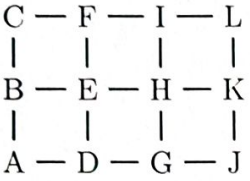
Fall 2019: CSE 221 | **Discrete Mathematics** | **Akdeniz University**

Tuesday 08/01/2020 | Final Exam | Duration: 90 minutes

Name: Solutions | Student No:

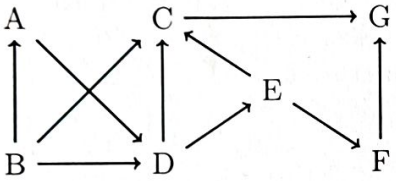
P1 [16 (1*16) points] Graph Definitions - Undirected Graph



Tick **ALL** appropriate definitions for each sequence:

Sequence	Walk	Path	Circuit	Cycle
A-D-E-F-I-H-E-B-A	✓		✓	
A-B-E-H-G-D-E-F	✓			
F-I-H-E	✓	✓		
A-B-C-F-I-L	✓	✓		

P2 [24 (2*8+8*1) points] Graph Basics - Directed Graph



Write the in-degrees of the vertices: A: 1 B: 0 C: 3 D: 2

Write the out-degrees of the vertices: E: 2 F: 1 G: 0

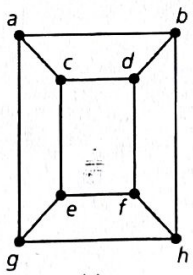
Is there a cycle in this graph? Yes · (No)

Give a topological order for this graph: - BADECFG or - BADEFCG.

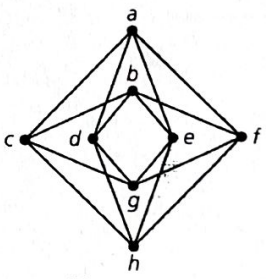
B → A → D → E → C → G ⇒ - BADECFG or - BADEFCG.

P3 [15 (5*3) points] Bipartite Graphs Are the graphs on the right bipartite? If yes, give a partition of vertices as reds and bluee (Like R: a,b,c,d B: e,f,g,h), if not explain why.

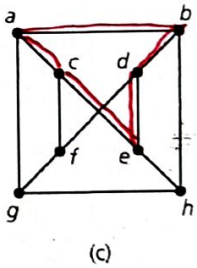
	REDS	BLUES
a)	a,d,e,h	b,c,f,g
b)	c,d,e,f	a,b,g,h
c)	not bipartite: there is an odd-length cycle: abdeca	



(a)

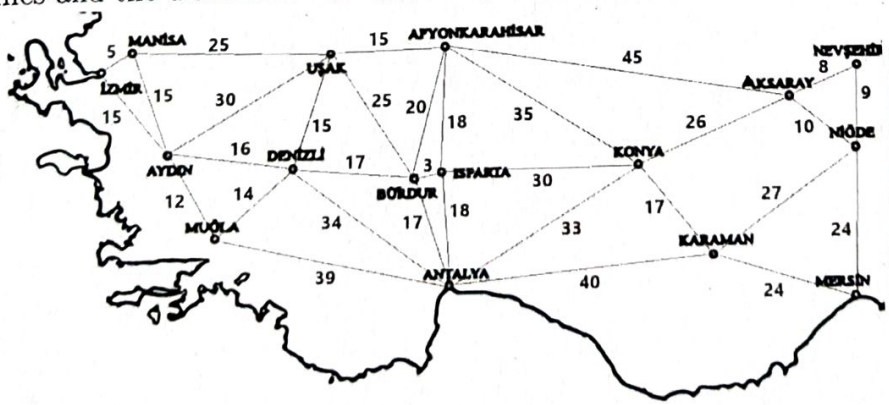


(b)



(c)

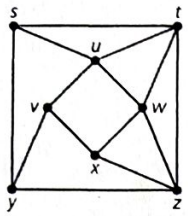
P4 [15 (1*15) points] Minimum Spanning Trees In the map below, find a minimal spanning tree by using Kruskal's Algorithm. Write the roads in the order you add them. (Use 3-letter short city names and the distance of the road. So, each line must contain a road description like Ant-Bur 17)



1	Bur-Isp 3	9	Uşa-Afy 15
2	Man-İzm 5	10	Den-Bur 17
3	Aks-Nev 8	11	Bur-Ant 17
4	Nev-Niğ 9	12	Kon-Kar 17
5	Ayd-Muğ 12	13	Kar-Mer 24
6	Muğ-Den 14	14	Niğ-Mer 24
7	İzm-Ayd 15	15	Isp-Kon 30
8	Den-Uşa 15		

P5 [20 (10*2) points] Counting paths

a) In the graph below, how many paths of length 2 are there? (You will count the paths which visit 2 edges 3 vertices. e.g. utw) (Do not count one-by-one, try to find an easy way so that you can also solve part b)



Observation: If vertex v has degree d_v, there are $\binom{d_v}{2}$ paths with middle point v.

