Appendix S1: Full set of governing equations and parameterizations considered for the water and thermal transfers in variably saturated porous media with freeze-thaw of the pore water.

Modified Richards equation with a source term accounting for actual evapotranspiration:

$$C_H(h)\frac{\partial h}{\partial t} = \nabla.(K_H(h,\, T)\nabla(h+z)) + Q_{AET}(h,\, T) \tag{A.1}$$

Generalized Darcy's law:

$$\mathbf{V}\,(h,\, T) = -K_H\,(h,\, T)\,\nabla\,(h+z) \tag{A.2}$$

Macro-scale heat transfer equation for the porous medium with a term of latent heat exchange:

$$\frac{\partial\left(\left(C_{T,\,eq}(h,T) + L\frac{\partial\theta_{ice}(h,\,T)}{\partial T}\right)T\right)}{\partial t} + \nabla.\,(\mathbf{V}\,(h,\, T)\,C_{T,\,liquid}T) = \nabla.\,(K_{T,\,eq}\,(h,\, T)\,\nabla T) \tag{A.3}$$

Apparent hydraulic conductivity:

$$K_H(h,\, T) = K_s K_{rel}(h) K_{freezing}(T) \tag{A.4}$$

Mualem-van Genuchten parameterization of soil retention curve:

$$\theta(h) = \begin{cases} \theta_s & if\ h \geq 0 \\ \theta_r + (\theta_s - \theta_r)\left(1 + (-\alpha h)^n\right)^{-(1-(1n))} & if\ h < 0 \end{cases} \tag{A.5}$$

$$C(h) = \begin{cases} S & if\ h \geq 0 \\ \frac{\partial\theta(h)}{\partial h} & if\ h < 0 \end{cases} \tag{A.6}$$

$$K_{rel}(h) =$$

$$\begin{cases} K_s \\ if\ h \geq 0 \\ \quad K_s\left(\left(1 + (-\alpha h)^n\right)^{-\left(1-\frac{1}{n}\right)}\right)^{\frac{1}{2}}\left(1 - \left(1 - \left(\left(1 + (-\alpha h)^n\right)^{-\left(1-\frac{1}{n}\right)}\right)^{\frac{n}{n-1}}\right)^{1-\frac{1}{n}}\right)^2 \\ if\ h < 0 \end{cases} \tag{A.7}$$

Relative hydraulic conductivity with respect to water freezing:

$$K_{freezing}(T) = max\left(10^{-\Omega\theta_{ice}}; K_{freezing\,min}\right) \tag{A.8}$$

SFC function:

$$\theta_{liquid}\,(h,\, T) = \begin{cases} \theta\,(h) & if\ T\ > T_{freeze} \\ \left(\theta_r + (\theta(h) - \theta_r)\exp\left(-\left(\left(T - T_{freeze}\right)/w\right)^2\right)\right) & if\ T\ \leq T_{freeze} \end{cases} \tag{A.9}$$

Ice volume fraction calculated from the total water volume fraction:

$$\theta_{ice}\left(h,\,T\right)=\theta\left(h\right)-\theta_{liquid}\left(h,\,T\right) \tag{A.10}$$

Estimator of actual evapotranspiration:

$$Q_{AET}(h,\,T)=$$

$$\begin{cases} Q_{PET} & if \quad \left(\dfrac{(\theta_{liquid}(h,\,T)-\theta_{WP})}{\Delta t}-Q_{PET}\right)\geqq 0 \\ \dfrac{(\theta_{liquid}(h,\,T)-\theta_{WP})}{\Delta t} & if \quad -Q_{PET}<\left(\dfrac{(\theta_{liquid}(h,\,T)-\theta_{WP})}{\Delta t}-Q_{PET}\right)<0 \\ 0 & if \quad \left(\dfrac{(\theta_{liquid}(h,\,T)-\theta_{WP})}{\Delta t}-Q_{PET}\right)\leqslant -Q_{PET} \end{cases} \tag{A.11}$$

$$Q_{PET}=PET\left(S_{soil}/V_{evapotranspiration}\right)\chi_{evapotranspiration} \tag{A.12}$$

