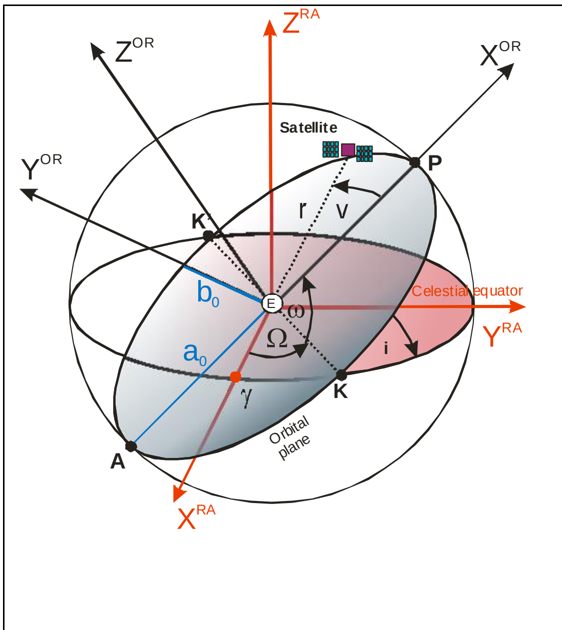
# Advanced Geodesy/Reference Systems

**Exercise 4: Transformation from Orbital Frame (OR) to Conventional Terrestrial Reference Frame (CT) and observer's frame called Horizontal (H).**

### 1. Satellite's position in space is determined by six parameters:

$a_0$, $e_0$ – define geometry of orbital ellipse , i.e. its size and shape
ω, v – define orientation of the orbit and satellite position in orbit
Ω, i – define orientation of the orbit in space (i.e. its orientation in RA)

We can define satellite's position in space, by defining this in its Orbital Frame (OR):
Central Point – Center of Earth masses
Primary axis - (z) – Normal to orbital frame
Secondary axis - (x) – Intersects the perigee
Third order axis - (y) - Orthogonal in right-handed frame




**K** – ascending node of the satellite
**K'** – descending node
**E** – Earth (coincides with one of the focal points of the ellipse)
**P** – perigee – point in orbit closest to Earth, located at main axis of orbital ellipse)
**A** – apogee – the most distant point of the orbit
**$a_0$** – semi-major axis of orbital ellipse
**$e_0$** – eccentricity of orbital ellipse
**i** – inclination angle between orbital plane and celestial equator
**Ω** – right-ascension of ascending node (RAAN)
**ω** – argument of perigee – angular distance of perigee from ascending node
**v** – true anomaly – angular distance of satellite from perigee
**r** – distance of the satellite from the center of Earth masses

### 2. Calculation of satellite position in Conventional Terrestrial Reference Frame

According to the second Kepler's Law, the motion of the satellite along the orbit is non-uniform. The position of the satellite in the orbital plane relatively to the perigee is determined by true anomaly **v** which is a function of time. The true anomaly changes unevenly over time, which means that **v** is a nonlinear function of time and is not convenient to use. An alternative way to describe the movement of the satellite is the so-called eccentric anomaly **E**. The concept of eccentric anomaly is similar to the concept of reduced latitude:

$$e^{OR} = \begin{bmatrix} r \cdot \cos v \\ r \cdot \sin v \\ 0 \end{bmatrix} = \begin{bmatrix} a_0(\cos E - e_0) \\ b_0 \sin E \\ 0 \end{bmatrix}$$

```
ST = [1:1:24]*hrs;
E = 2*ST;
eOR(:,:) = [a0*(cos(E)-e0); b0*sin(E); zeros(1, length(ST))];
```

