



# Rook POLYNOMIALS

1	2	3
4	5	6

C

In how many ways can we place  $x$  rooks so that they cannot take each other

$$x=0 \rightarrow 1$$

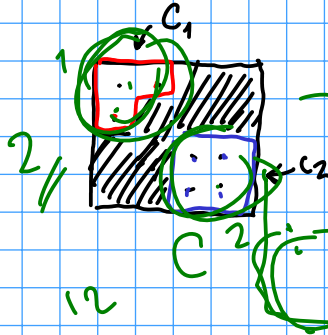
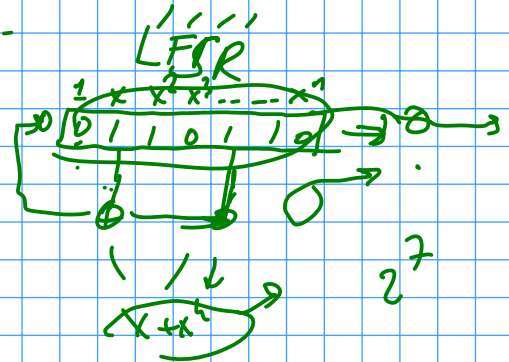
$$x=1 \rightarrow 6$$

$$x=2 \rightarrow 8$$

$$x=3 \rightarrow 2$$

$$r(C, x) = 1 + 6x + 8x^2 + 2x^3$$

$$\begin{pmatrix} 1 & 0 & 1 \\ * & 1 & 1 & 0 \end{pmatrix} (1+x^2)(1+x) \dots$$

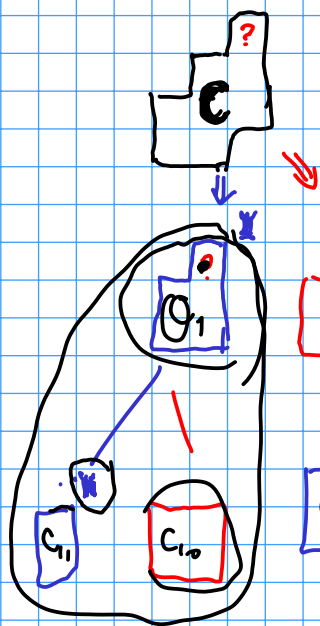


$$r(C_1, x) = 1 + 3x + x^2$$

$$r(C_2, x) = 1 + 4x + 2x^2$$

$$r(C, x) = (1 + 3x + x^2)(1 + 4x + 2x^2)$$

$$= 1 + 7x + 15x^2 + 10x^3 + 2x^4$$



$$P(C, x) = ? \quad 1 + 8x + 16x^2 + \dots$$

$$r(C, x) = x \cdot r(C_1, x) + r(C_0, x)$$

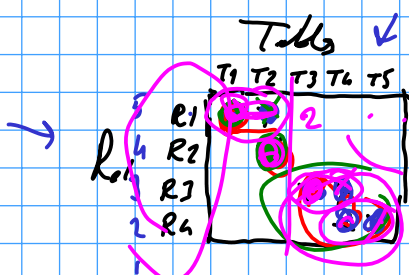
$$\square \quad 1+x$$

$$\square \quad 1+2x$$

$$\square \quad 1+4x+2x^2$$

$$\square \quad 1+3x+x^2$$

$$\begin{aligned} r(C_1, x) &= \underbrace{r(C_{11}, x)} \cdot x + \underbrace{r(C_{10}, x)} \\ &= (1+2x) \cdot x + 1+4x+2x^2 \\ &= x+2x^2 + 1+4x+2x^2 \\ &= 1+5x+4x^2 \end{aligned}$$



Let  $(c_i)$  denote  $R_i$  sits in a forbidden table

$$\bar{N} = S_0 - S_1 + S_2 - S_3 + S_4 \quad \text{by I.E.}$$

$$P(5,4) = \frac{5!}{1!} - \frac{7 \cdot 4!}{1!} + \frac{16 \cdot 3!}{2!} - \frac{13 \cdot 2!}{2!} + \frac{3 \cdot 1!}{1!} \quad \checkmark$$

$$S_0 = P(5,4) = 5!$$

$$S_1 = \frac{N(c_1)}{4! \cdot 1!} + \frac{N(c_2)}{4!} + \frac{N(c_3)}{4! \cdot 1!} + \frac{N(c_4)}{4! \cdot 1!} = 7 \cdot 4!$$

$$S_2 = \frac{N(c_1 c_2)}{3!} + \frac{N(c_1 c_3)}{4 \cdot 3!} + \frac{N(c_1 c_4)}{4 \cdot 3!} + \frac{N(c_2 c_3)}{2 \cdot 3!} + \frac{N(c_2 c_4)}{2 \cdot 3!} + \frac{N(c_3 c_4)}{3 \cdot 3!} = 16 \cdot 3!$$

$$S_3 = N(c_1 c_2 c_3) + N(c_1 c_2 c_4) + N(c_1 c_3 c_4) + N(c_2 c_3 c_4) = 2 \cdot 2! + 2 \cdot 2! + 6 \cdot 2! + 3 \cdot 2!$$

$$S_4 = N(c_1 c_2 c_3 c_4) = 3 \cdot 1! = 3$$