GCD

$\gcd(25, 16) = 1$ so 25 and 16 are coprime
relatively prime.

$\gcd(24, 16) = 8$

$\mathbb{Z}$ integers $\mathbb{Z}^-$

min. pos. int. that can be written as $\left| \underline{24x + 16y} \right|$ $= 8$

$\underline{x, y \in \mathbb{Z}}$

0, 8, 16, 24 ✓ are possible

$24x + 16y = 2$ impossible

because
$\cancel{8}(3x) + \cancel{8}(2y) = \cancel{8}k$
$3x + 2y = 1$
$3(+1) + 2(-1) = 1$

LCM

$\gcd(16, 24) = 8$ $\quad$ $\mathrm{lcm}(16, 24) = 48$

$\underset{a}{16} \cdot \underset{b}{24} = \underset{\gcd}{8} \cdot \underset{\mathrm{lcm}}{\cancel{48}}$ 3

gcd is very easy to find.
finding lcm is also easy

$\mathrm{lcm} = \dfrac{a.b}{\gcd}$

Let
$\gcd(a,b) = k$ then let $a = xk$ $b = yk$ such that $\gcd(x,y) = 1$

$\mathrm{lcm}(a,b) = \dfrac{a.b}{\gcd(a,b)} = \dfrac{xyk^2}{k} = xyk$

? Can we find a number $m < xyk$ which is a common multiple of a, b?

$t, u \in \mathbb{Z}^+$

Assume such a number m exists then $xyk = m = x.k.t = y.k.u$

$xt = yu$ $\quad$ $\dfrac{x}{y} = \dfrac{u}{t}$ $\quad u = x$, $t = y$

$\underbrace{56732}_{} = 2 \cdot \_ \cdot \_ \cdots$ unique

$\underline{a} \cdot b\_$

$2 \mid a^2 \quad 2 \mid a \cdot a$

$2 \mid a \vee 2 \mid a$

$\Rightarrow 2 \mid a$ //

Integer factorization problem for RSA numbers

$n = p \cdot q$ $\quad 17 \cdot 23 = \ldots\ldots = 391$ ✓

512 × 512 bit =

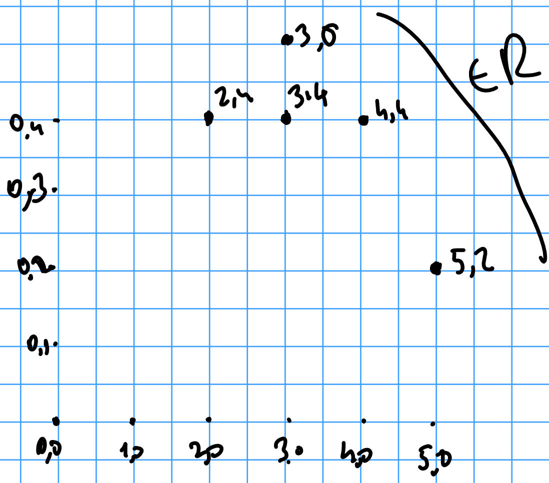
prime × prime

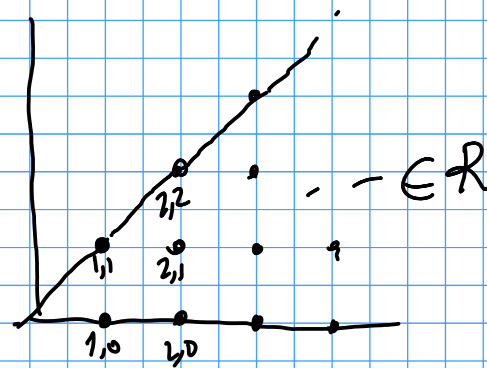
✓ RSA encryption

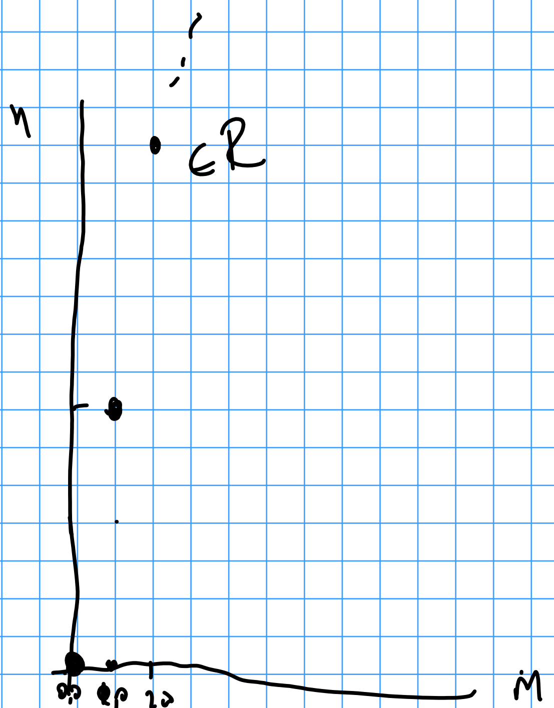
(Q....)

