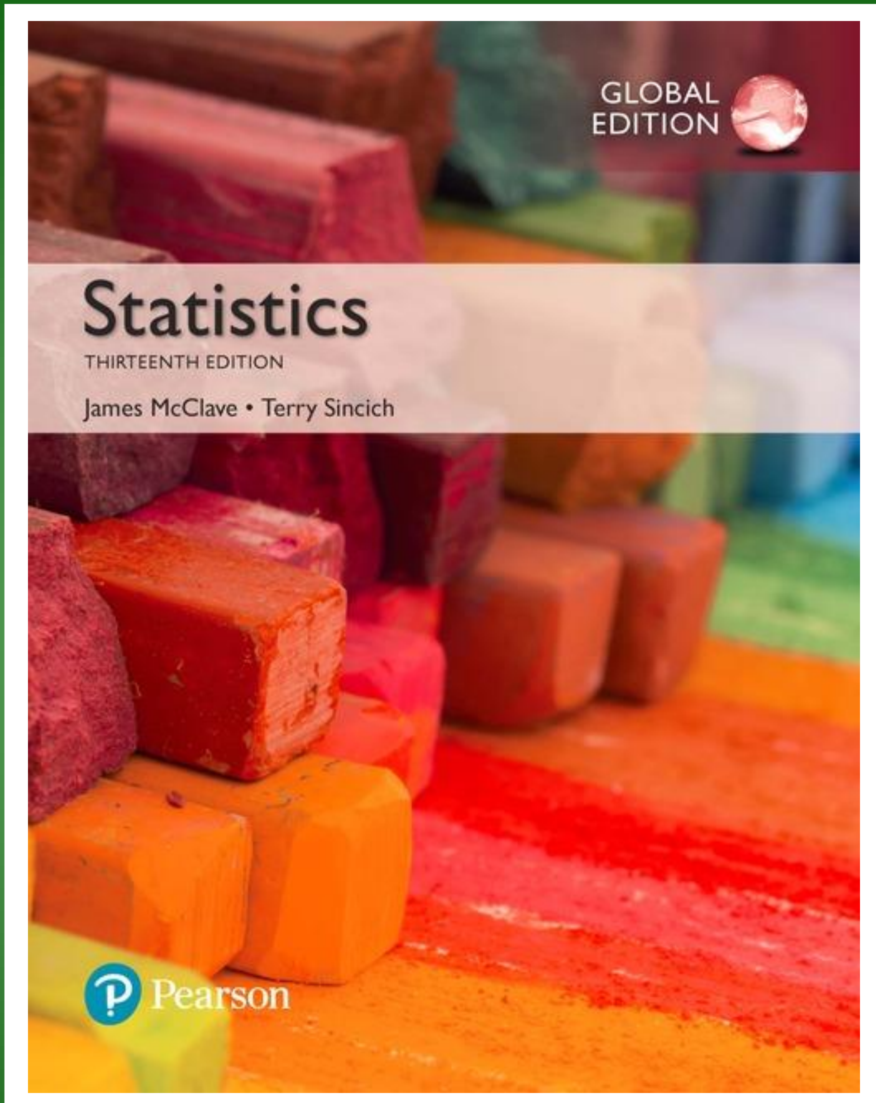



# Chapter 5

## Continuous Random Variables

# Recall that ...

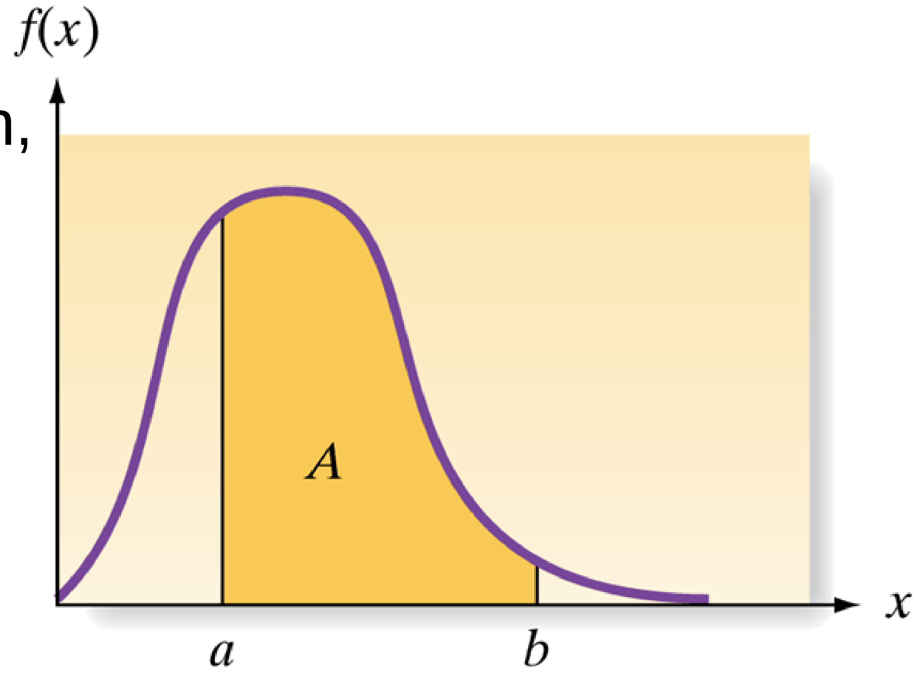
- A **continuous random variable** is a random variable that can assume any value within some interval or intervals.
  - the length of time between a person’s visits to a doctor
  - the thickness of sheets of steel produced in a rolling mill
  - the yield of wheat per acre of farmland

# 5.1

## Continuous Probability Distributions

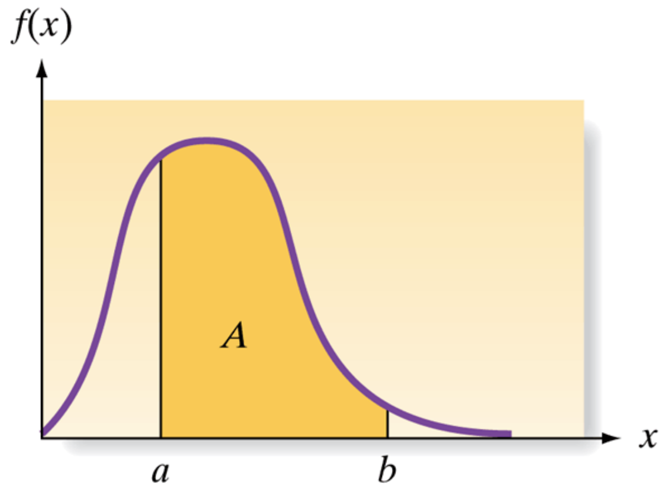
# **Figure 5.1** A probability distribution $f(x)$ for a continuous random variable $x$

- The graphical form of the probability distribution for a continuous random variable $x$ is a smooth curve
  - a probability density function,
  - a frequency function, or
  - a probability distribution.



## **Figure 5.1** A probability distribution $f(x)$ for a continuous random variable $x$

- Area $A$ beneath the curve is the probability that $x$ assumes a value between $a$ and $b$ $(a < x < b)$
- $P(a < x < b) = P(a \le x \le b)$



# Definition

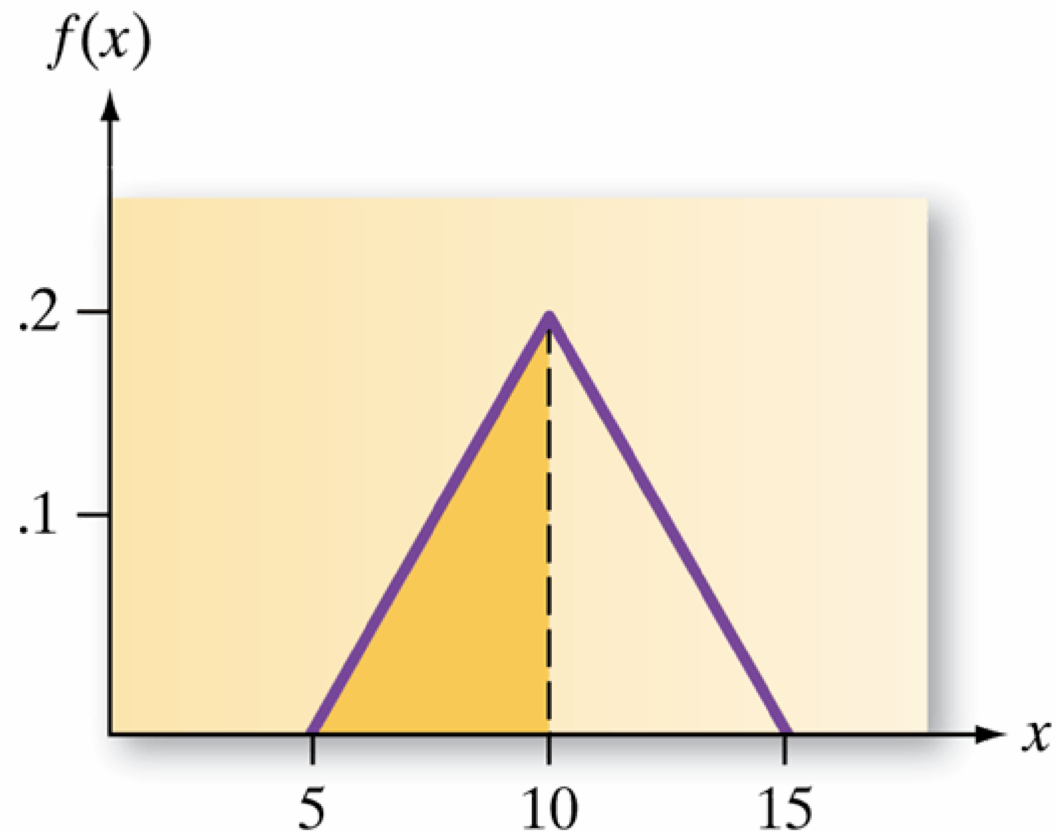
The **probability distribution for a continuous random variable, $x$,** can be represented by a smooth curve—a function of $x$, denoted $f(x)$. The curve is called a **density function** or **frequency function.** The probability that $x$ falls between two values, $a$ and $b$, i.e., $P(a < x < b)$, is the area under the curve between $a$ and $b$.

