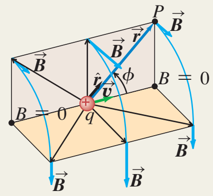
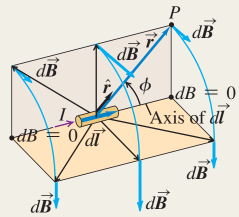
Magnetic field of a moving charge: The magnetic field $\vec{B}$ created by a charge q moving with velocity $\vec{v}$ depends on the distance r from the source point (the location of q) to the field point (where $\vec{B}$ is measured). The $\vec{B}$ field is perpendicular to $\vec{v}$ and to $\hat{r}$, the unit vector directed from the source point to the field point. The principle of superposition of magnetic fields states that the total $\vec{B}$ field produced by several moving charges is the vector sum of the fields produced by the individual charges. (See Example 28.1.)

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2} \tag{28.2}$$



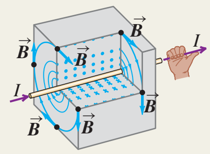
Magnetic field of a current-carrying conductor: The law of Biot and Savart gives the magnetic field $d\vec{B}$ created by an element $d\vec{l}$ of a conductor carrying current I. The field $d\vec{B}$ is perpendicular to both $d\vec{l}$ and $\hat{r}$, the unit vector from the element to the field point. The $\vec{B}$ field created by a finite current-carrying conductor is the integral of $d\vec{B}$ over the length of the conductor. (See Example 28.2.)

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l} \times \hat{r}}{r^2} \tag{28.6}$$



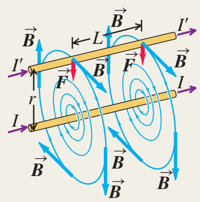
Magnetic field of a long, straight, current-carrying conductor: The magnetic field $\vec{B}$ at a distance r from a long, straight conductor carrying a current I has a magnitude that is inversely proportional to r. The magnetic field lines are circles coaxial with the wire, with directions given by the right-hand rule. (See Examples 28.3 and 28.4.)

$$B = \frac{\mu_0 I}{2\pi r} \tag{28.9}$$



Magnetic force between current-carrying conductors: Two long, parallel, current-carrying conductors attract if the currents are in the same direction and repel if the currents are in opposite directions. The magnetic force per unit length between the conductors depends on their currents I and I' and their separation r. The definition of the ampere is based on this relationship. (See Example 28.5.)

$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} \tag{28.11}$$



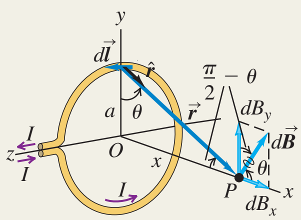
Magnetic field of a current loop: The law of Biot and Savart allows us to calculate the magnetic field produced along the axis of a circular conducting loop of radius a carrying current I. The field depends on the distance x along the axis from the center of the loop to the field point. If there are N loops, the field is multiplied by N. At the center of the loop, $x = 0$. (See Example 28.6.)

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}} \tag{28.15}$$

(circular loop)

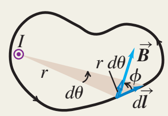
$$B_x = \frac{\mu_0 N I}{2a} \tag{28.17}$$

(center of N circular loops)



Ampere's law: Ampere's law states that the line integral of $\vec{B}$ around any closed path equals μ_0 times the net current through the area enclosed by the path. The positive sense of current is determined by a right-hand rule. (See Examples 28.7–28.10.)

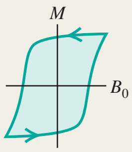
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \tag{28.20}$$



Magnetic fields due to current distributions: The table lists magnetic fields caused by several current distributions. In each case the conductor is carrying current I.

Current Distribution	Point in Magnetic Field	Magnetic-Field Magnitude
Long, straight conductor	Distance r from conductor	$B = \dfrac{\mu_0 I}{2\pi r}$
Circular loop of radius a	On axis of loop	$B = \dfrac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$
	At center of loop	$B = \dfrac{\mu_0 I}{2a}$ (for N loops, multiply these expressions by N)
Long cylindrical conductor of radius R	Inside conductor, $r < R$	$B = \dfrac{\mu_0 I}{2\pi} \dfrac{r}{R^2}$
	Outside conductor, $r > R$	$B = \dfrac{\mu_0 I}{2\pi r}$
Long, closely wound solenoid with n turns per unit length, near its midpoint	Inside solenoid, near center	$B = \mu_0 n I$
	Outside solenoid	$B \approx 0$
Tightly wound toroidal solenoid (toroid) with N turns	Within the space enclosed by the windings, distance r from symmetry axis	$B = \dfrac{\mu_0 N I}{2\pi r}$
	Outside the space enclosed by the windings	$B \approx 0$

Magnetic materials: When magnetic materials are present, the magnetization of the material causes an additional contribution to $\vec{B}$. For paramagnetic and diamagnetic materials, μ_0 is replaced in magnetic-field expressions by $\mu = K_m \mu_0$, where μ is the permeability of the material and K_m is its relative permeability. The magnetic susceptibility χ_m is defined as $\chi_m = K_m - 1$. Magnetic susceptibilities for paramagnetic materials are small positive quantities; those for diamagnetic materials are small negative quantities. For ferromagnetic materials, K_m is much larger than unity and is not constant. Some ferromagnetic materials are permanent magnets, retaining their magnetization even after the external magnetic field is removed. (See Examples 28.11 and 28.12.)



BRIDGING PROBLEM Magnetic Field of a Charged, Rotating Dielectric Disk

A thin dielectric disk with radius a has a total charge $+Q$ distributed uniformly over its surface. It rotates n times per second about an axis perpendicular to the surface of the disk and passing through its center. Find the magnetic field at the center of the disk.

SOLUTION GUIDE

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IDENTIFY and SET UP

1. Think of the rotating disk as a series of concentric rotating rings. Each ring acts as a circular current loop that produces a magnetic field at the center of the disk.
2. Use the results of Section 28.5 to find the magnetic field due to a single ring. Then integrate over all rings to find the total field.

EXECUTE

3. Find the charge on a ring with inner radius r and outer radius $r + dr$.
4. How long does it take the charge found in step 3 to make a complete trip around the rotating ring? Use this to find the current of the rotating ring.
5. Use a result from Section 28.5 to determine the magnetic field that this ring produces at the center of the disk.
6. Integrate your result from step 5 to find the total magnetic field from all rings with radii from $r = 0$ to $r = a$.

EVALUATE

7. Does your answer have the correct units?
8. Suppose all of the charge were concentrated at the rim of the disk (at $r = a$). Would this increase or decrease the field at the center of the disk?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q28.1 A topic of current interest in physics research is the search (thus far unsuccessful) for an isolated magnetic pole, or magnetic *monopole*. If such an entity were found, how could it be recognized? What would its properties be?

Q28.2 Streams of charged particles emitted from the sun during periods of solar activity create a disturbance in the earth's magnetic field. How does this happen?

Q28.3 The text discussed the magnetic field of an infinitely long, straight conductor carrying a current. Of course, there is no such thing as an infinitely long *anything*. How do you decide whether a particular wire is long enough to be considered infinite?

Q28.4 Two parallel conductors carrying current in the same direction attract each other. If they are permitted to move toward each other, the forces of attraction do work. From where does the energy come? Does this contradict the assertion in Chapter 27 that magnetic forces on moving charges do no work? Explain.

Q28.5 Pairs of conductors carrying current into or out of the power-supply components of electronic equipment are sometimes twisted together to reduce magnetic-field effects. Why does this help?

