$$a_0 \cancel{x^0} + a_1 x^1 + a_2 x^2$$

$$+ \quad -3a_0 x^1 - 3a_1 x^2$$

a_0

$$G(x) - 3xG(x) = 2$$

$$G(x) = \frac{2}{1-3x} = 2 \cdot 3^k x^k$$

$$a_n = -a_{n-1} + 6a_{n-2}$$

$$a_2 = -a_1 + 6a_0$$

$$a_1 + a_2 - 6a_0 = 0$$

$\frac{-2}{1+3x} \to -2 \quad -2(-3)^1 \quad -2(-3)^2 \quad \ldots$

$\frac{1}{1-2x} \to 1 \quad 2^1 \quad 2^2$

$G(x)$

$2^2 - 2(-3)^2$

$F_0 = 0 \quad F_1 = 1 \quad \boxed{F_n = F_{n-1} + F_{n-2}}$

$F_{1000} = ?$

i:	0	1	2	3	4	5	6	7	8	9	10
F_i:	0	1	1	2	3	5	8	13	21	34	55

$$- \; G_F(x) = \cancel{0x^0} + 1x^1 + 1x^2 + 2x^3 + 3x^4 + 5x^5 + \dots$$

$$xG_F(x) = \cancel{0x^1} + 1x^2 + 1x^3 + 2x^4 + 3x^5 + \dots$$

$$+ \; x^2 G_F(x) = 0x^2 + 1x^3 + 1x^4 + 2x^5 + \dots$$

$$G_F(x)(x^2+x-1) = -x$$

$$ax^2+bx+c=0 \qquad \frac{-b \mp \sqrt{b^2-4ac}}{2a}$$

$$G_F(x) = \frac{-x}{x^2+x-1} = \frac{A}{x - \frac{-1+\sqrt{5}}{2}} + \frac{B}{x - \frac{-1-\sqrt{5}}{2}}$$

$$= \frac{A}{x - \frac{\sqrt{5}-1}{2}} + \frac{B}{x + \frac{\sqrt{5}+1}{2}} = \frac{Ax + \frac{A(\sqrt{5}+1)}{2} + Bx - B\frac{(\sqrt{5}-1)}{2}}{x^2+x-1}$$

$-x =$

$A + B = -1$

$A\sqrt{5} + A - B\sqrt{5} + B = 0$

$(A-B)\sqrt{5} = 1$

$$A + B = -1$$
$$+ \; A - B = \frac{1}{\sqrt{5}}$$

$$2A = -1 + \frac{1}{\sqrt{5}}$$

$$A = \frac{-1}{2} + \frac{1}{2\sqrt{5}} \qquad B = \frac{-1}{2} - \frac{1}{2\sqrt{5}}$$

$$= \frac{-5}{10} + \frac{\sqrt{5}}{10} \qquad \frac{-5}{10} - \frac{\sqrt{5}}{10}$$

$$A = \frac{\sqrt{5}-5}{10} \qquad B = \frac{-\sqrt{5}-5}{10}$$

$$G_F(x) = \frac{\frac{\sqrt{5}-5}{10}}{\frac{5 \cdot}{5 \cdot} \frac{2x + 1 - \sqrt{5}}{2}} + \frac{\frac{-\sqrt{5}-5}{10}}{\frac{2x + 1 + \sqrt{5}}{2}}$$

$$= \frac{\sqrt{5}-5}{10x + 5 - 5\sqrt{5}} - \frac{5+\sqrt{5}}{10x + 5 + 5\sqrt{5}}$$

$$G_F(x) = \frac{1/\sqrt{5}}{\frac{10}{5-5\sqrt{5}} \cdot x + 1} - \frac{1/\sqrt{5}}{\frac{10}{5+5\sqrt{5}}x + 1}$$

$$= \frac{1}{\sqrt{5}} \left(\frac{1}{1 - \frac{10}{5\sqrt{5}-5}x} - \frac{1}{1 - \frac{-10}{5\sqrt{5}+5}x} \right) = \frac{1}{\sqrt{5}} \left(\frac{1}{1 - \frac{2}{\sqrt{5}-1}x} - \frac{1}{1 - \frac{-2}{\sqrt{5}+1}x} \right)$$