# Problem

- Researchers studied the friction that occurs in the paper-feeding process of a photocopier. The friction coefficient, $x$, is a continuous random variable that measures the degree of friction between two adjacent sheets of paper in the feeder stack. The random variable can be modeled using the smooth curve shown in Figure 5.2. Note that the friction coefficient, $x$, ranges between the values 5 and 15. Find the probability that the friction coefficient is less than 10.

# **Figure 5.2** Density Function for Friction Coefficient, Example 5.1

Find the probability that the friction coefficient is less than 10.



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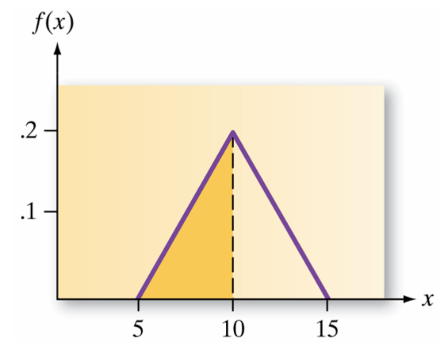
# Solution

- To find the probability that the friction coefficient is less than 10, we need to compute $P(5 < x < 10)$. This is the area between $x = 5$ and $x = 10$ shaded on the graph in Figure 5.2. Note that this area is represented by a right triangle with a base of length 5 and a height of .2. Since the area of a triangle is $(^1/_2)(base) \times (height)$, then

$$P(5 < x < 10) = (^1/_2)(base) \times (height)$$
$$= (.5)(5)(.2) = .5$$



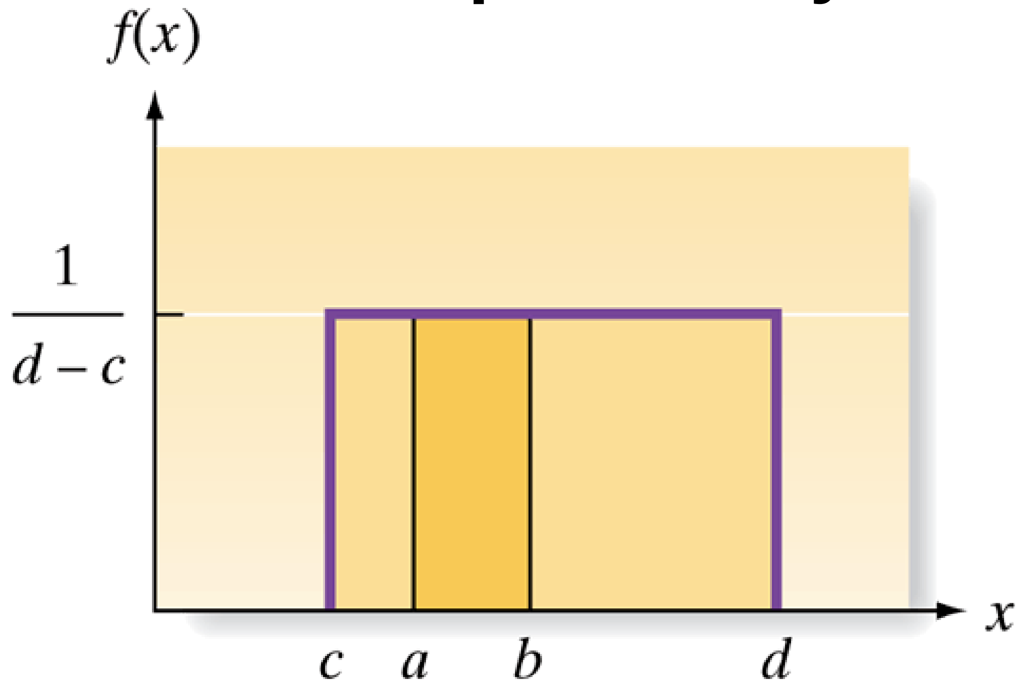
# 5.2

## The Uniform Distribution

# **Figure 5.3** The uniform probability distribution

- Continuous random variables that appear to have *equally likely outcomes over their range of possible values* possess a **uniform probability distribution**

$$Total\ area\ of\ rectangle\ = (Base)(Height)\ = (d-c)\left(\frac{1}{d-c}\right) = 1$$



# Definition

- The uniform distribution is sometimes referred to as the **randomness distribution,** since one way of generating a uniform random variable is to perform an experiment in which a point is *randomly selected* on the horizontal axis between the points *c* and *d*.

**Probability Distribution for a Uniform Random Variable $x$**

Probability density function: $f(x) = \dfrac{1}{d - c} \quad (c \leq x \leq d)$

Mean: $\mu = \dfrac{c + d}{2}$ Standard deviation: $\sigma = \dfrac{d - c}{\sqrt{12}}$

$P(a < x < b) = (b - a)/(d - c), c \leq a < b \leq d$

# Problem

- An unprincipled used-car dealer sells a car to an unsuspecting buyer, even though the dealer knows that the car will have a major breakdown within the next 6 months. The dealer provides a warranty of 45 days on all cars sold. Let $x$ represent the length of time until the breakdown occurs. Assume that $x$ is a uniform random variable with values between 0 and 6 months.
  a) Calculate and interpret the mean and standard deviation of $x$.
  b) Graph the probability distribution of $x$, and show the mean on the horizontal axis. Also show one- and two-standard-deviation intervals around the mean.
  c) Calculate the probability that the breakdown occurs while the car is still under warranty.

# Solution

a) To calculate the mean and standard deviation for $x$, we substitute 0 and 6 months for $c$ and $d$, respectively, in the formulas for uniform random variables. Thus,

$$\mu = \frac{c + d}{2} = \frac{0 + 6}{2} = 3 \text{ months}$$

$$\sigma = \frac{d - c}{\sqrt{12}} = \frac{6 - 0}{\sqrt{12}} = \frac{6}{3.464} = 1.73 \text{ months}$$

- The average length of time $x$ until breakdown for all similar used cars is $\mu = 3$ months. From Chebyshev's rule, we know that at least $75\%$ of the values of $x$ in the distribution will fall into the interval $\mu \pm 2\sigma = 3 \pm 2(1.73)$ $= 3 \pm 3.46$ or between $-0.46$ and $6.46$ months.



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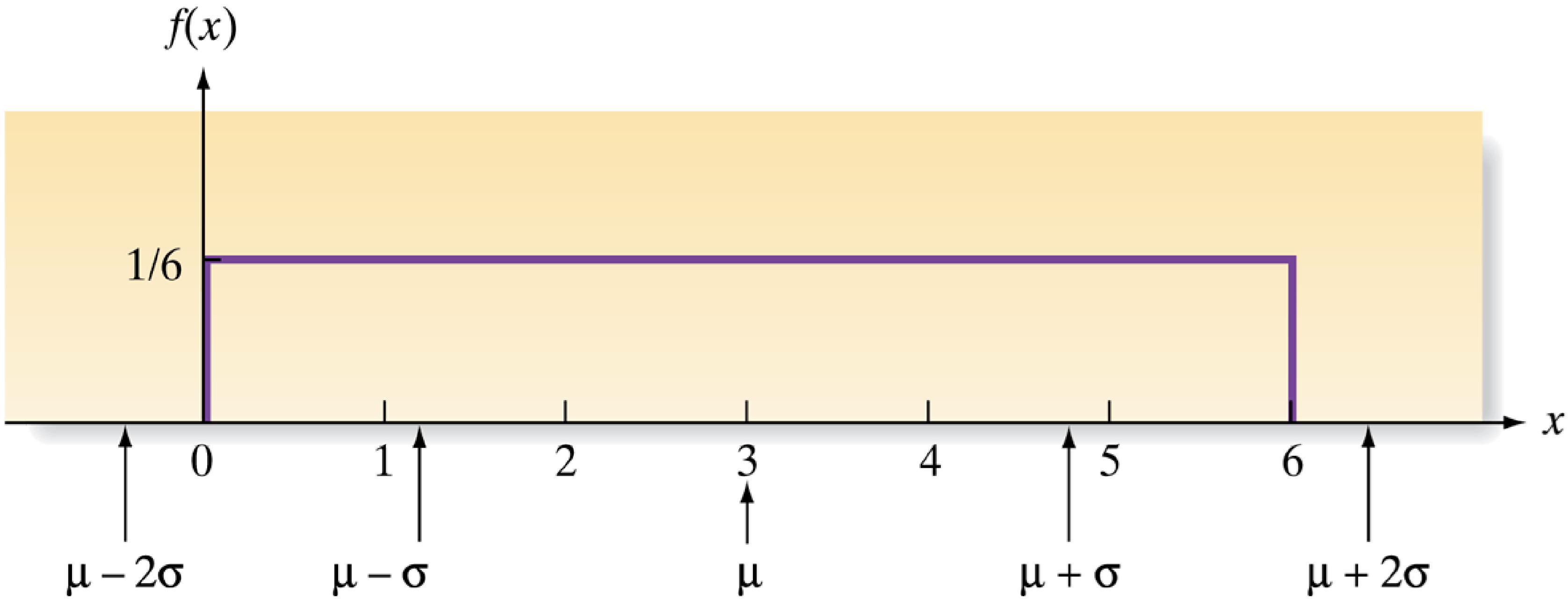
# Solution

b) The uniform probability distribution is →

$$f(x) = \frac{1}{d-c} = \frac{1}{6-0} = \frac{1}{6} \quad (0 \le x \le 6)$$

The mean and the one- and two-standard-deviation intervals around the mean are shown on the horizontal axis.