Smooth numbers $\quad 2^5 3^4 \ldots \ldots$ for .

Relation



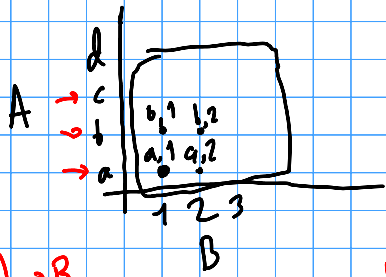




$0,0 \in R$
$1,7 \in R$
$2,14 \in R$
$3,21 \in R$
$\vdots$

Any subset of the Cartesian product is a relation

function

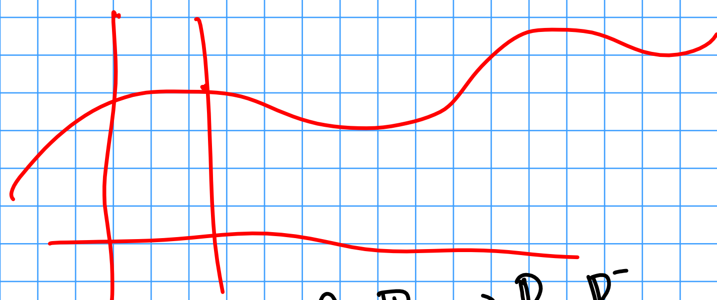


$f: A \to B$

twice $\Rightarrow$ ~~function~~

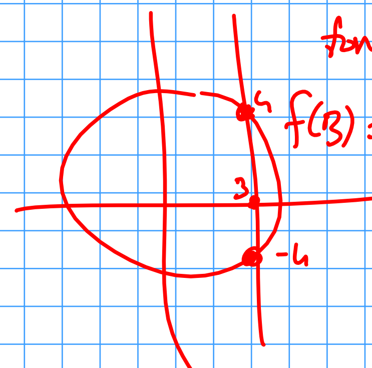
$R = \{(a,1), (a,2), (b,3)\}$

$R = \{(a,3), (b,2), (d,3), (c,1)\}$ is a function



fun

not a function



$f(3) = 4$

-4

$f: \mathbb{R} \to \mathbb{R} - \mathbb{R}^-$

$f(x) = x^2$

$f: \mathbb{R} \to \mathbb{R}$

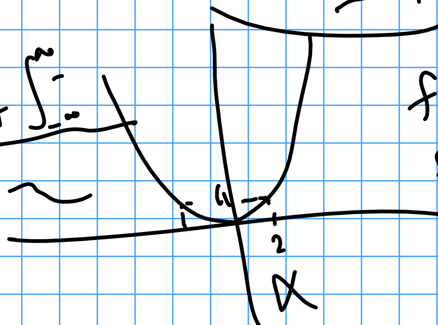
$\mathbb{R}$ domain

$\mathbb{R}$ codomain

$\mathbb{R} \setminus \mathbb{R}^-$ is the range

$f: A \to B$

$\sqrt{x^3 + \int_{-\infty}^{\infty}}$



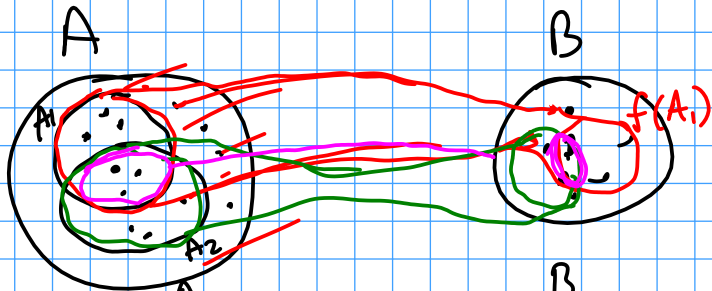
$f(2) = 4$

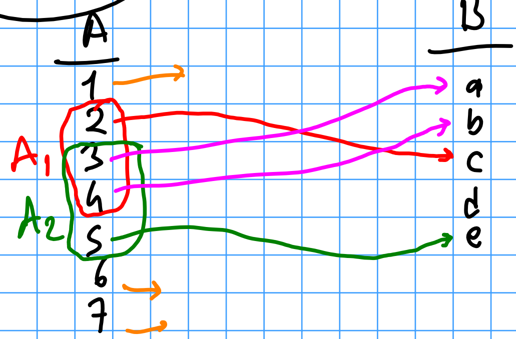
$f(-2) = 4$

$f(x) = -4$

$f(\mathbb{R}) = \mathbb{R} \setminus \mathbb{R}^-$

