

Q-1) a-)  $c(n)$  denotes the smallest number of items thief can steal using a bag capacity of  $n$

answer a-) Smallest number of items of denominations  $w_1, w_2, w_3, w_k$  needed to take from house for  $n$  weights thief have in bag there must be item  $w_i \leq n$  (total bag capacity) for the remain items the thief should take  $n - w_i$  should be optimal. So

$c(n) = 1 + c(n - w_i)$ , we don't know which item  $w_i$  lead us to optimal solution, we may check all item possibilities.

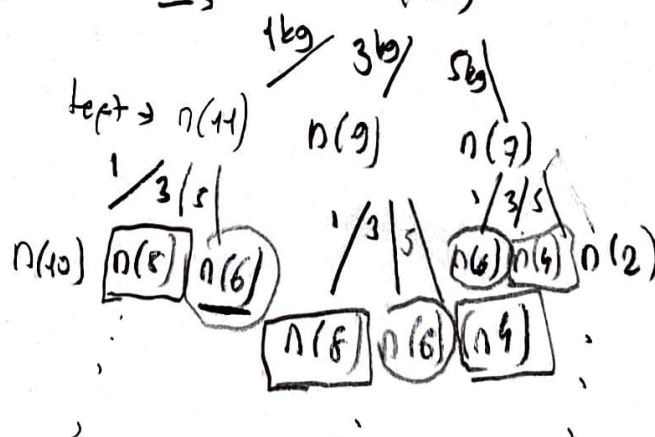
while  $w_i \leq n$ .

$$c[n] = \begin{cases} 0 & \text{if } n = 0 \\ \min_{1 \leq i \leq p} \{1 + c[n - w_i]\} & \text{if } n > 0 \end{cases}$$

b-) Show that this problem has overlapping subproblems property  
To show that, we need to assume the thief bag

$n = \text{some value}$ , for example  $n = 12 \text{ kg}$  so recursive approach

is lead us  $\Rightarrow$   $n(12)$  we have  $\{1 \text{ kg}, 3 \text{ kg}, 5 \text{ kg}\}$  3 different type items



$\Rightarrow$  at the second iteration we already proved that there are many overlapping subproblems. i.e.  $n(6), n(4), n(8) \dots$

Mert Karababo  
20/60807017

meclb

Undergraduate

c) Write recursive algorithm pseudocode.

before write code I need to define what we have in question

$N$  = bag of thief capacity. /  $N$  = size of the items array in have /  $W$  = weight of items

MinItems ( $W[]$ ,  $N$ ,  $n$ )

if  $n = 0$

return 0

else

total = 0

start  $\leftarrow \infty$

for  $i$  1 to  $N$  do

if  $W[i] \geq n$  then

total =  $1 + \text{minItems}(W, N, n - W[i])$

start =  $\min(\text{start}, \text{total})$

End if

return start

Mert Kordoba meeb Undergraduate 20160807012

Q-1) d) Greedy

We claim that the Weight of items are  $(1, 3, 4)$   
for example if bag of thief  $n$  can have 10  
 $n=10$ , greedy will choose first 4, then 4 and 1 and 1  
but this can be done better with only 4, 3, 3 weight items.  
So if we use greedy algorithm it will cause us to get  
more items which we don't want here.

met Korobko met Undergraduate  
2016 08 07 017

Q-1) (c.) Dynamic algorithms

MinItems (WC, dp, n, N)

for i 1 to N do  
dp[i] =  $\infty$  } initialize all values to infinity

dp[0] = 0 // base case

for i 1 to N do

// inner loop denotes the index of item array (w)

for j 0 to N do

// i  $\rightarrow$  sum weight

// j  $\rightarrow$  next item index

if (WC[j]  $\leq$  i)

// might include new items

dp[i] = min(dp[i], 1 + dp[i - WC[j]])

for i 1 to N do

return dp[N]

(F-) Complexity  $\Rightarrow$  N  $\rightarrow$  length items  
w  $\rightarrow$  weight of items  
 $O(N * w)$

} if the item is greater than 1 the inner loop will run (w - w<sub>i</sub>) iterations instead w, but need to consider worst case and ignore factors.



Q-2)  $\Rightarrow$  We can backup the array after every  $n$  insertions.

a) let  $C_i'$  be the charge  $i$ -th operation and  $C_i$  true cost then

$\sum_{i=1}^n C_i \leq 1 \sum_{i=1}^n C_i'$  for all  $n$  that the amortized time  $\sum_{i=1}^n C_i'$  for that sequence of  $n$  operations is a bound on the true time  $\sum_{i=1}^n C_i$ .

b) let  $\phi_i = i \bmod n$  then when  $i \bmod n = 0$ ,  $\phi_i = n + 0 - (n-1) = 1$ .

When  $i \bmod n \neq 0$ ,  $\phi_i = 1 + (i \bmod n) - ((i-1) \bmod n) = 2$ .

Q-3) find Common ID (Array A, Array B)

Array A length  $\leftarrow m$

Array B length  $\leftarrow n$

(if  $m \leq 0$  or  $n \leq 0$  break)  $\rightarrow$  base case check.

Quick sort (Array B, p, r)

// Defined in class materials  
So i don't write again.

$\Rightarrow$  We can sort the larger array.  
in this problem it is Array B  
with quick sort would lead us  
 $O(n \log n)$  solution.

for  $i = 0$  to  $m$  do

binary-search in Array A  $[i]$

if Array B  $[i] =$  Array A  $[i]$

print (Array B  $[i]$ , "common value")

or return

$\rightarrow$  we iterate in the smaller array, Array A and do binary-search of that element selected Array B.

$O(n \log n) + O(m \log n)$

$\Rightarrow$   $O(m \log n)$

Time Complexity  $\Rightarrow$  We get  $O(n \log n)$  complexity from quick sort  $O(n \log n)$  and  $O(m \log n)$  complexity from binary search  $\Rightarrow$   $m$  times iterate