Q28.6 Suppose you have three long, parallel wires arranged so that in cross section they are at the corners of an equilateral triangle. Is there any way to arrange the currents so that all three wires attract each other? So that all three wires repel each other? Explain.

Q28.7 In deriving the force on one of the long, current-carrying conductors in Section 28.4, why did we use the magnetic field due to only one of the conductors? That is, why didn't we use the *total* magnetic field due to *both* conductors?

Q28.8 Two concentric, coplanar, circular loops of wire of different diameter carry currents in the same direction. Describe the nature of the force exerted on the inner loop by the outer loop and on the outer loop by the inner loop.

Q28.9 A current was sent through a helical coil spring. The spring contracted, as though it had been compressed. Why?

Q28.10 What are the relative advantages and disadvantages of Ampere's law and the law of Biot and Savart for practical calculations of magnetic fields?

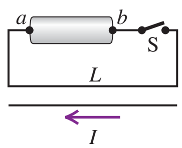
Q28.11 Magnetic field lines never have a beginning or an end. Use this to explain why it is reasonable for the field of a toroidal solenoid to be confined entirely to its interior, while a straight solenoid *must* have some field outside.

Q28.12 If the magnitude of the magnetic field a distance R from a very long, straight, current-carrying wire is B, at what distance from the wire will the field have magnitude $3B$?

Q28.13 Two very long, parallel wires carry equal currents in opposite directions. (a) Is there any place that their magnetic fields completely cancel? If so, where? If not, why not? (b) How would the answer to part (a) change if the currents were in the same direction?

Q28.14 In the circuit shown in Fig. Q28.14, when switch S is suddenly closed, the wire L is pulled toward the lower wire carrying current I. Which (a or b) is the positive terminal of the battery? How do you know?

Figure **Q28.14**



Q28.15 A metal ring carries a current that causes a magnetic field B_0 at the center of the ring and a field B at point P a distance x from the center along the axis of the ring. If the radius of the ring is doubled, find the magnetic field at the center. Will the field at point P change by the same factor? Why?

Q28.16 Why should the permeability of a paramagnetic material be expected to decrease with increasing temperature?

Q28.17 If a magnet is suspended over a container of liquid air, it attracts droplets to its poles. The droplets contain only liquid oxygen; even though nitrogen is the primary constituent of air, it is not attracted to the magnet. Explain what this tells you about the magnetic susceptibilities of oxygen and nitrogen, and explain why a magnet in ordinary, room-temperature air doesn't attract molecules of oxygen *gas* to its poles.

Q28.18 What features of atomic structure determine whether an element is diamagnetic or paramagnetic? Explain.

Q28.19 The magnetic susceptibility of paramagnetic materials is quite strongly temperature dependent, but that of diamagnetic materials is nearly independent of temperature. Why the difference?

Q28.20 A cylinder of iron is placed so that it is free to rotate around its axis. Initially the cylinder is at rest, and a magnetic field is applied to the cylinder so that it is magnetized in a direction parallel to its axis. If the direction of the *external* field is suddenly reversed, the direction of magnetization will also reverse and the cylinder will begin rotating around its axis. (This is called the *Einstein–de Haas effect*.) Explain why the cylinder begins to rotate.

Q28.21 The discussion of magnetic forces on current loops in Section 27.7 commented that no net force is exerted on a complete loop in a uniform magnetic field, only a torque. Yet magnetized materials that contain atomic current loops certainly *do* experience net forces in magnetic fields. How is this discrepancy resolved?

Q28.22 Show that the units $\mathrm{A \cdot m^2}$ and J/T for the Bohr magneton are equivalent.

EXERCISES

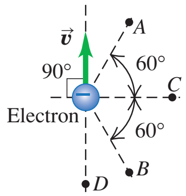
Section 26.1 Magnetic Field of a Moving Charge

28.1 •• A $+6.00\text{-}\mu\text{C}$ point charge is moving at a constant $8.00 \times 10^6\ \text{m/s}$ in the $+y$-direction, relative to a reference frame. At the instant when the point charge is at the origin of this reference frame, what is the magnetic-field vector $\vec{B}$ it produces at the following points: (a) $x = 0.500\ \text{m}$, $y = 0$, $z = 0$; (b) $x = 0$, $y = -0.500\ \text{m}$, $z = 0$; (c) $x = 0$, $y = 0$, $z = +0.500\ \text{m}$; (d) $x = 0$, $y = -0.500\ \text{m}$, $z = +0.500\ \text{m}$?

28.2 • **Fields Within the Atom.** In the Bohr model of the hydrogen atom, the electron moves in a circular orbit of radius $5.3 \times 10^{-11}\ \text{m}$ with a speed of $2.2 \times 10^6\ \text{m/s}$. If we are viewing the atom in such a way that the electron's orbit is in the plane of the paper with the electron moving clockwise, find the magnitude and direction of the electric and magnetic fields that the electron produces at the location of the nucleus (treated as a point).

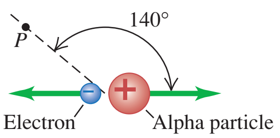
28.3 • An electron moves at $0.100c$ as shown in Fig. E28.3. Find the magnitude and direction of the magnetic field this electon produces at the following points, each 2.00 μm from the electron: (a) points A and B; (b) point C; (c) point D.

Figure **E28.3**



28.4 •• An alpha particle (charge $+2e$) and an electron move in opposite directions from the same point, each with the speed of 2.50×10^5 m/s (Fig. E28.4). Find the magnitude and direction of the total magnetic field these charges produce at point P, which is 1.75 nm from each of them.

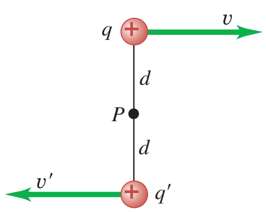
Figure **E28.4**



28.5 • A -4.80-μC charge is moving at a constant speed of 6.80×10^5 m/s in the $+x$-direction relative to a reference frame. At the instant when the point charge is at the origin, what is the magnetic-field vector it produces at the following points: (a) $x = 0.500$ m, $y = 0$, $z = 0$; (b) $x = 0$, $y = 0.500$ m, $z = 0$; (c) $x = 0.500$ m, $y = 0.500$ m, $z = 0$; (d) $x = 0$, $y = 0$, $z = 0.500$ m?

28.6 • Positive point charges $q = +8.00\ \mu$C and $q' = +3.00\ \mu$C are moving relative to an observer at point P, as shown in Fig. E28.6. The distance d is 0.120 m, $v = 4.50 \times 10^6$ m/s, and $v' = 9.00 \times 10^6$ m/s. (a) When the two charges are at the locations shown in the figure, what are the magnitude and direction of the net magnetic field they produce at point P? (b) What are the magnitude and direction of the electric and magnetic forces that each charge exerts on the other, and what is the ratio of the magnitude of the electric force to the magnitude of the magnetic force? (c) If the direction of $\vec{v}'$ is reversed, so both charges are moving in the same direction, what are the magnitude and direction of the magnetic forces that the two charges exert on each other?

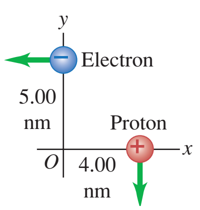
Figure **E28.6**



28.7 •• Figure E28.6 shows two point charges, q and q', moving relative to an observer at point P. Suppose that the lower charge is actually *negative*, with $q' = -q$. (a) Find the magnetic field (magnitude and direction) produced by the two charges at point P if (i) $v' = v/2$; (ii) $v' = v$; (iii) $v' = 2v$. (b) Find the direction of the magnetic force that q exerts on q', and find the direction of the magnetic force that q' exerts on q. (c) If $v = v' = 3.00 \times 10^5$ m/s, what is the ratio of the magnitude of the magnetic force acting on each charge to that of the Coulomb force acting on each charge?

