Adjustment

Observation Equations

$$L_1^{wyr} = L_1^{obs} + v_1 = F_1(x_1, x_2, \dots x_m) = F_1(x_1^0, x_2^0, \dots x_m^0) + \frac{\delta F_1}{\delta x_1^0} dx_1 + \frac{\delta F_1}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_1}{\delta x_m^0} dx_m$$

$$L_2^{wyr} = L_2^{obs} + v_2 = F_2(x_1, x_2, \dots x_m) = F_2(x_1^0, x_2^0, \dots x_m^0) + \frac{\delta F_2}{\delta x_1^0} dx_1 + \frac{\delta F_2}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_2}{\delta x_m^0} dx_m$$

....

$$L_n^{wyr} = L_n^{obs} + v_n = F_n(x_1, x_2, \dots x_m) = F_n(x_1^0, x_2^0, \dots x_m^0) + \frac{\delta F_n}{\delta x_1^0} dx_1 + \frac{\delta F_n}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_n}{\delta x_m^0} dx_m$$

$$n > m$$

Residuals equation system

$$v_1 = \frac{\delta F_1}{\delta x_1^0} dx_1 + \frac{\delta F_1}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_1}{\delta x_m^0} dx_m + F_1(x_1^0, x_2^0, \dots x_m^0) - L_1^{obs}$$

$$v_2 = \frac{\delta F_2}{\delta x_1^0} dx_1 + \frac{\delta F_2}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_2}{\delta x_m^0} dx_m + F_2(x_1^0, x_2^0, \dots x_m^0) - L_2^{obs}$$

....

$$v_n = \frac{\delta F_n}{\delta x_1^0} dx_1 + \frac{\delta F_n}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_n}{\delta x_m^0} dx_m + F_n(x_1^0, x_2^0, \dots x_m^0) - L_n^{obs}$$

or alternatively:

$$v_1 = \frac{\delta F_1}{\delta x_1^0} dx_1 + \frac{\delta F_1}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_1}{\delta x_m^0} dx_m - \left(L_1^{obs} - F_1(x_1^0, x_2^0, \dots x_m^0)\right)$$

$$v_2 = \frac{\delta F_2}{\delta x_1^0} dx_1 + \frac{\delta F_2}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_2}{\delta x_m^0} dx_m - \left(L_2^{obs} - F_2(x_1^0, x_2^0, \dots x_m^0)\right)$$

....

$$v_n = \frac{\delta F_n}{\delta x_1^0} dx_1 + \frac{\delta F_n}{\delta x_2^0} dx_2 + \dots + \frac{\delta F_n}{\delta x_m^0} dx_m - \left(L_n^{obs} - F_n(x_1^0, x_2^0, \dots x_m^0)\right)$$

Adjustment

System of residual equations – matrix form

$$V=AX+L \quad \text{or} \quad V=AX-L$$

observed - computed

$$V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} \qquad A = \begin{bmatrix} \frac{\delta F_1}{\delta x_1^0} & \frac{\delta F_1}{\delta x_2^0} & \cdots & \frac{\delta F_1}{\delta x_m^0} \\ \frac{\delta F_2}{\delta x_1^0} & \frac{\delta F_2}{\delta x_2^0} & \cdots & \frac{\delta F_2}{\delta x_m^0} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\delta F_n}{\delta x_1^0} & \frac{\delta F_n}{\delta x_2^0} & \cdots & \frac{\delta F_n}{\delta x_m^0} \end{bmatrix} \qquad X = \begin{bmatrix} dx_1 \\ dx_2 \\ \vdots \\ dx_m \end{bmatrix} \qquad L = \begin{bmatrix} L_1^{obs} - F_1\left(x_1^0, x_2^0, \ldots x_m^0\right) \\ L_2^{obs} - F_2\left(x_1^0, x_2^0, \ldots x_m^0\right) \\ \vdots \\ L_n^{obs} - F_n\left(x_1^0, x_2^0, \ldots x_m^0\right) \end{bmatrix}$$

$$\begin{aligned} V &= \ ? \\ X &= \ ? \end{aligned} \longrightarrow$$

n equations
m unknowns

Adjustment

Adjustment problem

$$V = AX - L$$
$$V^T V = \min$$

$$\Psi = V^T V = \min \longrightarrow$$

$$\frac{\delta \Psi}{\delta X} = 0$$

$$2V^T \frac{\delta V}{\delta X} = 0$$

$$V^T A = 0$$

$$A^T V = 0$$

$$A^T A X - A^T L = 0$$

system of normal equations

solution: $X = (A^T A)^{-1} A^T L$

Adjustment

Observation set with various accuracy

Adjustment problem

functional model

$V = AX - L$

$V^T P V = \min$

statistical model

$P = Q^{-1}$

$C = \sigma^2 Q$

$$\Psi = V^T P V = \min \longrightarrow$$

$$\frac{\delta \Psi}{\delta X} = 0$$

$$2 V^T P \frac{\delta V}{\delta X} = 0$$

$$V^T P A = 0$$

$$A^T P V = 0$$

$$\boxed{A^T P A X - A^T P L = 0}$$

normal equations system

solution: $X = (A^T P A)^{-1} A^T P L$