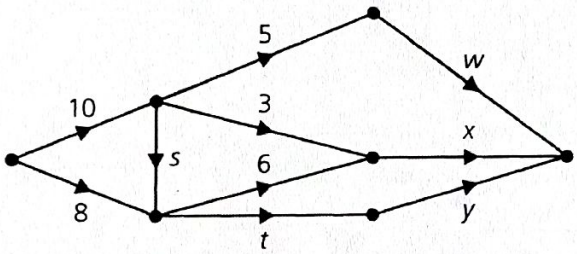
s, v, x, y has degree 3
t, u, w, z has degree 4

$\Rightarrow 4\cdot\binom{3}{2} + 4\binom{4}{2} = 12+24 = 36$ paths.

b) In a 5-regular graph with 56 vertices, how many paths of length 2 are there?

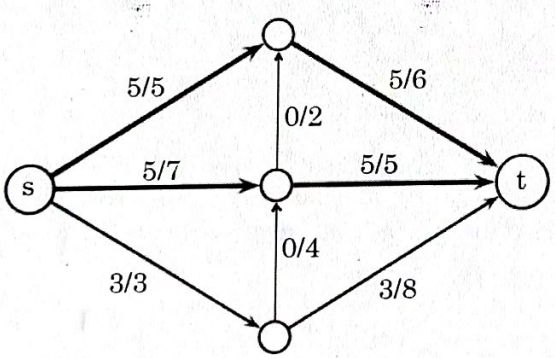
$56 \cdot \binom{5}{2} = 560$ paths.

P6 [10 (5*2) points] For the network shown below, let the capacity of each edge be 10. If each edge e in the figure is labeled by a function f, as shown, determine the values of s, t, w, x and y so that f is a flow in the network. (Obviously, leftmost vertex is the source and rightmost one is the sink.)

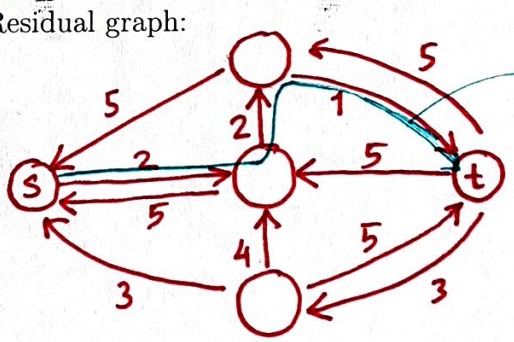


s	t	w	x	y
2	4	5	9	4

P7 (Bonus) [20 (8+8+4) points] Max flow A flow network is given below. Use the allocated spaces to 1) Draw the residual graph and find out whether the flow can be further improved (increased) 2) Update the flow accordingly 3) and show that it is indeed a maximum flow.

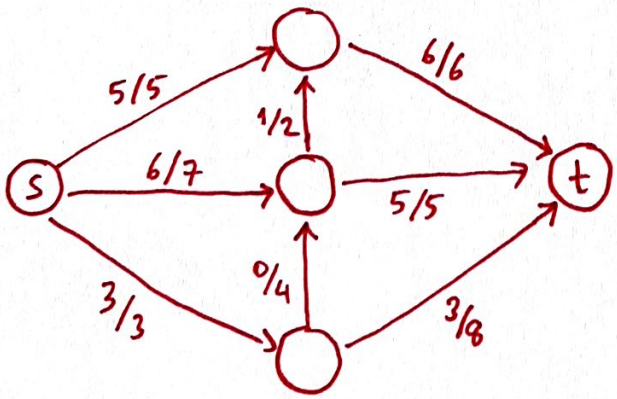


Residual graph:



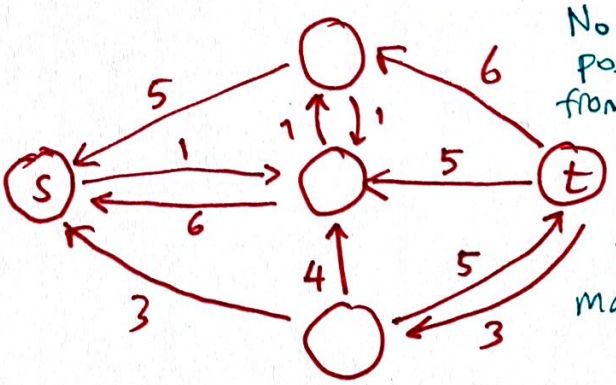
1 flow still possible.

New flow:



Proof of maximality:

New residual graph:



No flow possible from s to t now. So, the flow is maximal.