28.8 •• An electron and a proton are each moving at 845 km/s in perpendicular paths as shown in Fig. E28.8. At the instant when they are at the positions shown in the figure, find the magnitude and direction of (a) the total magnetic field they produce at the origin; (b) the magnetic field the electron produces at the location of the proton; (c) the total electric force and the total magnetic force that the electron exerts on the proton.

Figure **E28.8**



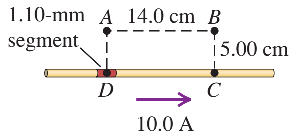
28.9 • A negative charge $q = -3.60 \times 10^{-6}$ C is located at the origin and has velocity $\vec{v} = (7.50 \times 10^4\ \text{m/s})\hat{\imath} + (-4.90 \times 10^4\ \text{m/s})\hat{\jmath}$. At this instant what are the magnitude and direction of the magnetic field produced by this charge at the point $x = 0.200$ m, $y = -0.300$ m, $z = 0$?

Section 28.2 Magnetic Field of a Current Element

28.10 • A short current element $d\vec{l} = (0.500\ \text{mm})\hat{\jmath}$ carries a current of 8.20 A in the same direction as $d\vec{l}$. Point P is located at $\vec{r} = (-0.730\ \text{m})\hat{\imath} + (0.390\ \text{m})\hat{k}$. Use unit vectors to express the magnetic field at P produced by this current element.

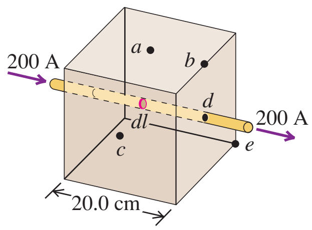
28.11 • A straight wire carries a 10.0-A current (Fig. E28.11). $ABCD$ is a rectangle with point D in the middle of a 1.10-mm segment of the wire and point C in the wire. Find the magnitude and direction of the magnetic field due to this segment at (a) point A; (b) point B; (c) point C.

Figure **E28.11**



28.12 • A long, straight wire, carrying a current of 200 A, runs through a cubical wooden box, entering and leaving through holes in the centers of opposite faces (Fig. E28.12). The length of each side of the box is 20.0 cm. Consider an element dl of the wire 0.100 cm long at the center of the box. Compute the magnitude dB of the magnetic field produced by this element at the points a, b, c, d, and e in Fig. E28.12. Points a, c, and d are at the centers of the faces of the cube; point b is at the midpoint of one edge; and point e is at a corner. Copy the figure and show the directions and relative magnitudes of the field vectors. (*Note:* Assume that the length dl is small in comparison to the distances from the current element to the points where the magnetic field is to be calculated.)

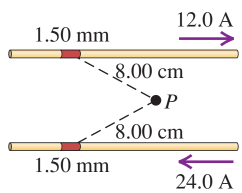
Figure **E28.12**



28.13 •• A long, straight wire lies along the z-axis and carries a 4.00-A current in the $+z$-direction. Find the magnetic field (magnitude and direction) produced at the following points by a 0.500-mm segment of the wire centered at the origin: (a) $x = 2.00$ m, $y = 0$, $z = 0$; (b) $x = 0$, $y = 2.00$ m, $z = 0$; (c) $x = 2.00$ m, $y = 2.00$ m, $z = 0$; (d) $x = 0$, $y = 0$, $z = 2.00$ m.

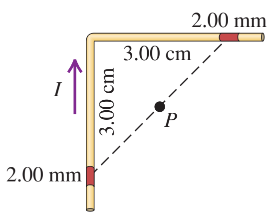
28.14 •• Two parallel wires are 5.00 cm apart and carry currents in opposite directions, as shown in Fig. E28.14. Find the magnitude and direction of the magnetic field at point P due to two 1.50-mm segments of wire that are opposite each other and each 8.00 cm from P.

Figure **E28.14**



28.15 • A wire carrying a 28.0-A current bends through a right angle. Consider two 2.00-mm segments of wire, each 3.00 cm from the bend (Fig. E28.15). Find the magnitude and direction of the magnetic field these two segments produce at point P, which is midway between them.

Figure **E28.15**



28.16 •• A square wire loop 10.0 cm on each side carries a clockwise current of 15.0 A. Find the magnitude and direction of the magnetic field at its center due to the four 1.20-mm wire segments at the midpoint of each side.

Section 28.3 Magnetic Field of a Straight Current-Carrying Conductor

28.17 • **The Magnetic Field from a Lightning Bolt.** Lightning bolts can carry currents up to approximately 20 kA. We can model such a current as the equivalent of a very long, straight wire. (a) If you were unfortunate enough to be 5.0 m away from such a lightning bolt, how large a magnetic field would you experience? (b) How does this field compare to one you would experience by being 5.0 cm from a long, straight household current of 10 A?

28.18 • A very long, straight horizontal wire carries a current such that 3.50×10^{18} electrons per second pass any given point going from west to east. What are the magnitude and direction of the magnetic field this wire produces at a point 4.00 cm directly above it?

28.19 • BIO **Currents in the Heart.** The body contains many small currents caused by the motion of ions in the organs and cells. Measurements of the magnetic field around the chest due to currents in the heart give values of about 10 μG. Although the actual currents are rather complicated, we can gain a rough understanding of their magnitude if we model them as a long, straight wire. If the surface of the chest is 5.0 cm from this current, how large is the current in the heart?

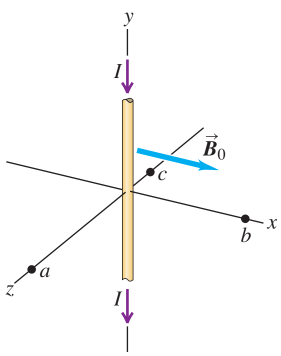
28.20 • BIO **Bacteria Navigation.** Certain bacteria (such as *Aquaspirillum magnetotacticum*) tend to swim toward the earth's geographic north pole because they contain tiny particles, called magnetosomes, that are sensitive to a magnetic field. If a transmission line carrying 100 A is laid underwater, at what range of distances would the magnetic field from this line be great enough to interfere with the migration of these bacteria? (Assume that a field less than 5 percent of the earth's field would have little effect on the bacteria. Take the earth's field to be 5.0×10^{-5} T and ignore the effects of the seawater.)

28.21 • (a) How large a current would a very long, straight wire have to carry so that the magnetic field 2.00 cm from the wire is equal to 1.00 G (comparable to the earth's northward-pointing magnetic field)? (b) If the wire is horizontal with the current running from east to west, at what locations would the magnetic field of the wire point in the same direction as the horizontal component of the earth's magnetic field? (c) Repeat part (b) except the wire is vertical with the current going upward.

28.22 • Two long, straight wires, one above the other, are separated by a distance $2a$ and are parallel to the x-axis. Let the $+y$-axis be in the plane of the wires in the direction from the lower wire to the upper wire. Each wire carries current I in the $+x$-direction. What are the magnitude and direction of the net magnetic field of the two wires at a point in the plane of the wires (a) midway between them; (b) at a distance a above the upper wire; (c) at a distance a below the lower wire?

