Mathematical model of the code observation

$$P_r^s = \left(t_r - t^s\right)c = \rho_r^s + c\left(dt_r - dt^s\right) + dT_r^s + dI_r^s + \varepsilon_P$$

P_r^s – code observation

t_r – signal reception time

t^s – signal emission time

c – speed of light

ρ_r^s – distance between the satellite s at epoch t^s and receiver at epoch t_r

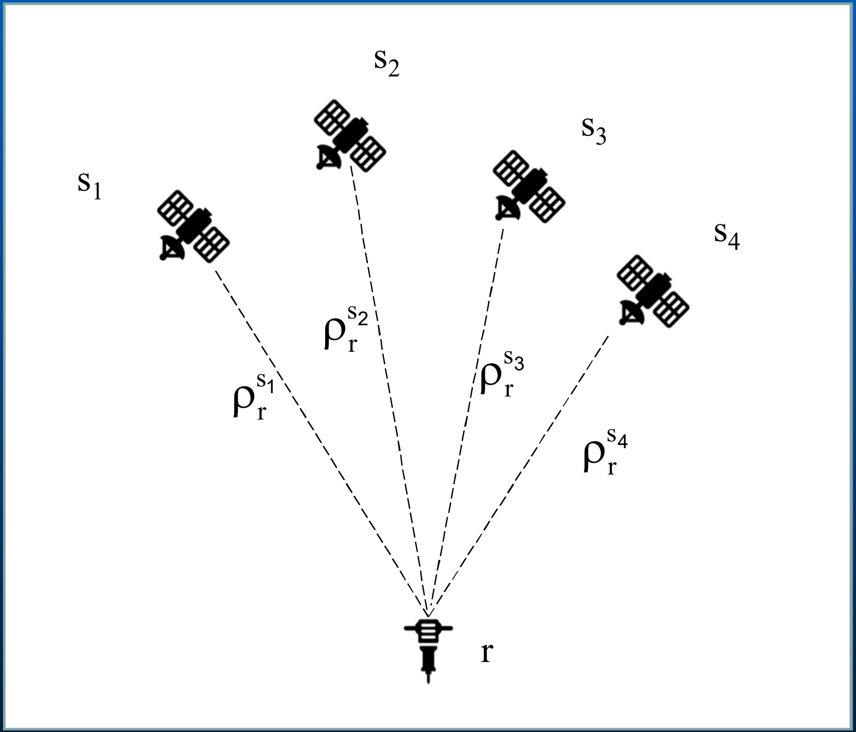
dt_r – receicer clock bias

dt^s – satellite clock bias

dT_r^s – tropospheric correction

dI_r^s – ionospheric correction

ε_P – unmodelled error



Observation equation of the code observation

$$P_r^s - dT_r^s - dI_r^s + v_P = \rho_r^s + cdt_r$$

Calculated in another computational process e.g. from iono- and tropospheric models

$$V_p = -\varepsilon_p$$

Linear form of the observation equation

$$P_r^s - dT_r^s - dI_r^s + v^s = \frac{\partial \rho_r^s}{\partial x^0} dx + \frac{\partial \rho_r^s}{\partial y^0} dy + \frac{\partial \rho_r^s}{\partial z^0} dz + \rho^0 + cdt_r$$

Linear form of the residual equation

$$v^s = \frac{\partial \rho_r^s}{\partial x^0} dx + \frac{\partial \rho_r^s}{\partial y^0} dy + \frac{\partial \rho_r^s}{\partial z^0} dz + cdt_r - \left(P_r^s - dT_r^s - dI_r^s - \rho^0\right)$$