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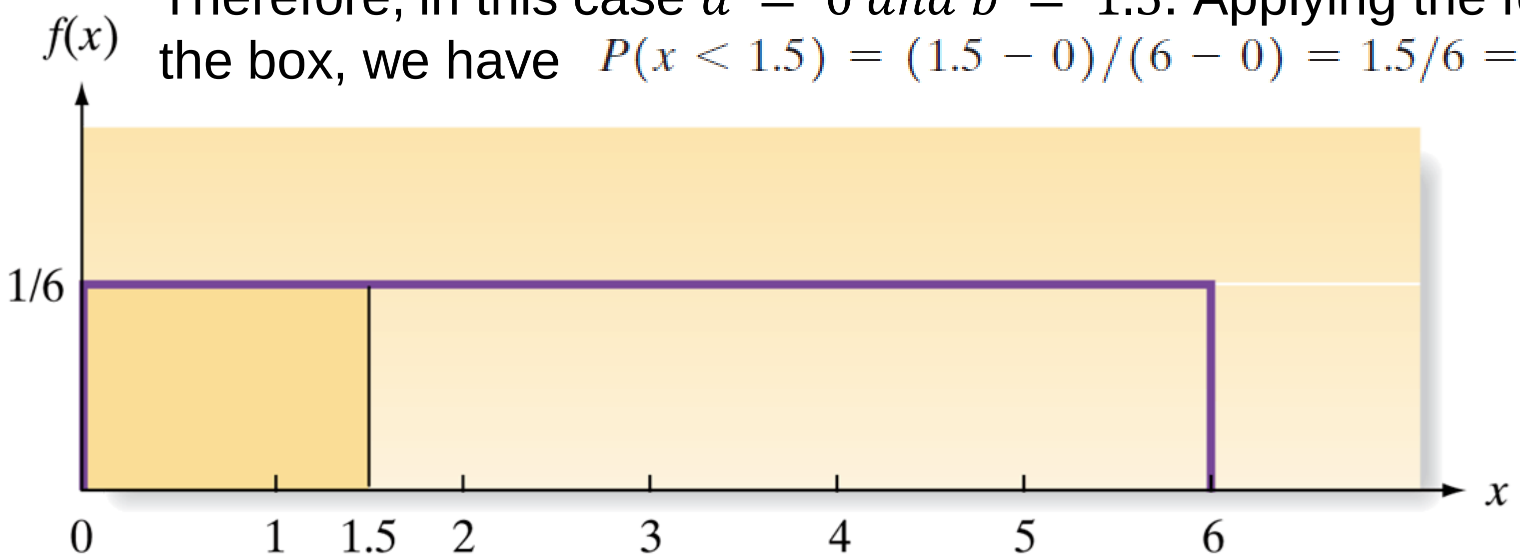
# Solution

c) To find the probability that the car is still under warranty when it breaks down, we must find the probability that $x$ is less than 45 days, or (about) 1.5 months. Therefore, we need to calculate the area under the frequency function $f(x)$ between the points $x = 0$ and $x = 1.5$. Therefore, in this case $a = 0$ *and* $b = 1.5$. Applying the formula in the box, we have $P(x < 1.5) = (1.5 - 0)/(6 - 0) = 1.5/6 = .25$



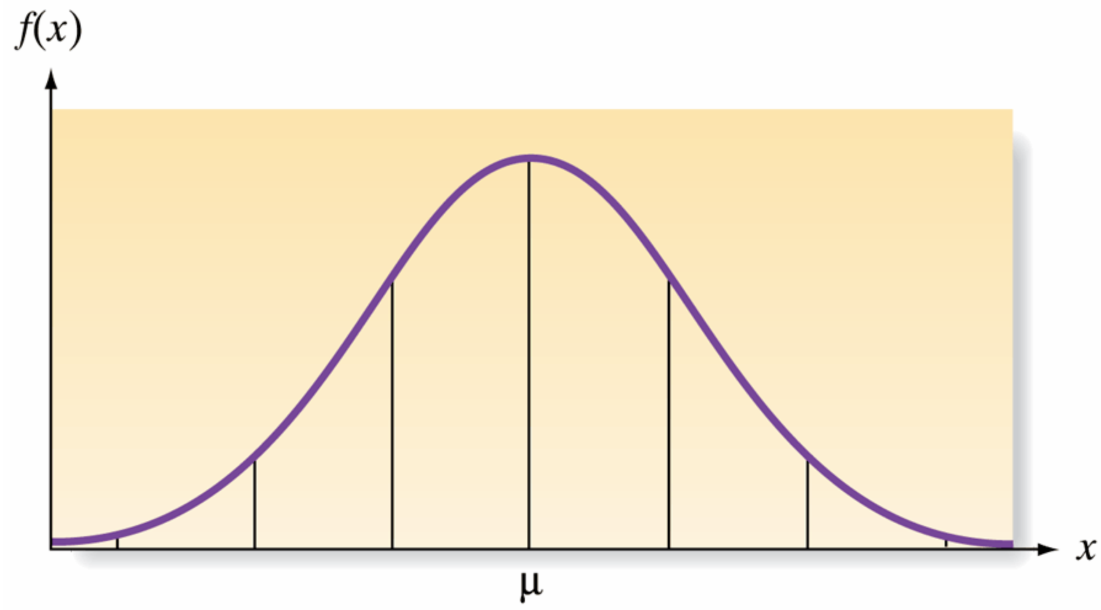

There is a 25% chance that the car will break down while under warranty.

# 5.3

## The Normal Distribution

# **Figure 5.6** A normal probability distribution

- One of the most commonly observed continuous random variables has a **bell-shaped probability distribution**.
- It is known as a **normal random variable** and its probability distribution is called a **normal distribution**.



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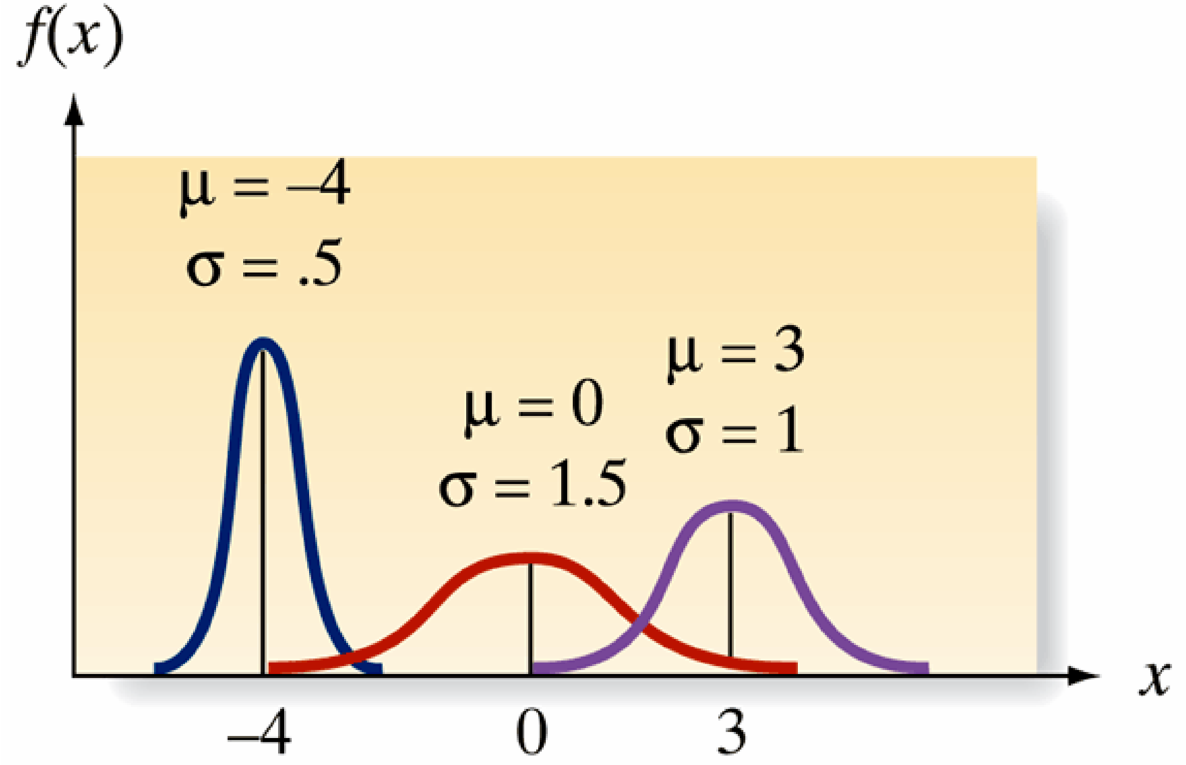
# Normal distribution is very important in the science of **statistical inference**

- Many phenomena generate random variables with probability distributions that are very well approximated by a normal distribution
  - the error made in measuring a person's blood pressure
  - the probability distribution for the yearly rainfall in a certain region

# **Figure 5.7** Several normal distributions with different means and standard deviations

- The normal distribution is perfectly symmetric about $\mu$.
- Its spread is determined by $\sigma$.



