

$$x_1 + x_2 + x_3 + x_4 = \underline{18} \quad 0 \leq x_i \leq 7 \quad i=1,2,3,4$$

$x_1 \geq 8, x_2 \geq 8, x_3 \geq 10$

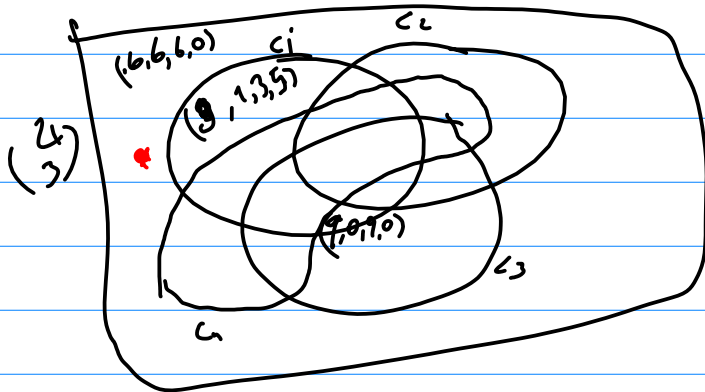
$= 2$

Let $c_i : x_i \geq 8$ then $\# = \bar{N} = S_0 - S_1 + S_2 - S_3 + S_4$

$= 0 \quad = 0$

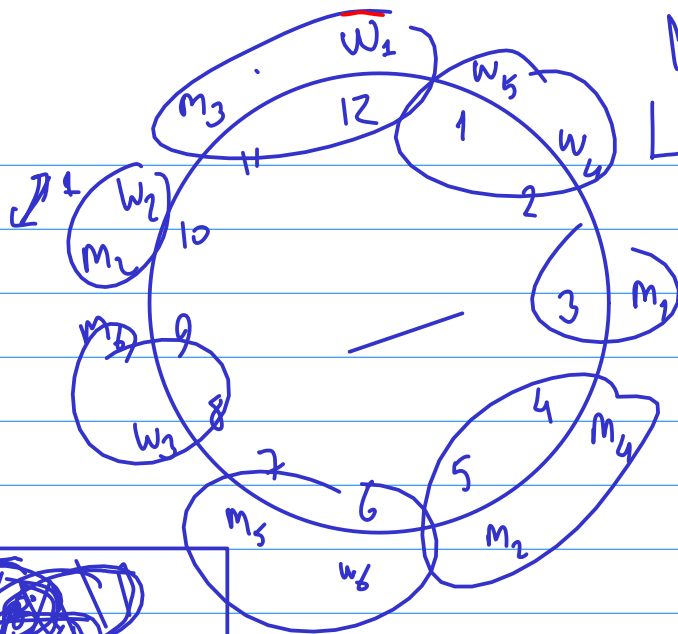
$$S_0 = N = \binom{4}{0} \binom{21}{3} \quad S_1 = \sum_i N(c_i) = \binom{4}{1} \binom{13}{3} \quad \dots S_2 = \binom{4}{2} \binom{5}{3}$$

$$\# = \binom{21}{3} - \binom{4}{1} \binom{13}{3} + \binom{4}{2} \binom{5}{3}$$



We can use P.I.E.

Let C_i : W_i & M_i sits next to each other.



$$\text{Ans} = \bar{N} = S_0 - S_1 + S_2 - S_3 + S_4 - S_5 + S_6$$

$$= 11! - \binom{6}{1}(2 \cdot 10!) + \binom{6}{2}(2^2 \cdot 9!)$$

$$- \binom{6}{3}(2^3 \cdot 8!) + \binom{6}{4}(2^4 \cdot 7!) -$$

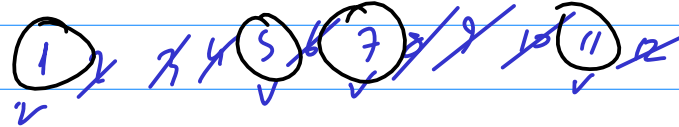
$$\binom{6}{5}(2^5 \cdot 6!) + \binom{6}{6}2^6 \cdot 5!$$

=

RSA $n = p \cdot q$ \downarrow $\Phi(n) = (p-1)(q-1)$ \downarrow $35 \cdot (1 - \frac{1}{5} - \frac{1}{7} + \frac{1}{35})$
 Euler's totient func. $35 \cdot (\frac{5-1}{5}) \cdot (\frac{7-1}{7})$

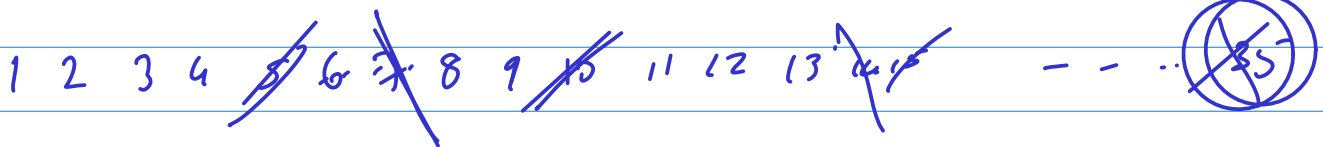
$\Phi(n)$ is the number of numbers $\leq n$, and coprime with n .

