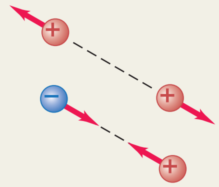
CHAPTER 21 SUMMARY

Electric charge, conductors, and insulators: The fundamental quantity in electrostatics is electric charge. There are two kinds of charge, positive and negative. Charges of the same sign repel each other; charges of opposite sign attract. Charge is conserved; the total charge in an isolated system is constant.

All ordinary matter is made of protons, neutrons, and electrons. The positive protons and electrically neutral neutrons in the nucleus of an atom are bound together by the nuclear force; the negative electrons surround the nucleus at distances much greater than the nuclear size. Electric interactions are chiefly responsible for the structure of atoms, molecules, and solids.

Conductors are materials in which charge moves easily; in insulators, charge does not move easily. Most metals are good conductors; most nonmetals are insulators.

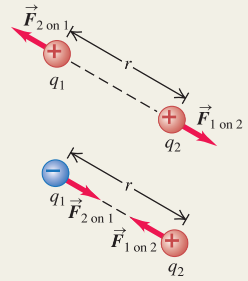


Coulomb's law: For charges q_1 and q_2 separated by a distance r, the magnitude of the electric force on either charge is proportional to the product q_1q_2 and inversely proportional to r^2. The force on each charge is along the line joining the two charges—repulsive if q_1 and q_2 have the same sign, attractive if they have opposite signs. In SI units the unit of electric charge is the coulomb, abbreviated C. (See Examples 21.1 and 21.2.)

When two or more charges each exert a force on a charge, the total force on that charge is the vector sum of the forces exerted by the individual charges. (See Examples 21.3 and 21.4.)

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} \qquad (21.2)$$

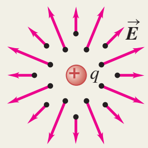
$$\frac{1}{4\pi\epsilon_0} = 8.988 \times 10^9\ \mathrm{N\cdot m^2/C^2}$$



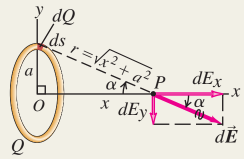
Electric field: Electric field $\vec{E}$, a vector quantity, is the force per unit charge exerted on a test charge at any point. The electric field produced by a point charge is directed radially away from or toward the charge. (See Examples 21.5–21.7.)

$$\vec{E} = \frac{\vec{F}_0}{q_0} \qquad (21.3)$$

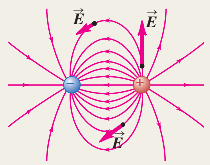
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r} \qquad (21.7)$$



Superposition of electric fields: The electric field $\vec{E}$ of any combination of charges is the vector sum of the fields caused by the individual charges. To calculate the electric field caused by a continuous distribution of charge, divide the distribution into small elements, calculate the field caused by each element, and then carry out the vector sum, usually by integrating. Charge distributions are described by linear charge density λ, surface charge density σ, and volume charge density ρ. (See Examples 21.8–21.12.)



Electric field lines: Field lines provide a graphical representation of electric fields. At any point on a field line, the tangent to the line is in the direction of $\vec{E}$ at that point. The number of lines per unit area (perpendicular to their direction) is proportional to the magnitude of $\vec{E}$ at the point.

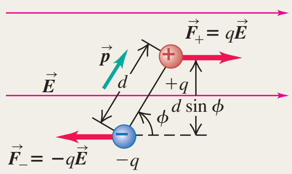


Electric dipoles: An electric dipole is a pair of electric charges of equal magnitude q but opposite sign, separated by a distance d. The electric dipole moment $\vec{p}$ has magnitude $p = qd$. The direction of $\vec{p}$ is from negative toward positive charge. An electric dipole in an electric field $\vec{E}$ experiences a torque $\vec{\tau}$ equal to the vector product of $\vec{p}$ and $\vec{E}$. The magnitude of the torque depends on the angle ϕ between $\vec{p}$ and $\vec{E}$. The potential energy U for an electric dipole in an electric field also depends on the relative orientation of $\vec{p}$ and $\vec{E}$. (See Examples 21.13 and 21.14.)

$$\tau = pE\sin\phi \qquad (21.15)$$

$$\vec{\tau} = \vec{p} \times \vec{E} \qquad (21.16)$$

$$U = -\vec{p}\cdot\vec{E} \qquad (21.18)$$



BRIDGING PROBLEM Calculating Electric Field: Half a Ring of Charge

Positive charge Q is uniformly distributed around a semicircle of radius a as shown in Fig. 21.34. Find the magnitude and direction of the resulting electric field at point P, the center of curvature of the semicircle.

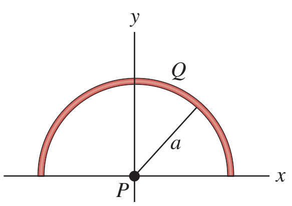
SOLUTION GUIDE

See MasteringPhysics® study area for a Video Tutor solution.

IDENTIFY and SET UP

1. The target variables are the components of the electric field at P.
2. Divide the semicircle into infinitesimal segments, each of which is a short circular arc of radius a and angle $d\theta$. What is the length of such a segment? How much charge is on a segment?

21.34



3. Consider an infinitesimal segment located at an angular position θ on the semicircle, measured from the lower right corner of the semicircle at $x = a$, $y = 0$. (Thus $\theta = \pi/2$ at $x = 0$, $y = a$ and $\theta = \pi$ at $x = -a$, $y = 0$.) What are the x- and y-components of the electric field at P (dE_x and dE_y) produced by just this segment?

EXECUTE

4. Integrate your expressions for dE_x and dE_y from $\theta = 0$ to $\theta = \pi$. The results will be the x-component and y-component of the electric field at P.
5. Use your results from step 4 to find the magnitude and direction of the field at P.

EVALUATE

6. Does your result for the electric-field magnitude have the correct units?
7. Explain how you could have found the x-component of the electric field using a symmetry argument.
8. What would be the electric field at P if the semicircle were extended to a full circle centered at P?

Problems

For instructor-assigned homework, go to www.masteringphysics.com

•, ••, •••: Problems of increasing difficulty. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q21.1 If you peel two strips of transparent tape off the same roll and immediately let them hang near each other, they will repel each other. If you then stick the sticky side of one to the shiny side of the other and rip them apart, they will attract each other. Give a plausible explanation, involving transfer of electrons between the strips of tape, for this sequence of events.

Q21.2 Two metal spheres are hanging from nylon threads. When you bring the spheres close to each other, they tend to attract. Based on this information alone, discuss all the possible ways that the spheres could be charged. Is it possible that after the spheres touch, they will cling together? Explain.

Q21.3 The electric force between two charged particles becomes weaker with increasing distance. Suppose instead that the electric force were *independent* of distance. In this case, would a charged comb still cause a neutral insulator to become polarized as in Fig. 21.8? Why or why not? Would the neutral insulator still be attracted to the comb? Again, why or why not?

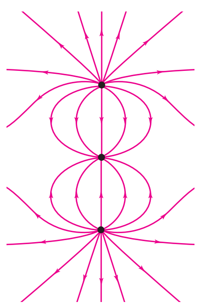
Q21.4 Your clothing tends to cling together after going through the dryer. Why? Would you expect more or less clinging if all your clothing were made of the same material (say, cotton) than if you dried different kinds of clothing together? Again, why? (You may want to experiment with your next load of laundry.)

Q21.5 An uncharged metal sphere hangs from a nylon thread. When a positively charged glass rod is brought close to the metal sphere, the sphere is drawn toward the rod. But if the sphere touches the rod, it suddenly flies away from the rod. Explain why the sphere is first attracted and then repelled.

Q21.6 The free electrons in a metal are gravitationally attracted toward the earth. Why, then, don't they all settle to the bottom of the conductor, like sediment settling to the bottom of a river?

Q21.7 • Figure Q21.7 shows some of the electric field lines due to three point charges arranged along the vertical axis. All three charges have the same magnitude. (a) What are the signs of the three charges? Explain your reasoning. (b) At what point(s) is the magnitude of the electric field the smallest? Explain your reasoning. Explain how the fields produced by each individual point charge combine to give a small net field at this point or points.

Figure **Q21.7**



Q21.8 Good electrical conductors, such as metals, are typically good conductors of heat; electrical insulators, such as wood, are typically poor conductors of heat. Explain why there should be a relationship between electrical conduction and heat conduction in these materials.