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# Procedure

**Probability Distribution for a Normal Random Variable $x$**

Probability density function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(1/2)[(x-\mu)/\sigma]^2}$

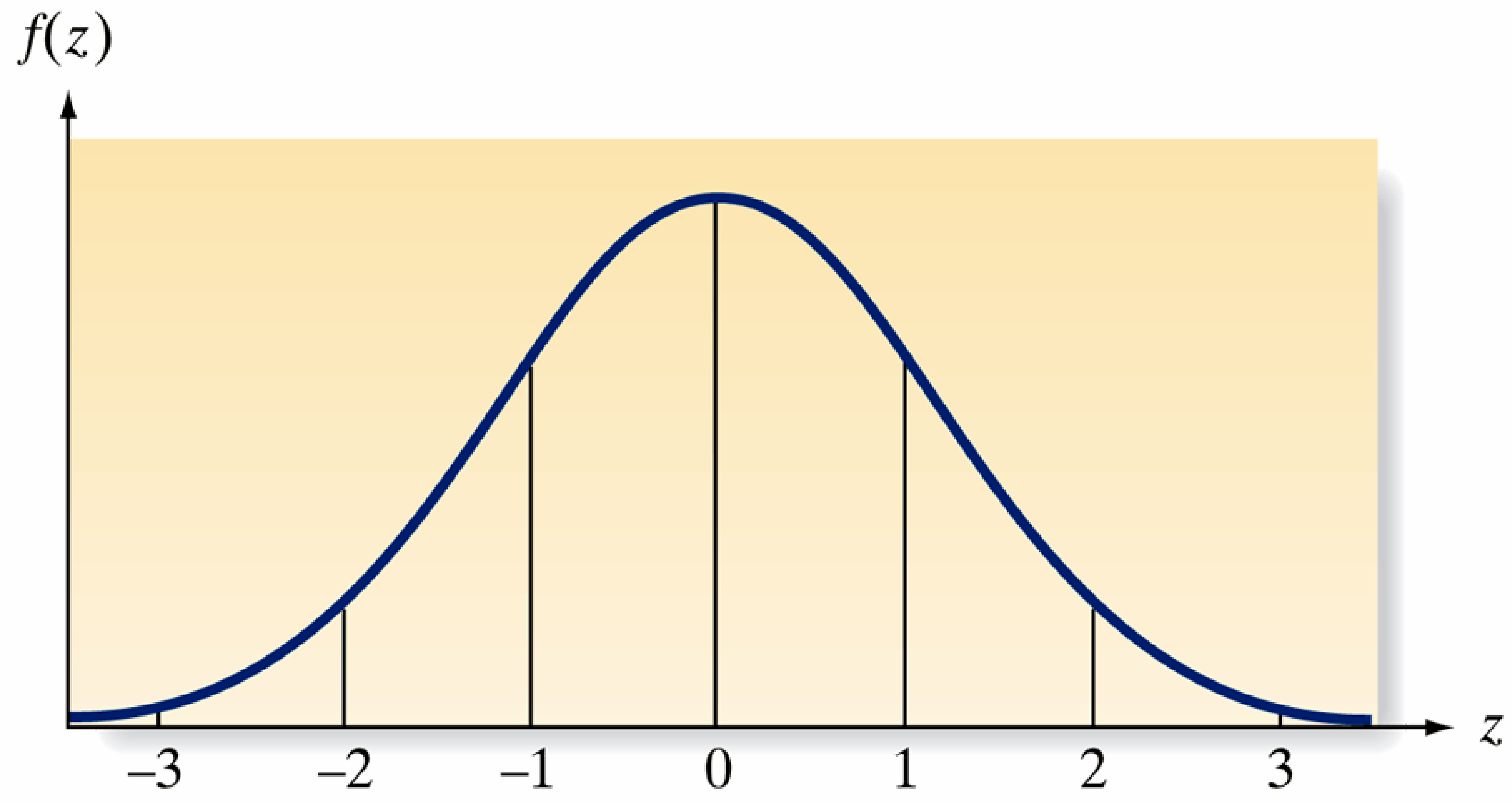
where

$$\begin{aligned} \mu &= \text{Mean of the normal random variable } x \\ \sigma &= \text{Standard deviation} \\ \pi &= 3.1416\ldots \\ e &= 2.71828\ldots \end{aligned}$$

$P(x < a)$ is obtained from a table of normal probabilities.

Note that the mean $\mu$ and standard deviation $\sigma$ appear in this formula, so that no separate formulas for $\mu$ and $\sigma$ are necessary. To graph the normal curve, we have to know the numerical values of $\mu$ and $\sigma$.

# **Figure 5.8** Several normal distributions: $\mu = 0, \sigma = 1$



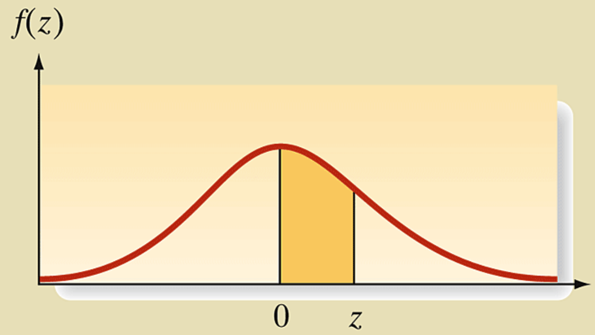
# Definition

The **standard normal distribution** is a normal distribution with $\mu = 0$ and $\sigma = 1$. A random variable with a standard normal distribution, denoted by the symbol $z$, is called a **standard normal random variable.**

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-(1/2)z^2}$$

# Table 5.1

**Table 5.1 Reproduction of Part of Table II in Appendix B**




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **.0** | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| **.1** | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| **.2** | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| **.3** | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| **.4** | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| **.5** | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| **.6** | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| **.7** | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| **.8** | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| **.9** | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| **1.0** | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| **1.1** | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| **1.2** | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| **1.3** | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| **1.4** | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| **1.5** | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |



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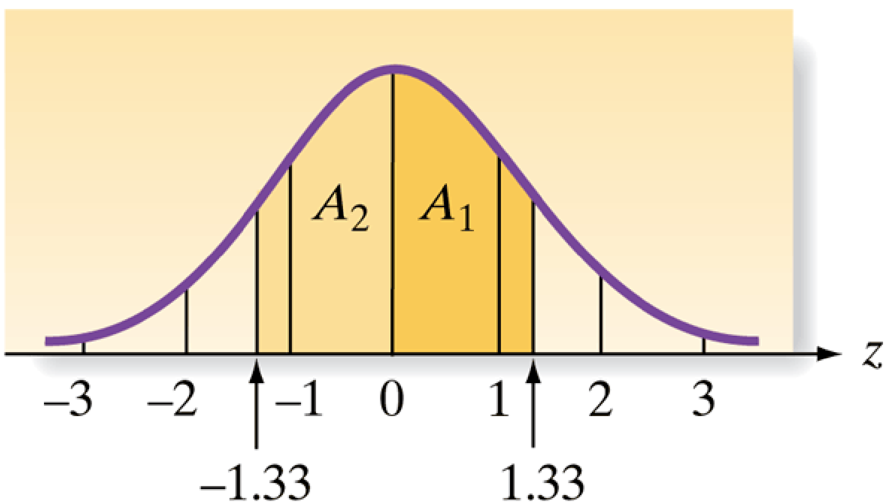
# Problem

- Find the probability that the standard normal random variable $z$ **falls between** $-1.33$ **and** $+1.33$.

| $z$ | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **.0** | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| **.1** | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| **.2** | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| **.3** | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| **.4** | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| **.5** | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| **.6** | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| **.7** | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| **.8** | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| **.9** | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| **1.0** | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| **1.1** | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| **1.2** | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| **1.3** | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| **1.4** | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| **1.5** | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |

# Solution

- Since all probabilities associated with standard normal random variables can be depicted as areas under the standard normal curve, you should always draw the curve and then equate the desired probability to an area.




$$A_1 = A_2 = .4082$$

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **.0** | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| **.1** | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| **.2** | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| **.3** | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| **.4** | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| **.5** | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| **.6** | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| **.7** | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| **.8** | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| **.9** | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| **1.0** | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| **1.1** | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| **1.2** | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| **1.3** | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| **1.4** | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| **1.5** | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |

$$P(-1.33 < z < -1.33) = P(-1.33 < z < 0) + P(0 < z \leq 1.33)$$
$$= A_1 + A_2 = .4082 + .4082 = .8164$$



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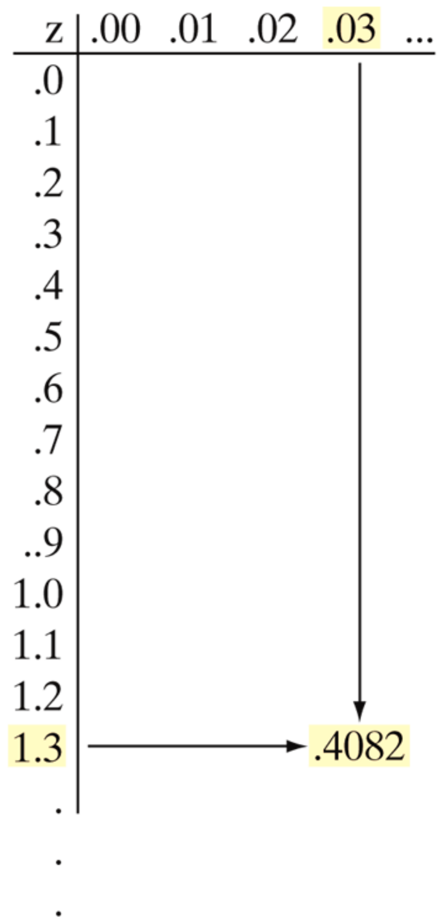
# **Figure 5.10** Finding $z$ =1.33 in the standard normal table, Example 5.3



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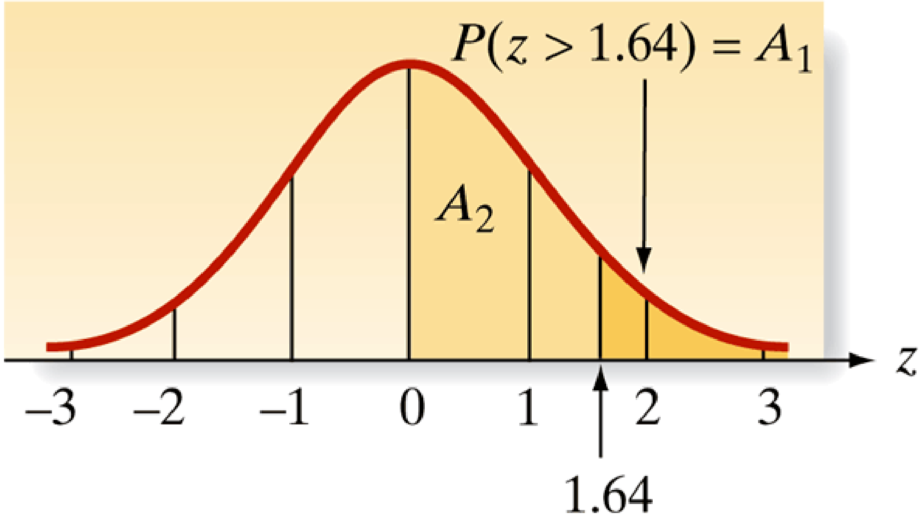
# Problem

- Find the probability that a standard normal random variable **exceeds** $\mathbf{1.64}$; that is, find $P(z > 1.64)$.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| .1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| .2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| .3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| .4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| .5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| .6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| .7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| .8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| .9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | $\boxed{.4495}$ | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |

# **Figure 5.11** Areas under the standard normal curve for Example 5.4

- The standard normal distribution is symmetric about its mean, $z = 0$.
- The total area under the standard normal probability distribution equals 1.



