FACULTY OF ENGINEERING
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING
ELE110 LINEAR ALGEBRA AND VECTOR ANALYSIS
MIDTERM EXAM
26 April 2023

IMPORTANT: In problems $1, 3, 5, 6$ consider your student ID as **a-b-c-d-e-f-g-h-i-j-k**.

1. Consider the following two figures.

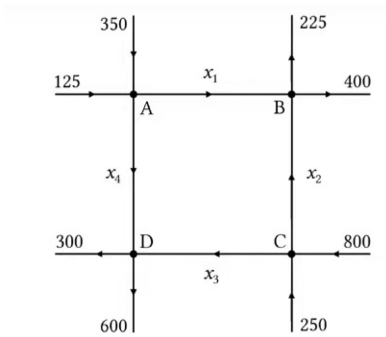
Figure 1: Three planes two of which are parallel

Figure 2: Three nonparallel planes that don't have a point in common

(a) Consider the equation $ax + cy + fz = k$. Write two more linear equations in x, y, z such that the planes corresponding to the resulting system is like in Figure 1.

(b) (a) Consider the equation $ax + cy + fz = j$. Write two more linear equations in x, y, z such that the planes corresponding to the resulting system is like in Figure 2.

2. Below is the diagram of a traffic network with four junctions. The indicated numbers are the count of cars entering or leaving each junction during a given day.



According to the diagram, construct the system of equations to calculate the number of cars that passed through adjacent pair of junctions during the day. Then solve this system. (There may be infinitely many solutions.) **(20 points)**

3. Solve the following linear system by using LU factorization, inverse matrix method or Cramer's rule. **(20 points)**

$$
\begin{aligned}
2x + y + z &= h \\
3x + z &= j \\
x - y - z &= k
\end{aligned}
$$

4. Prove the following.

Let V be a vector space and $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p, \mathbf{u}$ be vectors in V. If $\mathbf{u}$ is a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p$, then $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p\} = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_p, \mathbf{u}\}$. **(20 points)**

5. Consider the set $\{c + ax, j + kx + hx^2\}$. Add a polynomial of degree 2 or less to this set such that the resulting set is a basis for $\mathbb{P}_2$ (the set of polynomials of degree 2 or less). **(20 points)**

6. Consider the matrix $\mathbf{D} = \begin{bmatrix} 2 & 0 & -1 \\ 1 & -1 & 3 \\ 1 & 3 & -11 \end{bmatrix}$. Find a basis for the null space of $\mathbf{D}$. Then consider a vector $\mathbf{v}$ of length 3 whose first component is $jk + 1$, i.e. $\mathbf{v}$ is of the form $\begin{bmatrix} jk+1 \\ * \\ * \end{bmatrix}$. Fill the missing entries of $\mathbf{v}$ such that it becomes an element of the null space of D. **(20 points)**.

Remarks: The duration of the exam (including upload file) is 120 minutes. Attempt all the 6 questions. In order to get full marks, please make explanations whenever necessary.