In a donut store, there are 20 kinds of donuts. We'll buy 12 donuts. In how many ways can we do this?

$n = 20$

$r = 12$

$$x_1 + x_2 + \dots + x_{20} = 12$$

$$0 \le x_i$$

$$\binom{n+r-1}{r} = \binom{31}{12} = \binom{31}{19}$$

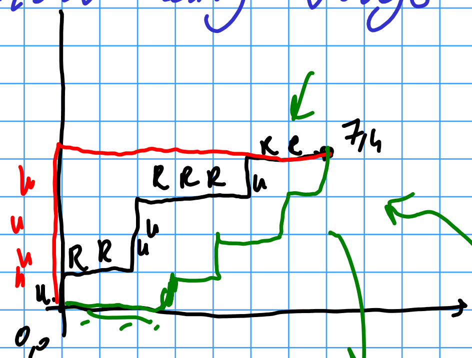
$D_1 \quad D_2 \quad D_3 \quad \dots$

$\cancel{20}$ 12 ← counts same configurations many times!!

$$\binom{10}{5} = \frac{10\cdot 9\cdot 8\cdot 7\cdot \cancel{6}}{5\cdot 4\cdot \cancel{3}\cdot \cancel{2}}$$

X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4
5	5	10	12	0	0	3	5

In how many ways can we go from 0,0 to 7,4 ?



$\uparrow u \quad \rightarrow R$

4U 7R

UUUURRRRRRR

URR UURRRRuRR

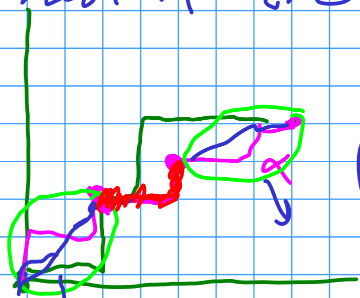
RRRURURRURU

$\binom{11}{4} = \binom{11}{7}$

What if the red path is prohibited?

pink paths

(Using all of it, not only some part of it)



$(0,0) \to (2,2)$

$\binom{4}{2}$

$\binom{4}{1}$

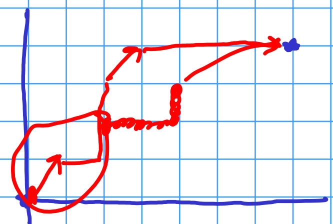
pink prohibited

$\binom{4}{2}\binom{4}{1}$

$$\binom{11}{4} - \binom{4}{2}\binom{4}{1}$$

	R	U
2D	0	0
1D	1	1
0D	2	2

$$\left(1+\frac{3!}{}+\frac{4!}{2!\,2!}\right)\left(\quad\right)$$



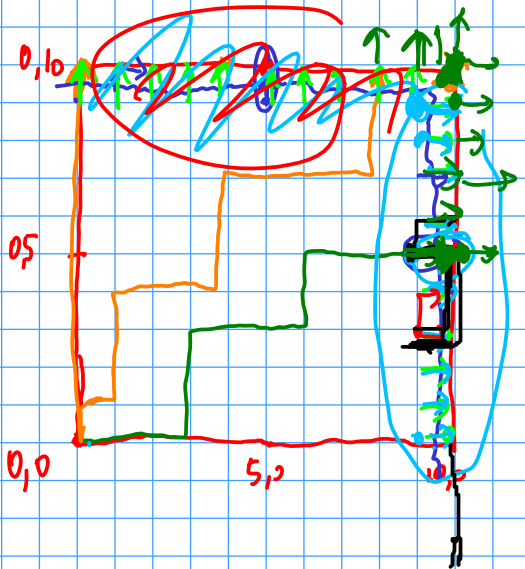
	R	U
4D	3	0
3D	4	1
2D	5	2
1D	6	3
0D	7	4

What if there is also a diagonal move?

$$\left(\frac{7!}{4!\,3!}+\frac{8!}{3!\,4!}+\frac{9!}{2!\,3!\,2!}+\frac{10!}{1!\,6!\,3!}+\frac{11!}{7!\,4!}\right)$$

Bursting balloon

$$\frac{\text{poss. we want}}{\text{all poss.}} = \frac{6}{6+0}$$

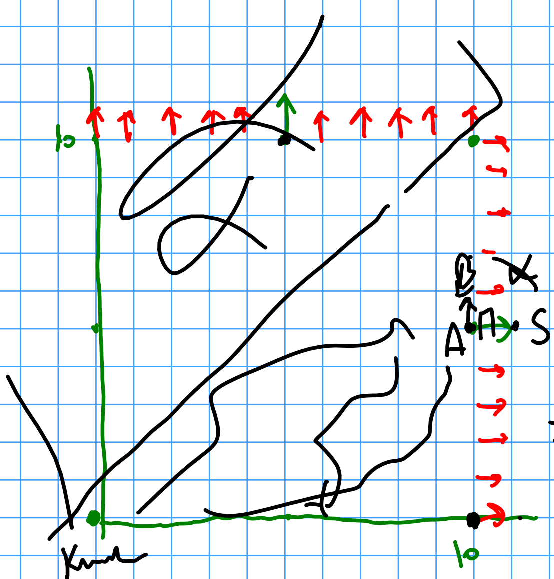


$$\left(\binom{15}{5}\right) \times \cancel{2}$$

$$\left(\binom{19}{10} + \binom{18}{9} + \binom{17}{8} + \binom{16}{7} + \dots \binom{9}{0}\right) \times \cancel{2}$$

$$3^{3,2}$$

$$\frac{\binom{15}{5}}{\binom{19}{10} + \binom{18}{9} + \binom{17}{8} + \dots + \binom{9}{0}}$$



$$= \frac{\binom{15}{5} \cdot \cancel{2}}{\left(\binom{10}{0} + \binom{11}{1} + \binom{12}{2} + \dots + \binom{20}{10}\right) \cdot \cancel{2}}$$