28.23 •• A long, straight wire lies along the y-axis and carries a current $I = 8.00$ A in the $-y$-direction (Fig. E28.23). In addition to the magnetic field due to the current in the wire, a uniform magnetic field $\vec{B}_0$ with magnitude 1.50×10^{-6} T is in the $+x$-direction What is the total field (magnitude and direction) at the following points in the xz-plane: (a) $x = 0$, $z = 1.00$ m; (b) $x = 1.00$ m, $z = 0$; (c) $x = 0$, $z = -0.25$ m?

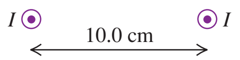
Figure **E28.23**



28.24 •• BIO **EMF.** Currents in dc transmission lines can be 100 A or more. Some people have expressed concern that the electromagnetic fields (EMFs) from such lines near their homes could cause health dangers. For a line with current 150 A and at a height of 8.0 m above the ground, what magnetic field does the line produce at ground level? Express your answer in teslas and as a percent of the earth's magnetic field, which is 0.50 gauss. Does this seem to be cause for worry?

28.25 • Two long, straight, parallel wires, 10.0 cm apart, carry equal 4.00-A currents in the same direction, as shown in Fig. E28.25. Find the magnitude and direction of the magnetic field at (a) point P_1, midway between the wires; (b) point P_2, 25.0 cm to the right of P_1; (c) point P_3, 20.0 cm directly above P_1.

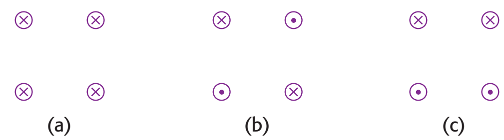
Figure **E28.25**



28.26 •• A rectangular loop with dimensions 4.20 cm by 9.50 cm carries current I. The current in the loop produces a magnetic field at the center of the loop that has magnitude 5.50×10^{-5} T and direction away from you as you view the plane of the loop. What are the magnitude and direction (clockwise or counterclockwise) of the current in the loop?

28.27 • Four, long, parallel power lines each carry 100-A currents. A cross-sectional diagram of these lines is a square, 20.0 cm on each side. For each of the three cases shown in Fig. E28.27, calculate the magnetic field at the center of the square.

Figure **E28.27**



28.28 • Four very long, current-carrying wires in the same plane intersect to form a square 40.0 cm on each side, as shown in Fig. E28.28. Find the magnitude and direction of the current I so that the magnetic field at the center of the square is zero.

Figure **E28.28**

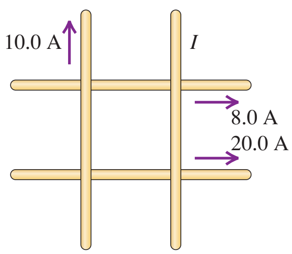
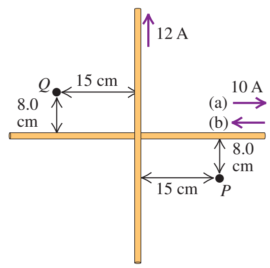


Figure **E28.29**

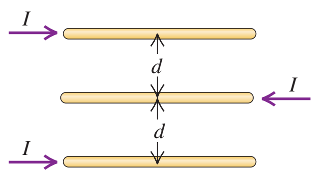


28.29 •• Two insulated wires perpendicular to each other in the same plane carry currents as shown in Fig. E28.29. Find the magnitude of the *net* magnetic field these wires produce at points P and Q if the 10.0 A-current is (a) to the right or (b) to the left.

Section 28.4 Force Between Parallel Conductors

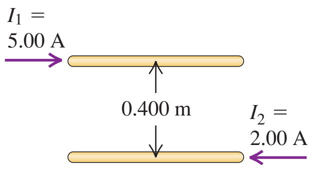
28.30 • Three parallel wires each carry current I in the directions shown in Fig. E28.30. If the separation between adjacent wires is d, calculate the magnitude and direction of the net magnetic force per unit length on each wire.

Figure **E28.30**



28.31 • Two long, parallel wires are separated by a distance of 0.400 m (Fig. E28.31). The currents I_1 and I_2 have the directions shown. (a) Calculate the magnitude of the force exerted by each wire on a 1.20-m length of the other. Is the force attractive or repulsive? (b) Each current is doubled, so that I_1 becomes 10.0 A and I_2 becomes 4.00 A. Now what is the magnitude of the force that each wire exerts on a 1.20-m length of the other?

Figure **E28.31**

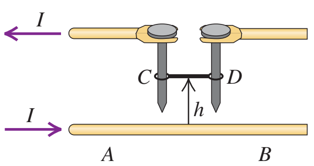


28.32 • Two long, parallel wires are separated by a distance of 2.50 cm. The force per unit length that each wire exerts on the other is 4.00×10^{-5} N/m, and the wires repel each other. The current in one wire is 0.600 A. (a) What is the current in the second wire? (b) Are the two currents in the same direction or in opposite directions?

28.33 • **Lamp Cord Wires.** The wires in a household lamp cord are typically 3.0 mm apart center to center and carry equal currents in opposite directions. If the cord carries current to a 100-W light bulb connected across a 120-V potential difference, what force per meter does each wire of the cord exert on the other? Is the force attractive or repulsive? Is this force large enough so it should be considered in the design of the lamp cord? (Model the lamp cord as a very long straight wire.)

28.34 • A long, horizontal wire *AB* rests on the surface of a table and carries a current *I*. Horizontal wire *CD* is vertically above wire *AB* and is free to slide up and down on the two vertical metal guides *C* and *D* (Fig. E28.34). Wire *CD* is connected through the sliding contacts to another wire that also carries a current *I*, opposite in direction to the current in wire *AB*. The mass per unit length of the wire *CD* is λ. To what equilibrium height *h* will the wire *CD* rise, assuming that the magnetic force on it is due entirely to the current in the wire *AB*?

Figure **E28.34**

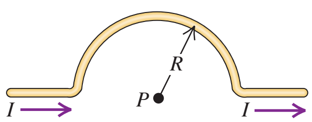


Section 28.5 Magnetic Field of a Circular Current Loop

28.35 • BIO **Currents in the Brain.** The magnetic field around the head has been measured to be approximately 3.0×10^{-8} G. Although the currents that cause this field are quite complicated, we can get a rough estimate of their size by modeling them as a single circular current loop 16 cm (the width of a typical head) in diameter. What is the current needed to produce such a field at the center of the loop?

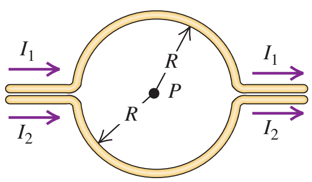
28.36 • Calculate the magnitude and direction of the magnetic field at point *P* due to the current in the semicircular section of wire shown in Fig. E28.36. (*Hint:* Does the current in the long, straight section of the wire produce any field at *P*?)

Figure **E28.36**



28.37 •• Calculate the magnitude of the magnetic field at point *P* of Fig. E28.37 in terms of R, I_1, and I_2. What does your expression give when $I_1 = I_2$?

Figure **E28.37**



28.38 •• A closely wound, circular coil with radius 2.40 cm has 800 turns. (a) What must the current in the coil be if the magnetic field at the center of the coil is 0.0580 T? (b) At what distance *x* from the center of the coil, on the axis of the coil, is the magnetic field half its value at the center?

28.39 •• A closely wound, circular coil with a diameter of 4.00 cm has 600 turns and carries a current of 0.500 A. What is the magnitude of the magnetic field (a) at the center of the coil and (b) at a point on the axis of the coil 8.00 cm from its center?

