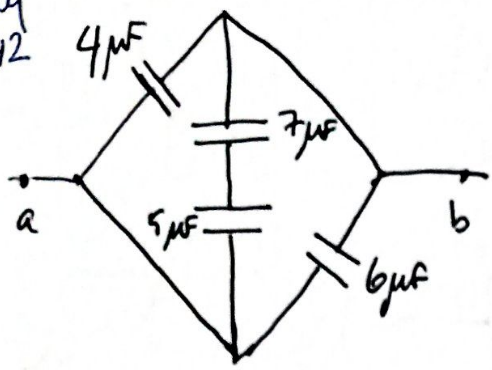
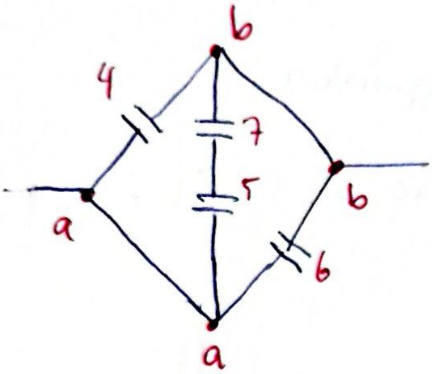
Serway 16.42



a) what's C_{eq} = ?

b) circuit is powered by V_{ab} potential.
if charge of 4μF Q_1
7μF Q_2
5μF Q_3
6μF Q_4

find the relations between charges?



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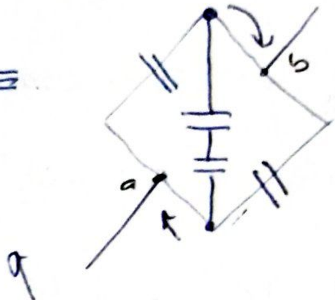
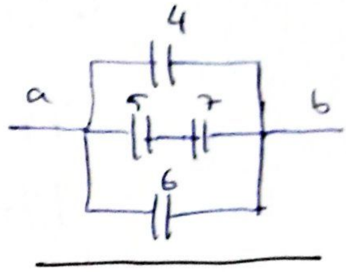
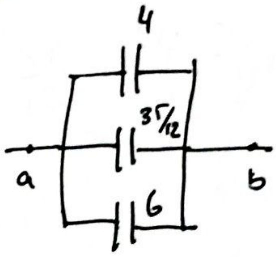


⇒



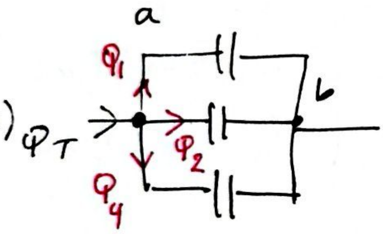
5 - 7μF series $\Rightarrow \left(\frac{1}{5}+\frac{1}{7}\right)^{-1} = \frac{35}{12}$ μF



} parallel

$$C_{eq} = 4+6+\frac{35}{12} = \frac{155}{12}\,\mu F$$

b)



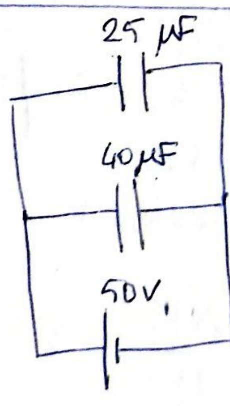
$Q_3 = Q_2$ since series

$$V_{ab} = \frac{Q_1}{4} = \frac{Q_2}{35/12} = \frac{Q_4}{6}$$

if $Q_1 = Q$ $\quad Q_4 = \frac{6}{4}Q = \frac{3}{2}Q$

$$Q_2 = Q_3 = \frac{35}{48}Q$$

16.37 Serway

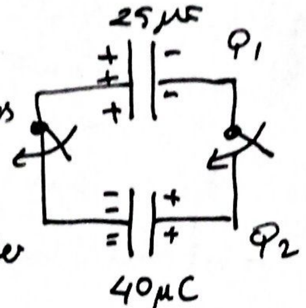


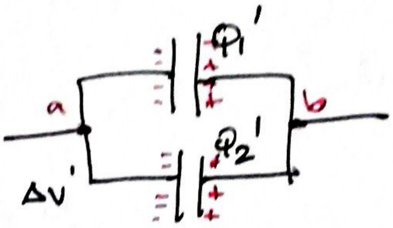
capacitors are charged with 50V.
battery is disconnected; and the capacitors are seperated;
if capacitors are connected oppositely
⇒ what are their final charge?
what " " " potential.

