At least

\# of people

pigeon

Pigeon

$$\left\lfloor \frac{82-1}{81} \right\rfloor + 1$$

people are from the same city

pigeonholes

\# cities

22 player $\quad 5+5 \quad \to$ 32 player

33 yellow card

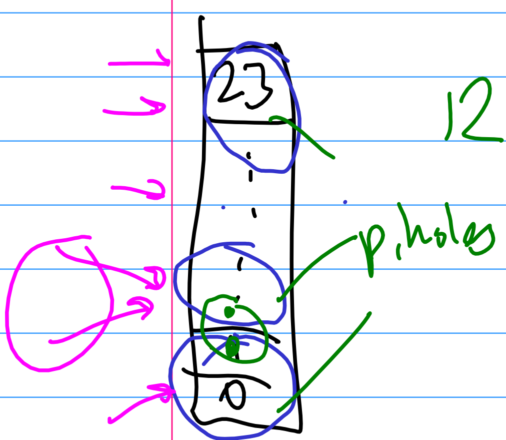
pign

pigeon

pigeonhs

At least $\left\lfloor \frac{33-1}{32} \right\rfloor + 1$ yellow cards is given to the same player

p.h.



pigeons : people

At least

$$\left\lfloor \frac{13-1}{12} \right\rfloor + 1 = 2$$

people are from consecutive floors

floor $\lfloor 56.999 \rfloor = 56$

John Doe → JD

Eng. $26^2 = 576$ If we have 577 people in the auditorium by the P.h.p. at least $\left\lfloor \frac{577-1}{576} \right\rfloor + 1$ of people has the same initials,

pigeons — people

pigeonhole initial combination

$S = \{3, 7, 11, 15, \ldots, 99, 103\}$ ... 51, 55, 59 ...

26.

(3) (55)

24 → 12

$12 + 2 = 14$ groups } P

pigeons? Numbers we select

p.holes? = $\{\{3\}, \{55\}, \{7, 103\}, \{11, 99\}, \ldots \{51, 59\}\}$

If we select x numbers from S, Ac. to P.H.P. at least $\left\lfloor \frac{x-1}{14} \right\rfloor + 1 = 2$ numbers will be in the same p.hole (number pair)

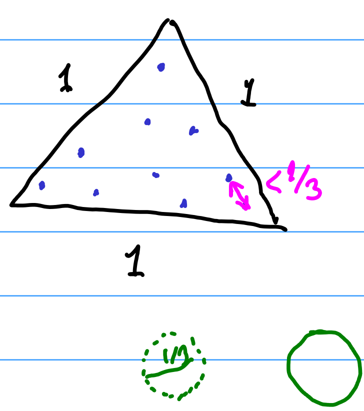
min x? ~~13~~, ~~14~~ $15 = x$.

$\{1, 2, \ldots, 9\}$ P.H: $\{(1,9), (2,8), (3,7) \ldots (4,6) (5)\}$

5 ↗

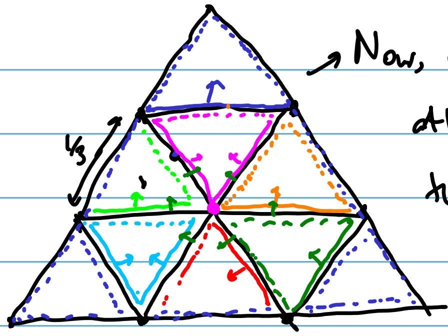
Pigeons 6 numbers

$\left\lfloor \frac{6-1}{5} \right\rfloor + 1 = 2$ numbers will be in the same p.hole, hence, their sum is 10.



Pigeons: points

pigeonholes: must be some exclusive regions such that whenever there happens to be two points in these regions, the distance between these points will be < 1/3



Now, we have 9 non-intersecting regions and, by the P.H.P.