Q21.9 • Suppose the charge shown in Fig. 21.28a is fixed in position. A small, positively charged particle is then placed at some point in the figure and released. Will the trajectory of the particle follow an electric field line? Why or why not? Suppose instead that the particle is placed at some point in Fig. 21.28b and released (the positive and negative charges shown in the figure are fixed in position). Will its trajectory follow an electric field line? Again, why or why not? Explain any differences between your answers for the two different situations.

Q21.10 Two identical metal objects are mounted on insulating stands. Describe how you could place charges of opposite sign but exactly equal magnitude on the two objects.

Q21.11 You can use plastic food wrap to cover a container by stretching the material across the top and pressing the overhanging material against the sides. What makes it stick? (*Hint:* The answer involves the electric force.) Does the food wrap stick to itself with equal tenacity? Why or why not? Does it work with metallic containers? Again, why or why not?

Q21.12 If you walk across a nylon rug and then touch a large metal object such as a doorknob, you may get a spark and a shock. Why does this tend to happen more on dry days than on humid days? (*Hint:* See Fig. 21.30.) Why are you less likely to get a shock if you touch a *small* metal object, such as a paper clip?

Q21.13 You have a negatively charged object. How can you use it to place a net negative charge on an insulated metal sphere? To place a net positive charge on the sphere?

Q21.14 When two point charges of equal mass and charge are released on a frictionless table, each has an initial acceleration a_0. If instead you keep one fixed and release the other one, what will be its initial acceleration: a_0, $2a_0$, or $a_0/2$? Explain.

Q21.15 A point charge of mass m and charge Q and another point charge of mass m but charge $2Q$ are released on a frictionless table. If the charge Q has an initial acceleration a_0, what will be the acceleration of $2Q$: a_0, $2a_0$, $4a_0$, $a_0/2$, or $a_0/4$? Explain.

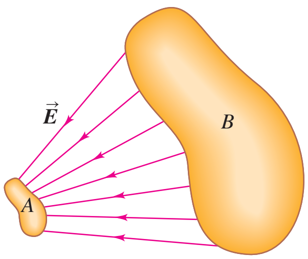
Q21.16 A proton is placed in a uniform electric field and then released. Then an electron is placed at this same point and released. Do these two particles experience the same force? The same acceleration? Do they move in the same direction when released?

Q21.17 In Example 21.1 (Section 21.3) we saw that the electric force between two α particles is of the order of 10^{35} times as strong as the gravitational force. So why do we readily feel the gravity of the earth but no electrical force from it?

Q21.18 What similarities do electrical forces have with gravitational forces? What are the most significant differences?

Q21.19 Two irregular objects A and B carry charges of opposite sign. Figure Q21.19 shows the electric field lines near each of these objects. (a) Which object is positive, A or B? How do you know? (b) Where is the electric field stronger, close to A or close to B? How do you know?

Figure **Q21.19**

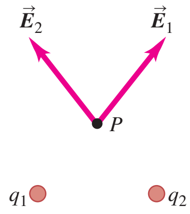


Q21.20 Atomic nuclei are made of protons and neutrons. This shows that there must be another kind of interaction in addition to gravitational and electric forces. Explain.

Q21.21 Sufficiently strong electric fields can cause atoms to become positively ionized—that is, to lose one or more electrons. Explain how this can happen. What determines how strong the field must be to make this happen?

Q21.22 The electric fields at point P due to the positive charges q_1 and q_2 are shown in Fig. Q21.22. Does the fact that they cross each other violate the statement in Section 21.6 that electric field lines never cross? Explain.

Figure **Q21.22**



Q21.23 The air temperature and the velocity of the air have different values at different places in the earth's atmosphere. Is the air velocity a vector field? Why or why not? Is the air temperature a vector field? Again, why or why not?

EXERCISES

Section 21.3 Coulomb's Law

21.1 •• Excess electrons are placed on a small lead sphere with mass 8.00 g so that its net charge is -3.20×10^{-9} C. (a) Find the number of excess electrons on the sphere. (b) How many excess electrons are there per lead atom? The atomic number of lead is 82, and its atomic mass is 207 g/mol.

21.2 • Lightning occurs when there is a flow of electric charge (principally electrons) between the ground and a thundercloud. The maximum rate of charge flow in a lightning bolt is about 20,000 C/s; this lasts for 100 μs or less. How much charge flows between the ground and the cloud in this time? How many electrons flow during this time?

21.3 •• **BIO** Estimate how many electrons there are in your body. Make any assumptions you feel are necessary, but clearly state what they are. (*Hint:* Most of the atoms in your body have equal numbers of electrons, protons, and neutrons.) What is the combined charge of all these electrons?

21.4 • **Particles in a Gold Ring.** You have a pure (24-karat) gold ring with mass 17.7 g. Gold has an atomic mass of 197 g/mol and an atomic number of 79. (a) How many protons are in the ring, and what is their total positive charge? (b) If the ring carries no net charge, how many electrons are in it?

21.5 • **BIO** **Signal Propagation in Neurons.** *Neurons* are components of the nervous system of the body that transmit signals as electrical impulses travel along their length. These impulses propagate when charge suddenly rushes into and then out of a part of the neuron called an *axon.* Measurements have shown that, during the inflow part of this cycle, approximately 5.6×10^{11} Na^+ (sodium ions) per meter, each with charge $+e$, enter the axon. How many coulombs of charge enter a 1.5-cm length of the axon during this process?

21.6 • Two small spheres spaced 20.0 cm apart have equal charge. How many excess electrons must be present on each sphere if the magnitude of the force of repulsion between them is 4.57×10^{-21} N?

21.7 •• An average human weighs about 650 N. If two such generic humans each carried 1.0 coulomb of excess charge, one positive and one negative, how far apart would they have to be for the electric attraction between them to equal their 650-N weight?

21.8 •• Two small aluminum spheres, each having mass 0.0250 kg, are separated by 80.0 cm. (a) How many electrons does each sphere contain? (The atomic mass of aluminum is 26.982 g/mol, and its atomic number is 13.) (b) How many electrons would have to be removed from one sphere and added to the other to cause an attractive force between the spheres of magnitude 1.00×10^4 N (roughly 1 ton)? Assume that the spheres may be treated as point charges. (c) What fraction of all the electrons in each sphere does this represent?

21.9 •• Two small plastic spheres are given positive electrical charges. When they are 15.0 cm apart, the repulsive force between them has magnitude 0.220 N. What is the charge on each sphere (a) if the two charges are equal and (b) if one sphere has four times the charge of the other?

21.10 •• **What If We Were Not Neutral?** A 75-kg person holds out his arms so that his hands are 1.7 m apart. Typically, a person's hand makes up about 1.0% of his or her body weight. For round numbers, we shall assume that all the weight of each hand is due to the calcium in the bones, and we shall treat the hands as point charges. One mole of Ca contains 40.18 g, and each atom has 20 protons and 20 electrons. Suppose that only 1.0% of the positive charges in each hand were unbalanced by negative charge. (a) How many Ca atoms does each hand contain? (b) How many coulombs of unbalanced charge does each hand contain? (c) What force would the person's arms have to exert on his hands to prevent them from flying off? Does it seem likely that his arms are capable of exerting such a force?

21.11 •• Two very small 8.55-g spheres, 15.0 cm apart from center to center, are charged by adding equal numbers of electrons to each of them. Disregarding all other forces, how many electrons would you have to add to each sphere so that the two spheres will accelerate at 25.0*g* when released? Which way will they accelerate?

21.12 •• **Just How Strong Is the Electric Force?** Suppose you had two small boxes, each containing 1.0 g of protons. (a) If one were placed on the moon by an astronaut and the other were left on the earth, and if they were connected by a very light (and very long!) string, what would be the tension in the string? Express your answer in newtons and in pounds. Do you need to take into account the gravitational forces of the earth and moon on the protons? Why? (b) What gravitational force would each box of protons exert on the other box?

21.13 • In an experiment in space, one proton is held fixed and another proton is released from rest a distance of 2.50 mm away. (a) What is the initial acceleration of the proton after it is released? (b) Sketch qualitative (no numbers!) acceleration–time and velocity–time graphs of the released proton's motion.

21.14 • A negative charge of $-0.550\ \mu C$ exerts an upward 0.200-N force on an unknown charge 0.300 m directly below it. (a) What is the unknown charge (magnitude and sign)? (b) What are the magnitude and direction of the force that the unknown charge exerts on the -0.550-μC charge?

21.15 •• Three point charges are arranged on a line. Charge $q_3 = +5.00$ nC and is at the origin. Charge $q_2 = -3.00$ nC and is at $x = +4.00$ cm. Charge q_1 is at $x = +2.00$ cm. What is q_1 (magnitude and sign) if the net force on q_3 is zero?

