P7 [15 points] Inclusion-Exclusion Principle
In an exam, there are 10 questions each worth 10 points. In how many different ways can a student get 50 points? (For example, the student can get $7, 10, 6, 2, 7, 0, 8, 0, 10, 0$ from questions 1 through 10, respectively. You need to count the number of such gradings that add up to 50.)

P8 [15 points] Generating Functions
   In how many ways can a farmer distribute 24 apples to four children so that each child gets at least three apples but no more than eight?

Table 1: Some generating functions that can be useful. For all $m, n \in \mathbb{Z}^+$, $a \in \mathbb{R}$

1) $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \cdots + \binom{n}{n}x^n$

2) $(1+ax)^n = \binom{n}{0} + \binom{n}{1}ax + \binom{n}{2}a^2x^2 + \cdots + \binom{n}{n}a^nx^n$

3) $(1+x^m)^n = \binom{n}{0} + \binom{n}{1}x^m + \binom{n}{2}x^{2m} + \cdots + \binom{n}{n}x^{nm}$

4) $(1 - x^{n+1})/(1-x) = 1 + x + x^2 + x^3 + \cdots + x^n$

5) $1/(1-x) = 1 + x + x^2 + x^3 + \cdots$

6) $1/(1-ax) = 1 + ax + a^2x^2 + a^3x^3 + \cdots$

7) $1/(1+x)^n = \binom{-n}{0} + \binom{-n}{1}x + \binom{-n}{2}x^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}x + (-1)^2\binom{n+2-1}{2}x^2 + \cdots$

8) $1/(1-x)^n = \binom{-n}{0} + \binom{-n}{1}(-x) + \binom{-n}{2}(-x)^2 + \cdots = 1 + (-1)\binom{n+1-1}{1}(-x) + (-1)^2\binom{n+2-1}{2}(-x)^2 + \cdots$