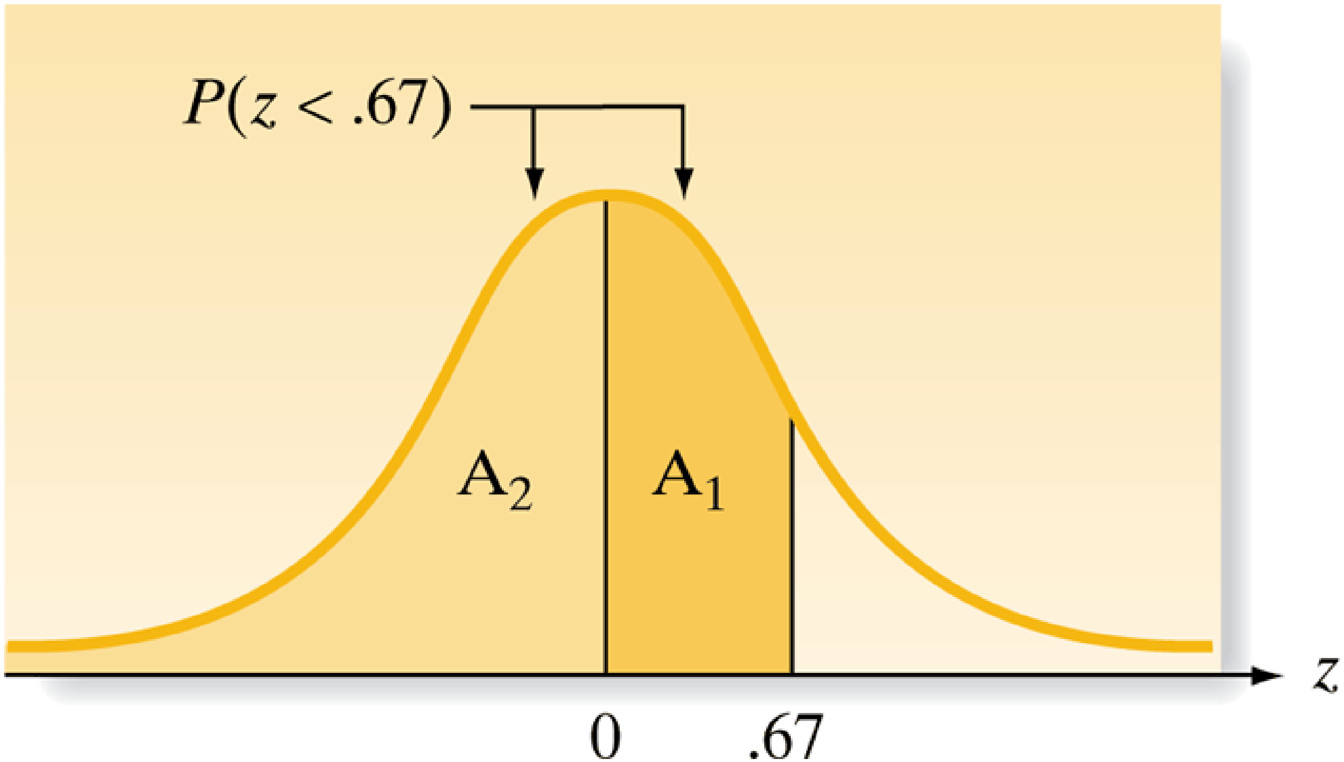

$$P(z > 1.64) = A_1 = .5 - A_2 = .5 - .4495 = .0505$$

# Problem

- Find the probability that a standard normal random variable lies **to the left of .67**.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| .1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| .2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| .3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| .4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| .5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| .6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| .7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| .8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| .9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |

# **Figure 5.12** Areas under the standard normal curve for Example 5.5




$$P(z < .67) = A_1 + A_2 = .2486 + .5 = .7486$$

# Problem

- Find the probability that a standard normal random variable exceeds **1.96 in absolute value**.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| .1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| .2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| .3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| .4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| .5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| .6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| .7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| .8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| .9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |
| 1.4 | .4192 | .4207 | .4222 | .4236 | .4251 | .4265 | .4279 | .4292 | .4306 | .4319 |
| 1.5 | .4332 | .4345 | .4357 | .4370 | .4382 | .4394 | .4406 | .4418 | .4429 | .4441 |
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |
| 2.1 | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| 2.2 | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| 2.3 | .4893 | .4896 | .4898 | .4901 | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| 2.4 | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| 2.5 | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |
| 2.6 | .4953 | .4955 | .4956 | .4957 | .4959 | .4960 | .4961 | .4962 | .4963 | .4964 |
| 2.7 | .4965 | .4966 | .4967 | .4968 | .4969 | .4970 | .4971 | .4972 | .4973 | .4974 |
| 2.8 | .4974 | .4975 | .4976 | .4977 | .4977 | .4978 | .4979 | .4979 | .4980 | .4981 |
| 2.9 | .4981 | .4982 | .4982 | .4983 | .4984 | .4984 | .4985 | .4985 | .4986 | .4986 |
| 3.0 | .4987 | .4987 | .4987 | .4988 | .4988 | .4989 | .4989 | .4989 | .4990 | .4990 |



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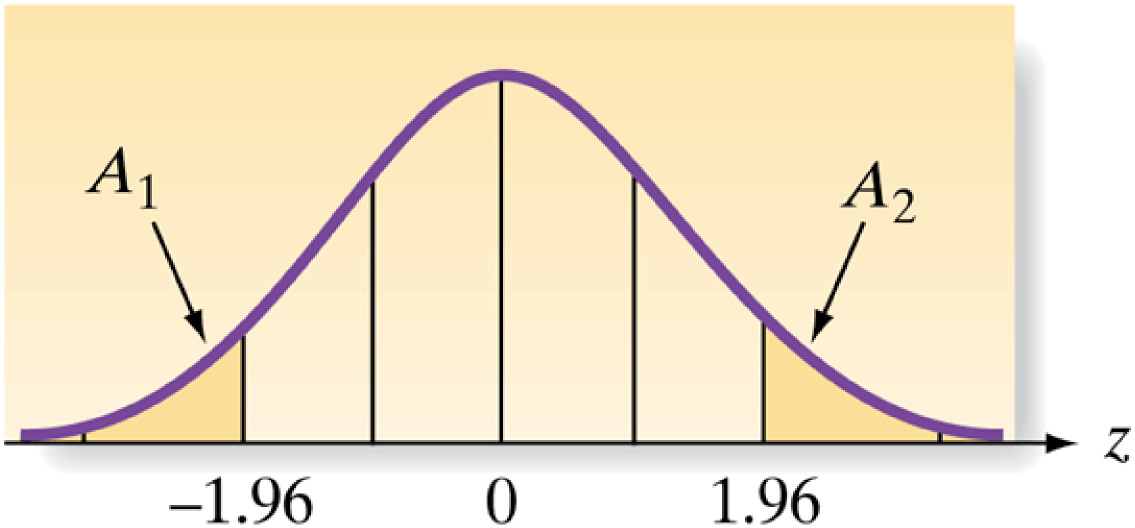
# **Figure 5.13** Areas under the standard normal curve for Example 5.6




$$P(|z| > 1.96) = P(z < -1.96 \text{ or } z > 1.96)$$

We look up $z = 1.96$ and find the area between $z = 0$ and $z = 1.96$ to be .4750. Then $A_2$, the area to the right of 1.96, is $.5 - .4750 = .0250$, so that

$$P(|z| > 1.96) = A_1 + A_2 = .0250 + .0250 = .05$$



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# Definition

**Converting a Normal Distribution to a Standard Normal Distribution**

If $x$ is a normal random variable with mean $\mu$ and standard deviation $\sigma$, then the random variable $z$ defined by the formula

$$z = \frac{x - \mu}{\sigma}$$

has a standard normal distribution. The value $z$ describes the number of standard deviations between $x$ and $\mu$.

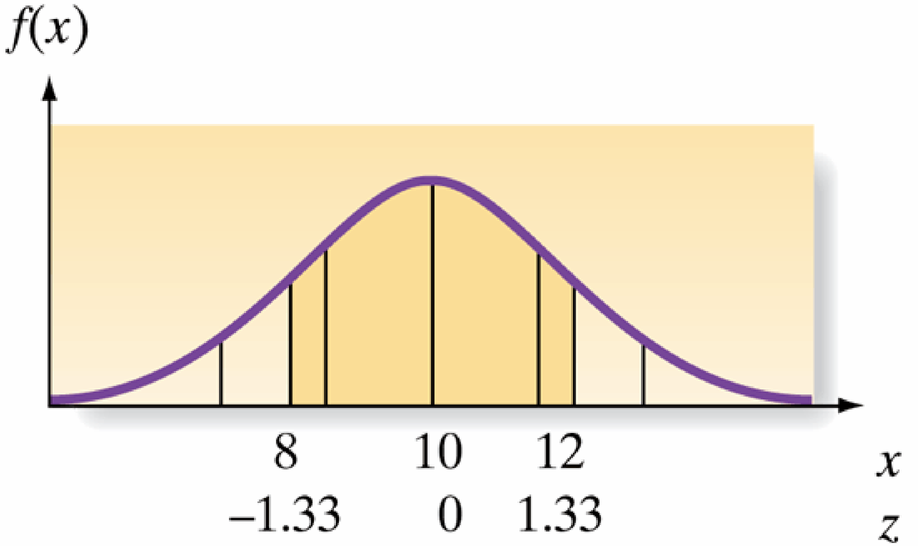
# Problem

- Assume that the length of time, $x$, between charges of a cellular phone is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours. Find the probability that the cell phone will last between 8 and 12 hours between charges.