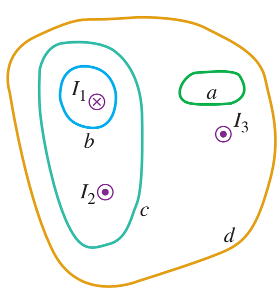
28.40 •• A closely wound coil has a radius of 6.00 cm and carries a current of 2.50 A. How many turns must it have if, at a point on the coil axis 6.00 cm from the center of the coil, the magnetic field is 6.39×10^{-4} T?

28.41 •• Two concentric circular loops of wire lie on a tabletop, one inside the other. The inner wire has a diameter of 20.0 cm and carries a clockwise current of 12.0 A, as viewed from above, and the outer wire has a diameter of 30.0 cm. What must be the magnitude and direction (as viewed from above) of the current in the outer wire so that the net magnetic field due to this combination of wires is zero at the common center of the wires?

Section 28.6 Ampere's Law

28.42 • Figure E28.42 shows, in cross section, several conductors that carry currents through the plane of the figure. The currents have the magnitudes $I_1 = 4.0$ A, $I_2 = 6.0$ A, and $I_3 = 2.0$ A, and the directions shown. Four paths, labeled *a* through *d*, are shown. What is the line integral $\oint \vec{B} \cdot d\vec{l}$ for each path? Each integral involves going around the path in the counterclockwise direction. Explain your answers.

Figure **E28.42**



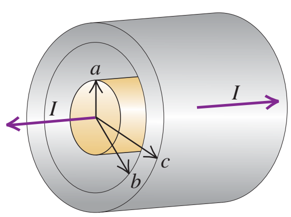
28.43 • A closed curve encircles several conductors. The line integral $\oint \vec{B} \cdot d\vec{l}$ around this curve is 3.83×10^{-4} T·m. (a) What is the net current in the conductors? (b) If you were to integrate around the curve in the opposite direction, what would be the value of the line integral? Explain.

Section 28.7 Applications of Ampere's Law

28.44 •• As a new electrical technician, you are designing a large solenoid to produce a uniform 0.150-T magnetic field near the center of the solenoid. You have enough wire for 4000 circular turns. This solenoid must be 1.40 m long and 2.80 cm in diameter. What current will you need to produce the necessary field?

28.45 • **Coaxial Cable.** A solid conductor with radius *a* is supported by insulating disks on the axis of a conducting tube with inner radius *b* and outer radius *c* (Fig. E28.45). The central conductor and tube carry equal currents *I* in opposite directions. The currents are distributed uniformly over the cross sections of each conductor. Derive an expression for the magnitude of the magnetic field (a) at points outside the central, solid conductor but inside the tube ($a < r < b$) and (b) at points outside the tube ($r > c$).

Figure **E28.45**



28.46 • Repeat Exercise 28.45 for the case in which the current in the central, solid conductor is I_1, the current in the tube is I_2, and these currents are in the same direction rather than in opposite directions.

28.47 • A long, straight, cylindrical wire of radius R carries a current uniformly distributed over its cross section. At what locations is the magnetic field produced by this current equal to half of its largest value? Consider points inside and outside the wire.

28.48 •• A 15.0-cm-long solenoid with radius 0.750 cm is closely wound with 600 turns of wire. The current in the windings is 8.00 A. Compute the magnetic field at a point near the center of the solenoid.

28.49 •• A solenoid is designed to produce a magnetic field of 0.0270 T at its center. It has radius 1.40 cm and length 40.0 cm, and the wire can carry a maximum current of 12.0 A. (a) What minimum number of turns per unit length must the solenoid have? (b) What total length of wire is required?

28.50 • A toroidal solenoid has an inner radius of 12.0 cm and an outer radius of 15.0 cm. It carries a current of 1.50 A. How many equally spaced turns must it have so that it will produce a magnetic field of 3.75 mT at points within the coils 14.0 cm from its center?

28.51 • A magnetic field of 37.2 T has been achieved at the MIT Francis Bitter National Magnetic Laboratory. Find the current needed to achieve such a field (a) 2.00 cm from a long, straight wire; (b) at the center of a circular coil of radius 42.0 cm that has 100 turns; (c) near the center of a solenoid with radius 2.40 cm, length 32.0 cm, and 40,000 turns.

28.52 • A toroidal solenoid (see Example 28.10) has inner radius $r_1 = 15.0$ cm and outer radius $r_2 = 18.0$ cm. The solenoid has 250 turns and carries a current of 8.50 A. What is the magnitude of the magnetic field at the following distances from the center of the torus: (a) 12.0 cm; (b) 16.0 cm; (c) 20.0 cm?

28.53 •• A wooden ring whose mean diameter is 14.0 cm is wound with a closely spaced toroidal winding of 600 turns. Compute the magnitude of the magnetic field at the center of the cross section of the windings when the current in the windings is 0.650 A.

Section 28.8 Magnetic Materials

28.54 •• A toroidal solenoid with 400 turns of wire and a mean radius of 6.0 cm carries a current of 0.25 A. The relative permeability of the core is 80. (a) What is the magnetic field in the core? (b) What part of the magnetic field is due to atomic currents?

28.55 • A toroidal solenoid with 500 turns is wound on a ring with a mean radius of 2.90 cm. Find the current in the winding that is required to set up a magnetic field of 0.350 T in the ring (a) if the ring is made of annealed iron $(K_m = 1400)$ and (b) if the ring is made of silicon steel $(K_m = 5200)$.

28.56 • The current in the windings of a toroidal solenoid is 2.400 A. There are 500 turns, and the mean radius is 25.00 cm. The toroidal solenoid is filled with a magnetic material. The magnetic field inside the windings is found to be 1.940 T. Calculate (a) the relative permeability and (b) the magnetic susceptibility of the material that fills the toroid.

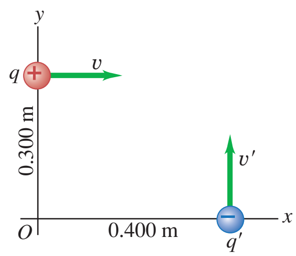
28.57 • A long solenoid with 60 turns of wire per centimeter carries a current of 0.15 A. The wire that makes up the solenoid is wrapped around a solid core of silicon steel $(K_m = 5200)$. (The wire of the solenoid is jacketed with an insulator so that none of the current flows into the core.) (a) For a point inside the core, find the magnitudes of (i) the magnetic field $\vec{B}_0$ due to the solenoid current; (ii) the magnetization $\vec{M}$; (iii) the total magnetic field $\vec{B}$. (b) In a sketch of the solenoid and core, show the directions of the vectors $\vec{B}$, $\vec{B}_0$, and $\vec{M}$ inside the core.

28.58 • When a certain paramagnetic material is placed in an external magnetic field of 1.5000 T, the field inside the material is measured to be 1.5023 T. Find (a) the relative permeability and (b) the magnetic permeability of this material.

PROBLEMS

28.59 •• A pair of point charges, $q = +8.00\ \mu$C and $q' = -5.00\ \mu$C, are moving as shown in Fig. P28.59 with speeds $v = 9.00 \times 10^4$ m/s and $v' = 6.50 \times 10^4$ m/s. When the charges are at the locations shown in the figure, what are the magnitude and direction of (a) the magnetic field produced at the origin and (b) the magnetic force that q' exerts on q?

