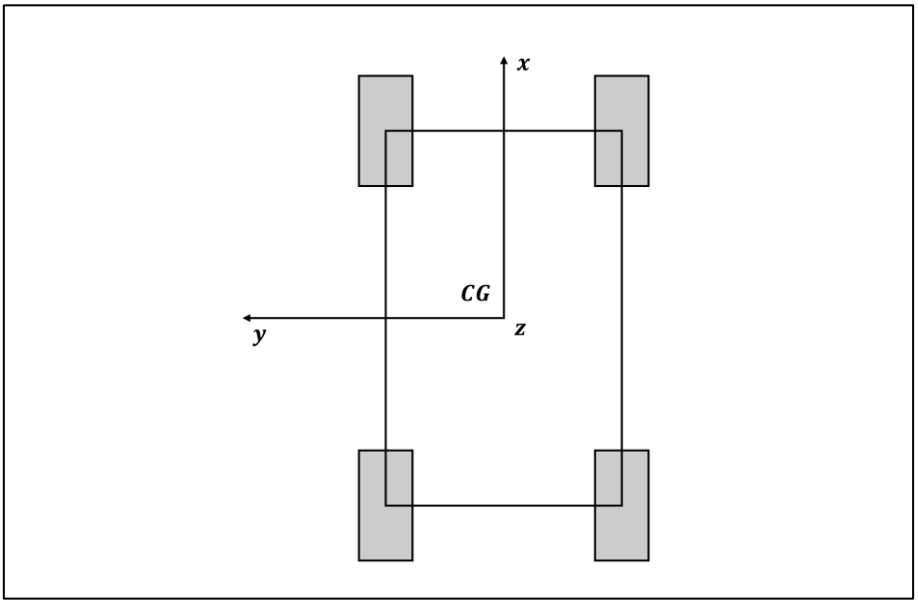
3-DOF RWD TV LPV Model

1. 모델 정의, 파라미터 및 변수

1.1. 상태공간 모델

$$\dot{x} = Ax + Bu + Dd + E$$

1.2. 좌표계 및 변수 정의



차량 무게중심(CG) 기준 좌표계 (x: 종방향, y: 횡방향, z: 수직방향)

상태변수 x

$$x = \begin{bmatrix} r \\ \beta \\ v_x \end{bmatrix}$$

r : 요 각속도 [rad/s]

β : 차체 슬립각 [rad]

v_x : 종방향 속도 [m/s]

제어 입력 u

$$u = \begin{bmatrix} T_{RL} \\ T_{RR} \end{bmatrix}$$

T_{RL}, T_{RR} : 후륜 좌/우 바퀴에 인가되는 구동 토크 [N·m]

외란 입력 d

$$d = [\delta]$$

δ: 운전자의 앞바퀴 조향각 [rad]

1.3. 파라미터

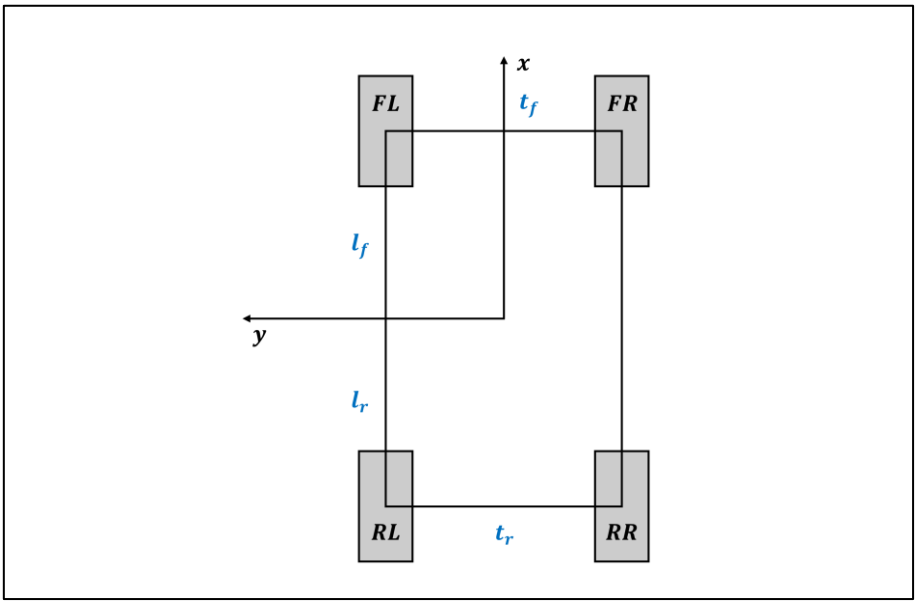
기호	의미	단위
m	차량 전체 질량	kg
I_z	무게중심 기준 요 관성 모멘트	kg·m²
l_f, l_r	무게중심에서 앞/뒤 차축까지의 거리	m
t_f, t_r	앞/뒤 바퀴의 윤거	m
r_e	타이어의 유효 구름 반경	m
$C_{\alpha f}$, $C_{\alpha r}$	앞/뒤 차축의 코너링 강성$(C_{\alpha f} = 2C_{\alpha fl}, C_{\alpha r} = 2C_{\alpha rl})$	N/rad
ρ	공기 밀도	kg/m³
A_f	차량 전면 투영 면적	m²
C_d	공기 저항 계수	-
f	구름 저항 계수	-
g	중력 가속도	m/s²

