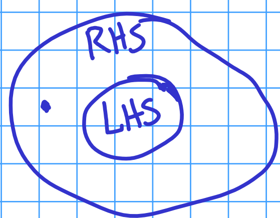
x	$p(x)$	$q(x)$
⋮		
Exists •	$\to$ T	$\to$ T
⋮		



LHS $\to$ RHS

x	$p(x)$	$q(x)$
⋮		
x_1	T	
⋮		
x_2		T
⋮		

x	p(x)	q(x)
1	1	1
2	1	1
3	1	1
4	1	1
5	1	1
6	1	1

$$\forall x\, p(x) \lor \forall x\, q(x) \overset{\not\Leftarrow}{\Rightarrow} \forall x\,(p(x) \lor q(x))$$

0

1

x	p(x)	q(x)
1	1	0
2	0	1
3	1	0
4	0	1
5	1	0

$$\neg\left(\forall x\,(p(x))\right) \iff \exists x\ \neg p(x)$$ ✓

x	$p(x)$
1	1
2	1
3	0
4	1
⋮	1
	⋮

find a zero

$\neg(\exists x\ p(x))$ ✓ $\Leftrightarrow$ $\forall x\ \neg(p(x))$ ✓

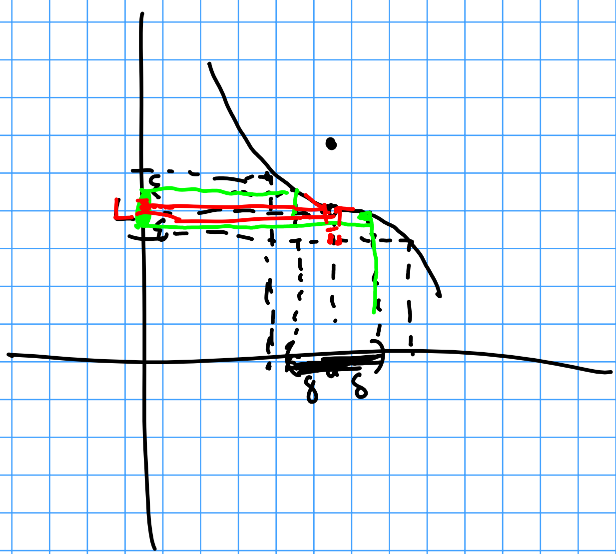
x	$p(x)$
1	0
2	0
3	$\cancel{1}$ 0
4	0
⋮	0
⋮	0
	⋮

$\exists x \exists y \exists z \ldots \Leftrightarrow \exists y \exists z \exists x \ldots$ B!

$\neg$ " $\forall x \left(\exists y \; (p(x,y) \wedge q(x,y)) \rightarrow r(x,y) \right)$ "

$\Leftrightarrow \exists x \left(\neg \exists y \; (p(x,y) \wedge q(x,y)) \rightarrow r(x,y) \right)$

$\Leftrightarrow \exists x \; \forall y \; \neg (p(x,y) \wedge q(x,y) \rightarrow r(x,y))$



1. $\forall n \ q(n) \to p(n)$ $*$

2. $\forall n \ q(n) \to p(n)$ $*$ } different

3. $\forall n \ \underline{p(n)} \to \underline{q(n)}$ —

4. $\exists n \ q(n)$ $\sim$

5. $\forall n \ q(n) \to p(n)$ $*$

6. $\forall n \ [\neg p(n) \to \neg q(n)]$

   $\Leftrightarrow \forall n \ q(n) \to p(n)$ $*$

7. $\forall n \ p(n) \to q(n)$ —

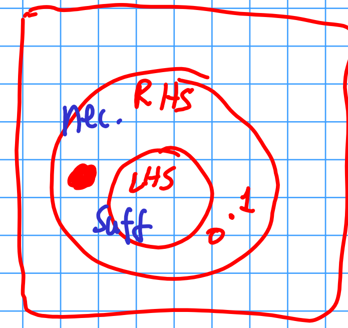
$S \to n$

$n \to S$

LHS $\to$ RHS

✓ ✓

<u>Suff.</u> <u>nec</u>



LHS 1

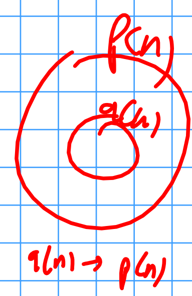
RHS 0

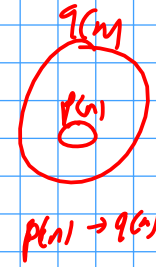
e e

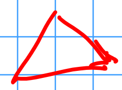
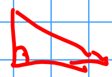
4 $\to$ 6

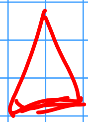
3 $\leftarrow$ 9

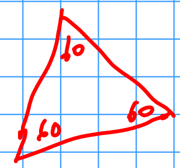
0 0

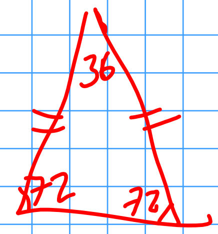




t   x 1, 2, 3, 4.







