

1. Given a rod of length n inches and an array $A[1 \dots n]$ of prices that includes prices of all pieces of discrete sizes smaller than or equal to n . Determine the maximum value obtainable by cutting up the rod and selling the pieces. For example, if the length of the rod is 8 and the values of different pieces are given as the following, then the maximum obtainable value is 22

a) Give a mathematical recursive formulation for $R(n)$ which denotes the maximum possible prices value you can get for a rod of length n

b) Overlapping

c) Dynamic program. And max.

d) Provide the running time of your dynamic algorithm program.

You are given an array of jobs where every job takes single unit. Also every job has a deadline.

a 2 100

b 1 19

c 2 27

d 1 25

e 3 15

Deadline Profits

The max profit sequence of jobs is a, c, e

Thus, the min possible deadline for any job is 1. If only one job can be scheduled at a time, the job sequencing problem is to maximize total profit

→ greedy alg. → for job sequencing

→ choice is optimal proof

③ Assume you are creating an array data structure that has a fixed size of n . You want to backup this array after every n insertion operations. Unfortunately, the backup operation is quite expensive, it takes n time to do the backup. Insertions without a backup just take 1 time unit. Show that you can do backups in $O(1)$ amortized time. Use potential method.

④ Given an unsorted array, the array has this property that every element in array is at most k distance from its position in sorted array where k is a variable smaller than size of array. For example, let us consider k is 2 an element at index 7 in sorted array, can be at indexes 5, 6, 7, 8, 9 in the given array. We can sort such arrays more efficiently with the help of Heap data structure. Following is the sorting algorithm that uses Min-Heap:

- Create min-heap of size $k+1$ with first $k+1$ elements

- One by one remove min element from heap, put it in result array, and add a new element to heap from remaining elements.

Provide a tight asymptotic upper bound for this algorithm. Explain briefly.