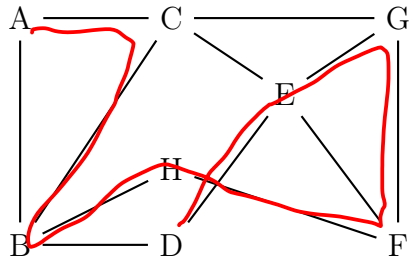
Tuesday 11/01/2022 Final Exam Duration: 90 minutes

Name: Student No:

P1 [25 points] If exists in the graph, give an example of the following. If impossible, write impossible. (Ex. If **path** was asked, a correct answer would be: A-C-E)



Eulerian trail: G C A B C E D B H F E G F

Eulerian cycle: Imp. because there are odd-deg vertices. G, F.

Hamiltonian path: D E G F H B C A

P2 [10 points] Choose the correct option:

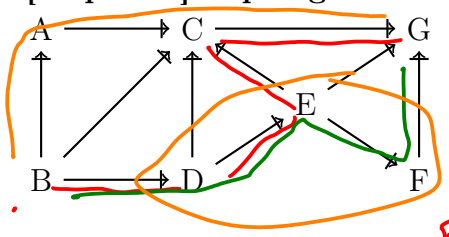
What is the chromatic number of $K_{3,4}$?

A) 2 B)3 C)4 D)6 E)12

What is the chromatic number of K_9?

A) 3 B)6 C)8 D)9 E)10

P3 [20 points] Topological Sort & Counting



B D A E F C G 7

B.D.E.F.C.G

B.D.E.C.F.G

Give a topological order for the graph:

B D A E F C G

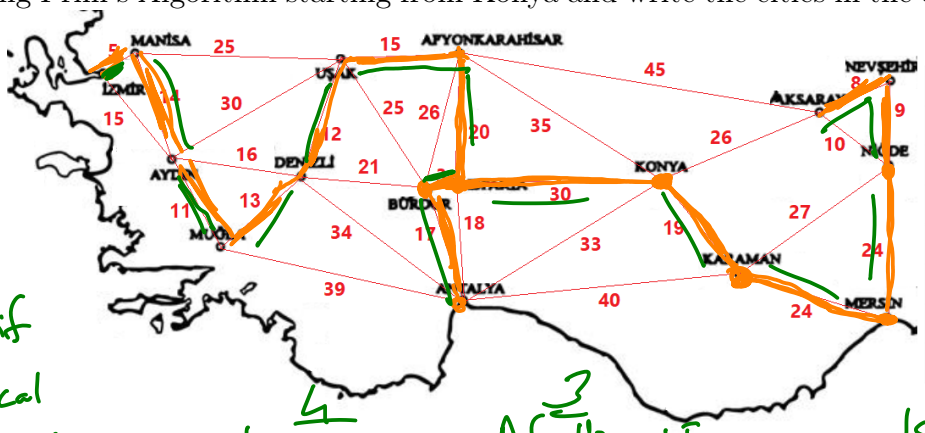
B A D E C F G

What is the number of possible topological orders?

A→F 6

F→A 1

P4 [15 points] Minimum Spanning Tree In the map below, draw a minimum spanning tree by using Prim's Algorithm starting from Konya and write the cities in the order you add them to the MST.



1	Konya	9	Ant
2	Kar	10	Kay
3	Mer	11	Usa
4	Niğ	12	Den
5	Nev	13	Muğ
6	Afy	14	Ayd
7		15	Manis
8	Isp Bur	16	Izm

what if Kruskal

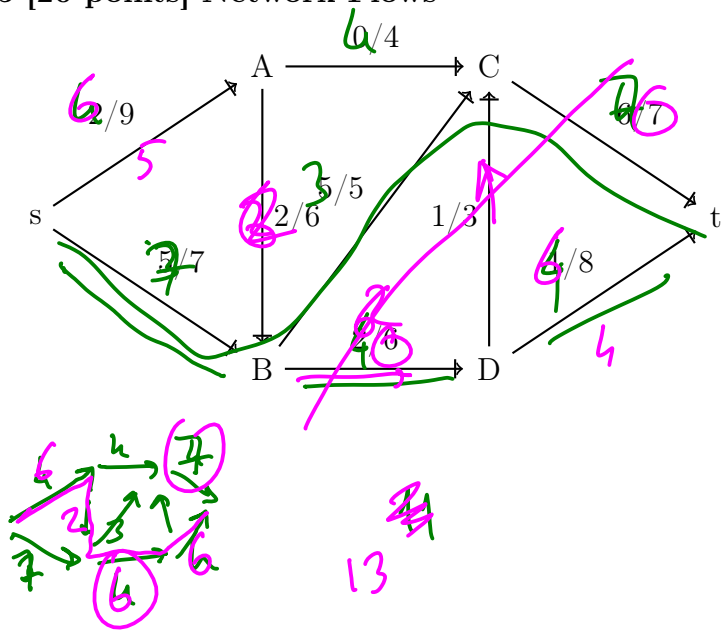
Kon Mer 24
Nig Mer 24
Kon Isp 25

Afy Usa 15
Bur Ant 17
Kon Kon Afy Isp 20

Isp-Bur 3
Man-Izm 5
Nev Aks 8
Nev Nig 9

Ayd-Muğ 11
Usa Den 12
Muğ Den 13
Man Ayd 14

P5 [20 points] Network Flows



A network and a flow F on this network are given on the left.

1. According to the capacities, what is the maximum flow of this network?

2. Draw the residual graph.

P6 [15 points] Generating Functions & Combinations Solve this question using generating functions (Build the polynomial, determine the coefficient to look for, and calculate the final result) [Recall that $1/(1-x) = 1 + x + x^2 + x^3 + \ldots$]

How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ with the restriction that all of $x_i \geq 1$ where two of them are odd and the remaining three are even integers?