Pigeons = points

atleast $\left\lfloor \frac{10-1}{9} \right\rfloor + 1$ points will fall into the same region. And no matter which region, their distance will be $< 1/3$

pigeonhole (regions we have defined)

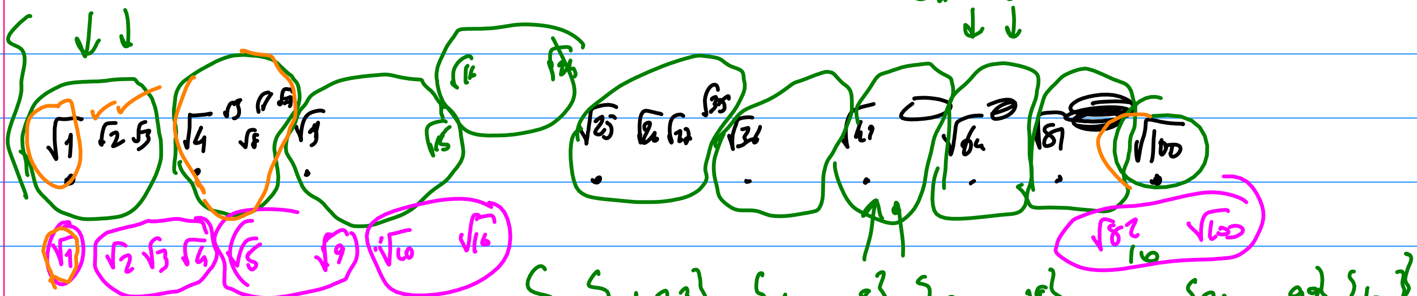
$S = \{1, 2, 3, \dots, 100\}$

Pigeons : Numbers we select

pigeonholes :

Subsets $\{1\}\ \{4\} \dots \{\ \}$

idea

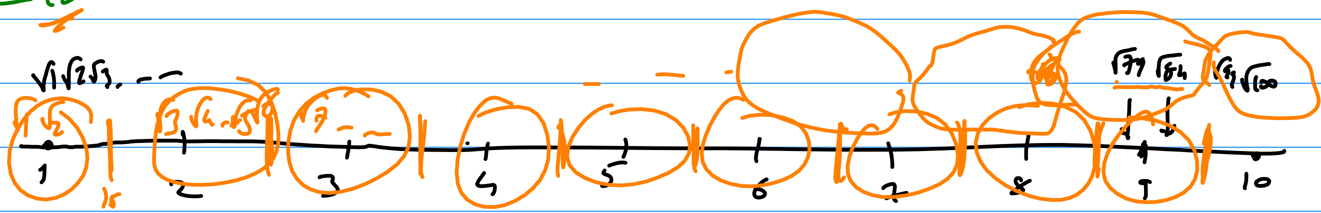


Let our pigeonholes be the subsets $\{\{1,2,3\}, \{4,\dots,8\}, \{9,\dots,15\}, \dots, \{81,\dots,99\}, \{100\}\}$

Then, since there are 10 pigeonholes, by P.H.P. if we select 11 numbers at least

$\left\lfloor \frac{11-1}{10} \right\rfloor + 1 = 2$ numbers x, y will be in the same subset and $\sqrt{x} - \sqrt{y} < 1$.

QED.



Thm. Let $m \in \mathbb{Z}^+$, m is odd. Prove that $\exists n$: $m \mid 2^n - 1$

Proof. $2^1-1,\ 2^2-1,\ 2^3-1,\ 2^4-1,\ 2^5-1,\ 2^6-1, \ldots, 2^{m+1}-1$

By P.H.P. $\exists\ s,t \in \{1,\ldots,m+1\}$: $2^s - 1 \overset{\text{mod } m}{=} 2^t - 1 \bmod m.$ $s \neq t$ w.l.o.g. $s > t$

m divides $(2^s-1)-(2^t-1) = 2^s - 2^t = 2^t \cdot 2^{s-t} - 2^t = 2^t(2^{s-t}-1)$

So m divides $2^{s-t}-1$

So we are done. QED.

$n = s-t$

distinct number.

12 0 5 1 7 3 8 4 6 9 10

$\exists$ either an increasing or decreasing subsequence of length 4.

n^2+1 any 5 26

$n+1$ $\uparrow$ $\downarrow$ ✓ 5 $n=2$ 6

3

Erdős-Szekeres

7