Transformation from Orbital Frame (OR) to Conventional Terrestrial Reference Frame (CT) and Observer's Frame Called Horizontal (H).

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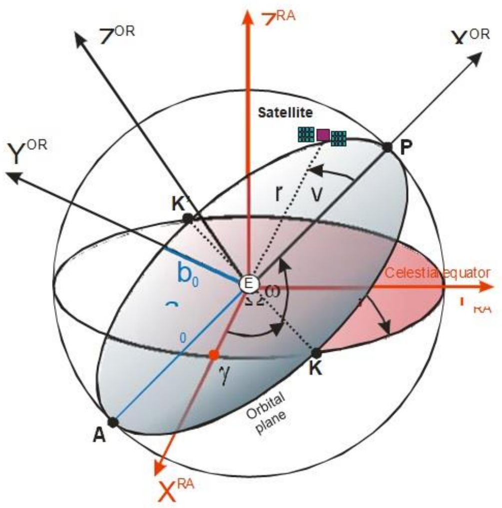
- a0, e0 – define geometry of orbital ellipse , i.e. its size and shape
- ω, v – define orientation of the orbit and satellite position in orbit
- Ω, i – define orientation of the orbit in space (i.e. its orientation in RA)

We can define satellite's position in space, by defining this in its Orbital Frame (OR): Central Point – Center of Earth masses.

- Primary axis - (z) – Normal to orbital frame Secondary axis - (x) – Intersects the perigee.
- Secondary axis - (x) – Intersects the perigee.
- Third order axis(y) – Ortogonal in right handed frame.



K – ascending node of the satellite
K' – descending node
E – Earth (coincides with one of the focal points of the ellipse)
P – perigee – point in orbit closest to Earth, located at main axis of orbital ellipse)
A – apogee – the most distant point of the orbit
$\mathbf{a_0}$ – semi-major axis of orbital ellipse
$\mathbf{e_0}$ – eccentricity of orbital ellipse
i – inclination angle between orbital plane and celestial equator
Ω – right-ascension of ascending node (RAAN)
ω – argument of perigee – angular distance of perigee from ascending node
v – true anomaly – angular distance of satellite from perigee
r – distance of the satellite from the center of Earth masses

According to the second Kepler's Law, the motion of the satellite along the orbit is non-uniform. The position of the satellite in the orbital plane relatively to the perigee is determined by true anomaly v which is a function of time.

The true anomaly changes unevenly over time, which means that v is a nonlinear function of time and is not convenient to use. An alternative way to describe the movement of the satellite is the so-called eccentric anomaly E.

```
clc
clear
rad = pi/180
iq = (55 * pi/180);
ST = [0:.2:24] * pi/12;
E = (2*ST);
a0 = 26560; % semi-major axis GRS80
e0 = 0.02; % first eccentricity GRS80
b0 = a0*sqrt(1-e0);
OM = 150 * rad;
om = 45 * rad;
R = 6371; %approximately
phi = 53 * rad;
P2=[1 0 0;0 -1 0;0 0 1];

eOR(:,:) = [a0*(cos(E)-e0); b0*sin(E); zeros(1, length(ST))]
eRA = spatial3(-OM)*spatial1(-iq)*spatial3(-om)*eOR

%eIT = spatial3(GAST) * eRA;
%eCT = spatial2(-xp) * spatial1(-yp) * eIT; %Conventional Terrestrial CT
%eCT = spatial2(-xp) * spatial1(-yp) * spatial3(GAST) * spatial3(-OM) * spatial1(-iq) * spatial3(-om) * eOR;

eHAobs = [R * cos(phi); 0; R * sin(phi)];
%eHAST = P2 * spatial3(ST) * eRA
%zh = 90 - atan(z/p)
```