1.4. 중간 계산 변수

기호	의미	단위
α_i	각 타이어의 슬립각	rad
$F_{x,i}$	각 타이어의 종방향 힘	N
$F_{y,i}$	각 타이어의 횡방향 힘	N
F_{aero}	공기 저항력	N
R_{roll}	구름 저항력	N

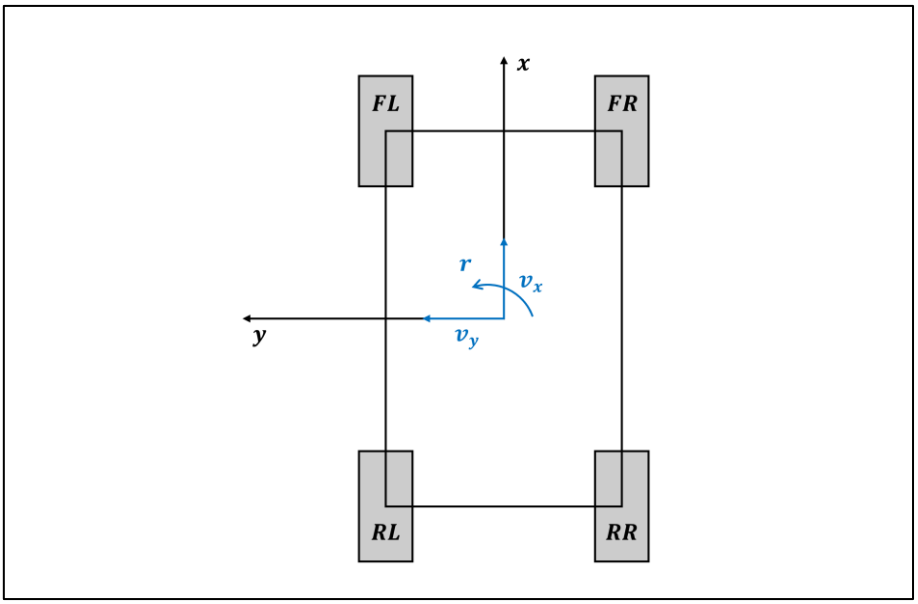
2. 타이어 속도 및 힘 모델링

2.1. 각 타이어 속도 (차체 좌표계 기준)



