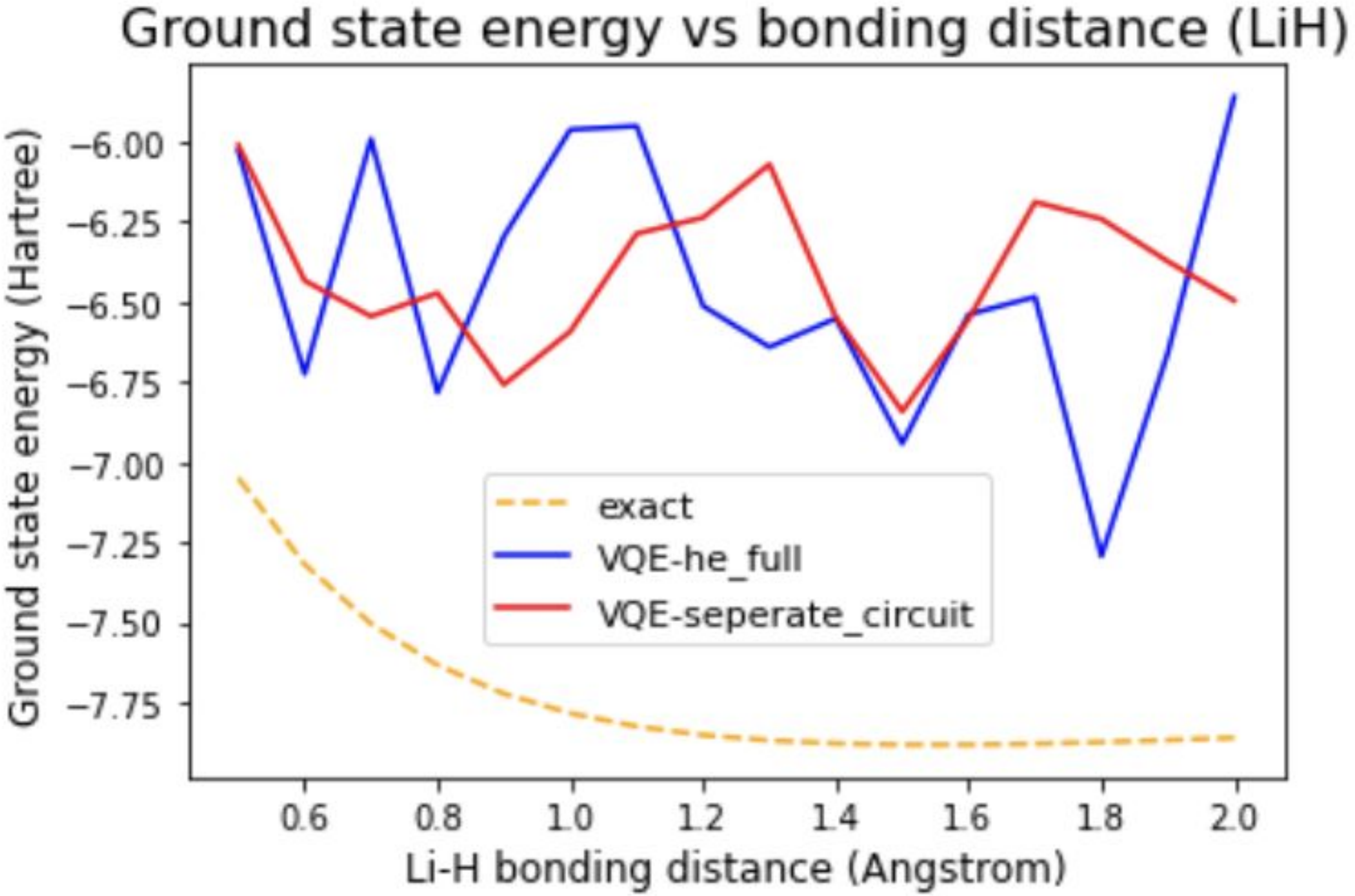
LiH molecule VQE

12 qubits, depth=4

6 qubits, depth=4

+

6 qubits, depth=4



Conclusion

01 We can solve larger max cut problem by using divide-and-conquer QAOA or tensor decomposition.

02 We can still apply tensor decomposition on solving molecular problems; however, it might be more important to add depth on H-E circuits or to choose a better ansatz to decompose.

