

Tuesday 11/01/2022

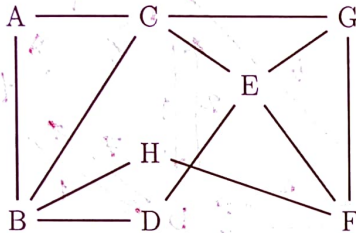
Final Exam

Duration: 90 minutes

Name:

Student No:

P1 [25 points] If exists in the graph, give an example of the following. If impossible, write impossible.  
(Ex. If path was asked, a correct answer would be: A-C-E)

Eulerian trail:  $G-C-A-B-D-E-G-F-E-C-B-H-F$ 

Eulerian cycle: Impossible. G &amp; F has odd degree.

Hamiltonian path:  $A-C-E-D-B-H-F-G$ 

P2 [10 points] Choose the correct option:

What is the chromatic number of  $K_{3,4}$ ?

A) 2 B) 3 C) 4 D) 6 E) 12

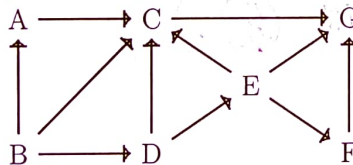
Every complete bipartite graph has ch. num. 2

What is the chromatic number of  $K_9$ ?

A) 3 B) 6 C) 8 D) 9 E) 10

" " " " " " = |V|

P3 [20 points] Topological Sort & Counting



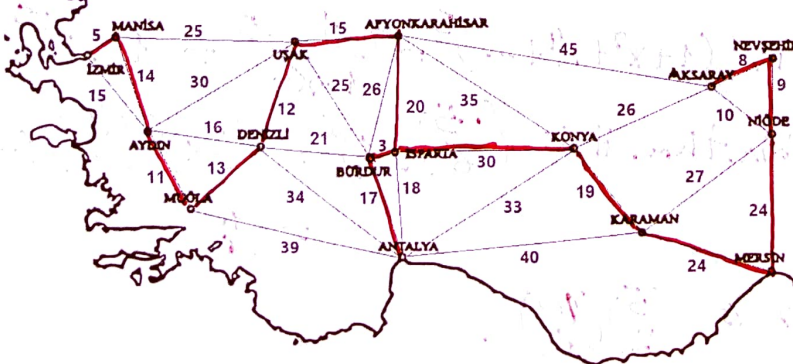
Give a topological order for the graph:

 $B A D E F C G$ 

What is the number of possible topological orders?

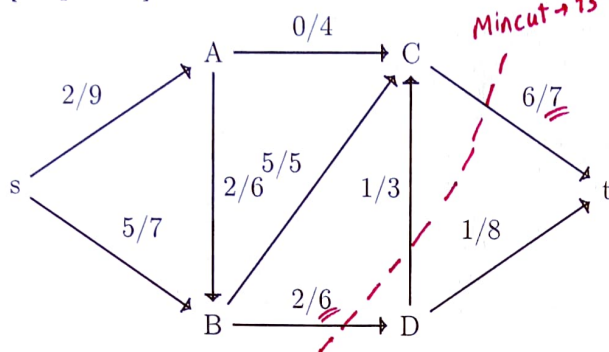
$B \rightarrow D \rightarrow E \rightarrow C \rightarrow G$   $A \rightarrow F: 3 \times 2 = 6 \rightarrow 7$  in total.  
 $F \rightarrow A: 1$

P4 [15 points] Minimum Spanning Tree In the map below, draw a minimum spanning tree by using Prim's Algorithm starting from Konya and write the cities in the order you add them to the MST.



1	Konya	9	Antalya
2	Karaman	10	Afyon
3	Mersin	11	Uşak
4	Niğde	12	Denizli
5	Nevşehir	13	Mugla
6	Aksaray	14	Aydın
7	Isparta	15	Manisa
8	Burdur	16	izmir

P5 [20 points] Network Flows

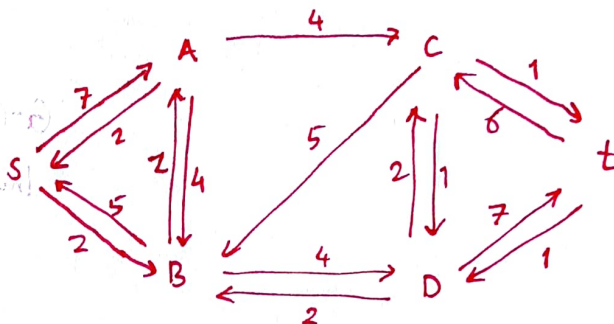


A network and a flow  $F$  on this network are given on the left.

1. According to the capacities, what is the maximum flow of this network?

*Mincut given gives us  $6 + 7 = 13$*

2. Draw the residual graph.



P6 [15 points] Generating Functions & Combinations Solve this question using generating functions (Build the polynomial, determine the coefficient to look for, and calculate the final result) [Recall that  $1/(1-x) = 1 + x + x^2 + x^3 + \dots$ ]

How many integer solutions are there to the equation  $x_1 + x_2 + x_3 + x_4 + x_5 = 20$  with the restriction that all of  $x_i \geq 1$  where two of them are odd and the remaining three are even integers?

*Without loss of generality, suppose that  $x_1$  &  $x_2$  are the odd ones. (At the end we will take this back by multiplying with  $\binom{5}{2}$ .)*

*Now,  $\underbrace{x_1 + x_2}_{\text{odd}} + \underbrace{x_3 + x_4 + x_5}_{\text{even}} = 20$*

*Polynomial:  $(x + x^3 + x^5 + \dots)^2 (x^2 + x^4 + x^6 + \dots)^3 = x^8 (1 + x^2 + x^4 + \dots)^5$*

*Find the coefficient of  $x^{20}$ . But  $x^8$  can be cancelled:*

*Find  $[x^{12}]$  in  $(1 + x^2 + x^4 + \dots)^5 = \left(\frac{1}{1-x^2}\right)^5 = (1-x^2)^{-5}$*

*By extended bin. thm. it is  $\binom{-5}{6} (-1)^6 = (-1)^6 \binom{10}{6} (-1)^6 = \binom{10}{6}$*

*Thus, the answer is  $\binom{5}{2} \binom{10}{6}$ .*