21.16 •• In Example 21.4, suppose the point charge on the *y*-axis at $y = -0.30$ m has negative charge $-2.0\ \mu C$, and the other charges remain the same. Find the magnitude and direction of the net force on Q. How does your answer differ from that in Example 21.4? Explain the differences.

21.17 •• In Example 21.3, calculate the net force on charge q_1.

21.18 •• In Example 21.4, what is the net force (magnitude and direction) on charge q_1 exerted by the other two charges?

21.19 •• Three point charges are arranged along the *x*-axis. Charge $q_1 = +3.00\ \mu C$ is at the origin, and charge $q_2 = -5.00\ \mu C$ is at $x = 0.200$ m. Charge $q_3 = -8.00\ \mu C$. Where is q_3 located if the net force on q_1 is 7.00 N in the $-x$-direction?

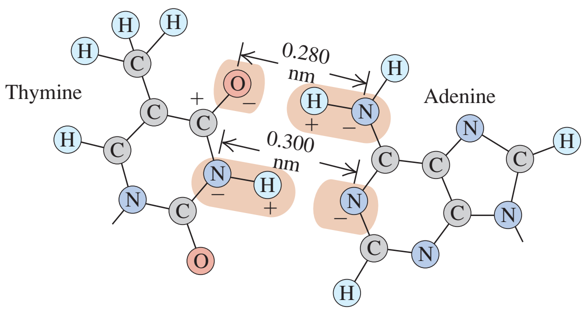
21.20 •• Repeat Exercise 21.19 for $q_3 = +8.00\ \mu C$.

21.21 •• Two point charges are located on the *y*-axis as follows: charge $q_1 = -1.50$ nC at $y = -0.600$ m, and charge $q_2 = +3.20$ nC at the origin ($y = 0$). What is the total force (magnitude and direction) exerted by these two charges on a third charge $q_3 = +5.00$ nC located at $y = -0.400$ m?

21.22 •• Two point charges are placed on the *x*-axis as follows: Charge $q_1 = +4.00$ nC is located at $x = 0.200$ m, and charge $q_2 = +5.00$ nC is at $x = -0.300$ m. What are the magnitude and direction of the total force exerted by these two charges on a negative point charge $q_3 = -6.00$ nC that is placed at the origin?

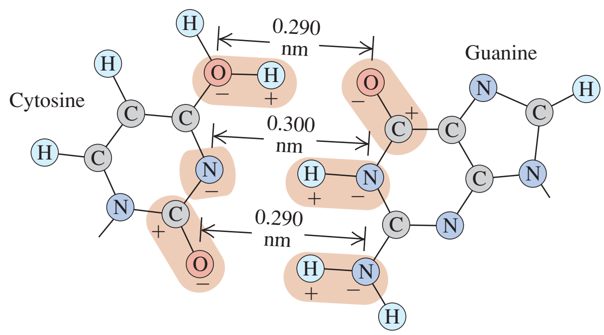
21.23 •• **BIO Base Pairing in DNA, I.** The two sides of the DNA double helix are connected by pairs of bases (adenine, thymine, cytosine, and guanine). Because of the geometric shape of these molecules, adenine bonds with thymine and cytosine bonds with guanine. Figure E21.23 shows the thymine–adenine bond. Each charge shown is $\pm e$, and the H—N distance is 0.110 nm. (a) Calculate the *net* force that thymine exerts on adenine. Is it attractive or repulsive? To keep the calculations fairly simple, yet reasonable, consider only the forces due to the O—H—N and the N—H—N combinations, assuming that these two combinations are parallel to each other. Remember, however, that in the O—H—N set, the O^- exerts a force on both the H^+ and the N^-, and likewise along the N—H—N set. (b) Calculate the force on the electron in the hydrogen atom, which is 0.0529 nm from the proton. Then compare the strength of the bonding force of the electron in hydrogen with the bonding force of the adenine–thymine molecules.

Figure **E21.23**



21.24 •• **BIO Base Pairing in DNA, II.** Refer to Exercise 21.23. Figure E21.24 shows the bonding of the cytosine and guanine molecules. The O—H and H—N distances are each 0.110 nm. In this case, assume that the bonding is due only to the forces along the O—H—O, N—H—N, and O—H—N combinations, and assume also that these three combinations are parallel to each other. Calculate the *net* force that cytosine exerts on guanine due to the preceding three combinations. Is this force attractive or repulsive?

Figure **E21.24**



Section 21.4 Electric Field and Electric Forces

21.25 • **CP** A proton is placed in a uniform electric field of 2.75×10^3 N/C. Calculate: (a) the magnitude of the electric force felt by the proton; (b) the proton's acceleration; (c) the proton's speed after 1.00 μs in the field, assuming it starts from rest.

21.26 • A particle has charge -3.00 nC. (a) Find the magnitude and direction of the electric field due to this particle at a point 0.250 m directly above it. (b) At what distance from this particle does its electric field have a magnitude of 12.0 N/C?

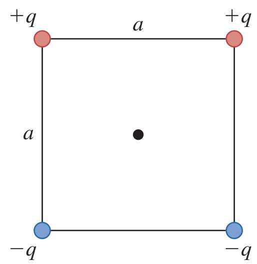
21.27 • **CP** A proton is traveling horizontally to the right at 4.50×10^6 m/s. (a) Find the magnitude and direction of the weakest electric field that can bring the proton uniformly to rest over a distance of 3.20 cm. (b) How much time does it take the proton to stop after entering the field? (c) What minimum field (magnitude and direction) would be needed to stop an electron under the conditions of part (a)?

21.28 • **CP** An electron is released from rest in a uniform electric field. The electron accelerates vertically upward, traveling 4.50 m in the first 3.00 μs after it is released. (a) What are the magnitude and direction of the electric field? (b) Are we justified in ignoring the effects of gravity? Justify your answer quantitatively.

21.29 •• (a) What must the charge (sign and magnitude) of a 1.45-g particle be for it to remain stationary when placed in a downward-directed electric field of magnitude 650 N/C? (b) What is the magnitude of an electric field in which the electric force on a proton is equal in magnitude to its weight?

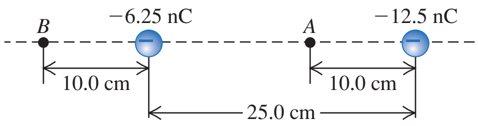
21.30 •• A point charge is placed at each corner of a square with side length a. The charges all have the same magnitude q. Two of the charges are positive and two are negative, as shown in Fig. E21.30. What is the direction of the net electric field at the center of the square due to the four charges, and what is its magnitude in terms of q and a?

Figure **E21.30**



21.31 • Two point charges are separated by 25.0 cm (Fig. E21.31). Find the net electric field these charges produce at (a) point A and (b) point B. (c) What would be the magnitude and direction of the electric force this combination of charges would produce on a proton at A?

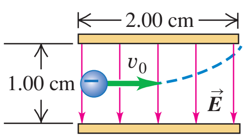
Figure **E21.31**



21.32 •• **Electric Field of the Earth.** The earth has a net electric charge that causes a field at points near its surface equal to 150 N/C and directed in toward the center of the earth. (a) What magnitude and sign of charge would a 60-kg human have to acquire to overcome his or her weight by the force exerted by the earth's electric field? (b) What would be the force of repulsion between two people each with the charge calculated in part (a) and separated by a distance of 100 m? Is use of the earth's electric field a feasible means of flight? Why or why not?

21.33 •• **CP** An electron is projected with an initial speed $v_0 = 1.60 \times 10^6$ m/s into the uniform field between the parallel plates in Fig. E21.33. Assume that the field between the plates is uniform and directed vertically downward, and that the field outside the plates is zero. The electron enters the field at a point midway between the plates. (a) If the electron just misses the upper plate as it emerges from the field, find the magnitude of the electric field. (b) Suppose that in Fig. E21.33 the electron is replaced by a proton with the same initial speed v_0. Would the proton hit one of the plates? If the proton would not hit one of the plates, what would be the magnitude and direction of its vertical displacement as it exits the region between the plates? (c) Compare the paths traveled by the electron and the proton and explain the differences. (d) Discuss whether it is reasonable to ignore the effects of gravity for each particle.