03 The max cut problem has applications in VLSI design. Therefore, these methods can assist the process of VLSI design.

THANK YOU

Semidefinite program

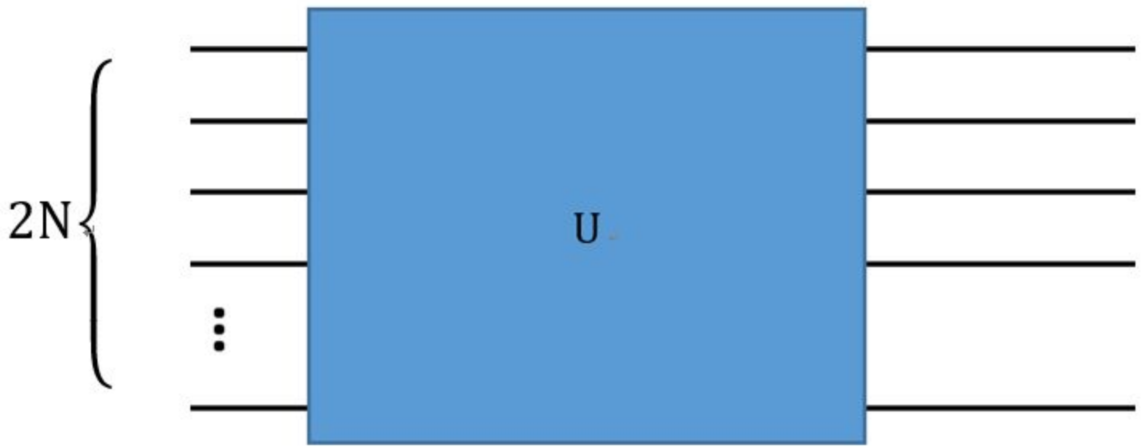
Goemans-William
son Max-Cut
Algorithm
87%

```
import cvxpy as cp
x = cp.Variable((5, 5), symmetric=True )
constraints = [x>>0]
constraints += [
    x[i, i] == 1 for i in range(5)
]
objective = sum( 0.5*(1-x[i,j]) for (i,j) in edges)
prob = cp.Problem(cp.Maximize(objective), constraints)
prob.solve()
```

