Tuesday 11/01/2022   Final Exam   Duration: 90 minutes
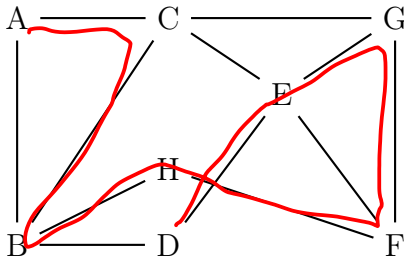
Name: [   ]   Student No: [   ]

P1 [25 points] If exists in the graph, give an example of the following. If impossible, write impossible. (Ex. If **path** was asked, a correct answer would be: A-C-E)

Eulerian trail: *GCABCEDB H F EGF*

Eulerian cycle: *Imp. because there are odd deg vertices.* *G, F.*

Hamiltonian path: *DEGFHBCA*

P2 [10 points] Choose the correct option:
What is the chromatic number of $K_{3,4}$?
A) 2   B)3   C)4   D)6   E)12   *(A circled)*
What is the chromatic number of K_9?
A) 3   B)6   C)8   D)9   E)10   *(D circled)*

P3 [20 points] Topological Sort & Counting
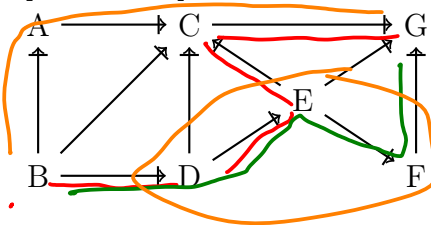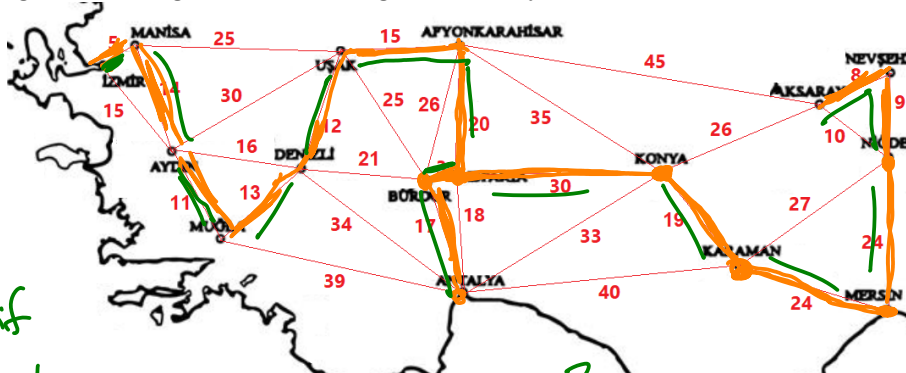

Give a topological order for the graph: *BDAEFCG*

What is the number of possible topological orders?

B D E F G → B.D.E.F.CG
B.D.E.C.F.G → 7
BAD ECF6
A→F 6
F→A 1

P4 [15 points] Minimum Spanning Tree In the map below, draw a minimum spanning tree by using Prim's Algorithm starting from Konya and write the cities in the order you add them to the MST.

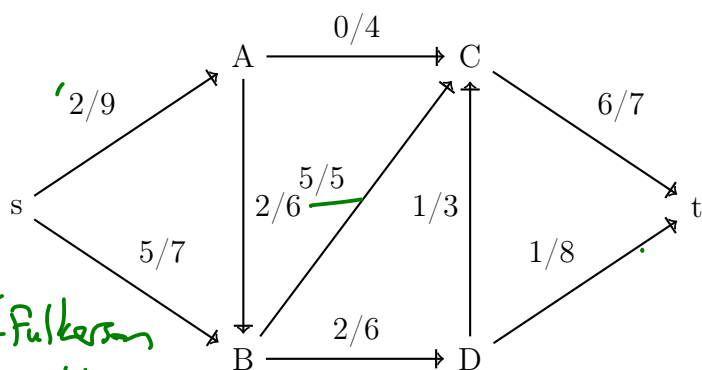| 1 | Konya | 9 | Ant |
2	Karam	10	Kış
3	Mers	11	Uşak
4	Nığ	12	Den.
5	Nev	13	Muğ
6	Aks	14	Ayd
7	Isp	15	Manis
8	Bur	16	Izm

What if Kruskal

4   Kar Mer 24
Nığ Mer 26
Kor Isp 5

3   Afy Uşak 15
Bur Ant 17
Kon Kor 19
Afy Isp 20

1   Isp-Bur 3
Mar-Izm 5
Nev Aks 8
Nev Niğ 9

2   Ayd-Muğ 11
Uşak Den 12
Muğ Den 13
Mar. Ayd 14

P5 [20 points] Network Flows
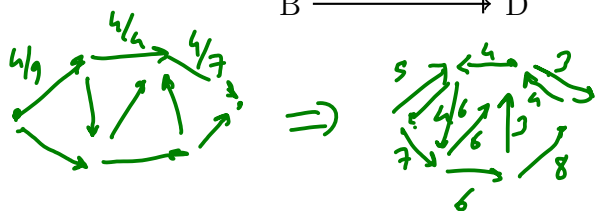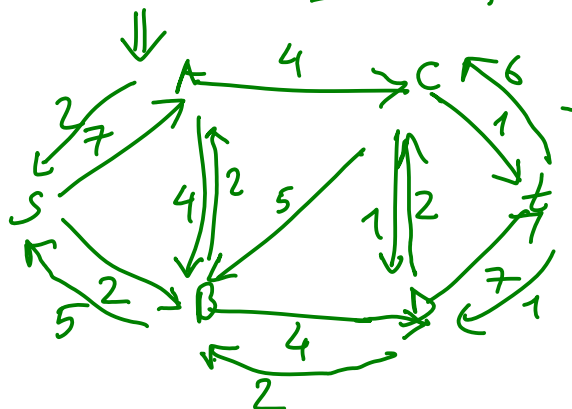
A network and a flow F on this network are given on the left.

1. According to the capacities, what is the maximum flow of this network?

2. Draw the residual graph. ~~for the given flow.~~

P6 [15 points] Generating Functions & Combinations Solve this question using generating functions (Build the polynomial, determine the coefficient to look for, and calculate the $\underline{\text{final}}$ result) [Recall that $1/(1-x) = 1 + x + x^2 + x^3 + \ldots$]

How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ with the restriction that all of $x_i \geq 1$ where two of them are $\underline{\text{odd}}$ and the remaining three are even integers?

- Assume $\underline{x_1}$ & $\underline{x_2}$ are the odd ones, solve the ques $\frac{6}{}$ mult by $\binom{5}{2}$ $\quad \binom{5}{2}\binom{10}{6}$

$$\left(x + x^3 + x^5 + \cdots\right)^2 \left(x^2 + x^4 + x^6 + \cdots\right)^3 \quad \text{coef}[x^{20}] = ?$$

$$x^2\left(1 + x^2 + x^4 + \cdots\right)^2 \quad x^6 \cdot \left(1 + x^2 + x^4 + \cdots\right)^3$$

$$x^8 \underline{\left(1 + x^2 + x^4 + \cdots\right)^5} \quad [x^{20}] = ?$$

$$\left(1 + x^2 + x^4 + \cdots\right)^5 \quad [x^{12}] = ?$$

$$\to \quad \left(\frac{1}{1 - x^2}\right)^5 = (1 - x^2)^{-5} \quad \binom{-5}{6}(-1)^6$$

$$\binom{10}{6}\cdots(-1)$$