# Quiz 4: Depth-First-Search (DFS)

Name: _____ Solution _____

Consider any DFS run on a directed graph $G = (V, E)$. Prove or disprove the following claim: For any edge $uv \in E$, if there is a path from v to u in G, then uv cannot be a cross edge.

Note: If you will disprove, you must provide a counter-example with an explanation of why the example shows that the statement is false. If you will prove, you must provide a rigorous formal argument to show that the statement always holds. To do so, you can refer to the theorems in the book (Parentheses Theorem, White Path Theorem) as appropriate (e.g., "we know by the parentheses theorem that...").

Solution: The statement is false. As a counter-example, consider the following graph:
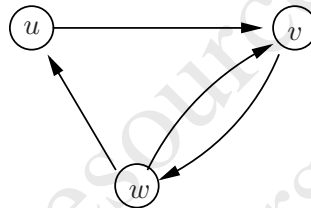
In this graph, there is a path from v to u.

Assume that a DFS run on this graph discovers w before it discovers u and v (which is always possible since the outer for loop of generic DFS considers the vertices in arbitrary order). Then u and v will be white at time $w.d$. Now assume that DFS explores edge wv before edge wu. Then wv and wu will be tree edges, which will make uv a cross edge.

Grading Key	
Proof is satisfactory:	3
Proof is sound, but not complete:	2
Proof is not satisfactory:	1
Student is absent:	0