MAT222
LINEAR ALGEBRA
HOMEWORK ASSIGNMENT 1
Final Delivery Date: March 11, 2024, 17.30

(1) Suppose that the row echelon form of the augmented matrix of a given linear system is

$$\left[\begin{array}{ccc:c} 1 & 2 & 3 & 0 \\ 0 & m & m & m^2-m \\ 0 & 0 & m^2-m & m \end{array}\right]$$

Determine (and explain your reasoning) whether

(a) the system has infinitely many solutions depending on one parameter if $m = 0$

(b) the system has infinitely many solutions depending on one parameter if $m = 1$

(c) the system is inconsistent for $m = 1$

(d) the system has infinitely many solutions for $m = 0$ and $m = 1$

(e) the system has exactly one solution for $m \neq 0$

(2) Determine the values of k for which the system

$$\begin{aligned} y + 2kz &= 0 \\ x + 2y + 6z &= 2 \\ kx + 2z &= 1 \end{aligned}$$

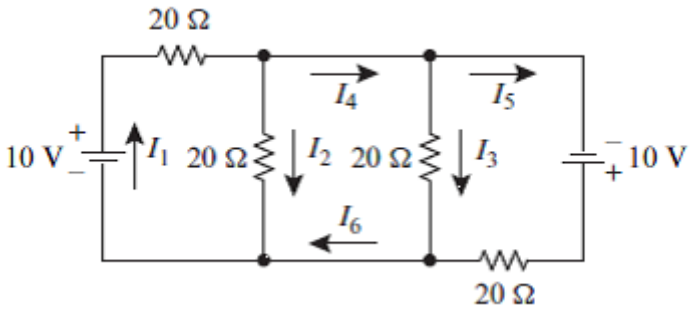
has no solution.

(3) A conic section is a curve in $\mathbb{R}^2$ that can be described by an equation of the form

$$f(x, y) = c_1 + c_2x + c_3y + c_4x^2 + c_5xy + c_6y^2 = 0,$$

where at least one of the coefficients c_k is nonzero. Find all conics through the points $(1, 0)$, $(2, 0)$, $(2, 2)$, $(5, 2)$, and $(5, 6)$ using Gauss-Jordan elimination.

(4) Analyze the given electrical circuit by finding the unknown currents.



(5) Let A be the matrix of size 4×10 with entries $a_{ij} = \dfrac{j}{j+i}$ and B be the matrix of size 10×3 with entries $b_{ij} = \dfrac{j}{i^2+i}$. If $AB = C$ with entries c_{ij}, find c_{21}.