Friday 18/12/2020 Midterm Duration: 90 minutes

P1 [10 points]

(a) A group of 15 students want to have a futsal tournament. Since a futsal team has 5 players, they will form three teams. In how many ways can they do this? 15! / (5!5!5!)

(b) What will be the answer if there is a twin couple who have to be in the same team?

P2 [20 points]

(a) Five friends decide to buy a ball for 50 TL. Since each of them must pay at least 5 TL and they can only pay a multiple of 1 TL, in how many different ways can they pay (Regarding who pays how much)?

(b) What if nobody should pay more than 15 TL? (So each of them will pay at least 5 TL, at most 15 TL)

P3 [20 points]

Establish the validity of the argument:

$\forall x \ \ (p(x) \lor q(x))$
$\forall x \ \ (\neg q(x) \lor u(x))$
$\forall x \ \ (t(x) \to \neg u(x))$
$\exists x \ \ (\neg p(x))$
$\therefore \exists x \ \ (\neg t(x))$

Proof:
1.
2.
3.
4.
5. ...

P4 [30 points] Prove the following statement by using mathematical induction:

(a) Fibonacci numbers are defined as follows:
$F_0 = 0, F_1 = 1$
$F_n = F_{n-1} + F_{n-2}$ for $n \in \mathbb{Z}^+$ with $n \geq 2$
Prove that:

$$F_0F_1 + F_1F_2 + ... + F_{2n-1}F_{2n} = {F_{2n}}^2$$

(b) By looking at first few values of n, find a formula for $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \ldots + \frac{1}{n(n+1)}$ for $n \geq 1$. Then, prove that your formula is correct.

P5 [20 points] When using the Pigeonhole Principle, clearly indicate pigeons and pigeonholes.

(a) Show that if we choose nine points having integer coordinates from the 3D Cartesian space, the midpoint of at least one pair of these points has to have integer coordinates.

(b) Suppose that there are t people in a room having ages $a_1, a_2, \ldots, a_t$ who want candles for their birthday cake. To celebrate their birthday, each person needs as many candles as their age. Show that if we randomly distribute $a_1 + a_2 + \ldots + a_t - t + 1$ candles to these people, at least one person can celebrate his/her birthday.