Figure **E21.33**



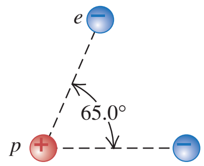
21.34 •• Point charge $q_1 = -5.00$ nC is at the origin and point charge $q_2 = +3.00$ nC is on the x-axis at $x = 3.00$ cm. Point P is on the y-axis at $y = 4.00$ cm. (a) Calculate the electric fields $\vec{E}_1$ and $\vec{E}_2$ at point P due to the charges q_1 and q_2. Express your results in terms of unit vectors (see Example 21.6). (b) Use the results of part (a) to obtain the resultant field at P, expressed in unit vector form.

21.35 •• **CP** In Exercise 21.33, what is the speed of the electron as it emerges from the field?

21.36 • (a) Calculate the magnitude and direction (relative to the $+x$-axis) of the electric field in Example 21.6. (b) A -2.5-nC point charge is placed at point P in Fig. 21.19. Find the magnitude and direction of (i) the force that the -8.0-nC charge at the origin exerts on this charge and (ii) the force that this charge exerts on the -8.0-nC charge at the origin.

21.37 •• If two electrons are each 1.50×10^{-10} m from a proton, as shown in Fig. E21.37, find the magnitude and direction of the net electric force they will exert on the proton.

Figure **E21.37**

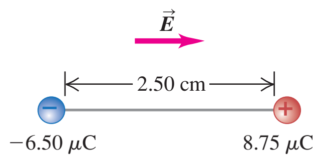


21.38 •• **CP** A uniform electric field exists in the region between two oppositely charged plane parallel plates. A proton is released from rest at the surface of the positively charged plate and strikes the surface of the opposite plate, 1.60 cm distant from the first, in a time interval of 1.50×10^{-6} s. (a) Find the magnitude of the electric field. (b) Find the speed of the proton when it strikes the negatively charged plate.

21.39 • A point charge is at the origin. With this point charge as the source point, what is the unit vector $\hat{r}$ in the direction of (a) the field point at $x = 0$, $y = -1.35$ m; (b) the field point at $x = 12.0$ cm, $y = 12.0$ cm; (c) the field point at $x = -1.10$ m, $y = 2.60$ m? Express your results in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

21.40 •• A $+8.75$-μC point charge is glued down on a horizontal frictionless table. It is tied to a -6.50-μC point charge by a light, nonconducting 2.50-cm wire. A uniform electric field of magnitude 1.85×10^8 N/C is directed parallel to the wire, as shown in Fig. E21.40. (a) Find the tension in the wire. (b) What would the tension be if both charges were negative?

Figure **E21.40**

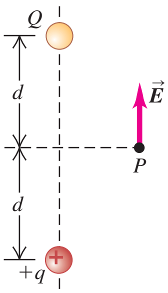


21.41 •• (a) An electron is moving east in a uniform electric field of 1.50 N/C directed to the west. At point A, the velocity of the electron is 4.50×10^5 m/s toward the east. What is the speed of the electron when it reaches point B, 0.375 m east of point A? (b) A proton is moving in the uniform electric field of part (a). At point A, the velocity of the proton is 1.90×10^4 m/s, east. What is the speed of the proton at point B?

Section 21.5 Electric-Field Calculations

21.42 • Two point charges Q and $+q$ (where q is positive) produce the net electric field shown at point P in Fig. E21.42. The field points parallel to the line connecting the two charges. (a) What can you conclude about the sign and magnitude of Q? Explain your reasoning. (b) If the lower charge were negative instead, would it be possible for the field to have the direction shown in the figure? Explain your reasoning.

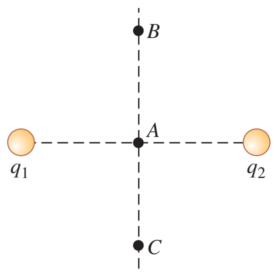
Figure **E21.42**



21.43 •• Two positive point charges q are placed on the x-axis, one at $x = a$ and one at $x = -a$. (a) Find the magnitude and direction of the electric field at $x = 0$. (b) Derive an expression for the electric field at points on the x-axis. Use your result to graph the x-component of the electric field as a function of x, for values of x between $-4a$ and $+4a$.

21.44 • The two charges q_1 and q_2 shown in Fig. E21.44 have equal magnitudes. What is the direction of the net electric field due to these two charges at points A (midway between the charges), B, and C if (a) both charges are negative, (b) both charges are positive, (c) q_1 is positive and q_2 is negative.

Figure **E21.44**

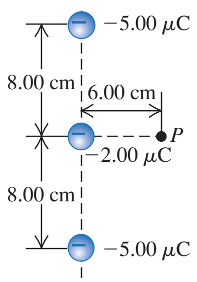


21.45 • A $+2.00$-nC point charge is at the origin, and a second -5.00-nC point charge is on the x-axis at $x = 0.800$ m. (a) Find the electric field (magnitude and direction) at each of the following points on the x-axis: (i) $x = 0.200$ m; (ii) $x = 1.20$ m; (iii) $x = -0.200$ m. (b) Find the net electric force that the two charges would exert on an electron placed at each point in part (a).

21.46 •• Repeat Exercise 21.44, but now let $q_1 = -4.00$ nC.

21.47 • Three negative point charges lie along a line as shown in Fig. E21.47. Find the magnitude and direction of the electric field this combination of charges produces at point P, which lies 6.00 cm from the -2.00-μC charge measured perpendicular to the line connecting the three charges.

Figure **E21.47**



21.48 •• **BIO** **Electric Field of Axons.** A nerve signal is transmitted through a neuron when an excess of Na^+ ions suddenly enters the axon, a long cylindrical part of the neuron. Axons are approximately 10.0 μm in diameter, and measurements show that about 5.6×10^{11} Na^+ ions per meter (each of charge $+e$) enter during this process. Although the axon is a long cylinder, the charge does not all enter everywhere at the same time. A plausible model would be a series of point charges moving along the axon. Let us look at a 0.10-mm length of the axon and model it as a point charge. (a) If the charge that enters each meter of the axon gets distributed uniformly along it, how many coulombs of charge enter a 0.10-mm length of the axon? (b) What electric field (magnitude and direction) does the sudden influx of charge produce at the surface of the body if the axon is 5.00 cm below the skin? (c) Certain sharks can respond to electric fields as weak as 1.0 μN/C. How far from this segment of axon could a shark be and still detect its electric field?

21.49 • In a rectangular coordinate system a positive point charge $q = 6.00 \times 10^{-9}$ C is placed at the point $x = +0.150$ m, $y = 0$, and an identical point charge is placed at $x = -0.150$ m, $y = 0$. Find the x- and y-components, the magnitude, and the direction of the electric field at the following points: (a) the origin; (b) $x = 0.300$ m, $y = 0$; (c) $x = 0.150$ m, $y = -0.400$ m; (d) $x = 0$, $y = 0.200$ m.

21.50 •• A point charge $q_1 = -4.00$ nC is at the point $x = 0.600$ m, $y = 0.800$ m, and a second point charge $q_2 = +6.00$ nC is at the point $x = 0.600$ m, $y = 0$. Calculate the magnitude and direction of the net electric field at the origin due to these two point charges.

21.51 •• Repeat Exercise 21.49 for the case where the point charge at $x = +0.150$ m, $y = 0$ is positive and the other is negative, each with magnitude 6.00×10^{-9} C.

21.52 •• A very long, straight wire has charge per unit length 1.50×10^{-10} C/m. At what distance from the wire is the electric-field magnitude equal to 2.50 N/C?

21.53 • A ring-shaped conductor with radius $a = 2.50$ cm has a total positive charge $Q = +0.125$ nC uniformly distributed around it, as shown in Fig. 21.23. The center of the ring is at the origin of coordinates O. (a) What is the electric field (magnitude and direction) at point P, which is on the x-axis at $x = 40.0$ cm? (b) A point charge $q = -2.50$ μC is placed at the point P described in part (a). What are the magnitude and direction of the force exerted *by* the charge q *on* the ring?

21.54 •• A straight, nonconducting plastic wire 8.50 cm long carries a charge density of $+175$ nC/m distributed uniformly along its length. It is lying on a horizontal tabletop. (a) Find the magnitude and direction of the electric field this wire produces at a point 6.00 cm directly above its midpoint. (b) If the wire is now bent into a circle lying flat on the table, find the magnitude and direction of the electric field it produces at a point 6.00 cm directly above its center.

21.55 •• A charge of -6.50 nC is spread uniformly over the surface of one face of a nonconducting disk of radius 1.25 cm. (a) Find the magnitude and direction of the electric field this disk produces at a point P on the axis of the disk a distance of 2.00 cm from its center. (b) Suppose that the charge were all pushed away from the center and distributed uniformly on the outer rim of the disk. Find the magnitude and direction of the electric field at point P. (c) If the charge is all brought to the center of the disk, find the magnitude and direction of the electric field at point P. (d) Why is the field in part (a) stronger than the field in part (b)? Why is the field in part (c) the strongest of the three fields?