Figure **P28.59**



28.60 •• At a particular instant, charge $q_1 = +4.80 \times 10^{-6}$ C is at the point $(0, 0.250\text{ m}, 0)$ and has velocity $\vec{v}_1 = (9.20 \times 10^5\text{ m/s})\hat{\imath}$. Charge $q_2 = -2.90 \times 10^{-6}$ C is at the point $(0.150\text{ m}, 0, 0)$ and has velocity $\vec{v}_2 = (-5.30 \times 10^5\text{ m/s})\hat{\jmath}$. At this instant, what are the magnitude and direction of the magnetic force that q_1 exerts on q_2?

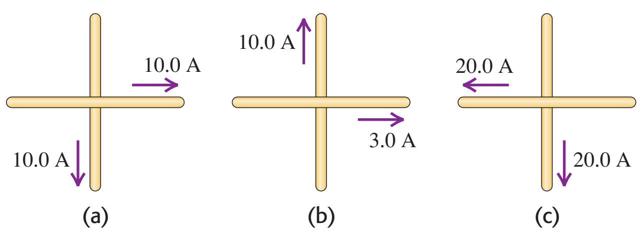
28.61 ••• Two long, parallel transmission lines, 40.0 cm apart, carry 25.0-A and 75.0-A currents. Find all locations where the net magnetic field of the two wires is zero if these currents are in (a) the same direction and (b) the opposite direction.

28.62 • A long, straight wire carries a current of 5.20 A. An electron is traveling in the vicinity of the wire. At the instant when the electron is 4.50 cm from the wire and traveling with a speed of 6.00×10^4 m/s directly toward the wire, what are the magnitude and direction (relative to the direction of the current) of the force that the magnetic field of the current exerts on the electron?

28.63 • **CP** A long, straight wire carries a 13.0-A current. An electron is fired parallel to this wire with a velocity of 250 km/s in the same direction as the current, 2.00 cm from the wire. (a) Find the magnitude and direction of the electron's initial acceleration. (b) What should be the magnitude and direction of a uniform electric field that will allow the electron to continue to travel parallel to the wire? (c) Is it necessary to include the effects of gravity? Justify your answer.

28.64 • Two very long, straight wires carry currents as shown in Fig. P28.64. For each case, find all locations where the net magnetic field is zero.

Figure **P28.64**



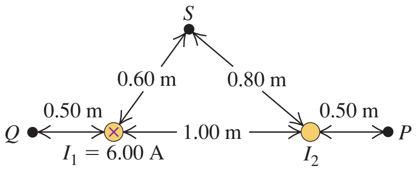
28.65 •• **CP** Two identical circular, wire loops 40.0 cm in diameter each carry a current of 3.80 A in the same direction. These loops are parallel to each other and are 25.0 cm apart. Line ab is normal to the plane of the loops and passes through their centers.

A proton is fired at 2400 km/s perpendicular to line *ab* from a point midway between the centers of the loops. Find the magnitude of the magnetic force these loops exert on the proton just after it is fired.

28.66 • A negative point charge $q = -7.20\ \text{mC}$ is moving in a reference frame. When the point charge is at the origin, the magnetic field it produces at the point $x = 25.0\ \text{cm}$, $y = 0$, $z = 0$ is $\vec{B} = (6.00\ \mu\text{T})\hat{\jmath}$, and its speed is 800 m/s. (a) What are the x-, y-, and z-components of the velocity $\vec{v}_0$ of the charge? (b) At this same instant, what is the magnitude of the magnetic field that the charge produces at the point $x = 0$, $y = 25.0\ \text{cm}$, $z = 0$?

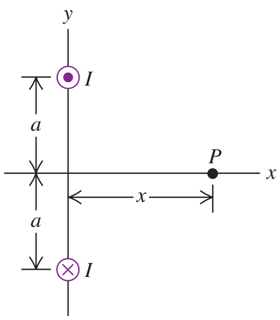
28.67 • Two long, straight, parallel wires are 1.00 m apart (Fig. P28.67). The wire on the left carries a current I_1 of 6.00 A into the plane of the paper. (a) What must the magnitude and direction of the current I_2 be for the net field at point P to be zero? (b) Then what are the magnitude and direction of the net field at Q? (c) Then what is the magnitude of the net field at S?

Figure **P28.67**



28.68 •• Figure P28.68 shows an end view of two long, parallel wires perpendicular to the xy-plane, each carrying a current I but in opposite directions. (a) Copy the diagram, and draw vectors to show the $\vec{B}$ field of each wire and the net $\vec{B}$ field at point P. (b) Derive the expression for the magnitude of $\vec{B}$ at any point on the x-axis in terms of the x-coordinate of the point. What is the direction of $\vec{B}$? (c) Graph the magnitude of $\vec{B}$ at points on the x-axis. (d) At what value of x is the magnitude of $\vec{B}$ a maximum? (e) What is the magnitude of $\vec{B}$ when $x \gg a$?

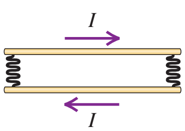
Figure **P28.68**



28.69 • Refer to the situation in Problem 28.68. Suppose that a third long, straight wire, parallel to the other two, passes through point P (see Fig. P28.68) and that each wire carries a current $I = 6.00\ \text{A}$. Let $a = 40.0\ \text{cm}$ and $x = 60.0\ \text{cm}$. Find the magnitude and direction of the force per unit length on the third wire, (a) if the current in it is directed into the plane of the figure, and (b) if the current in it is directed out of the plane of the figure.

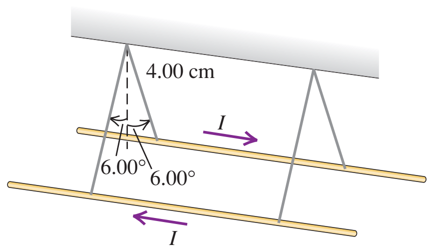
28.70 •• **CP** A pair of long, rigid metal rods, each of length L, lie parallel to each other on a perfectly smooth table. Their ends are connected by identical, very light conducting springs of force constant k (Fig. P28.70) and negligible unstretched length. If a current I runs through this circuit, the springs will stretch. At what separation will the rods remain at rest? Assume that k is large enough so that the separation of the rods will be much less than L.

Figure **P28.70**



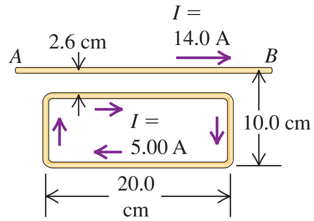
28.71 ••• **CP** Two long, parallel wires hang by 4.00-cm-long cords from a common axis (Fig. P28.71). The wires have a mass per unit length of 0.0125 kg/m and carry the same current in opposite directions. What is the current in each wire if the cords hang at an angle of 6.00° with the vertical?

Figure **P28.71**



28.72 • The long, straight wire AB shown in Fig. P28.72 carries a current of 14.0 A. The rectangular loop whose long edges are parallel to the wire carries a current of 5.00 A. Find the magnitude and direction of the net force exerted on the loop by the magnetic field of the wire.

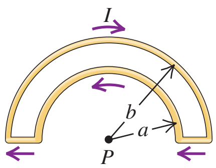
Figure **P28.72**



28.73 •• **CP** A flat, round iron ring 5.00 cm in diameter has a current running through it that produces a magnetic field of 75.4 μT at its center. This ring is placed in a uniform external magnetic field of 0.375 T. What is the maximum torque the external field can exert on the ring? Show how the ring should be oriented relative to the field for the torque to have its maximum value.

28.74 • The wire semicircles shown in Fig. P28.74 have radii a and b. Calculate the net magnetic field (magnitude and direction) that the current in the wires produces at point P.