The transformation from the OR orbital frame to the right-ascension frame RA reads:

$$e^{RA} = R_3(-\Omega) \cdot R_1(-i) \cdot R_3(-\omega) \cdot e^{OR}$$

```
eRA = R3r(-OM)*R1r(-iq)*R3r(-om)*eOR;
```

then to Instantaneous Terrestrial **IT**

$$e^{IT} = R_3(GAST)\cdot e^{RA}$$

and to Conventional Terrestrial **CT**

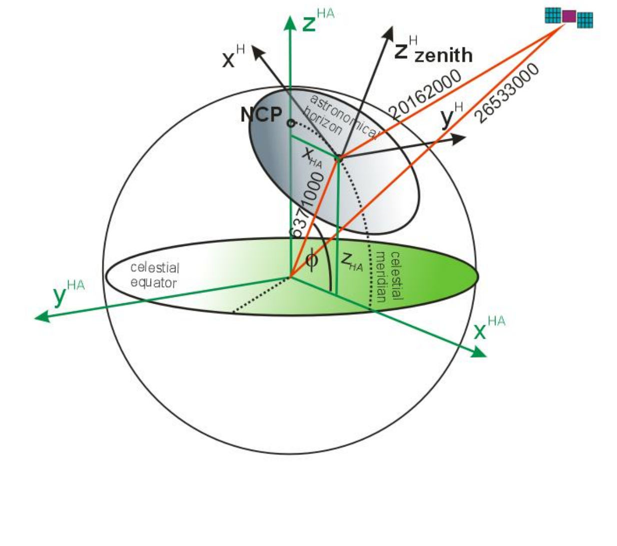
$$e^{CT} = R_2(-x_P)\cdot R_1(-y_P)\cdot e^{IT}$$

The overall transformation is therefore as follows:

$$e^{CT} = R_2(-x_P)\cdot R_1(-y_P)\cdot R_3(GAST)\cdot R_3(-\Omega)\cdot R_1(-i)\cdot R_3(-\omega)\cdot e^{OR}$$

### 3. Calculation of the satellite position in the observer's frame (horizontal - H)

The position in OR frame can also be transformed to the observer's (horizontal) H system.
We will then know the satellite's visibility in the local sky




The starting steps are the same as in the previous transformation (transformation to **RA**).
Then we recalculate to **HA** frame (iterations for ST):

$$e^{HA}_{ST} = P_2\cdot R_3(ST)\cdot e^{RA}$$

Subsequently we need to reduce the difference in directions and distances due to the distance between the Earth's mass center and the observer. For this purpose, we find the position of the observer in the **HA** system. It does not depend on time and does not require iterations.

$$e^{HA}_{obs.} = \begin{bmatrix} R\cdot cos\varphi \\ 0 \\ R\cdot sin\varphi \end{bmatrix}$$

Then we must use the position of the observer in HA, for reducing the vector to the satellite, and transform it to H:

$$e^{H}_{ST} = R_3(\pi)\cdot R_2\left(\frac{\pi}{2}-\phi\right)\cdot\left(e^{HA}_{ST} - e^{HA}obs.\right)$$

### 4. Exercise.

Having given the position of the satellite in OR, check its availability on the visible part of the celestial sphere at the given moment of sidereal time (ST). Use a loop to calculate the satellite track during the day. Start with eOR, then transform to RA, which is possible without the loop. Next, using the loop and the parameters variable in time (ST, E) calculate $e^{HA}_{ST}$, $e^{HA}_{obs.}$, and finally $e^{H}_{ST}$. Start the loop as follows, but extend it to calcualtion of eH, A, Z:

```
for i=1:length(ST)
        eHA(:,i)  = P2*R3r(ST(i))*eRA(:,i);
```

Calculate **A, Z (for zenital distance use Z = 90° - atan(z/p) to avoid negative Z below the horizon) in degrees minutes and seconds**). Make polar plot for A in radians and Z in decimal degrees.

$a_0$ = 26 560 km
$e_0$ = 0.02
ST = 0 to 24 h
E = 0° - 720° (2*ST)
i = 55°
Ω = 150°
ω = 45°
ϕ = 53°
R = 6 371 km (approximately)

Use polar plot for drawing the observer's hemisphere:

```
for z=1:length(zh)
    if zh(z)>90;
        zh(z)=NaN;
    end
end
polar(ah,zh,'-bs');% ah is in radians
view(90,-90)
```