System of the residual equations

$$v^{s1} = \frac{\partial \rho_r^{s1}}{\partial x^0} dx + \frac{\partial \rho_r^{s1}}{\partial y^0} dy + \frac{\partial \rho_r^{s1}}{\partial z^0} dz + cdt_r - \left(P_r^{s1} - dT_r^{s1} - dI_r^{s1} - \rho_r^{0s1}\right)$$

$$v^{s2} = \frac{\partial \rho_r^{s2}}{\partial x^0} dx + \frac{\partial \rho_r^{s2}}{\partial y^0} dy + \frac{\partial \rho_r^{s2}}{\partial z^0} dz + cdt_r - \left(P_r^{s2} - dT_r^{s2} - dI_r^{s2} - \rho_r^{0s2}\right)$$

$$\vdots$$

$$\underbrace{v^{sn}}_{V} = \underbrace{\frac{\partial \rho_r^{sn}}{\partial x^0} dx + \frac{\partial \rho_r^{sn}}{\partial y^0} dy + \frac{\partial \rho_r^{sn}}{\partial z^0} dz + cdt_r}_{AX} - \underbrace{\left(P_r^{sn} - dT_r^{sn} - dI_r^{sn} - \rho_r^{0sn}\right)}_{L}$$

$$V = AX - L$$

Matrix form

System of the residual equations

$$\rho_r^s = \sqrt{\left(x^s - x_r\right)^2 + \left(y^s - y_r\right)^2 + \left(z^s - z_r\right)^2}$$

partial derivatives

$$\frac{\partial \rho_r^{si}}{\partial x^0}$$

Matrices

$$V = \begin{bmatrix} v^{s1} \\ v^{s2} \\ \vdots \\ v^{sn} \end{bmatrix} \quad A = \begin{bmatrix} \frac{x_r - x^{s1}}{\rho_r^{0s1}} & \frac{y_r - y^{s1}}{\rho_r^{0s1}} & \frac{z_r - z^{s1}}{\rho_r^{0s1}} & 1 \\ \frac{x_r - x^{s2}}{\rho_r^{0s2}} & \frac{y_r - y^{s2}}{\rho_r^{0s2}} & \frac{z_r - z^{s2}}{\rho_r^{0s2}} & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \frac{x_r - x^{sn}}{\rho_r^{0sn}} & \frac{y_r - y^{sn}}{\rho_r^{0sn}} & \frac{z_r - z^{sn}}{\rho_r^{0sn}} & 1 \end{bmatrix} \quad X = \begin{bmatrix} dx_r \\ dy_r \\ dz_r \\ cdt_r \end{bmatrix}$$

$$P = \begin{bmatrix} m_{P_r^{s1}}^{-2} & & & \\ & m_{P_r^{s2}}^{-2} & & \\ & & \ddots & \\ & & & m_{P_r^{sn}}^{-2} \end{bmatrix} \quad L = \begin{bmatrix} \left(P_r^{s1} - dT_r^{s1} - dI_r^{s1} - \rho_r^{0s1}\right) \\ \left(P_r^{s2} - dT_r^{s2} - dI_r^{s2} - \rho_r^{0s2}\right) \\ \vdots \\ \left(P_r^{sn} - dT_r^{sn} - dI_r^{sn} - \rho_r^{0sn}\right) \end{bmatrix}$$

Ex 3 Single Point Positioning

Data:

x^s, y^s, z^s – satellite coordinates

x_r, y_r, z_r – approximate point position

P^{si} – code observation set

m = 30 cm – mean error of code observation (for forming weight matrix)

Task:

Prepare Matlab function for forming matrices: A, L, P and then conducting least squares adjustment process

Results:

X – increments to coordinates of approximate point position

C_x – covariance matrix of the X vector

m_0^2 – variance factor

Some useful Matlab function

[nr,x,y, …] = textread('filename','%d%f%f…') - read data from text file
nr is an integer vector; x, y, … are real number vectors

Matrix operations:

a+b; a-b; a*b – addition, subtraction, multiplication
a = [1 2 3;4 5 6;7 8 9] b=a(:,1) = [1;4;7] c=a(2,:)=[2 5 8] d=a(2:3,2) = [5;8]
e = d'= [5 8] (transpose)
h = inv(g) (inverse)
n = norm(d) – 2-norm of the d vector (useful for calculating distance from coordinates)
f = diag(p) – get diagonal elements or create diagonal matrix