$= ($non negative real numbers$)$





$f(A_1) = \{a, b, c\}$

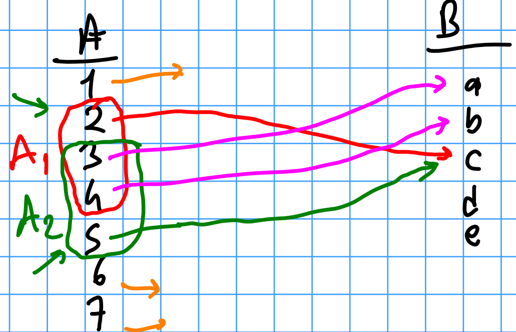
$\cup$

$f(A_2) = \{a, b, e\}$

$\cap$

$f(A_1 \cup A_2) = \{a, b, c, e\}$

$f(A_1 \cap A_2) = \{a, b\}$



$f(A_1) = \{a, b, c\}$

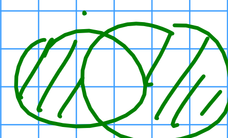
$\cap \quad \{a, b, c\}$

$f(A_2) = \{a, b, c\}$

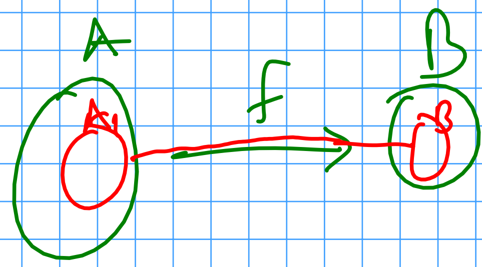
$\not\subseteq$

$f\{A_1 \cap A_2\} = \{a, b\}$

$A \Delta B = (A \cup B) \setminus (A \cap B)$



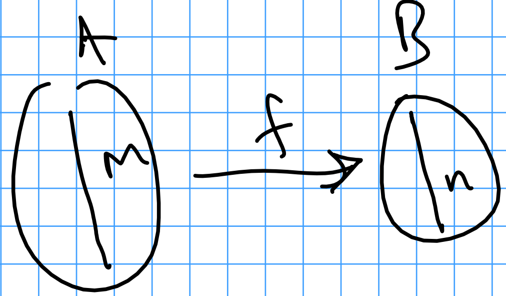
symmetric diff.



$f: \mathbb{C} \to \mathbb{C} \quad f(x) = \sqrt{x}$

$f|_{\mathbb{R}} : \mathbb{R} \to \mathbb{C} \quad f(x) = \sqrt{x}$

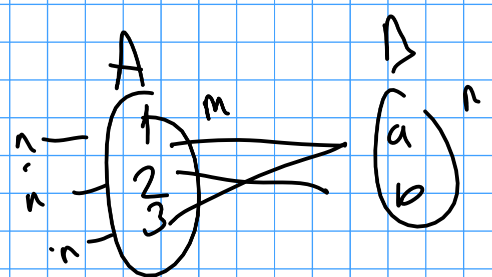
$\sqrt{-5} = \sqrt{5}\,i$



$f: \mathbb{R} \to \mathbb{R} - \mathbb{R}^- \quad f(x) = \sqrt{x}$

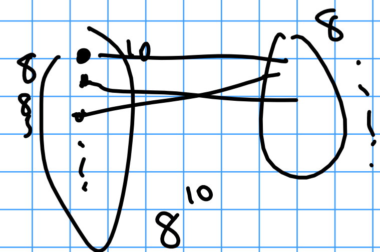
$g(x): \mathbb{C} \to \mathbb{C} \quad g(x) = f(x) = \sqrt{x}$