$\frac{Q}{C} = V$ $\quad Q_1 = (50)\,25\mu\,FV = 1250\mu C$

$Q_2 = 50(40)\mu FV = 2000\mu C$

opposite charges connected to each other





$V_{ab} = 50V$

$V_{ab}' = ?$

Q total will be shared between capacitors.

$Q_T = Q_1 - Q_2 = 1250 - 2000 = -750\mu C$

$|Q_T| = 750\mu C = Q_1' + Q_2'$

$\Delta V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{25} = \frac{Q_2'}{40}$ $\quad Q_1' = \frac{25}{40} Q_2' = \frac{5}{8} Q_2'$

$Q_1' + Q_2' = 750 = Q_T$

$\frac{5}{8} Q_2' + Q_2' = \frac{13 Q_2'}{8} = 750$

$\left\{ \begin{array}{l} Q_2' = 461.5\mu C \\ Q_1' = 288.5\mu C \end{array} \right.$

summary $\quad V_0 = 50V;$ $\quad V' = 11.5V$

$\Delta V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2}$

$\Delta V' = \frac{288.5}{25} = 11.5V$

or $\frac{461.5}{40} = 11.5V$

$\Rightarrow$ ratio of energy stored before & after?

$U_T = U_1 + U_2 = \frac{1}{2} C_1 \Delta V^2 + \frac{1}{2} C_2 \Delta V^2 = \frac{1}{2}(25+40)\,50^2 = 81250\mu C$

$U_T' = U_1' + U_2' = \frac{1}{2} C_1 \Delta V'^2 + \frac{1}{2} C_2 \Delta V'^2 = \frac{1}{2}(25+40)\,11.5^2 = 4298\mu J$

$81250 - 4298 = 76952$

Lost energy $=$ ~~[illegible]~~ $-$ ~~[illegible]~~ $-$ ~~[illegible]~~ $\mu J = |\Delta U| = |U_T' - U_T|$

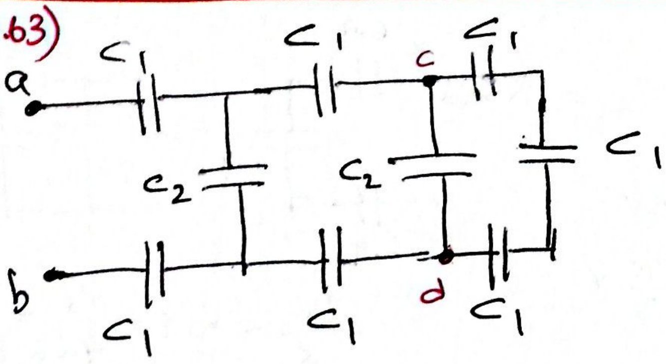
percent of lost energy to initial $= \frac{\Delta U}{U_T} = 0.95 \Rightarrow \%95$ LOST

$\Delta U = \frac{1}{2}(C_1 + C_2)\left[(\Delta V')^2 - \Delta V^2\right]$

$= \frac{1}{2}(C_1 + C_2)\,\Delta V^2$

$\left|\frac{11.5^2 - 50^2}{50^2}\right| = 0.947 = \%95$

(24.63)