Section 21.7 Electric Dipoles

21.56 • The ammonia molecule (NH_3) has a dipole moment of 5.0×10^{-30} C·m. Ammonia molecules in the gas phase are placed in a uniform electric field $\vec{E}$ with magnitude 1.6×10^6 N/C. (a) What is the change in electric potential energy when the dipole moment of a molecule changes its orientation with respect to $\vec{E}$ from parallel to perpendicular? (b) At what absolute temperature T is the average translational kinetic energy $\frac{3}{2}kT$ of a molecule equal to the change in potential energy calculated in part (a)? (*Note:* Above this temperature, thermal agitation prevents the dipoles from aligning with the electric field.)

21.57 • Point charges $q_1 = -4.5$ nC and $q_2 = +4.5$ nC are separated by 3.1 mm, forming an electric dipole. (a) Find the electric dipole moment (magnitude and direction). (b) The charges are in a uniform electric field whose direction makes an angle of 36.9° with the line connecting the charges. What is the magnitude of this field if the torque exerted on the dipole has magnitude 7.2×10^{-9} N · m?

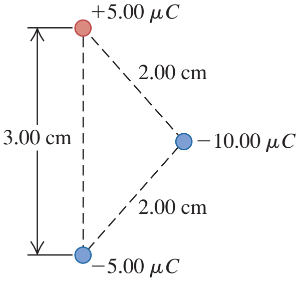
21.58 • The dipole moment of the water molecule (H_2O) is 6.17×10^{-30} C · m. Consider a water molecule located at the origin whose dipole moment $\vec{p}$ points in the +x-direction. A chlorine ion (Cl^-), of charge -1.60×10^{-19} C, is located at $x = 3.00 \times 10^{-9}$ m. Find the magnitude and direction of the electric force that the water molecule exerts on the chlorine ion. Is this force attractive or repulsive? Assume that x is much larger than the separation d between the charges in the dipole, so that the approximate expression for the electric field along the dipole axis derived in Example 21.14 can be used.

21.59 • **Torque on a Dipole.** An electric dipole with dipole moment $\vec{p}$ is in a uniform electric field $\vec{E}$. (a) Find the orientations of the dipole for which the torque on the dipole is zero. (b) Which of the orientations in part (a) is stable, and which is unstable? (*Hint:* Consider a small displacement away from the equilibrium position and see what happens.) (c) Show that for the stable orientation in part (b), the dipole's own electric field tends to oppose the external field.

21.60 •• Consider the electric dipole of Example 21.14. (a) Derive an expression for the magnitude of the electric field produced by the dipole at a point on the x-axis in Fig. 21.33. What is the direction of this electric field? (b) How does the electric field at points on the x-axis depend on x when x is very large?

21.61 • Three charges are at the corners of an isosceles triangle as shown in Fig. E21.61. The ± 5.00-μC charges form a dipole. (a) Find the force (magnitude and direction) the -10.00-μC charge exerts on the dipole. (b) For an axis perpendicular to the line connecting the ± 5.00-μC charges at the midpoint of this line, find the torque (magnitude and direction) exerted on the dipole by the -10.00-μC charge.

Figure **E21.61**



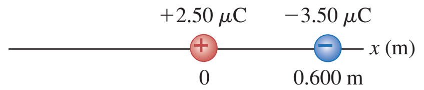
21.62 • A dipole consisting of charges $\pm e$, 220 nm apart, is placed between two very large (essentially infinite) sheets carrying equal but opposite charge densities of 125 $\mu C/m^2$. (a) What is the maximum potential energy this dipole can have due to the sheets, and how should it be oriented relative to the sheets to attain this value? (b) What is the maximum torque the sheets can exert on the dipole, and how should it be oriented relative to the sheets to attain this value? (c) What net force do the two sheets exert on the dipole?

PROBLEMS

21.63 ••• Four identical charges Q are placed at the corners of a square of side L. (a) In a free-body diagram, show all of the forces that act on one of the charges. (b) Find the magnitude and direction of the total force exerted on one charge by the other three charges.

21.64 ••• Two charges, one of 2.50 μC and the other of -3.50 μC, are placed on the x-axis, one at the origin and the other at $x = 0.600$ m, as shown in Fig. P21.64. Find the position on the x-axis where the net force on a small charge $+q$ would be zero.

Figure **P21.64**



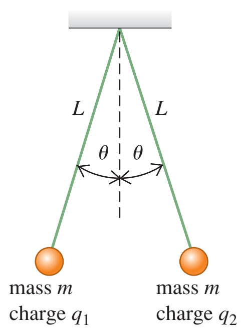
21.65 •• Three point charges are arranged along the x-axis. Charge $q_1 = -4.50$ nC is located at $x = 0.200$ m, and charge $q_2 = +2.50$ nC is at $x = -0.300$ m. A positive point charge q_3 is located at the origin. (a) What must the value of q_3 be for the net force on this point charge to have magnitude 4.00 μN? (b) What is the direction of the net force on q_3? (c) Where along the x-axis can q_3 be placed and the net force on it be zero, other than the trivial answers of $x = +\infty$ and $x = -\infty$?

21.66 •• A charge $q_1 = +5.00$ nC is placed at the origin of an xy-coordinate system, and a charge $q_2 = -2.00$ nC is placed on the positive x-axis at $x = 4.00$ cm. (a) If a third charge $q_3 = +6.00$ nC is now placed at the point $x = 4.00$ cm, $y = 3.00$ cm, find the x- and y-components of the total force exerted on this charge by the other two. (b) Find the magnitude and direction of this force.

21.67 •• **CP** Two positive point charges Q are held fixed on the x-axis at $x = a$ and $x = -a$. A third positive point charge q, with mass m, is placed on the x-axis away from the origin at a coordinate x such that $|x| \ll a$. The charge q, which is free to move along the x-axis, is then released. (a) Find the frequency of oscillation of the charge q. (*Hint:* Review the definition of simple harmonic motion in Section 14.2. Use the binomial expansion $(1 + z)^n = 1 + nz + n(n-1)z^2/2 + \cdots$, valid for the case $|z| < 1$.) (b) Suppose instead that the charge q were placed on the y-axis at a coordinate y such that $|y| \ll a$, and then released. If this charge is free to move anywhere in the xy-plane, what will happen to it? Explain your answer.

21.68 •• **CP** Two identical spheres with mass m are hung from silk threads of length L, as shown in Fig. P21.68. Each sphere has the same charge, so $q_1 = q_2 = q$. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. Show that if the angle θ is small, the equilibrium separation d between the spheres is $d = (q^2L/2\pi\epsilon_0 mg)^{1/3}$. (*Hint:* If θ is small, then $\tan\theta \cong \sin\theta$.)

Figure **P21.68**



21.69 ••• **CP** Two small spheres with mass $m = 15.0$ g are hung by silk threads of length $L = 1.20$ m from a common point (Fig. P21.68). When the spheres are given equal quantities of negative charge, so that $q_1 = q_2 = q$, each thread hangs at $\theta = 25.0°$ from the vertical. (a) Draw a diagram showing the forces on each sphere. Treat the spheres as point charges. (b) Find the magnitude of q. (c) Both threads are now shortened to length $L = 0.600$ m, while the charges q_1 and q_2 remain unchanged. What new angle will each thread make with the vertical? (*Hint:* This part of the problem can be solved numerically by using trial values for θ and adjusting the values of θ until a self-consistent answer is obtained.)

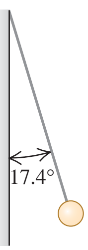
21.70 •• **CP** Two identical spheres are each attached to silk threads of length $L = 0.500$ m and hung from a common point (Fig. P21.68). Each sphere has mass $m = 8.00$ g. The radius of each sphere is very small compared to the distance between the spheres, so they may be treated as point charges. One sphere is given positive charge q_1, and the other a different positive charge q_2; this causes the spheres to separate so that when the spheres are in equilibrium, each thread makes an angle $\theta = 20.0°$ with the vertical. (a) Draw a free-body diagram for each sphere when in equilibrium, and label all the forces that act on each sphere. (b) Determine the magnitude of the electrostatic force that acts on each sphere, and determine the tension in each thread. (c) Based on the information you have been given, what can you say about the magnitudes of q_1 and q_2? Explain your answers. (d) A small wire is now connected between the spheres, allowing charge to be transferred from one sphere to the other until the two spheres have equal charges; the wire is then removed. Each thread now makes an angle of 30.0° with the vertical. Determine the original charges. (*Hint:* The total charge on the pair of spheres is conserved.)