$\neg r(c)$

$\forall t\, [p(t) \to q(t))$

$\forall t\, (q(t) \to r(t))$

$\therefore \neg p(c)$

Proof

Steps	Reason
1. $\neg r(c)$	Premise –
2. $\forall t\, (p(t) \to q(t))$	Premise –
3. $\forall t\, (q(t) \to r(t))$	Premise –
4. $q(c) \to r(c)$	R. univ. spec. St. 3
5. $\neg q(c)$	M. Tollens St 1, 4
6. $p(c) \to q(c)$	R. univ. sp. St 2
7. $\neg p(c)$	M. T. St. 5, 6

how do we bind together?

$\neg r(c)$

$\forall t\, (p(t) \to q(t))$

$\forall t\, (q(t) \to r(t))$

$\therefore \neg p(c)$

<u>MT</u> = Cont + MP

Proof

Steps	Reason
1. $\neg r(c)$	Premise
2. $\forall t\, (p(t) \to q(t))$	Premise
3. $\forall t\, (q(t) \to r(t))$	Premise
4. $\forall t\, (p(t) \to r(t))$	L of Syl.S. 2,3
5. $p(c) \to r(c)$	R. Uni. sp. 4
6. $\neg r(c) \to \neg p(c)$	Contrap. of 5
7. $\neg p(c)$	M. Ponens 1,6

how do we bind together?

$\neg r(c)$

$\forall t\, [p(t) \to q(t))$

$\forall t\, (q(t) \to r(t))$

$\therefore \neg p(c)$

Proof

Steps	Reason
1. $\neg r(c)$	Premise
2. $\forall t\, (p(t) \to q(t))$	Premise
3. $\forall t\, (q(t) \to r(t))$	Premise
4. $p(c)$	Assumption
5. $p(c) \to q(c)$	R. Un. Sp. 2
6. $q(c)$	MP 4, 5
7. $q(c) \to r(c)$	R. Un. Sp. 3
8. $r(c)$	MP. 6, 7.
9. $r(c) \wedge \neg r(c) = F_0$	Contr.
10. $\neg p(c)$	Proof by Contradiction 4-9

how do we bind together?

Quarantena

1. $\forall x\ [p(x) \vee q(x)]$ Premise

2. $\exists x\ \neg p(x)$ Premise

3. $\forall x\ [\neg q(x) \vee r(x)]$ Premise

4. $\forall x\ [s(x) \to \neg r(x)]$ Premise

Conc. $\therefore \exists x\ \neg s(x)$

$\neg \Downarrow$

$\forall x\ s(x)$

5. $\forall x\ \ s(x)$ Assumption — Arbitrary

6. $\neg p(c)$ (for particular c) R. Existential Spec. St 2. — Particular

7. $p(c) \vee q(c)$ R. Un. Spec. St 1

8. $q(c)$ R. Disj. Syll S. 6, 7.

9. $\neg q(c) \vee r(c)$ R. un. sp. S. 3

10. $r(c)$ R. Disj. Syl. S. 8, 9

11. $s(c)$ R. un. sp. S 5

12. $s(c) \to \neg r(c)$ R. un. sp S. 4

13. $\neg r(c)$ M. Pon S 11, 12

14. $r(c) \wedge \neg r(c) = F_0$ R. of Conj. S. 10, 13

15. $\exists x\ \neg s(x)$

Important to remember

$\forall x\, p(x) \longrightarrow p(c)$ for arbitrary c — Universal Spec.

$p(c) \longrightarrow \forall x\, p(x)$ (if c is arbitrary) — Universal Generalization

$\exists x\, p(x) \longrightarrow p(c)$ for particular c — Existential Spec.

$p(c) \longrightarrow \exists x\, p(x)$ (if c is a particular value) — Existential generalization

1. $\forall x \; p(x) \to \neg q(x)$ Premise

Conc. $\therefore \exists x \; q(x) \to r(x)$

2. $p(c)$ part.

3. $p(c) \to \neg q(c)$ (part. c) Un. Sp. S1

4. $\neg q(c)$ MP. 2,3

5. $\neg q(c) \vee r(c)$ R. of disj ampl. S4

6. $q(c) \to r(c)$ Eqv. to S5

7. $\exists x \; q(x) \to r(x)$ R. Ex. gen. S6.

$- \forall x \; \neg q(x)$ for none

$p \to q \Leftrightarrow \exists x \; q(x)$ for some

$\neg p \vee q \quad \forall x \; q(x)$ for all

8.

1 2 Con

$\exists x$ — ∅