$C_1 = 6.9\mu F$

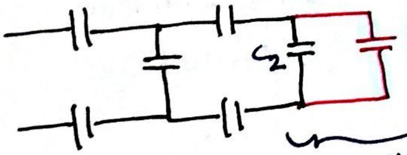
$C_2 = 4.6\mu F$

$V_{ab} = 420\ V$

a) $C_{eq} = ?$ $C_{ab} = ?$

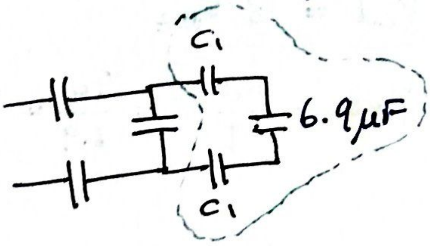
b) charge distribution / charge on each?

c) $V_{cd} = ?$

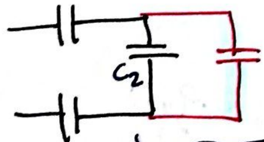


$\frac{C_1}{3} = 2.3\mu F$

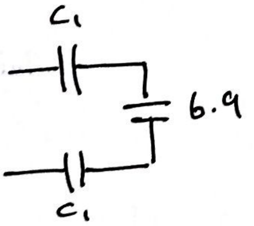
parallel $= C_2 + \frac{C_1}{3} = 4.6 + 2.3 = 6.9\mu F \equiv C_1$



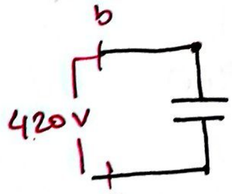
again series $\left(\frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_1}\right)^{-1} = \frac{C_1}{3} = 2.3\mu F$



paralel $4.6 + 2.3 = 6.9\mu F$

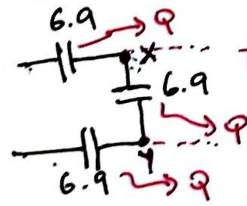


series $\frac{C_1}{3} = \frac{6.9}{3} = 2.3\mu F = C_{ab} = C_{eq}$ //



$\frac{Q}{\Delta V} = C \Rightarrow Q = 966\mu C \equiv$ total charge

series: Q same, V shared



series will have same Q

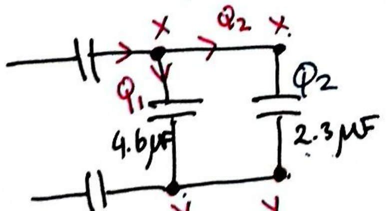
ΔV for $6.9\mu F$

$3\Delta V = 420$

$\Delta V = 140\ V = V_{xy}$

$\frac{Q}{6.9\mu F} = 140V = V_{xy}$

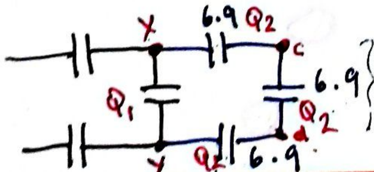
parallel: V same, Q shared



$V_{xy} = 140V = \frac{Q_1}{4.6\mu F} = \frac{Q_2}{2.3\mu F}$

$Q_1 + Q_2 = 966\mu C$

$Q_1 = 644\mu C$ $Q_2 = 322\mu C$

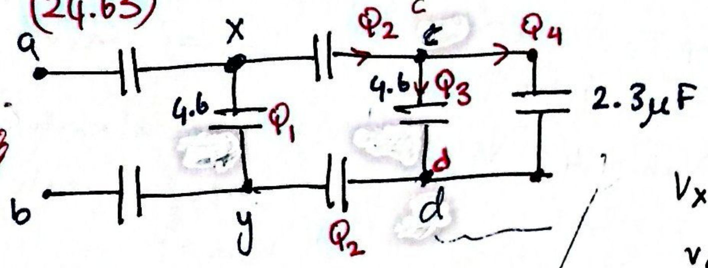


each has Q_2 charges

6.9 } V_{cd}

$V_{cd} = ?$ $\frac{Q_2}{6.9\mu F} = 46.7V$

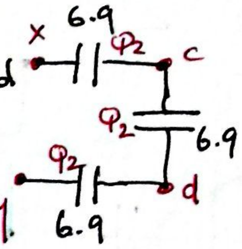
(24.63)