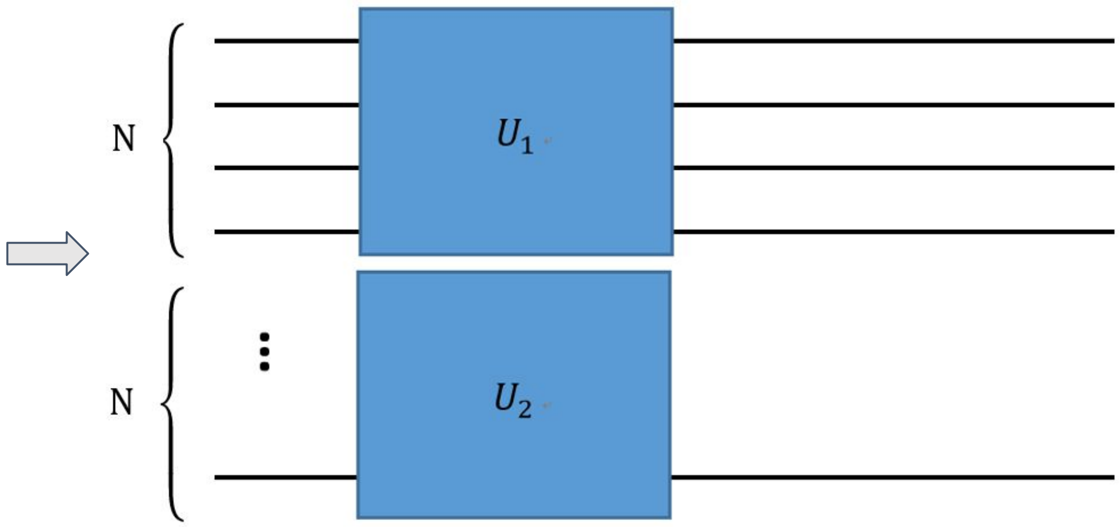
```
5.185492438628357
```

sdp and max cut: https://www.youtube.com/watch?v=aFVnWq3RHYU

Tensor Decomposition



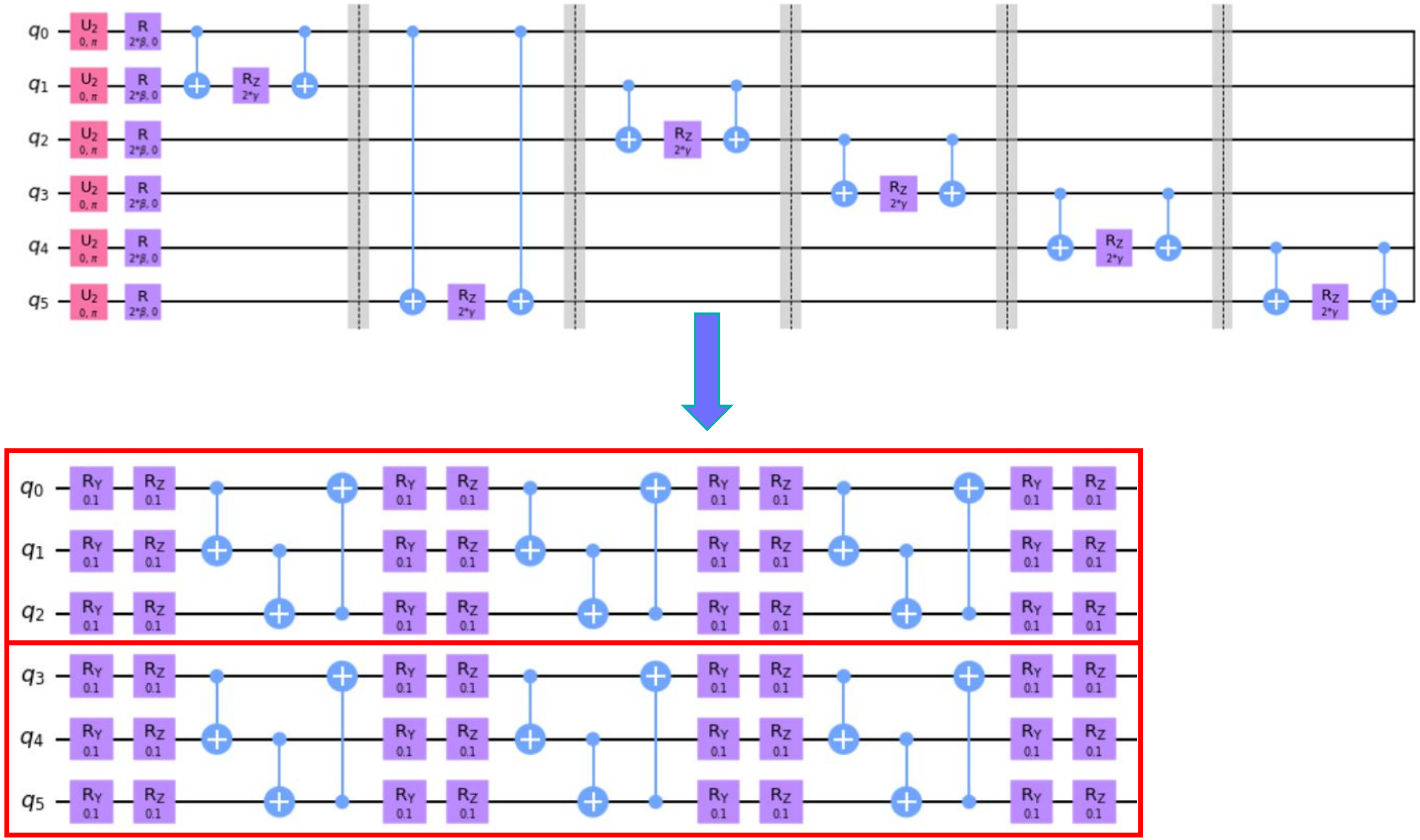
original circuit



$$[\mathrm{U}]_{(2*N)\times(2*N)} \approx [U_1]_{N\times N} \otimes [U_2]_{N\times N}$$

Tensor Decomposition

$$U(\beta,\gamma)= U_1(\theta_1) \otimes U_2(\theta_2)$$



6 qubits

3 qubits

Application of Max cut

Theoretical physics

the Max Cut problem is equivalent to minimizing the Hamiltonian of a spin glass model

Design of the circuit

The max cut problem has applications in VLSI design.

Conclusion

01 We can solve larger max cut problem by using divide-and-conquer QAOA or tensor decomposition.

02 We can still apply tensor decomposition on solving molecular problems; however, it might be more important to choose a good ansatz to decompose.

03 The max cut problem has applications in VLSI design. Therefore, our methods can assist the process of VLSI design.

04 Theoretically, all max cut problems