21.71 •• Sodium chloride (NaCl, ordinary table salt) is made up of positive sodium ions (Na^+) and negative chloride ions (Cl^-). (a) If a point charge with the same charge and mass as all the Na^+ ions in 0.100 mol of NaCl is 2.00 cm from a point charge with the same charge and mass as all the Cl^- ions, what is the magnitude of the attractive force between these two point charges? (b) If the positive point charge in part (a) is held in place and the negative point charge is released from rest, what is its initial acceleration? (See Appendix D for atomic masses.) (c) Does it seem reasonable that the ions in NaCl could be separated in this way? Why or why not? (In fact, when sodium chloride dissolves in water, it breaks up into Na^+ and Cl^- ions. However, in this situation there are additional electric forces exerted by the water molecules on the ions.)

21.72 •• A -5.00-nC point charge is on the x-axis at $x = 1.20$ m. A second point charge Q is on the x-axis at -0.600 m. What must be the sign and magnitude of Q for the resultant electric field at the origin to be (a) 45.0 N/C in the $+x$-direction, (b) 45.0 N/C in the $-x$-direction?

21.73 •• **CP** A small 12.3-g plastic ball is tied to a very light 28.6-cm string that is attached to the vertical wall of a room (Fig. P21.73). A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of $-1.11\ \mu$C, you observe that it remains suspended, with the string making an angle of 17.4° with the wall. Find the magnitude and direction of the electric field in the room.

Figure **P21.73**

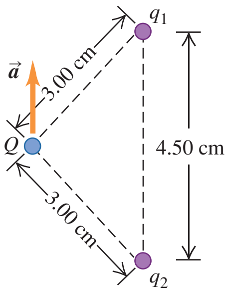


21.74 •• **CP** At $t = 0$ a very small object with mass 0.400 mg and charge $+9.00\ \mu$C is traveling at 125 m/s in the $-x$-direction. The charge is moving in a uniform electric field that is in the $+y$-direction and that has magnitude $E = 895$ N/C. The gravitational force on the particle can be neglected. How far is the particle from the origin at $t = 7.00$ ms?

21.75 •• Two particles having charges $q_1 = 0.500$ nC and $q_2 = 8.00$ nC are separated by a distance of 1.20 m. At what point along the line connecting the two charges is the total electric field due to the two charges equal to zero?

21.76 ••• Two point charges q_1 and q_2 are held in place 4.50 cm apart. Another point charge $Q = -1.75\ \mu$C of mass 5.00 g is initially located 3.00 cm from each of these charges (Fig. P21.76) and released from rest. You observe that the initial acceleration of Q is 324 m/s^2 upward, parallel to the line connecting the two point charges. Find q_1 and q_2.

Figure **P21.76**



21.77 • Three identical point charges q are placed at each of three corners of a square of side L. Find the magnitude and direction of the net force on a point charge $-3q$ placed (a) at the center of the square and (b) at the vacant corner of the square. In each case, draw a free-body diagram showing the forces exerted on the $-3q$ charge by each of the other three charges.

21.78 ••• Three point charges are placed on the y-axis: a charge q at $y = a$, a charge $-2q$ at the origin, and a charge q at $y = -a$. Such an arrangement is called an electric quadrupole. (a) Find the magnitude and direction of the electric field at points on the positive x-axis. (b) Use the binomial expansion to find an approximate expression for the electric field valid for $x \gg a$. Contrast this behavior to that of the electric field of a point charge and that of the electric field of a dipole.

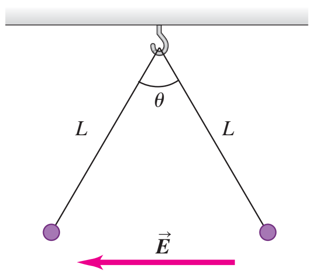
21.79 •• **CP** **Strength of the Electric Force.** Imagine two 1.0-g bags of protons, one at the earth's north pole and the other at the south pole. (a) How many protons are in each bag? (b) Calculate the gravitational attraction and the electrical repulsion that each bag exerts on the other. (c) Are the forces in part (b) large enough for you to feel if you were holding one of the bags?

21.80 • **Electric Force Within the Nucleus.** Typical dimensions of atomic nuclei are of the order of 10^{-15} m (1 fm). (a) If two protons in a nucleus are 2.0 fm apart, find the magnitude of the electric force each one exerts on the other. Express the answer in newtons and in pounds. Would this force be large enough for a person to feel? (b) Since the protons repel each other so strongly, why don't they shoot out of the nucleus?

21.81 •• **If Atoms Were Not Neutral . . .** Because the charges on the electron and proton have the same absolute value, atoms are electrically neutral. Suppose this were not precisely true, and the absolute value of the charge of the electron were less than the charge of the proton by 0.00100%. (a) Estimate what the net charge of this textbook would be under these circumstances. Make any assumptions you feel are justified, but state clearly what they are. (*Hint:* Most of the atoms in this textbook have equal numbers of electrons, protons, and neutrons.) (b) What would be the magnitude of the electric force between two textbooks placed 5.0 m apart? Would this force be attractive or repulsive? Estimate what the acceleration of each book would be if the books were 5.0 m apart and there were no non-electric forces on them. (c) Discuss how the fact that ordinary matter is stable shows that the absolute values of the charges on the electron and proton must be identical to a *very* high level of accuracy.

21.82 ••• **CP** Two tiny spheres of mass 6.80 mg carry charges of equal magnitude,

Figure **P21.82**



72.0 nC, but opposite sign. They are tied to the same ceiling hook by light strings of length 0.530 m. When a horizontal uniform electric field E that is directed to the left is turned on, the spheres hang at rest with the angle θ between the strings equal to 50.0° (Fig. P21.82). (a) Which ball (the one on the right or the one on the left) has positive charge? (b) What is the magnitude E of the field?

21.83 •• **CP** Consider a model of a hydrogen atom in which an electron is in a circular orbit of radius $r = 5.29 \times 10^{-11}$ m around a stationary proton. What is the speed of the electron in its orbit?

21.84 •• **CP** A small sphere with mass 9.00 μg and charge $-4.30\ \mu$C is moving in a circular orbit around a stationary sphere that has charge $+7.50\ \mu$C. If the speed of the small sphere is 5.90×10^3 m/s, what is the radius of its orbit? Treat the spheres as point charges and ignore gravity.

21.85 •• Two small copper spheres each have radius 1.00 mm. (a) How many atoms does each sphere contain? (b) Assume that each copper atom contains 29 protons and 29 electrons. We know that electrons and protons have charges of exactly the same magnitude, but let's explore the effect of small differences (see also Problem 21.81). If the charge of a proton is $+e$ and the magnitude of the charge of an electron is 0.100% smaller, what is the net charge of each sphere and what force would one sphere exert on the other if they were separated by 1.00 m?

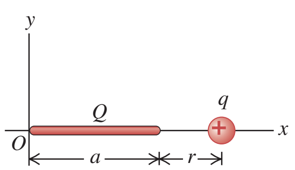
21.86 ••• **CP** **Operation of an Inkjet Printer.** In an inkjet printer, letters are built up by squirting drops of ink at the paper from a rapidly moving nozzle. The ink drops, which have a mass of 1.4×10^{-8} g each, leave the nozzle and travel toward the paper at 20 m/s, passing through a charging unit that gives each drop a positive charge q by removing some electrons from it. The drops then pass between parallel deflecting plates 2.0 cm long where there is a uniform vertical electric field with magnitude 8.0×10^4 N/C. If a drop is to be deflected 0.30 mm by the time it reaches the end of the deflection plates, what magnitude of charge must be given to the drop?

21.87 •• **CP** A proton is projected into a uniform electric field that points vertically upward and has magnitude E. The initial velocity of the proton has a magnitude v_0 and is directed at an angle α below the horizontal. (a) Find the maximum distance h_{max} that the proton descends vertically below its initial elevation. You can ignore gravitational forces. (b) After what horizontal distance d does the proton return to its original elevation? (c) Sketch the trajectory of the proton. (d) Find the numerical values of h_{max} and d if $E = 500$ N/C, $v_0 = 4.00 \times 10^5$ m/s, and $\alpha = 30.0^{\circ}$.

