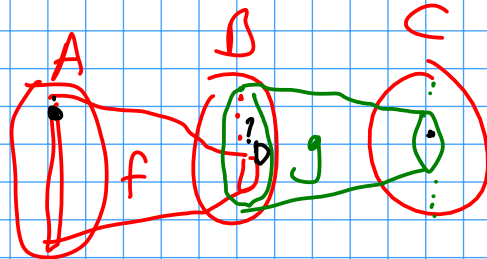
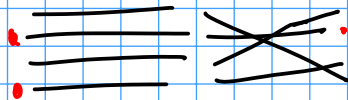


$$f(x) = x^2 \quad g(x) = x + 3$$

$|A| \geq |B|$ onto = surjective = *öften*
 $|A| \leq |B|$ one to one = injective = *bijectiv*
 bijective = *bijectiv*
 $|A| = |B|$

$$\rightarrow f: ? \rightarrow ? \rightarrow g: ? \rightarrow ?$$



$$A = \{1, 2, 3\}$$

$$B = \{a, b\}$$

	1	2	3
1	a	b	b
2	b	a	a
3			

$$g(f(x))$$

$$f(1,1) = a$$

$$f(2,1) = b$$

$$f(3,1) = b..$$

The set of integers

$$\mathbb{Z}$$

is closed under addition

$$f(x) = \sqrt{x} \quad f: \mathbb{C} \rightarrow \mathbb{C}$$

$$\text{add } f(5, 2) = 7$$

$$\text{mult}(7, 2) = 14$$

$$\text{div}(30, 17) = \text{not int}$$

\mathbb{Z} is not closed under div

$$f: \mathbb{Q} \setminus \{0\} \times \mathbb{Q} \setminus \{0\} \rightarrow \mathbb{Q} \setminus \{0\} \quad f(a, b) = \frac{a}{b}$$

$\mathbb{Q} \setminus \{0\}$ is closed under div

$$f(7, 2) = f(2, 7)$$

$$7 \times 2 = 2 \times 7$$

$$f(a, b) = a - b$$

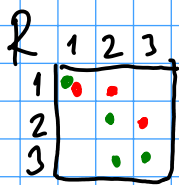
$$(5 - 3) - 2$$

$$0$$

$$5 - (3 - 2)$$

$$4$$

$$\frac{24/8/2}{3/2} ? \quad \times$$



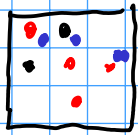
$$A = \{1, 2, 3\}$$

$$R: A \rightarrow A$$

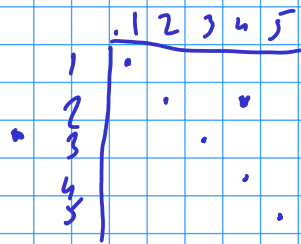
$$R \subseteq A \times A$$

$$R = \{(1,1), (1,2), (2,3)\} \text{ not reflexive}$$

$$R = \{(1,1), (2,2), (3,3), (3,2)\} \text{ is reflexive}$$

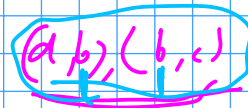
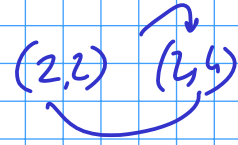


✓ α



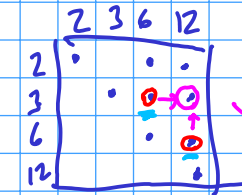
not sym.

trans.



(a,c)

$$R = \{(2,2), (2,6), (2,12), (3,3), (3,6), (3,12), (6,12), (6,6)\}$$



$$A = \{2, 3, 6, 12\}$$

" \geq " is an antisym. rel

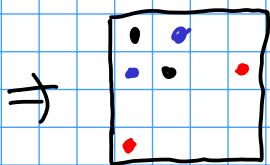
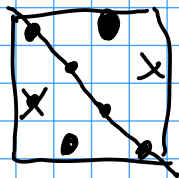
$$a \geq b \quad b \geq a \rightarrow a = b$$

$$7 \geq 5 \quad 5 \geq 7$$

$$\begin{array}{cc} p & q \\ 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{array}$$

$$p \rightarrow q$$

a, b b, a



Sym & antisym at the same time

$$R = \{(1,1), (2,2)\}$$

$$R = \{\}$$

neither sym nor antisym.

$$\Rightarrow R = \{(2,1), (4,3), (3,4)\}$$

$$5^{(1)} \geq 5^{(2)} \geq 2 \geq 1$$

$$5^{(2)} \geq 5^{(1)} \geq 2 \geq 1$$

$$\mathbb{Z}^+$$

divisibility

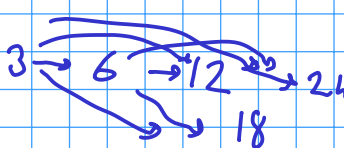
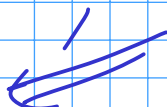
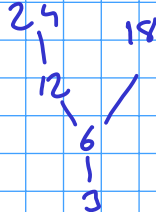
prim

$$24 \mid 18 \mid 12 \mid 6 \mid 3$$

$$x \mid x \text{ red.}$$

$$x \mid y \wedge y \mid x \rightarrow y = x \text{ ant.}$$

$$x \mid y \wedge y \mid z \rightarrow x \mid z \text{ trans.}$$



P.H.P.

n : # of pigeons m : # p.holes

at least one p.h. has $\lfloor \frac{n-1}{m} \rfloor + 1$ pigeons

$n=32 \quad m=5$

$\frac{32-1}{5} + 1 = 7$ pigeons

min x

$\frac{28-1}{28} + 1 = 2$

$\lfloor \frac{82-1}{81} \rfloor + 1 = 2$

pigeons: people
p.holes: cities

players: p.holes $22+6 = 28$
y. cards: pigeons. $28 \rightarrow 29$



$\frac{24}{2} = 12$ p.h.

$\lfloor \frac{13-1}{12} \rfloor + 1 = 2$

Rule of thumb keep pigeonholes
exclusive.
non-intersect

Pigeons: humans

p.holes. cons. floors $(2n, 2n+1)$ for $n=0, 1, \dots, 11$

$(25+1)(25+1)$

$\begin{array}{r} 675 \\ 50 \\ 1 \\ \hline 676 \end{array}$

$\left(\frac{677-1}{676} \right) + 1$

pigeons: people

677

p.hole: init. hab. am. = 676