# **Figure 5.14** Areas under the normal curve for Example 5.7

- The normal distribution with mean $\mu = 10$ and $\sigma = 1.5$ is shown in Figure 5.14. The desired probability that the cell phone lasts between 8 and 12 hours is highlighted.
- We must first convert the distribution to a standard normal distribution, which we do by calculating the *z*-score:

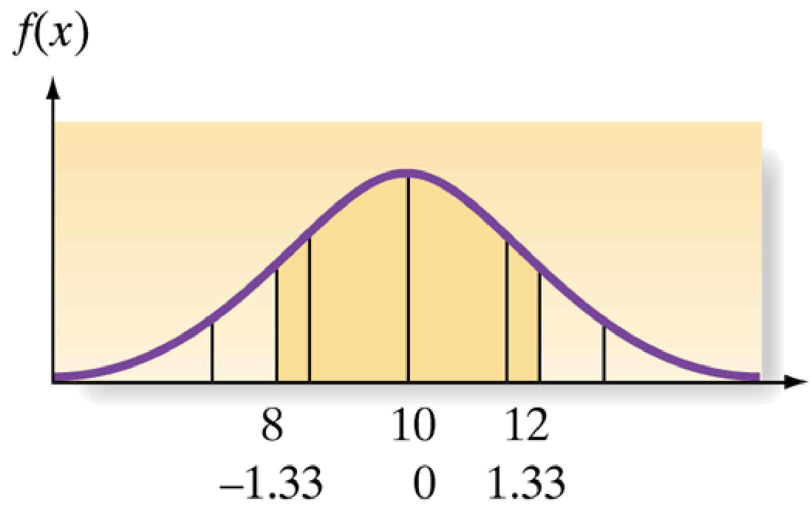
$$z = \frac{x - \mu}{\sigma}$$



| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .0000 | .0040 | .0080 | .0120 | .0160 | .0199 | .0239 | .0279 | .0319 | .0359 |
| .1 | .0398 | .0438 | .0478 | .0517 | .0557 | .0596 | .0636 | .0675 | .0714 | .0753 |
| .2 | .0793 | .0832 | .0871 | .0910 | .0948 | .0987 | .1026 | .1064 | .1103 | .1141 |
| .3 | .1179 | .1217 | .1255 | .1293 | .1331 | .1368 | .1406 | .1443 | .1480 | .1517 |
| .4 | .1554 | .1591 | .1628 | .1664 | .1700 | .1736 | .1772 | .1808 | .1844 | .1879 |
| .5 | .1915 | .1950 | .1985 | .2019 | .2054 | .2088 | .2123 | .2157 | .2190 | .2224 |
| .6 | .2257 | .2291 | .2324 | .2357 | .2389 | .2422 | .2454 | .2486 | .2517 | .2549 |
| .7 | .2580 | .2611 | .2642 | .2673 | .2704 | .2734 | .2764 | .2794 | .2823 | .2852 |
| .8 | .2881 | .2910 | .2939 | .2967 | .2995 | .3023 | .3051 | .3078 | .3106 | .3133 |
| .9 | .3159 | .3186 | .3212 | .3238 | .3264 | .3289 | .3315 | .3340 | .3365 | .3389 |
| 1.0 | .3413 | .3438 | .3461 | .3485 | .3508 | .3531 | .3554 | .3577 | .3599 | .3621 |
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | .3997 | .4015 |
| 1.3 | .4032 | .4049 | .4066 | $\boxed{.4082}$ | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |

# Solution

- The z-scores corresponding to the important values of $x$ are shown beneath the $x$ values on the horizontal axis.
- Note that $z = 0$ corresponds to the mean of $m = 10$ hours, whereas the $x$ values 8 and 12 yield z-scores of $-1.33$ and $+1.33$, respectively.




$$P(8 \leq x \leq 12) = P(-1.33 \leq z \leq 1.33) = 2(.4082) = .8164$$



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# Procedure

**Steps for Finding a Probability Corresponding to a Normal Random Variable**

1. Sketch the normal distribution and indicate the mean of the random variable $x$. Then shade the area corresponding to the probability you want to find.
2. Convert the boundaries of the shaded area from $x$ values to standard normal random variable $z$ values by using the formula

$$z = \frac{x - \mu}{\sigma}$$

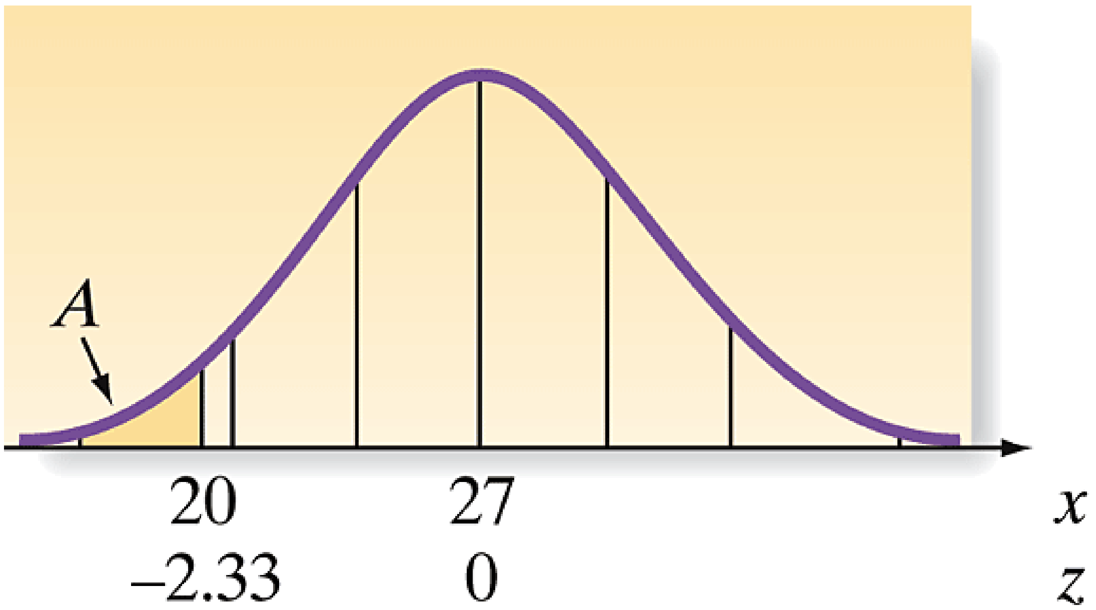
   Show the $z$ values under the corresponding $x$ values on your sketch.
3. Use technology or Table II in Appendix B to find the areas corresponding to the $z$ values. If necessary, use the symmetry of the normal distribution to find areas corresponding to negative $z$ values and the fact that the total area on each side of the mean equals .5 to convert the areas from Table II to the probabilities of the event you have shaded.

# Problem

- Suppose an automobile manufacturer introduces a new model that has an advertised mean in-city mileage of 27 miles per gallon. Although such advertisements seldom report any measure of variability, suppose you write the manufacturer for the details of the tests and you find that the standard deviation is 3 miles per gallon. This information leads you to formulate a probability model for the random variable $x$, the in-city mileage for this car model. You believe that the probability distribution of $x$ can be approximated by a normal distribution with **a mean of 27** and **a standard deviation of 3**.
  - If you were to buy this model of automobile, what is the probability that you would purchase one that averages less than 20 miles per gallon for in-city driving? In other words, find $P(x < 20)$.
  - Suppose you purchase one of these new models and it does get less than 20 miles per gallon for in-city driving. Should you conclude that your probability model is incorrect?

# **Figure 5.16** Area under the standard normal curve for Example 5.8

- The probability model proposed for $x$, the in-city mileage, is shown in Figure 5.16.



# Solution

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| **2.1** | .4821 | .4826 | .4830 | .4834 | .4838 | .4842 | .4846 | .4850 | .4854 | .4857 |
| **2.2** | .4861 | .4864 | .4868 | .4871 | .4875 | .4878 | .4881 | .4884 | .4887 | .4890 |
| **2.3** | .4893 | .4896 | .4898 | $\boxed{.4901}$ | .4904 | .4906 | .4909 | .4911 | .4913 | .4916 |
| **2.4** | .4918 | .4920 | .4922 | .4925 | .4927 | .4929 | .4931 | .4932 | .4934 | .4936 |
| **2.5** | .4938 | .4940 | .4941 | .4943 | .4945 | .4946 | .4948 | .4949 | .4951 | .4952 |