Equivalent thermal conductivity of the medium - geometric mean:

$$K_{T,\,eq}=K_{T,\,liquid}^{\theta_{liquid}(h,\,T)}K_{T,\,ice}^{\theta_{ice}(h,\,T)}K_{T,\,soil}^{(1-\theta_s)}K_{T,\,air}^{(\theta_s-\theta_{liquid}(h,\,T)-\theta_{ice}(h,\,T))} \tag{A.13}$$

Equivalent heat capacity of the medium – arithmetic mean:

$$C_{T,\,eq}=$$

$$C_{T,\,liquid}\theta_{liquid}\left(h,\,T\right)+C_{T,\,ice}\theta_{ice}\left(h,\,T\right)+C_{T,\,soil}\left(1-\theta_s\right)$$

$$+C_{T,\,air}\left(\theta_s-\theta_{liquid}\left(h,\,T\right)-\theta_{ice}\left(h,\,T\right)\right) \tag{A.14}$$

Symbols - roman letters:

C_H the field of capillary capacity (Mualem-van Genuchten parameterization) [m^{-1}]

$C_{T,\,air}$ the heat capacity of air [$kg.m^{-1}.s^{-2}.K^{-1}$]

$C_{T,\,eq}$ the field of equivalent heat capacity [$kg.m^{-1}.s^{-2}.K^{-1}$]

$C_{T,\,ice}$ the heat capacity of liquid water [$kg.m^{-1}.s^{-2}.K^{-1}$]

$C_{T,\,liquid}$ the heat capacity of liquid water [$kg.m^{-1}.s^{-2}.K^{-1}$]

$C_{T,\,soil}$ the field of heat capacity of the soil [$kg.m^{-1}.s^{-2}.K^{-1}$]

h the field of water pressure (the primary variable of Richards equation (A.1)) [m]

$K_{freezing}$ the field of relative hydraulic conductivity with respect to water freezing [-]

$K_{freezing\,min}$ the minimal value of the relative hydraulic conductivity with respect to water freezing [-]

K_H the field of apparent hydraulic conductivity [m.s^{-1}]

K_{rel} the field of relative hydraulic conductivity with respect to water saturation (Mualem-van Genuchten parametrization) [-]

K_s the field of saturated hydraulic conductivity [m.s^{-1}]

$K_{T,\,air}$ the thermal conductivity of air [kg.m.s^{-3}.K^{-1}]

$K_{T,\,eq}$ the field of equivalent thermal conductivity [kg.m.s^{-3}.K^{-1}]

$K_{T,\,ice}$ the thermal conductivity of ice [kg.m.s^{-3}.K^{-1}]

$K_{T,\,liquid}$ the thermal conductivity of liquid water [kg.m.s^{-3}.K^{-1}]

$K_{T,\,soil}$ the field of thermal conductivity of the soil [kg.m.s^{-3}.K^{-1}]

L the latent heat of fusion of ice [kg.m^{-1}.s^{-2}]

n the field of the second Mualem-van Genuchten fitting parameter [-]

PET the potential evapotranspiration [m.s^{-1}]

Q_{AET} the field of actual evapotranspiration volumetric rate [s^{-1}]

Q_{PET} the field of potential evapotranspiration volumetric rate [s^{-1}]

S the field of storage coefficient [m^{-1}]

S_{soil} the total surface of the soil [m^2]

t is the temporal coordinate [s]

T the field of temperature (the primary variable of heat transfer equation (A.2)) [K]

T_{freeze} the temperature of starting of the freezing of the water phase [K]

$\mathbf{V}$ the field of Darcy velocity [m.s^{-1}]

$V_{evapotranspiration}$ the total volume of the evapotranspiration layer [m^3]

w the fitting parameter for the freezing function [K]

z the vertical coordinate, oriented upward [m]

Symbol - greek letters:

α the field of the first Mualem-van Genuchten fitting parameter [m^{-1}]

Δt the length of the current time step of numerical solution [s]

θ the field of soil volumetric water content (Mualem-van Genuchten parameterization) [-]

θ_{ice} the field of volumetric frozen water content [-]

θ_{liquid} the field of volumetric liquid water content [-]

θ_s the field of volumetric water content at saturation [-]

θ_r the field of volumetric residual water content [-]

θ_{WP} the field of soil volumetric water content at the wilting point [-]

$\chi_{evapotranspiration}$ the indicatrix of the evapotranspiration layer (equal to 1 inside the evapotranspiration layer and 0 outside) [-]

Ω the field of empirical thermal impedance factor [-]