$2^3 = 8$



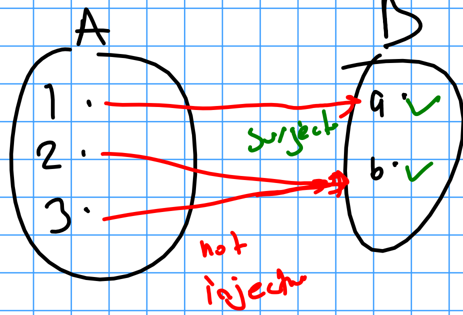
$F = \{ \{(1,a)\ (2,b)\ (3,a)\},\ \{(1,b)\ (2,a)\ (3,a)\},\ \ldots \}$ ✓

n^m



8^{10}

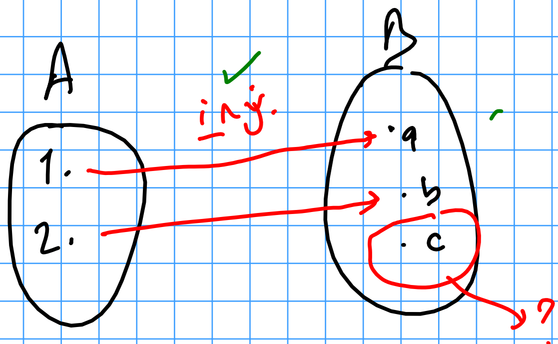
$\overbrace{n \cdot n \cdot n \cdot n}^{m} = n^m \qquad n=2^{m=3} \qquad |B|^{|A|}$



$$f = \{ (1,a), (2,b), (3,b) \}$$

$$f^{-1} = \{ (a,1), (b,2), (b,3) \}$$ not a function

Surjectn
$\wedge$
+ Injectn
bijection



$A \to B$

$$f = \{ (1,a), (2,b) \}$$ function ✓

$B \to A$

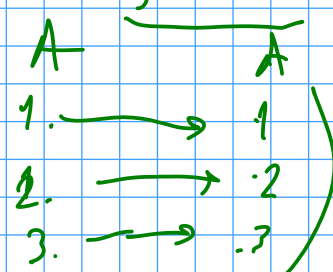
$$f^{-1} = \{ (a,1), (b,2) \}$$ not a fn

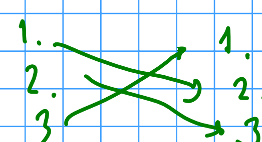
c is not mapped.

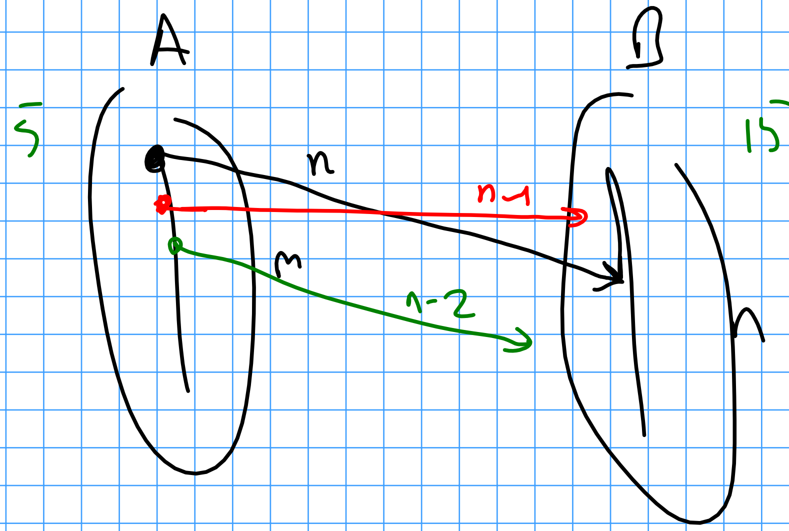
- Surjective → örten (içine olmayan)
- Injective → birebir
- bijective → birebir ve örten.

Identity

$f(x) = x$

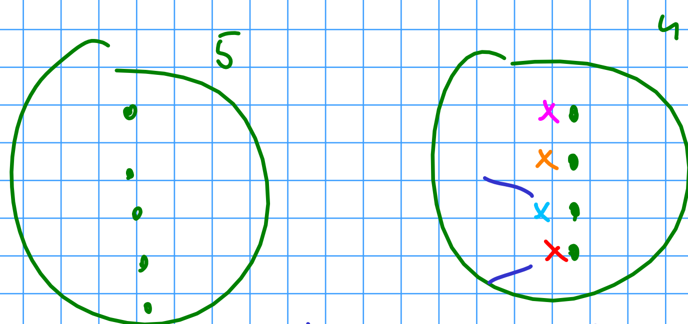






$15 \cdot 14 \cdot 13 \cdot 12 \cdot 11$

$P(15,5)$



$$\overbrace{4^5}^{inc}\ \overbrace{-\,3^5 - 3^5 - 3^5 - 3^5}^{exc}\ + \binom{4}{2}2^5$$

$$\binom{4}{0}4^5 - \binom{4}{1}3^5 + \binom{4}{2}2^5 - \binom{4}{3}1^5 + \binom{4}{4}0^5$$