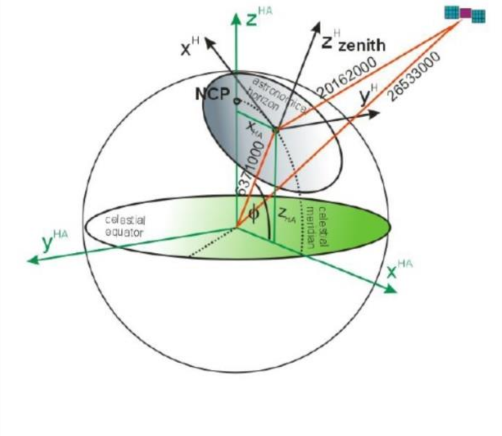
The transformation from the OR orbital frame to the right-ascension frame **RA** reads with **eRA** in the code.   The position in **OR** frame can also be transformed to the observer's (horizontal) H system. We will then know the satellite's visibility in the local sky.

```
for i=1:length(ST)

  eHA(:,i) = P2 * spatial3(ST(i))*eRA(:,i);
  eHST(:,i) = spatial3(pi) * spatial2((pi/2) - phi) * (eHA(:,i) - eHAobs)

  ah(i) = azymut1(eHST(2,i),eHST(1,i))
  rrr(i) = sqrt(eHST(1,i)^2+eHST(2,i)^2);
  zh(i) = 90-atan(eHST(3,i)/rrr(i))/rad;

end

for z=1:length(zh)
  if zh(z)>90;
     zh(z)=NaN;
  end
end

polar(ah*rad,zh,'-bs');% ah is in radians
%view(90,-90)
```

We created a loop for calculading A, Z (for zenital distance use **Z = 90° - atan(z/p)** to avoid negative Z below the horizon).

Having given the position of the satellite in OR, check its availability on the visible part of the celestial sphere at the given moment of sidereal time (ST). Use a loop to calculate the satellite track during the day.

We made polar plot for A in radians and Z in decimal degrees.



The starting steps are the same as in the previous transformation (transformation to **RA**).
Then we recalculate to **HA** frame (iterations for ST):

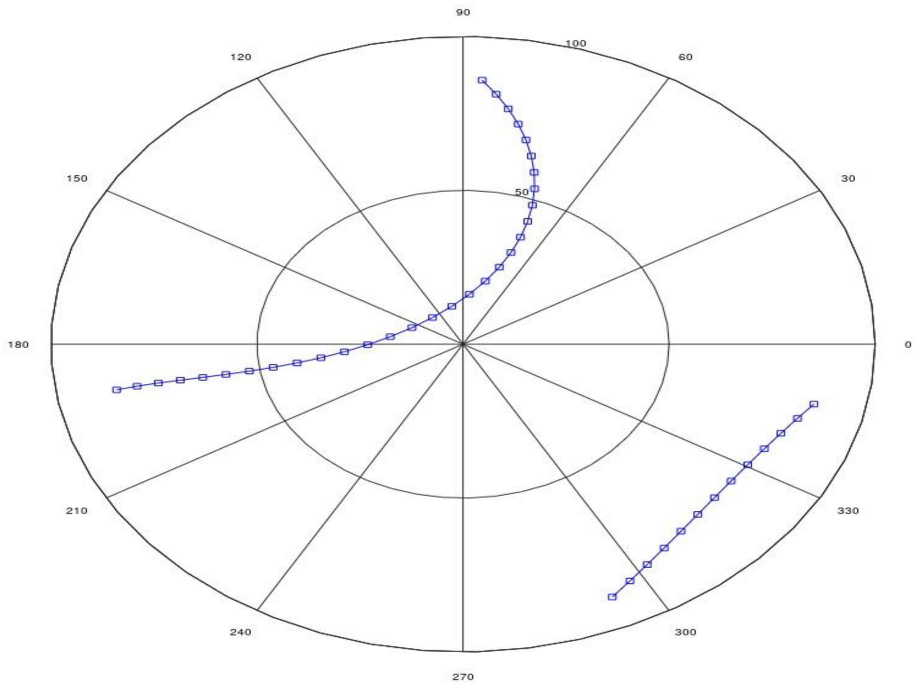
$$e_{ST}^{HA} = P_2 \cdot R_3(ST) \cdot e^{RA}$$

Subsequently we need to reduce the difference in directions and distances due to the distance between the Earth's mass center and the observer. For this purpose, we find the position of the observer in the **HA** system. It does not depend on time and does not require iterations.

$$e_{obs.}^{HA} = \begin{bmatrix} R \cdot cos\varphi \\ 0 \\ R \cdot sin\varphi \end{bmatrix}$$

Then we must use the position of the observer in HA, for reducing the vector to the satellite, and transform it to H:

$$e_{ST}^{H} = R_3(\pi) \cdot R_2\left(\frac{\pi}{2} - \phi\right) \cdot \left(e_{ST}^{HA} - e^{HA}obs.\right)$$



We can see the path the satellite has drawn in the plot. We can change the **phi** value for another angle.

END