$\Phi(12) = 4$



$\Phi(35) = 5 \cdot 7 = 4 \cdot 6 = 24$

$35 - \frac{35}{5} - \frac{35}{7} + 1$



$\Phi(n) = ?$

$n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_t^{\alpha_t}$

$\Phi(n) = n (1 - \frac{1}{p_1}) (1 - \frac{1}{p_2}) \dots (1 - \frac{1}{p_t})$

$2^3 3^5 5^1 = \dots$

$\Phi(n) \dots$
 $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_t^{\alpha_t}$ (1 2 3 ... -)

$\frac{16}{n} \rightarrow \frac{45}{3 \cdot 3 \cdot 5}$

$C_i: \text{div by } p_i \quad S = N(C_1) + N(C_2) + N(C_3) + \dots + N(C_t)$

$\Phi(n) = \bar{N} = S_0 - S_1 + \dots + (-1)^t S_t$

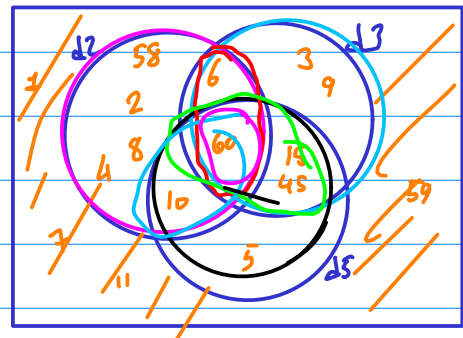
$S_1 = \sum_i N(C_i)$

$= n - \left(\frac{n}{p_1} + \frac{n}{p_2} + \dots + \frac{n}{p_t} \right) + \left(\frac{n}{p_1 p_2} + \dots + \frac{n}{p_{t-1} p_t} \right) - \left(\frac{n}{p_1 p_2} + \dots \right) \dots + (-1)^t \left(\frac{n}{p_1 p_2 \dots p_t} \right)$

$\Phi(n) = n \left(1 - \frac{1}{p_1} \right) \left(1 - \frac{1}{p_2} \right) \dots \left(1 - \frac{1}{p_t} \right)$

$\# = \bar{N} = S_0 - S_1 + S_2 - S_3$
 $= 60 - 30 - 20 - 12 + 10 + 4 + 6 - 2$
 $= 60 \left(1 - \frac{1}{2} \right) \left(1 - \frac{1}{3} \right) \left(1 - \frac{1}{5} \right)$
 $= 60$

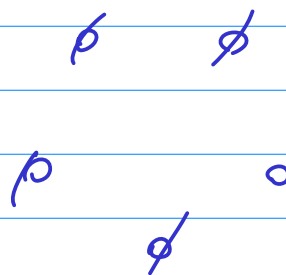
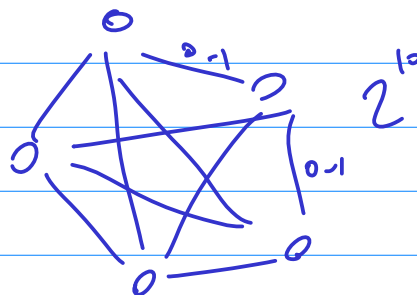
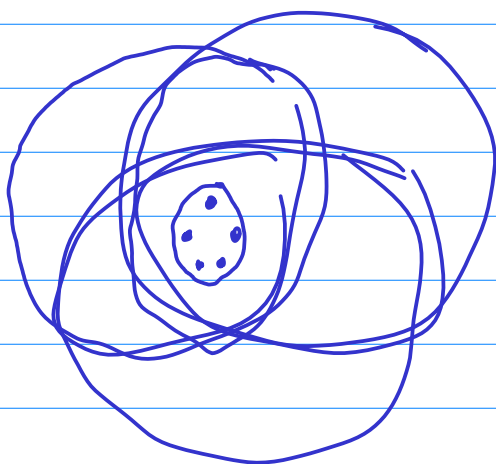
$60 = 2^2 \cdot 3^1 \cdot 5^1$



Let $c_i : v_i$ is isolated

$$\text{Then the num} = \bar{N} = S_0 - S_1 + S_2 - S_3 + S_4 - S_5$$

$$= 2^{\binom{5}{2}} - \binom{5}{1} 2^{\binom{4}{2}} + \binom{5}{2} 2^{\binom{3}{2}} - \binom{5}{3} 2^{\binom{2}{2}} + \binom{5}{4} \cdot 1 - \binom{5}{5} \cdot 1$$



$$\#V=3$$

$$\bar{N} = S_0 - S_1 + S_2 - S_3$$

$$= \binom{3}{0} 2^{\binom{3}{2}} - \binom{3}{1} 2^{\binom{2}{2}} + \binom{3}{2} 1 - \binom{3}{3} 1$$

$$= 8 - 3 \cdot 2 + 3 - 1$$

$$= 4$$

