Discrete Mathematics Handout

1 Laws of Logic

For any primitive statements p, q, r, any tautology T_0, and any contradiction F_0,

$\neg\neg p \Leftrightarrow p$	Double Negation Law
$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$ $\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$	DeMorgan's Laws
$p \vee q \Leftrightarrow q \vee p$ $p \wedge q \Leftrightarrow q \wedge p$	Commutative Laws
$p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r$ $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r$	Associative Laws
$p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$	Distributive Laws
$p \vee p \Leftrightarrow p$ $p \wedge p \Leftrightarrow p$	Idempotent Laws
$p \vee F_0 \Leftrightarrow p$ $p \wedge T_0 \Leftrightarrow p$	Identity Laws
$p \vee \neg p \Leftrightarrow T_0$ $p \wedge \neg p \Leftrightarrow F_0$	Inverse Laws
$p \vee T_0 \Leftrightarrow T_0$ $p \wedge F_0 \Leftrightarrow F_0$	Domination Laws
$p \vee (p \wedge q) \Leftrightarrow p$ $p \wedge (p \vee q) \Leftrightarrow p$	Absorption Laws
$\neg p \vee q \Leftrightarrow p \rightarrow q$	(Equivalence)
$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$	Contrapositive

2 Laws of Set Theory

For any sets A, B, and C, taken from a universe $\mathcal{U}$,

$\overline{\overline{A}} = A$	Double Comp.
$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$	DeMorgan's Laws
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Laws
$A \cup (B \cup C) = (A \cup B) \cup C$ $A \cap (B \cap C) = (A \cap B) \cap C$	Associative Laws
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive Laws
$A \cup A = A$ $A \cap A = A$	Idempotent Laws
$A \cup \emptyset = A$ $A \cap \mathcal{U} = A$	Identity Laws
$A \cup \overline{A} = \mathcal{U}$ $A \cap \overline{A} = \emptyset$	Inverse Laws
$A \cup \mathcal{U} = \mathcal{U}$ $A \cap \emptyset = \emptyset$	Domination Laws
$A \cup (A \cap B) = A$ $A \cap (A \cup B) = A$	Absorption Laws

3 Rules of Inference

Rule of Inference	Name of the Rule
1) $\dfrac{\begin{array}{l} p \\ p \rightarrow q \end{array}}{\therefore q}$	Rule of Detachment (Modus Ponens)
2) $\dfrac{\begin{array}{l} p \rightarrow q \\ q \rightarrow r \end{array}}{\therefore p \rightarrow r}$	Law of Syllogism
3) $\dfrac{\begin{array}{l} p \rightarrow q \\ \neg q \end{array}}{\therefore \neg p}$	Modus Tollens
4) $\dfrac{\begin{array}{l} p \\ q \end{array}}{\therefore p \wedge q}$	Rule of Conjunction
5) $\dfrac{\begin{array}{l} p \vee q \\ \neg p \end{array}}{\therefore q}$	Rule of Disjunctive Syllogism
6) $\dfrac{\neg p \rightarrow F_0}{\therefore p}$	Rule of Contradiction
7) $\dfrac{p \wedge q}{\therefore p}$	Rule of Conjunctive Simplification
8) $\dfrac{p}{\therefore p \vee q}$	Rule of Disjunctive Amplification
9) $\dfrac{\begin{array}{l} p \wedge q \\ p \rightarrow (q \rightarrow r) \end{array}}{\therefore r}$	Rule of Conditional Proof
10) $\dfrac{\begin{array}{l} p \rightarrow r \\ q \rightarrow r \end{array}}{\therefore (p \vee q) \rightarrow r}$	Proof by Cases
11) $\dfrac{\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ p \vee r \end{array}}{\therefore q \vee s}$	Rule of the Constructive Dilemma
12) $\dfrac{\begin{array}{l} p \rightarrow q \\ r \rightarrow s \\ \neg q \vee \neg s \end{array}}{\therefore \neg p \vee \neg r}$	Rule of Destructive Dilemma

4 Quantifier Rules

Rule	Name of the Rule
1) $\dfrac{\forall x\ p(x)}{\therefore p(a) \quad \text{[arbitrary a]}}$	Rule of Universal Specification
2) $\dfrac{p(a) \quad \text{[arbitrary a]}}{\therefore \forall x\ p(x)}$	Rule of Universal Generalization
3) $\dfrac{\exists x\ p(x)}{\therefore p(a) \quad \text{[particular a]}}$	Rule of Existential Specification
4) $\dfrac{p(a) \quad \text{[particular a]}}{\therefore \exists x\ p(x)}$	Rule of Existential Generalization