- We are interested in finding the area A to the left of 20, since that area corresponds to the probability that a measurement chosen from this distribution falls below 20. In other words, if this model is correct, the area A represents the fraction of cars that can be expected to get less than 20 miles per gallon for in-city driving. To find A, we first calculate the *z* value corresponding to $x = 20$. That is,

$$z = \frac{x - \mu}{\sigma} = \frac{20 - 27}{3} = -\frac{7}{3} = -2.33$$

$$P(x < 20) = P(z < -2.33)$$

$$P(x < 20) = A = .5 - .4901 = .0099 \approx .01$$

# Solution

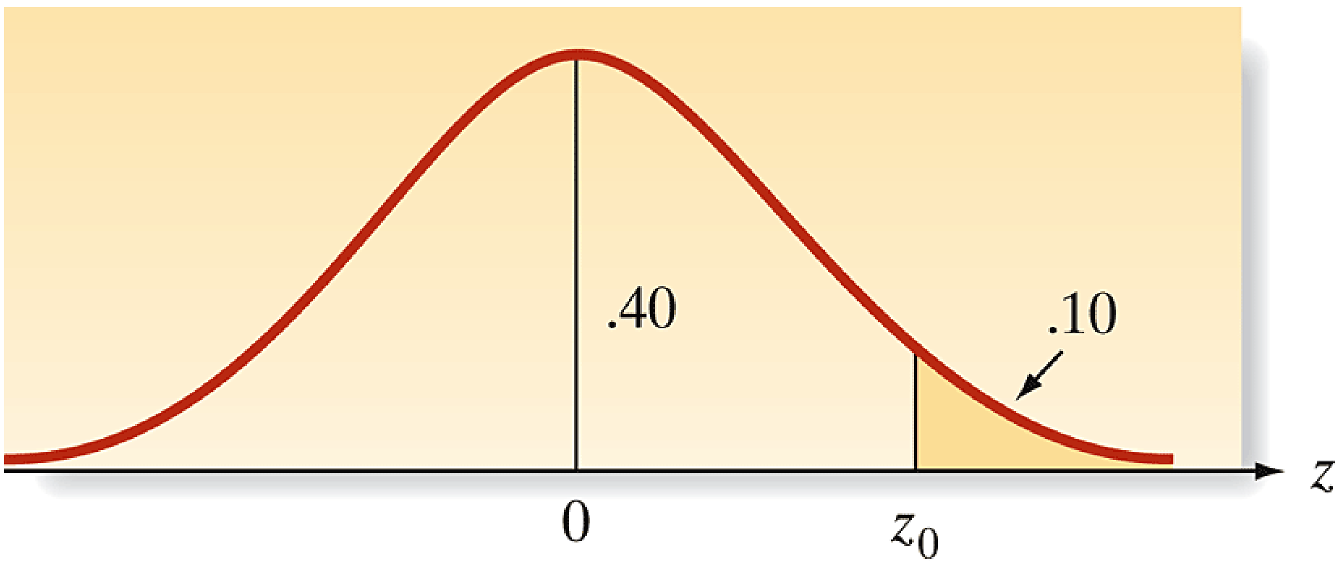
- Now you are asked to make an inference based on a sample: the car you purchased. You are getting less than 20 miles per gallon for in-city driving. What do you infer?
- We think you will agree that one of two possibilities exists:
  - 1. The probability model is correct. You simply were unfortunate to have purchased one of the cars in the 1% that get less than 20 miles per gallon in the city.
  - 2. The probability model is incorrect. Perhaps the assumption of a normal distribution is unwarranted, or the mean of 27 is an overestimate, or the standard deviation of 3 is an underestimate, or some combination of these errors occurred. At any rate, the form of the actual probability model certainly merits further investigation.

# Problem

- Find the value of $z$—call it $z_0$—in the standard normal distribution that will be exceeded only 10% of the time. That is, find $z_0$ such that $\mathrm{P}(z \geq z_0) = .10$.

# **Figure 5.17** Areas under the standard normal curve for Example 5.9

- In this case, we are given a probability, or an area, and are asked to find the value of the standard normal random variable that corresponds to the area. Specifically, we want to find the value $z_0$ such that only 10% of the standard normal distribution exceeds $z_0$. (See Figure 5.17.)



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| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.1 | .3643 | .3665 | .3686 | .3708 | .3729 | .3749 | .3770 | .3790 | .3810 | .3830 |
| 1.2 | .3849 | .3869 | .3888 | .3907 | .3925 | .3944 | .3962 | .3980 | $\boxed{.3997}$ | .4015 |
| 1.3 | .4032 | .4049 | .4066 | .4082 | .4099 | .4115 | .4131 | .4147 | .4162 | .4177 |

# Solution

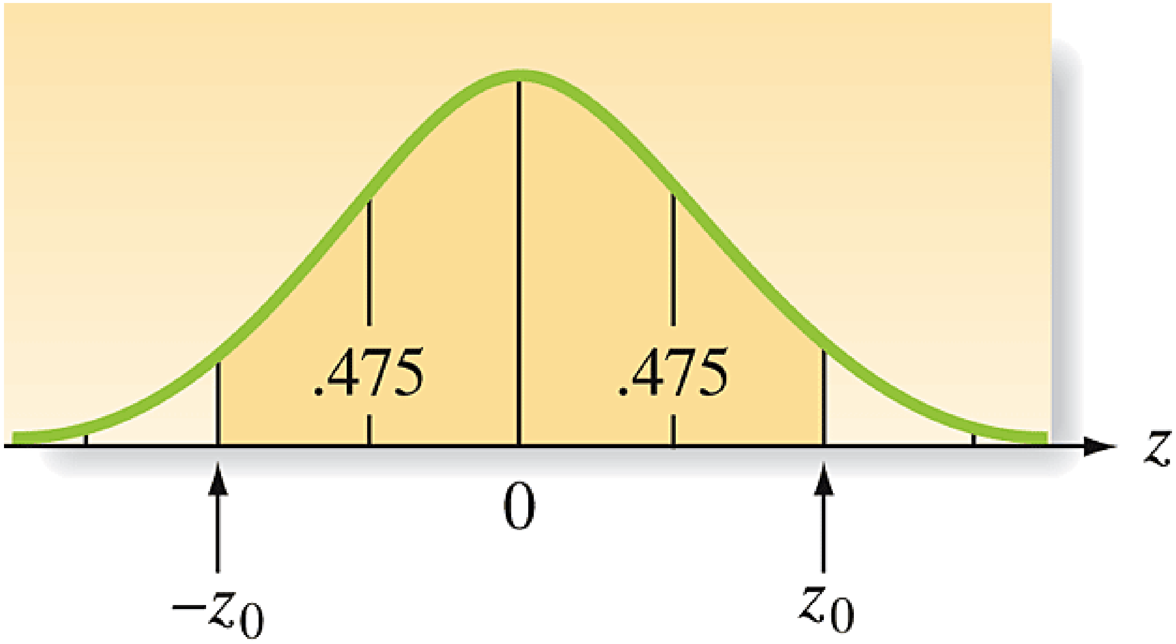
- We know that the total area to the right of the mean, $z = 0$, is .5, which implies that $z_0$ must lie to the right of 0 $(z > 0)$.
- To pinpoint the value, we use the fact that the area to the right of $z_0$ is .10, which implies that the area between $z = 0$ and $z_0$ is $.5 - .1 = .4$.
- But areas between $z = 0$ and some other $z$ value are exactly the types given in Table II.
- Therefore, we look up the area .4000 in the body of Table II and find that the corresponding $z$ value is $z_0 = 1.28$.
- The implication is that the point 1.28 standard deviations above the mean is the 90th percentile of a normal distribution.

# Problem

- Find the value of $z_0$ such that 95% of the standard normal $z$ values lie between $-z_0$ and $+z_0$; that is, find $\mathrm{P}(-z_0 \le z \le z_0) = .95$

# **Figure 5.19** Areas under the standard normal curve for Example 5.10

- Here we wish to move an equal distance $z_0$ in the positive and negative directions from the mean $z = 0$ until 95% of the standard normal distribution is enclosed.



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| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1.6 | .4452 | .4463 | .4474 | .4484 | .4495 | .4505 | .4515 | .4525 | .4535 | .4545 |
| 1.7 | .4554 | .4564 | .4573 | .4582 | .4591 | .4599 | .4608 | .4616 | .4625 | .4633 |
| 1.8 | .4641 | .4649 | .4656 | .4664 | .4671 | .4678 | .4686 | .4693 | .4699 | .4706 |
| 1.9 | .4713 | .4719 | .4726 | .4732 | .4738 | .4744 | .4750 | .4756 | .4761 | .4767 |
| 2.0 | .4772 | .4778 | .4783 | .4788 | .4793 | .4798 | .4803 | .4808 | .4812 | .4817 |

# Solution

- This means that the area on each side of the mean will be equal to .475.
- Since the area between $z = 0$ and $z_0$ is 0.475, we look up 0.475 in the body of Table II to find the value $z_0 = 1.96$.
- Thus, 95% of a normal distribution lies between $+1.96$ and $-1.96$ standard deviations of the mean.



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# 5.4

## Descriptive Methods for Assessing Normality

# Procedure

**Determining whether the Data Are from an Approximately Normal Distribution**

1. Construct either a histogram or stem-and-leaf display for the data, and note the shape of the graph. If the data are approximately normal, the shape of the histogram or stem-and-leaf display will be similar to the normal curve shown in Figure 5.6 (i.e., the display will be mound shaped and symmetric about the mean).
2. Compute the intervals $\bar{x} \pm s$, $\bar{x} \pm 2s$, and $\bar{x} \pm 3s$, and determine the percentage of measurements falling into each. If the data are approximately normal, the percentages will be approximately equal to 68%, 95%, and 100%, respectively.
3. Find the interquartile range IQR and standard deviation $s$ for the sample, and then calculate the ratio IQR/$s$. If the data are approximately normal, then $IQR/s \approx 1.3$.
4. Construct a *normal probability plot* for the data. If the data are approximately normal, the points will fall (approximately) on a straight line.

# Definition

A **normal probability plot** for a data set is a scatterplot with the ranked data values on one axis and their corresponding expected $z$-scores from a standard normal distribution on the other axis. [*Note:* Computation of the expected standard normal $z$-scores is beyond the scope of this text. Therefore, we will rely on available statistical software packages to generate a normal probability plot.]

