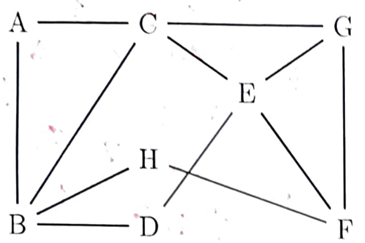
Fall 2021: CSE 221 | Discrete Mathematics | Akdeniz University

Tuesday 11/01/2022 | Final Exam | Duration: 90 minutes

Name:

Student No:

P1 [25 points] If exists in the graph, give an example of the following. If impossible, write impossible. (Ex. If **path** was asked, a correct answer would be: A-C-E)



Eulerian trail: G-C-A-B-D-E-G-F-E-C-B-H-F

Eulerian cycle: Impossible. G & F has odd degree.

Hamiltonian path: A-C-E-D-B-H-F-G

P2 [10 points] Choose the correct option:

What is the chromatic number of $K_{3,4}$?

A) 2 B)3 C)4 D)6 E)12

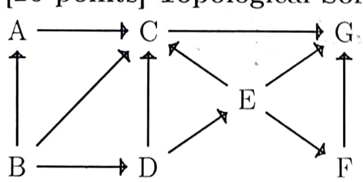
Every complete bipartite graph has ch. num. 2

What is the chromatic number of K_9?

A) 3 B)6 C)8 D)9 E)10

" " " " " " n =|V|

P3 [20 points] Topological Sort & Counting



Give a topological order for the graph:

B A D E F C G

What is the number of possible topological orders?

A

B → D → E → C → G

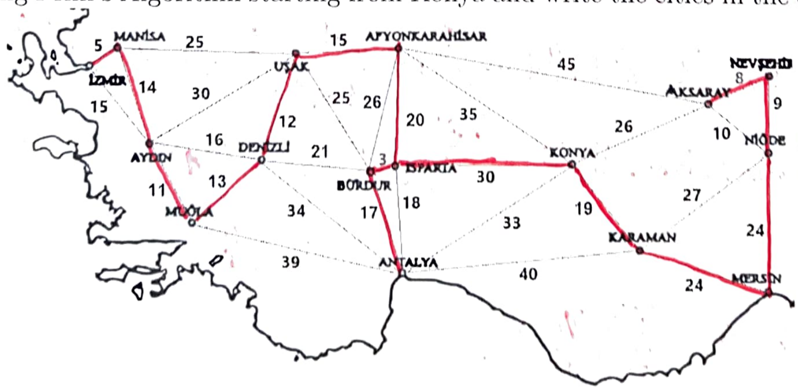
F

A→F: 3×2=6

F→A: 1

→ 7 in total.

P4 [15 points] Minimum Spanning Tree In the map below, draw a minimum spanning tree by using Prim's Algorithm starting from Konya and write the cities in the order you add them to the MST.

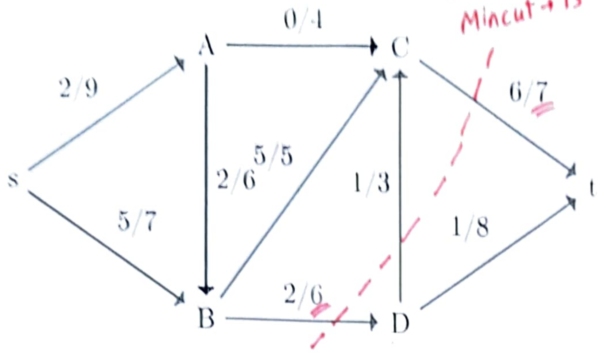


1	Konya	9	Antalya
2	Karaman	10	Afyon
3	Mersin	11	Uşak
4	Niğde	12	Denizli
5	Nevşehir	13	Muğla
6	Aksaray	14	Aydın
7	Isparta	15	Manisa
8	Burdur	16	izmir

P5 [20 points] Network Flows

(The minimum cut possible)

Mincut → 13 = 6+7

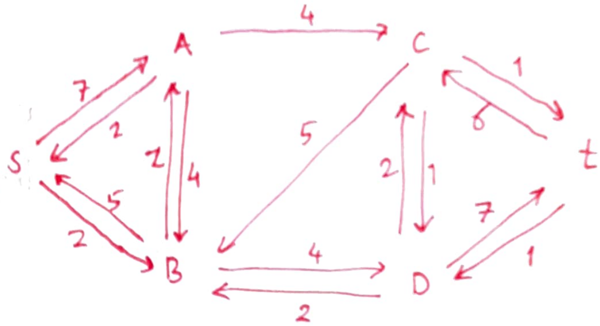


A network and a flow F on this network are given on the left.

1. According to the capacities, what is the maximum flow of this network?

   Mincut given gives us 6+7=13

2. Draw the residual graph.



P6 [15 points] Generating Functions & Combinations Solve this question using generating functions (Build the polynomial, determine the coefficient to look for, and calculate the final result) [Recall that $1/(1-x) = 1 + x + x^2 + x^3 + \ldots$]

How many integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 20$ with the restriction that all of $x_i \geq 1$ where two of them are odd and the remaining three are even integers?

Without loss of generality, suppose that x_1 & x_2 are the odd ones.
(At the end we will take this back by multiplying with $\binom{5}{2}$.)

Now, $\underbrace{x_1+x_2}_{\text{odd}}+\underbrace{\cdots+x_5}_{\text{even}} = 20$

Polynomial: $(x+x^3+x^5+\ldots)^2 (x^2+x^4+x^6+\ldots)^3 = x^8(1+x^2+x^4+\ldots)^5$

Find the coefficient of x^{20}. But x^8 can be cancelled:

∴ Find $[x^{12}]$ in $(1+x^2+x^4+\ldots)^5 = \left(\frac{1}{1-x^2}\right)^5 = (1-x^2)^{-5}$

By extended bin. thm. it is $\binom{-5}{6}(-1)^6 = (-1)^6\binom{10}{6}(-1)^6$

$= \binom{10}{6}$

Thus, the answer is $\binom{5}{2}\binom{10}{6}$.