Figure **P28.74**



28.75 • **CALC** **Helmholtz Coils.** Figure 28.75 is a sectional view of two circular coils with radius a, each wound with N turns of wire carrying a current I, circulating in the same direction in both coils. The coils are separated by a distance a equal to their radii. In this configuration the coils are called Helmholtz coils; they produce a very uniform magnetic field in the region between them. (a) Derive the expression for the magnitude B of the magnetic field at a point on the axis a distance x to the right of point P, which is midway between the coils. (b) Graph B versus x for $x = 0$ to $x = a/2$. Compare this graph to one for the magnetic field due to the right-hand coil alone. (c) From part (a), obtain an expression for the magnitude of the magnetic field at point P. (d) Calculate the magnitude of the magnetic field at P if $N = 300$ turns, $I = 6.00\ \text{A}$, and $a = 8.00\ \text{cm}$. (e) Calculate dB/dx and d^2B/dx^2 at $P(x = 0)$. Discuss how your results show that the field is very uniform in the vicinity of P.

Figure **P28.75**

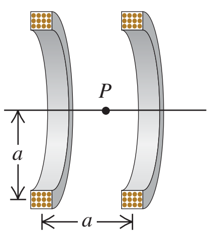
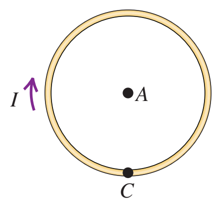


Figure **P28.76**

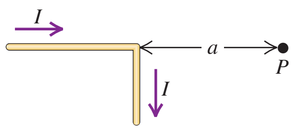


28.76 • A circular wire of diameter D lies on a horizontal table and carries a current I. In Fig. P28.76 point A marks the center of the circle and point C is on its rim. (a) Find the magnitude and direction of the magnetic field at point A. (b) The wire is now unwrapped so it is straight, centered on point C, and perpendicular to the line AC, but the same current is maintained in it. Now find the magnetic field at point A. (c) Which field is greater: the one in part (a) or in part (b)? By what factor? Why is this result physically reasonable?

28.77 • **CALC** A long, straight wire with a circular cross section of radius R carries a current I. Assume that the current density is not constant across the cross section of the wire, but rather varies as $J = \alpha r$, where α is a constant. (a) By the requirement that J integrated over the cross section of the wire gives the total current I, calculate the constant α in terms of I and R. (b) Use Ampere's law to calculate the magnetic field $B(r)$ for (i) $r \le R$ and (ii) $r \ge R$. Express your answers in terms of I.

28.78 • **CALC** The wire shown in Fig. P28.78 is infinitely long and carries a current I. Calculate the magnitude and direction of the magnetic field that this current produces at point P.

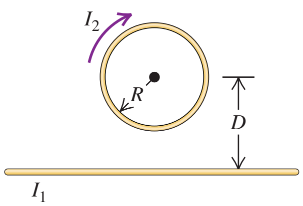
Figure **P28.78**



28.79 • A conductor is made in the form of a hollow cylinder with inner and outer radii a and b, respectively. It carries a current I uniformly distributed over its cross section. Derive expressions for the magnitude of the magnetic field in the regions (a) $r < a$; (b) $a < r < b$; (c) $r > b$.

28.80 • A circular loop has radius R and carries current I_2 in a clockwise direction (Fig. P28.80). The center of the loop is a distance D above a long, straight wire. What are the magnitude and direction of the current I_1 in the wire if the magnetic field at the center of the loop is zero?

Figure **P28.80**



28.81 • **CALC** A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetric about the cylinder axis, is not constant but varies according to the relationship

$$\vec{J} = \frac{2I_0}{\pi a^2}\left[1 - \left(\frac{r}{a}\right)^2\right]\hat{k} \quad \text{for } r \le a$$
$$= \mathbf{0} \quad \text{for } r \ge a$$

where a is the radius of the cylinder, r is the radial distance from the cylinder axis, and I_0 is a constant having units of amperes. (a) Show that I_0 is the total current passing through the entire cross section of the wire. (b) Using Ampere's law, derive an expression for the magnitude of the magnetic field $\vec{B}$ in the region $r \ge a$. (c) Obtain an expression for the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis. (d) Using Ampere's law, derive an expression for the magnitude of the magnetic field $\vec{B}$ in the region $r \le a$. How do your results in parts (b) and (d) compare for $r = a$?

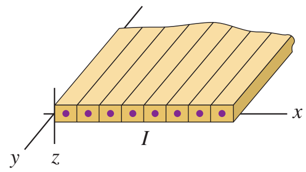
28.82 • A long, straight, solid cylinder, oriented with its axis in the z-direction, carries a current whose current density is $\vec{J}$. The current density, although symmetric about the cylinder axis, is not constant and varies according to the relationship

$$\vec{J} = \left(\frac{b}{r}\right)e^{(r-a)/\delta}\hat{k} \quad \text{for } r \le a$$
$$= \mathbf{0} \quad \text{for } r \ge a$$

where the radius of the cylinder is $a = 5.00$ cm, r is the radial distance from the cylinder axis, b is a constant equal to 600 A/m, and δ is a constant equal to 2.50 cm. (a) Let I_0 be the total current passing through the entire cross section of the wire. Obtain an expression for I_0 in terms of b, δ, and a. Evaluate your expression to obtain a numerical value for I_0. (b) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \ge a$. Express your answer in terms of I_0 rather than b. (c) Obtain an expression for the current I contained in a circular cross section of radius $r \le a$ and centered at the cylinder axis. Express your answer in terms of I_0 rather than b. (d) Using Ampere's law, derive an expression for the magnetic field $\vec{B}$ in the region $r \le a$. (e) Evaluate the magnitude of the magnetic field at $r = \delta$, $r = a$, and $r = 2a$.

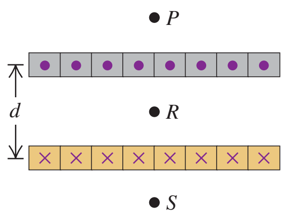
28.83 • **An Infinite Current Sheet.** Long, straight conductors with square cross sections and each carrying current I are laid side by side to form an infinite current sheet (Fig. P28.83). The conductors lie in the xy-plane, are parallel to the y-axis, and carry current in the $+y$-direction There are n conductors per unit length measured along the x-axis. (a) What are the magnitude and direction of the magnetic field a distance a below the current sheet? (b) What are the magnitude and direction of the magnetic field a distance a above the current sheet?

Figure **P28.83**



28.84 • Long, straight conductors with square cross section, each carrying current I, are laid side by side to form an infinite current sheet with current directed out of the plane of the page (Fig. P28.84). A second infinite current sheet is a distance d below the first and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length. (Refer to Problem 28.83.) Calculate the magnitude and direction of the net magnetic field at (a) point P (above the upper sheet); (b) point R (midway between the two sheets); (c) point S (below the lower sheet).

Figure **P28.84**



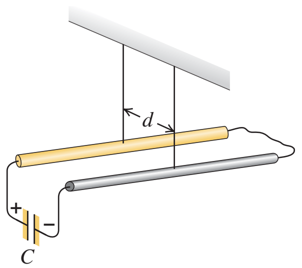
28.85 • **CP** A piece of iron has magnetization $M = 6.50 \times 10^4$ A/m. Find the average magnetic dipole moment *per atom* in this piece of iron. Express your answer both in $A \cdot m^2$ and in Bohr magnetons. The density of iron is given in Table 14.1, and the atomic mass of iron (in grams per mole) is given in Appendix D. The chemical symbol for iron is Fe.