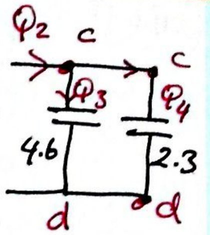
$V_{xy} = 3\,V_{cd}$

$V_{xy} = 140\text{V}$

$V_{cd} = \frac{140}{3}\text{ V}$



$\Rightarrow$

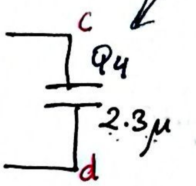


$$\frac{140}{3} = V_{cd} = \frac{Q_3}{4.6\mu} = \frac{Q_4}{2.3\mu}$$

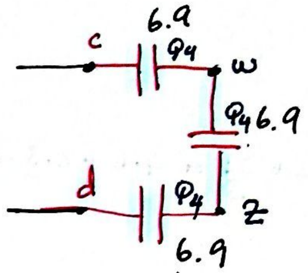
$Q_3 = 214.7\,\mu C$

$Q_4 = 107.3\,\mu C$

Lastly



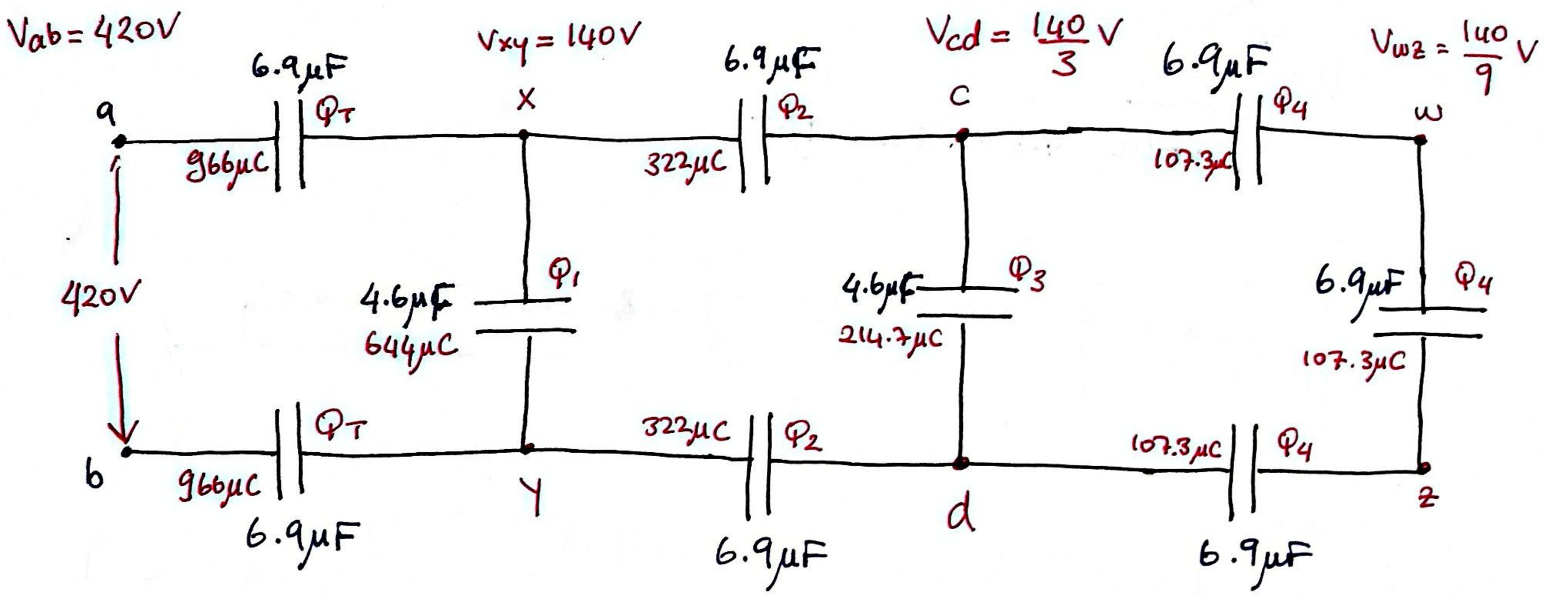
$\Rightarrow$



$V_{cd} = 3\,V_{wz}$

$$V_{wz} = \frac{140}{9}\text{ V} = \frac{Q_4}{6.9} = 15.56\text{ V}$$

Summary



$V_{ab} = 420\text{V}$

$V_{xy} = 140\text{V}$

$V_{cd} = \frac{140}{3}\text{ V}$

$V_{wz} = \frac{140}{9}\text{ V}$

$Q_T = 966\,\mu C$

$Q_1 = 644\,\mu C$

$Q_2 = 322\,\mu C$

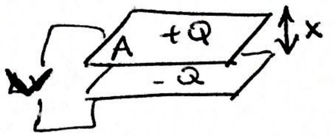
$Q_3 = 214.7\,\mu C$

$Q_4 = 107.3\,\mu C$

$C_{eq} = 2.3\,\mu F$

24.27) A parallel plate with area A; separation X has +Q, −Q charges on its plates. The capacitor is disconnected from battery. & charges are fixed.

a) what's the total energy stored in the capacitor?

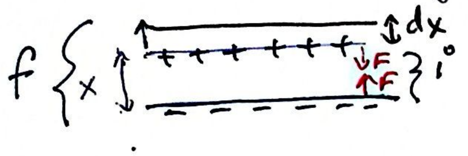


$C = \frac{\varepsilon_0 A}{d}$ $\quad U = \frac{q^2}{2C} = \frac{qV}{2} = \frac{1}{2}CV^2$