21.88 • A negative point charge $q_1 = -4.00$ nC is on the x-axis at $x = 0.60$ m. A second point charge q_2 is on the x-axis at $x = -1.20$ m. What must the sign and magnitude of q_2 be for the net electric field at the origin to be (a) 50.0 N/C in the $+x$-direction and (b) 50.0 N/C in the $-x$-direction?

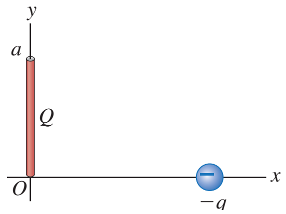
21.89 •• **CALC** Positive charge Q is distributed uniformly along the x-axis from $x = 0$ to $x = a$. A positive point charge q is located on the positive x-axis at $x = a + r$, a distance r to the right of the end of Q (Fig. P21.89). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis where $x > a$. (b) Calculate the force (magnitude and direction) that the charge distribution Q exerts on q. (c) Show that if $r \gg a$, the magnitude of the force in part (b) is approximately $Qq/4\pi\epsilon_0 r^2$. Explain why this result is obtained.

Figure **P21.89**



21.90 •• **CALC** Positive charge Q is distributed uniformly along the positive y-axis between $y = 0$ and $y = a$. A negative point charge $-q$ lies on the positive x-axis, a distance x from the origin (Fig. P21.90). (a) Calculate the x- and y-components of the electric field produced by the charge distribution Q at points on the positive x-axis. (b) Calculate the x- and y-components of the force that the charge distribution Q exerts on q. (c) Show that if $x \gg a$, $F_x \cong -Qq/4\pi\epsilon_0 x^2$ and $F_y \cong +Qqa/8\pi\epsilon_0 x^3$. Explain why this result is obtained.

Figure **P21.90**



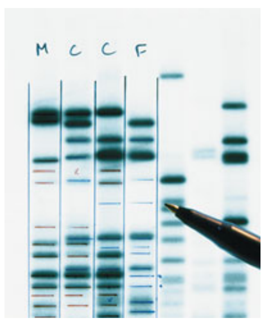
21.91 •• A charged line like that shown in Fig. 21.24 extends from $y = 2.50$ cm to $y = -2.50$ cm. The total charge distributed uniformly along the line is -7.00 nC. (a) Find the electric field (magnitude and direction) on the x-axis at $x = 10.0$ cm. (b) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 10.0 cm from a point charge that has the same total charge as this finite line of charge? In terms of the approximation used to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.9), explain why this is so. (c) At what distance x does the result for the finite line of charge differ by 1.0% from that for the point charge?

21.92 • **CP** **A Parallel Universe.** Imagine a parallel universe in which the electric force has the same properties as in our universe but there is no gravity. In this parallel universe, the sun carries charge Q, the earth carries charge $-Q$, and the electric attraction between them keeps the earth in orbit. The earth in the parallel universe has the same mass, the same orbital radius, and the same orbital period as in our universe. Calculate the value of Q. (Consult Appendix F as needed.)

21.93 ••• A uniformly charged disk like the disk in Fig. 21.25 has radius 2.50 cm and carries a total charge of 7.0×10^{-12} C. (a) Find the electric field (magnitude and direction) on the x-axis at $x = 20.0$ cm. (b) Show that for $x \gg R$, Eq. (21.11) becomes $E = Q/4\pi\epsilon_0 x^2$, where Q is the total charge on the disk. (c) Is the magnitude of the electric field you calculated in part (a) larger or smaller than the electric field 20.0 cm from a point charge that has the same total charge as this disk? In terms of the approximation used in part (b) to derive $E = Q/4\pi\epsilon_0 x^2$ for a point charge from Eq. (21.11), explain why this is so. (d) What is the percent difference between the electric fields produced by the finite disk and by a point charge with the same charge at $x = 20.0$ cm and at $x = 10.0$ cm?

21.94 •• **BIO** **Electrophoresis.** Electrophoresis is a process used by biologists to separate different biological molecules (such as proteins) from each other according to their ratio of charge to size. The materials to be separated are in a viscous solution that produces a drag force F_D proportional to the size and speed of the molecule. We can express this relationship as $F_D = KRv$, where R is the radius of the molecule (modeled as being spherical), v is its speed, and K is a constant that depends on the viscosity of the

Figure **P21.94**



solution. The solution is placed in an external electric field E so that the electric force on a particle of charge q is $F = qE$. (a) Show that when the electric field is adjusted so that the two forces (electric and viscous drag) just balance, the ratio of q to R is Kv/E. (b) Show that if we leave the electric field on for a time T, the distance x that the molecule moves during that time is $x = (ET/k)(q/R)$. (c) Suppose you have a sample containing three different biological molecules for which the molecular ratio q/R for material 2 is twice that of material 1 and the ratio for material 3 is three times that of material 1. Show that the distances migrated by these molecules after the same amount of time are $x_2 = 2x_1$ and $x_3 = 3x_1$. In other words, material 2 travels twice as far as material 1, and material 3 travels three times as far as material 1. Therefore, we have separated these molecules according to their ratio of charge to size. In practice, this process can be carried out in a special gel or paper, along which the biological molecules migrate. (Fig. P21.94). The process can be rather slow, requiring several hours for separations of just a centimeter or so.

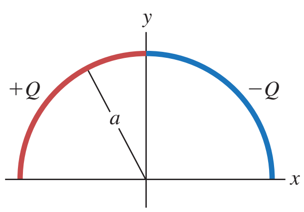
21.95 • **CALC** Positive charge $+Q$ is distributed uniformly along the $+x$-axis from $x = 0$ to $x = a$. Negative charge $-Q$ is distributed uniformly along the $-x$-axis from $x = 0$ to $x = -a$. (a) A positive point charge q lies on the positive y-axis, a distance y from the origin. Find the force (magnitude and direction) that the positive and negative charge distributions together exert on q. Show that this force is proportional to y^{-3} for $y \gg a$. (b) Suppose instead that the positive point charge q lies on the positive x-axis, a distance $x > a$ from the origin. Find the force (magnitude and direction) that the charge distribution exerts on q. Show that this force is proportional to x^{-3} for $x \gg a$.

21.96 •• **CP** A small sphere with mass m carries a positive charge q and is attached to one end of a silk fiber of length L. The other end of the fiber is attached to a large vertical insulating sheet that has a positive surface charge density σ. Show that when the sphere is in equilibrium, the fiber makes an angle equal to arctan $(q\sigma/2mg\epsilon_0)$ with the vertical sheet.

21.97 •• **CALC** Negative charge $-Q$ is distributed uniformly around a quarter-circle of radius a that lies in the first quadrant, with the center of curvature at the origin. Find the x- and y-components of the net electric field at the origin.

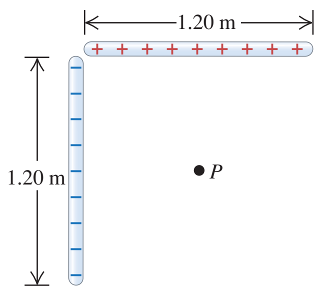
21.98 •• **CALC** A semicircle of radius a is in the first and second quadrants, with the center of curvature at the origin. Positive charge $+Q$ is distributed uniformly around the left half of the semicircle, and negative charge $-Q$ is distributed uniformly around the right half of the semicircle (Fig. P21.98). What are the magnitude and direction of the net electric field at the origin produced by this distribution of charge?

Figure **P21.98**



21.99 •• Two 1.20-m nonconducting wires meet at a right angle. One segment carries $+2.50\ \mu C$ of charge distributed uniformly along its length, and the other carries $-2.50\ \mu C$ distributed uniformly along it, as shown in Fig. P21.99. (a) Find the magnitude and direction of the electric field these wires produce at point P, which is 60.0 cm from each wire. (b) If an electron is released at P, what are the magnitude and direction of the net force that these wires exert on it?

Figure **P21.99**



21.100 • Two very large parallel sheets are 5.00 cm apart. Sheet A carries a uniform surface charge density of $-9.50 \mu C/m^2$, and sheet B, which is to the right of A, carries a uniform charge density of $-11.6\ \mu C/m^2$. Assume the sheets are large enough to be treated as infinite. Find the magnitude and direction of the net electric field these sheets produce at a point (a) 4.00 cm to the right of sheet A; (b) 4.00 cm to the left of sheet A; (c) 4.00 cm to the right of sheet B.

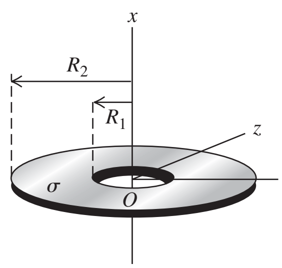
21.101 • Repeat Problem 21.100 for the case where sheet B is positive.