CHALLENGE PROBLEMS

28.86 ••• A wide, long, insulating belt has a uniform positive charge per unit area σ on its upper surface. Rollers at each end move the belt to the right at a constant speed v. Calculate the magnitude and direction of the magnetic field produced by the moving belt at a point just above its surface. (*Hint:* At points near the surface and far from its edges or ends, the moving belt can be considered to be an infinite current sheet like that in Problem 28.83.)

28.87 ••• **CP** Two long, straight conducting wires with linear mass density λ are suspended from cords so that they are each horizontal, parallel to each other, and a distance d apart. The back ends of the wires are connected to each other by a slack, low-resistance connecting wire. A charged capacitor (capacitance C) is now added to the system; the positive plate of the capacitor (initial charge $+Q_0$) is connected to the front end of one of the wires, and the negative plate of the capacitor (initial charge $-Q_0$) is connected to the front end of the other wire (Fig. P28.87). Both of these connections are also made by slack, low-resistance wires. When the connection is made, the wires are pushed aside by the repulsive force between the wires, and each wire has an initial horizontal velocity of magnitude v_0. Assume that the time constant for the capacitor to discharge is negligible compared to the time it takes for any appreciable displacement in the position of the wires to occur. (a) Show that the initial speed v_0 of either wire is given by

Figure **P28.87**

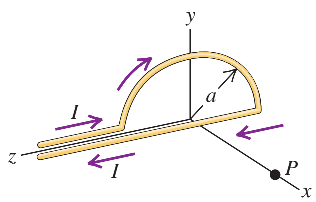


$$v_0 = \frac{\mu_0 Q_0^2}{4\pi \lambda RCd}$$

where R is the total resistance of the circuit. (b) To what height h will each wire rise as a result of the circuit connection?

28.88 ••• **CALC** A wire in the shape of a semicircle with radius a is oriented in the yz-plane with its center of curvature at the origin (Fig. P28.88). If the current in the wire is I, calculate the magnetic-field components produced at point P, a distance x out along the x-axis. (*Note:* Do not forget the contribution from the straight wire at the bottom of the semicircle that runs from $z = -a$ to $z = +a$. You may use the fact that the fields of the two antiparallel currents at $z > a$ cancel, but you must explain *why* they cancel.)

Figure **P28.88**



Answers

Chapter Opening Question ?

There would be *no* change in the magnetic field strength. From Example 28.9 (Section 28.7), the field inside a solenoid has magnitude $B = \mu_0 nI$, where n is the number of turns of wire per unit length. Joining two solenoids end to end doubles both the number of turns and the length, so the number of turns per unit length is unchanged.

Test Your Understanding Questions

28.1 Answers: (a) (i), (b) (ii) The situation is the same as shown in Fig. 28.2 except that the upper proton has velocity $\vec{v}$ rather than $-\vec{v}$. The magnetic field due to the lower proton is the same as shown in Fig. 28.2, but the direction of the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$ on the upper proton is reversed. Hence the magnetic force is attractive. Since the speed v is small compared to c, the magnetic force is much smaller in magnitude than the repulsive electric force and the net force is still repulsive.

28.2 Answer: (i) and (iii) (tie), (iv), (ii) From Eq. (28.5), the magnitude of the field dB due to a current element of length dl carrying current I is $dB = (\mu/4\pi)(I\,dl \sin\phi/r^2)$. In this expression r is the distance from the element to the field point, and ϕ is the angle between the direction of the current and a vector from the current element to the field point. All four points are the same distance $r = L$ from the current element, so the value of dB is proportional to the value of $\sin\phi$. For the four points the angle is (i) $\phi = 90°$, (ii) $\phi = 0$, (iii) $\phi = 90°$, and (iv) $\phi = 45°$, so the values of $\sin\phi$ are (i) 1, (ii) 0, (iii) 1, and (iv) $1/\sqrt{2}$.

28.3 Answer: A This orientation will cause current to flow clockwise around the circuit. Hence current will flow south through the wire that lies under the compass. From the right-hand rule for the magnetic field produced by a long, straight, current-carrying conductor, this will produce a magnetic field that points to the left at the position of the compass (which lies atop the wire). The combination of the northward magnetic field of the earth and the westward field produced by the current gives a net magnetic field to the northwest, so the compass needle will swing counterclockwise to align with this field.

28.4 Answers: (a) (i), (b) (iii), (c) (ii), (d) (iii) Current flows in the same direction in adjacent turns of the coil, so the magnetic forces between these turns are attractive. Current flows in opposite directions on opposite sides of the same turn, so the magnetic forces between these sides are repulsive. Thus the magnetic forces on the solenoid turns squeeze them together in the direction along its axis but push them apart radially. The *electric* forces are zero because the wire is electrically neutral, with as much positive charge as there is negative charge.

28.5 Answers: (a) (ii), (b) (v) The vector $d\vec{B}$ is in the direction of $d\vec{l} \times \vec{r}$. For a segment on the negative y-axis, $d\vec{l} = -\hat{k}\,dl$ points in the negative z-direction and $\vec{r} = x\hat{\imath} + a\hat{\jmath}$. Hence $d\vec{l} \times \vec{r} = (a\,dl)\hat{\imath} - (x\,dl)\hat{\jmath}$, which has a positive x-component, a negative y-component, and zero z-component. For a segment on the negative z-axis, $d\vec{l} = \hat{\jmath}\,dl$ points in the positive y-direction and $\vec{r} = x\hat{\imath} + a\hat{k}$. Hence $d\vec{l} \times \vec{r} = 1(a\,dl)\hat{\imath} - 1(x\,dl)\hat{k}$, which has a positive x-component, zero y-component, and a negative z-component.

28.6 Answer: (ii) Imagine carrying out the integral $\oint \vec{B} \cdot d\vec{l}$ along an integration path that goes counterclockwise around the red magnetic field line. At each point along the path the magnetic field $\vec{B}$ and the infinitesimal segment $d\vec{l}$ are both tangent to the path, so $\vec{B} \cdot d\vec{l}$ is positive at each point and the integral $\oint \vec{B} \cdot d\vec{l}$ is likewise positive. It follows from Ampere's law $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ and the right-hand rule that the integration path encloses a current directed out of the plane of the page. There are no currents in the empty space outside the magnet, so there must be currents inside the magnet (see Section 28.8).

28.7 Answer: (iii) By symmetry, any $\vec{B}$ field outside the cable must circulate around the cable, with circular field lines like those surrounding the solid cylindrical conductor in Fig. 28.20. Choose an integration path like the one shown in Fig. 28.20 with radius $r > R$, so that the path completely encloses the cable. As in Example 28.8, the integral $\oint \vec{B} \cdot d\vec{l}$ for this path has magnitude $B(2\pi r)$. From Ampere's law this is equal to $\mu_0 I_{encl}$. The net enclosed current I_{encl} is zero because it includes two currents of equal magnitude but opposite direction: one in the central wire and one in the hollow cylinder. Hence $B(2\pi r) = 0$, and so $B = 0$ for any value of r outside the cable. (The field is nonzero *inside* the cable; see Exercise 28.45.)

28.8 Answer: (i), (iv) Sodium and uranium are paramagnetic materials and hence are attracted to a magnet, while bismuth and lead are diamagnetic materials that are repelled by a magnet. (See Table 28.1.)

Bridging Problem

Answer: $B = \dfrac{\mu_0 n Q}{a}$