$$U = \frac{Q^2}{2\varepsilon_0 \frac{A}{x}} = \frac{Q^2 x}{2\varepsilon_0 A} = \frac{QV}{2}$$

b) The plates are pulled apart an additional ~~force~~, displacement $\underline{\underline{dx}}$

What's the change in the stored energy? $\Delta U = ?$

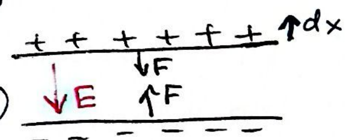


$U_i \to U_F$ $\qquad \Delta U = U_F - U_i$

$\frac{Q^2 x}{2\varepsilon_0 A} \to \frac{Q^2(x+dx)}{2\varepsilon_0 A}$; $\quad = \frac{Q^2 dx}{2\varepsilon_0 A} = dU$

c) The plates are attracting each other with force F, what's F?

$W = \vec{F}\cdot d\vec{x} < 0$



$W = -\Delta U$ (phys I)

$-F dx = -dU$

$dW = F dx$ amount of work for F to move dx.

$dW = dU$ work done = change in pot. energy

$dW = F dx = dU$, $\quad F = \frac{dU}{dx} = \frac{Q^2 dx / 2\varepsilon_0 A}{dx}$

$$\boxed{F = \frac{Q^2}{2\varepsilon_0 A}}$$ (Force between capacitor plates)

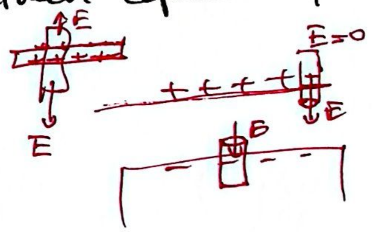
remember:

for metal surface $E = \frac{\sigma}{\varepsilon_0} = \frac{Q/A}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

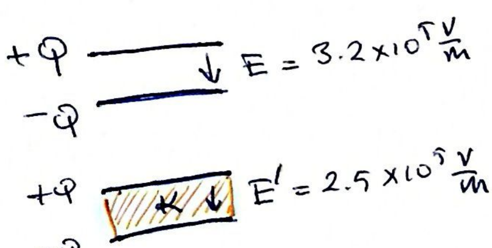
$\frac{Q^2}{2\varepsilon_0 A} = F \neq QE = \frac{Q^2}{\varepsilon_0 A}$