21.102 • Two very large horizontal sheets are 4.25 cm apart and carry equal but opposite uniform surface charge densities of magnitude σ. You want to use these sheets to hold stationary in the region between them an oil droplet of mass 324 μg that carries an excess of five electrons. Assuming that the drop is in vacuum, (a) which way should the electric field between the plates point, and (b) what should σ be?

21.103 •• An infinite sheet with positive charge per unit area σ lies in the xy-plane. A second infinite sheet with negative charge per unit area $-\sigma$ lies in the yz-plane. Find the net electric field at all points that do not lie in either of these planes. Express your answer in terms of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

21.104 •• **CP** A thin disk with a circular hole at its center, called an *annulus,* has inner radius R_1 and outer radius R_2 (Fig. P21.104). The disk has a uniform positive surface charge density σ on its surface. (a) Determine the total electric charge on the annulus. (b) The annulus lies in the yz-plane, with its center at the origin. For an arbitrary point on the x-axis (the axis of the annulus), find the magnitude and direction of the electric field $\vec{E}$. Consider points both above and below the annulus in Fig. P21.104. (c) Show that at points on the x-axis that are sufficiently close to the origin, the magnitude of the electric field is approximately proportional to the distance between the center of the annulus and the point. How close is "sufficiently close"? (d) A point particle with mass m and negative charge $-q$ is free to move along the x-axis (but cannot move off the axis). The particle is originally placed at rest at $x = 0.01R_1$ and released. Find the frequency of oscillation of the particle. (*Hint:* Review Section 14.2. The annulus is held stationary.)

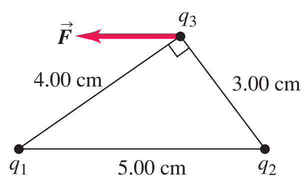
Figure **P21.104**



CHALLENGE PROBLEMS

21.105 ••• Three charges are placed as shown in Fig. P21.105. The magnitude of q_1 is 2.00 μC, but its sign and the value of the charge q_2 are not known. Charge q_3 is $+4.00\ \mu C$, and the net force $\vec{F}$ on q_3 is entirely in the negative x-direction. (a) Considering the different possible signs of q_1, there are four possible force diagrams representing the forces $\vec{F}_1$ and $\vec{F}_2$ that q_1 and q_2 exert on q_3. Sketch these four possible force configurations.

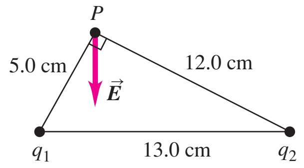
Figure **P21.105**



(b) Using the sketches from part (a) and the direction of $\vec{F}$, deduce the signs of the charges q_1 and q_2. (c) Calculate the magnitude of q_2. (d) Determine F, the magnitude of the net force on q_3.

21.106 ••• Two charges are placed as shown in Fig. P21.106. The magnitude of q_1 is 3.00 μC, but its sign and the value of the charge q_2 are not known. The direction of the net electric field $\vec{E}$ at point P is entirely in the negative y-direction. (a) Considering the different possible signs of q_1 and q_2, there are four possible diagrams that could represent the electric fields $\vec{E}_1$ and $\vec{E}_2$ produced by q_1 and q_2. Sketch the four possible electric-field configurations. (b) Using the sketches from part (a) and the direction of $\vec{E}$, deduce the signs of q_1 and q_2. (c) Determine the magnitude of $\vec{E}$.

Figure **P21.106**



21.107 ••• **CALC** Two thin rods of length L lie along the x-axis, one between $x = a/2$ and $x = a/2 + L$ and the other between $x = -a/2$ and $x = -a/2 - L$. Each rod has positive charge Q distributed uniformly along its length. (a) Calculate the electric field produced by the second rod at points along the positive x-axis. (b) Show that the magnitude of the force that one rod exerts on the other is

$$F = \frac{Q^2}{4\pi\epsilon_0 L^2}\ln\left[\frac{(a+L)^2}{a(a+2L)}\right]$$

(c) Show that if $a \gg L$, the magnitude of this force reduces to $F = Q^2/4\pi\epsilon_0 a^2$. (*Hint:* Use the expansion $\ln(1+z) = z - z^2/2 + z^3/3 - \cdots$, valid for $|z| \ll 1$. Carry *all* expansions to at least order L^2/a^2.) Interpret this result.

Answers

Chapter Opening Question ?

Water molecules have a permanent electric dipole moment: One end of the molecule has a positive charge and the other end has a negative charge. These ends attract negative and positive ions, respectively, holding the ions apart in solution. Water is less effective as a solvent for materials whose molecules do not ionize (called *nonionic* substances), such as oils.

Test Your Understanding Questions

21.1 Answers: (a) the plastic rod weighs more, (b) the glass rod weighs less, (c) the fur weighs less, (d) the silk weighs more The plastic rod gets a negative charge by taking electrons from the fur, so the rod weighs a little more and the fur weighs a little less after the rubbing. By contrast, the glass rod gets a positive charge by giving electrons to the silk. Hence, after they are rubbed together, the glass rod weighs a little less and the silk weighs a little more. The weight change is *very* small: The number of electrons transferred is a small fraction of a mole, and a mole of electrons has a mass of only $(6.02 \times 10^{23}\text{ electrons})(9.11 \times 10^{-31}\text{ kg/electron}) = 5.48 \times 10^{-7}\text{ kg} = 0.548$ milligram!

21.2 Answers: (a) (i), (b) (ii) Before the two spheres touch, the negatively charged sphere exerts a repulsive force on the electrons in the other sphere, causing zones of positive and negative induced charge (see Fig. 21.7b). The positive zone is closer to the negatively charged sphere than the negative zone, so there is a net force of attraction that pulls the spheres together, like the comb and insulator in Fig. 21.8b. Once the two metal spheres touch, some of the excess electrons on the negatively charged sphere will flow onto the other sphere (because metals are conductors). Then both spheres will have a net negative charge and will repel each other.

21.3 Answer: (iv) The force exerted by q_1 on Q is still as in Example 21.4. The magnitude of the force exerted by q_2 on Q is still equal to $F_{1\text{ on }Q}$, but the direction of the force is now *toward* q_2 at an angle α below the x-axis. Hence the x-components of the two forces cancel while the (negative) y-components add together, and the total electric force is in the negative y-direction.

21.4 Answers: (a) (ii), (b) (i) The electric field $\vec{E}$ produced by a positive point charge points directly away from the charge (see Fig. 21.18a) and has a magnitude that depends on the distance r from the charge to the field point. Hence a second, negative point charge $q < 0$ will feel a force $\vec{F} = q\vec{E}$ that points directly toward the positive charge and has a magnitude that depends on the distance r between the two charges. If the negative charge moves directly toward the positive charge, the direction of the force remains the same but the force magnitude increases as the distance r decreases. If the negative charge moves in a circle around the positive charge, the force magnitude stays the same (because the distance r is constant) but the force direction changes.

21.5 Answer: (iv) Think of a pair of segments of length dy, one at coordinate $y > 0$ and the other at coordinate $-y < 0$. The upper segment has a positive charge and produces an electric field $d\vec{E}$ at P that points away from the segment, so this $d\vec{E}$ has a positive x-component and a negative y-component, like the vector $d\vec{E}$ in Fig. 21.24. The lower segment has the same amount of negative charge. It produces a $d\vec{E}$ that has the same magnitude but points *toward* the lower segment, so it has a negative x-component and a negative y-component. By symmetry, the two x-components are equal but opposite, so they cancel. Thus the total electric field has only a negative y-component.

21.6 Answer: yes If the field lines are straight, $\vec{E}$ must point in the same direction throughout the region. Hence the force $\vec{F} = q\vec{E}$ on a particle of charge q is always in the same direction. A particle released from rest accelerates in a straight line in the direction of $\vec{F}$, and so its trajectory is a straight line along a field line.

21.7 Answer: (ii) Equations (21.17) and (21.18) tell us that the potential energy for a dipole in an electric field is $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between the directions of $\vec{p}$ and $\vec{E}$. If $\vec{p}$ and $\vec{E}$ point in opposite directions, so that $\phi = 180°$, we have $\cos\phi = -1$ and $U = +pE$. This is the maximum value that U can have. From our discussion of energy diagrams in Section 7.5, it follows that this is a situation of unstable equilibrium.

Bridging Problem

Answer: $E = 2kQ/\pi a^2$ in the $-y$-direction