$$G_F(x) = \frac{1}{\sqrt{5}} \left(\frac{1}{1 - \underset{a}{\frac{2}{\sqrt{5}-1}} x} - \frac{1}{1 - \underset{a}{\frac{-2}{\sqrt{5}+1}} x} \right)$$

$$\Rightarrow F_n = \frac{1}{\sqrt{5}} \left[\left(\frac{2}{\sqrt{5}-1} \right)^n - \left(\frac{-2}{\sqrt{5}+1} \right)^n \right]$$

$$x_1 + x_2 + \cdots + x_{10} = 50 \qquad 0 \le x_i \le 10$$

For the purpose of using I.E, let $c_i : x_i \geq 11$. Then, the num is

$$\bar{N} = S_0 - S_1 + S_2 - \cdots + S_{10}$$

$$= \binom{59}{9} - \binom{10}{1}\binom{48}{9} + \binom{10}{2}\binom{37}{9} - \binom{10}{3}\binom{26}{9} + \binom{10}{4}\binom{15}{9}$$

$$S_0 = \binom{50+9}{9}$$

$$S_1 = \sum_{i=1}^{10} N(c_i) = \binom{10}{1}\binom{48}{9}$$

$$\left(1 + x + x^2 + \cdots + x^{10}\right)^{10} \to ?\,x^{50}$$

$$\left(\frac{1-x^{11}}{1-x}\right)^{10} \to (1-x^{11})^{10}(1-x)^{-10}$$

$$\binom{59}{50} x^{50}$$

$$1 \cdot \binom{-10}{50}(-x)^{50} \qquad \binom{10+50-1}{50}(-1)^{50} \cdot (-1)^{50} \cdot x^{50}$$

$$\binom{10}{1} - x^{11} \cdot \binom{-10}{39}(-x)^{39} \qquad -\binom{48}{39}\binom{10}{1}$$

$$\binom{10}{2}(-x^{11})^2 \cdot \binom{-10}{28}(-x)^{28} \qquad \binom{37}{28}x^{28}\binom{10}{2}$$

$$\binom{10}{3}(-x^{11})^3 \cdot \binom{-10}{17}(-x)^{17} \qquad -\binom{26}{17}x^{17}\binom{10}{3}$$

$$\binom{10}{4}(-x^{11})^4 \cdot \binom{-10}{6}(-x)^6 \qquad \binom{15}{6}x^6\binom{10}{4}$$

$$\left(x^3+x^4+\cdots+x^8\right)^4 \qquad ?\ x^{24}$$

$$\cancel{(x^3)^4}\left(1+x+\cdots+x^5\right)^4 \qquad ?\ x^{12}$$

$$\left(\frac{1-x^6}{1-x}\right)^4 = \left(1-x^6\right)^4\cdot(1-x)^{-4}$$

$$\binom{4}{0}1 \qquad \binom{-4}{12}(-x)^{12} \qquad \binom{4}{0}\binom{15}{12}(-1)^{12}\cancel{(-1)^{12}}x^{12}$$

$$\binom{-n}{r} = \binom{n+r-1}{r}(-1)^r$$

$$\binom{4}{1}\cdot -x^6 \qquad \binom{-4}{6}(-x)^6 \qquad -\binom{4}{1}\binom{9}{6}(-1)^6\cancel{(-1)^6}\cdot x^6$$

$$\binom{4}{2}(-x^6)^2 \qquad \binom{-4}{0}(-x)^0 \qquad \binom{4}{2}\binom{3}{0}(-1)^0\ 1$$

$$\left[\binom{4}{0}\binom{15}{3}-\binom{4}{1}\binom{9}{3}+\binom{4}{2}\binom{3}{0}\right]x^{12}$$

$x_1+x_2+\cdots x_4 = 24$

$y_1+y_2\cdots y_4 = 12$

$c_i : x_i \geq 6$

$$\binom{15}{3}-\binom{4}{1}\binom{9}{3}+\binom{4}{2}\binom{3}{3}$$