$\left\{ F = qE = \frac{QQ}{\varepsilon_0 A} \right.$

$F \neq QE$; $\quad F = \frac{QE}{2}$



24.37 Two parallel plates have equal opposite charges. In vacuum E is $E = 3.2\times10^5 \frac{V}{m}$; when the space between plates is filled with a dielectric K $E' = 2.5\times10^5 \frac{V}{m}$ (with K). a) what's σ_i (induced) charge density on the dielectric surface. b) what's K?



$E' = \frac{E}{K} \Rightarrow K = \frac{3.2\times10^5}{2.5\times10^5} = 1.28 = K$

$E = \frac{\sigma}{\varepsilon_0}$ $\qquad E' = \frac{\sigma'}{\varepsilon_0} = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0 K} \Rightarrow \sigma_i = \sigma\left(1-\frac{1}{K}\right)$

$\sigma_i = \sigma\left(1-\frac{1}{K}\right) = E\varepsilon_0\left(1-\frac{1}{1.28}\right)$

also $\sigma = \varepsilon_0 E = 28.3\times10^{-7}\ C/m^2$

$\sigma = 2830\ nC/m^2$

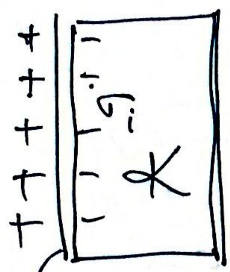
$\sigma' = \frac{\sigma}{K} = 2213\ \frac{nC}{m^2}$

$\sigma_i = 3.2\times10^5 \times 8.85\times10^{-12}\left(\frac{0.28}{1.28}\right)$

$\sigma_i \cong 6.2\times10^{-7}\ C/m^2$

$\sigma_i = 620\times10^{-9} C = 620\ nC/m^2$

explaination



$$E = \frac{\sigma}{\varepsilon_0} \qquad E' = \frac{\sigma'}{\varepsilon_0} = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

$3.2 \times 10^{+5}$ $\qquad$ 8.85×10^{-12}

$$E' = \frac{\sigma - \sigma_i}{\varepsilon_0}$$

σ

$$\sigma' = \sigma - \sigma_i$$

$$2.5 \times 10^{5} \times 8.85 \times 10^{-12} = \sigma - \sigma_i$$

$K = 1$ vacum $\qquad$ $K > 1$

E $\qquad$ $E' = \frac{E}{K}$

V $\qquad$ $V' = \frac{V}{K}$

σ $\qquad$ $\sigma' = \frac{\sigma}{K} = [\sigma - \sigma_i]$

$$\sigma_i = \sigma - \frac{\sigma}{K} = \sigma\left(1 - \frac{1}{K}\right)$$

$K = 2$

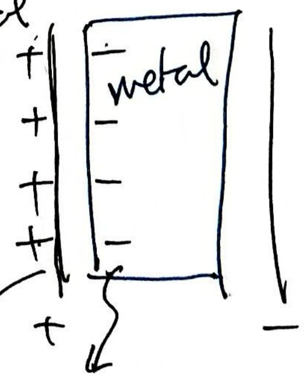
$$\sigma_i = \sigma\left(\frac{2-1}{2}\right) = \frac{\sigma}{2}$$

$$\boxed{\sigma_i = \sigma\left(\frac{k-1}{k}\right)}$$

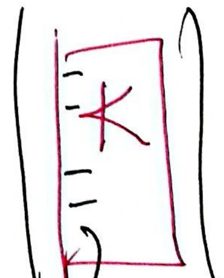
K → capacity to be able to charge on its surface

half of the initial charge is induced on the ~~plat~~ dielectric material

metal



σ $\qquad$ $\sigma_i = \sigma$



$\sigma_i < \sigma$

$K > 1$