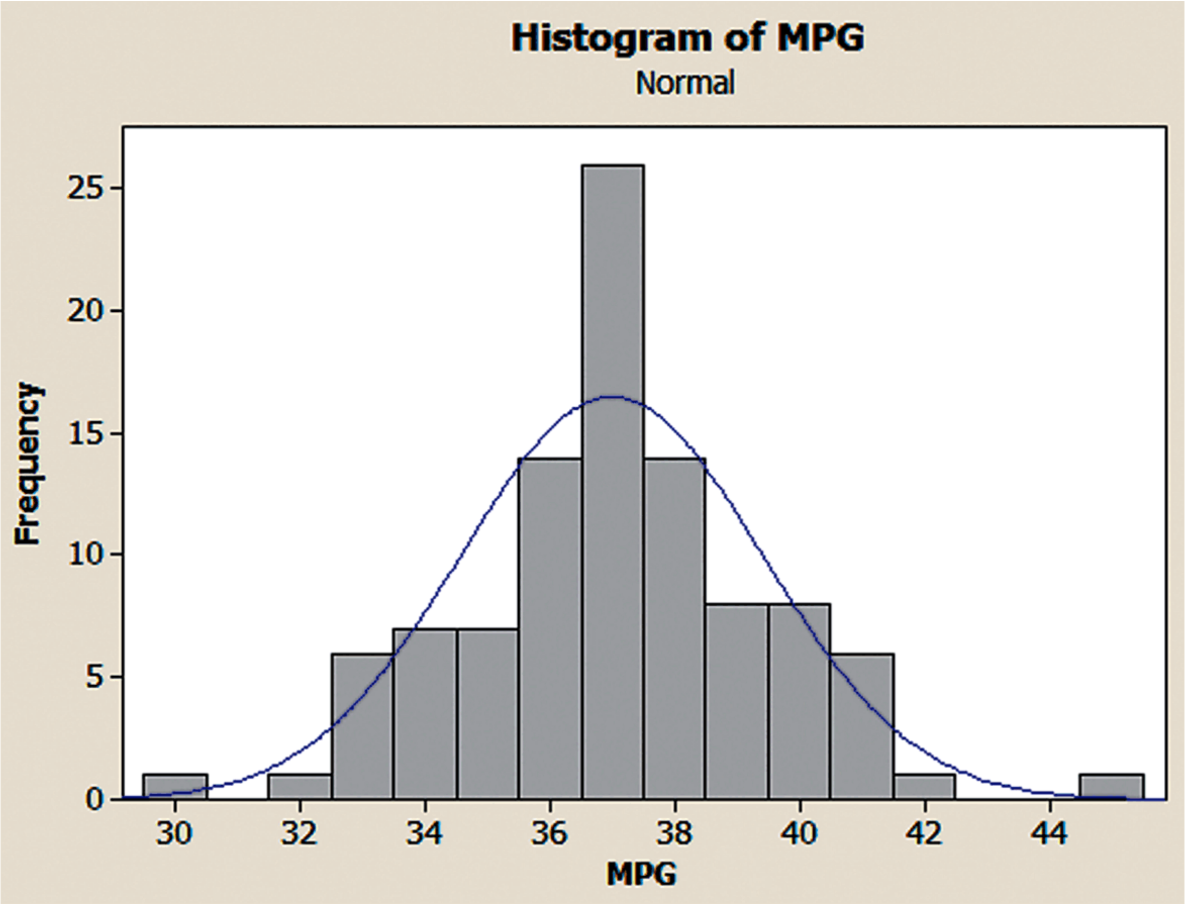
# Problem

- The EPA mileage ratings on 100 cars, first presented in Chapter 2 (p. 71), are reproduced in Table 5.2. Numerical and graphical descriptive measures for the data are shown on the MINITAB and SPSS printouts presented in Figure 5.21a–c. Determine whether the EPA mileage ratings are from an approximate normal distribution.

# Table 5.2

| Table 5.2 | EPA Gas Mileage Ratings for 100 Cars (miles per gallon) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36.3 | 41.0 | 36.9 | 37.1 | 44.9 | 36.8 | 30.0 | 37.2 | 42.1 | 36.7 |
| 32.7 | 37.3 | 41.2 | 36.6 | 32.9 | 36.5 | 33.2 | 37.4 | 37.5 | 33.6 |
| 40.5 | 36.5 | 37.6 | 33.9 | 40.2 | 36.4 | 37.7 | 37.7 | 40.0 | 34.2 |
| 36.2 | 37.9 | 36.0 | 37.9 | 35.9 | 38.2 | 38.3 | 35.7 | 35.6 | 35.1 |
| 38.5 | 39.0 | 35.5 | 34.8 | 38.6 | 39.4 | 35.3 | 34.4 | 38.8 | 39.7 |
| 36.3 | 36.8 | 32.5 | 36.4 | 40.5 | 36.6 | 36.1 | 38.2 | 38.4 | 39.3 |
| 41.0 | 31.8 | 37.3 | 33.1 | 37.0 | 37.6 | 37.0 | 38.7 | 39.0 | 35.8 |
| 37.0 | 37.2 | 40.7 | 37.4 | 37.1 | 37.8 | 35.9 | 35.6 | 36.7 | 34.5 |
| 37.1 | 40.3 | 36.7 | 37.0 | 33.9 | 40.1 | 38.0 | 35.2 | 34.8 | 39.5 |
| 39.9 | 36.9 | 32.9 | 33.8 | 39.8 | 34.0 | 36.8 | 35.0 | 38.1 | 36.9 |

# **Figure 5.21a** MINITAB histogram for gas mileage data



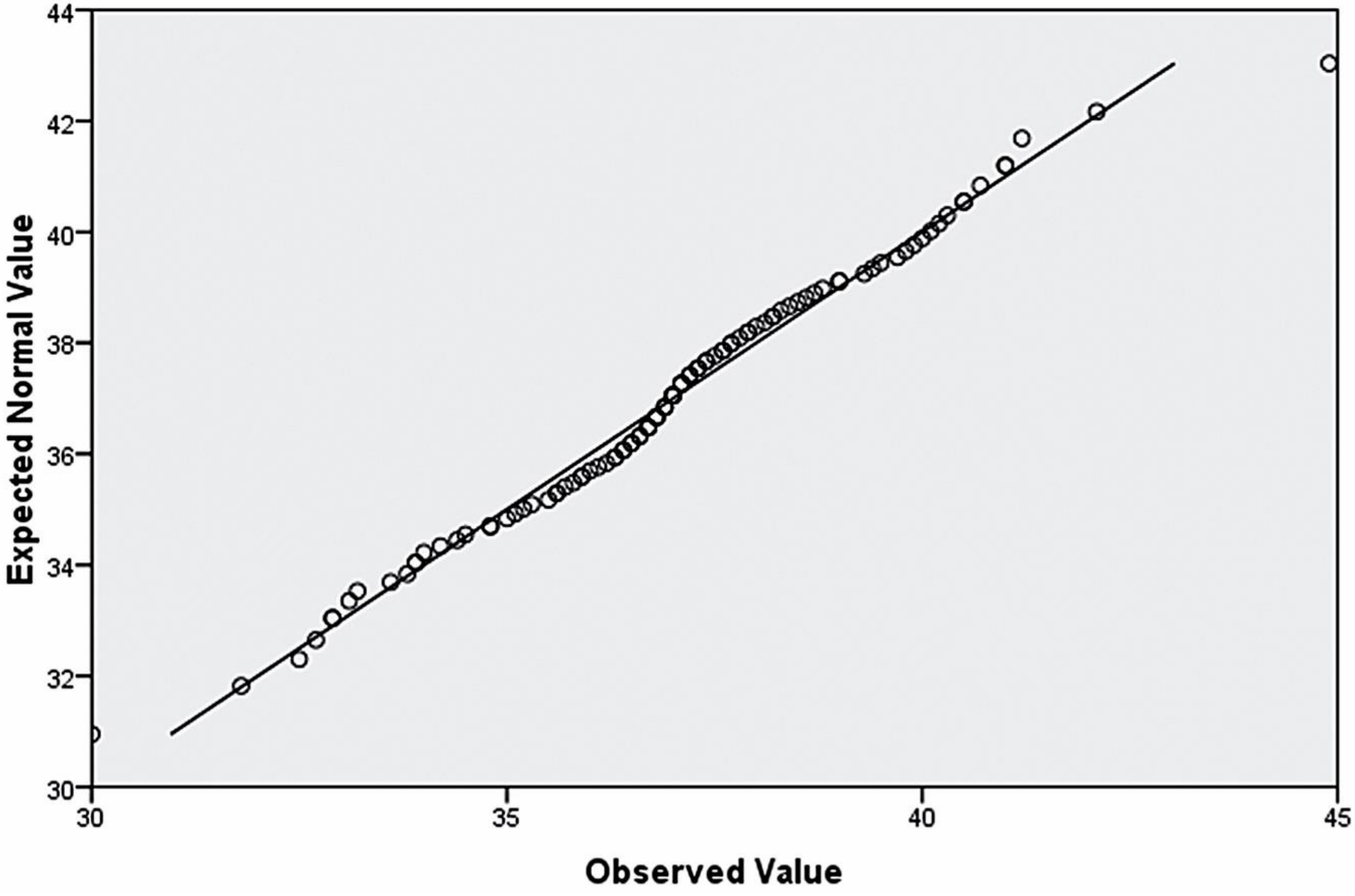
# **Figure 5.21b** MINITAB Descriptive statistics for gas mileage data

**Descriptive Statistics: MPG**

| Variable | N | Mean | StDev | Minimum | Q1 | Median | Q3 | Maximum |
|---|---|---|---|---|---|---|---|---|
| MPG | 100 | 36.994 | 2.418 | 30.000 | 35.625 | 37.000 | 38.375 | 44.900 |

$$\frac{IQR}{s} = \frac{38.375 - 35.625}{2.418} = 1.137$$

# **Figure 5.21c** SPSS normal probability plot for gas mileage data



# Table 5.3

| Table 5.3 Describing the 100 EPA Mileage Ratings | |
|---|---|
| Interval | Percentage in Interval |
| $\bar{x} \pm s = (34.6, 39.4)$ | 68 |
| $\bar{x} \pm 2s = (32.2, 41.8)$ | 96 |
| $\bar{x} \pm 3s = (29.8, 44.2)$ | 99 |



PEARSON