$$vy \approx vx \cdot \beta$$

$$(tan\beta \approx \beta)$$



$$FL = \left(l_f, \frac{t_f}{2}\right)$$

$$v_{x,FL} = v_x - \frac{rt_f}{2}$$

$$v_{y,FL} = v_y + rl_f = \beta v_x + rl_f$$

$$FR = \left(l_f, -\frac{t_f}{2}\right)$$

$$v_{x,FR} = v_x + \frac{rt_f}{2}$$

$$v_{y,FR} = v_y + rl_f = \beta v_x + rl_f$$

$$RL = \left(-l_r, \frac{t_r}{2}\right)$$

$$v_{x,RL} = v_x - \frac{rt_r}{2}$$

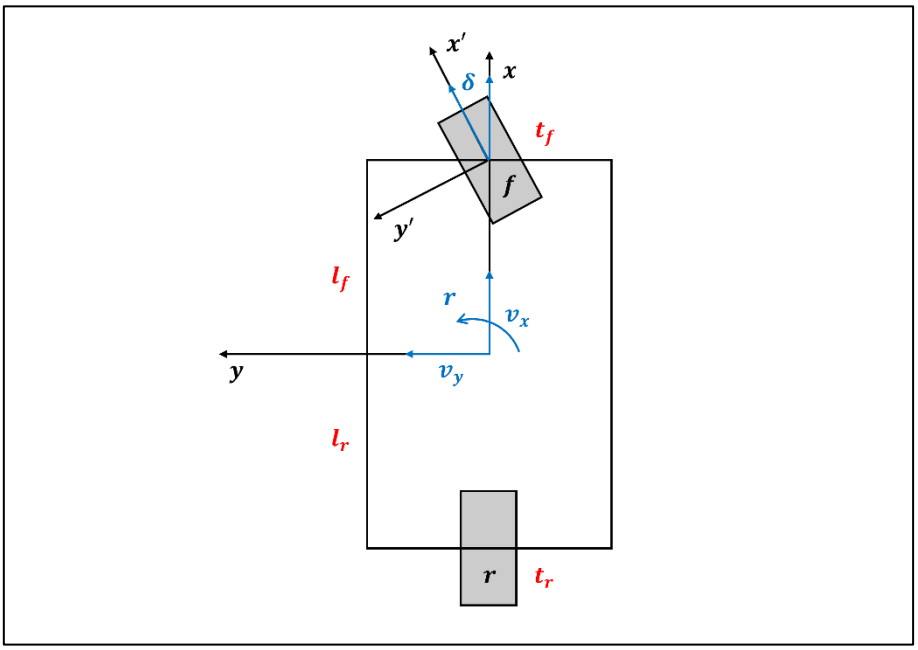
$$v_{y,RL} = v_y - rl_r = \beta v_x - rl_r$$

$$RR = \left(-l_r, -\frac{t_r}{2}\right)$$

$$v_{x,RR} = v_x + \frac{rt_r}{2}$$

$$v_{y,RR} = v_y - rl_r = \beta v_x - rl_r$$

2.2. 타이어 슬립각



$$\alpha_i \approx \tan^{-1}\left(\frac{v_{y',i}}{v_{x',i}}\right) \approx \frac{v_{y',i}}{v_{x',i}}$$

$$(v_{y'} \approx -v_x \cdot \sin\delta + v_y \cdot \cos\delta)$$

$$(\sin\delta \approx \delta,\ \cos\delta \approx 1)$$

$$\alpha_{fl} \approx \frac{v_{y,FL} - v_{x,FL} \cdot \delta}{v_{x,FL}} = \frac{(\beta v_x + r l_f) - \left(v_x - \frac{r t_f}{2}\right) \cdot \delta}{v_x - \frac{r t_f}{2}}$$

$$\alpha_{fr} \approx \frac{v_{y,FR} - v_{x,FR} \cdot \delta}{v_{x,FR}} = \frac{(\beta v_x + r l_f) - \left(v_x + \frac{r t_f}{2}\right) \cdot \delta}{v_x + \frac{r t_f}{2}}$$

$$v_x \approx v_{x,i}$$

$$v_x \gg \frac{r t_f}{2}$$

$$\alpha_f \approx \beta + \frac{r l_f}{v_x} - \delta$$

$$\delta = 0 \text{ (뒷바퀴)}$$

$$\alpha_{rl} \approx \frac{v_{y,RL}}{v_{x,RL}} = \frac{\beta v_x - r l_r}{v_x - \frac{r t_r}{2}}$$

$$\alpha_{rr} \approx \frac{v_{y,RR}}{v_{x,RR}} = \frac{\beta v_x - r l_r}{v_x + \frac{r t_r}{2}}$$

$$\alpha_r \approx \beta - \frac{r l_r}{v_x}$$

2.3. 타이어 힘

$$F_y = -C_\alpha \cdot \alpha$$

$$F_{yf} = -C_{\alpha f} \cdot \alpha_f = -C_{\alpha f} \cdot \left(\beta + \frac{r l_f}{v_x} - \delta\right)$$

$$F_{yr} = -C_{\alpha r} \cdot \alpha_r = -C_{\alpha r} \cdot \left(\beta - \frac{r l_r}{v_x}\right)$$

2.4 종방향 힘

$$F_{x,RL} = \frac{T_{RL}}{r_e}$$

$$F_{x,RR} = \frac{T_{RR}}{r_e}$$

3. 차량 운동방정식 유도

3.1. 요 운동

$$I_z \cdot \dot{r} = (F_{y,FL} + F_{y,FR}) \cdot l_f - (F_{y,RL} + F_{y,RR}) \cdot l_r + (F_{x,RR} - F_{x,RL}) \cdot \frac{t_r}{2}$$

$$I_z \cdot \dot{r} = F_{yf} \cdot l_f - F_{yr} \cdot l_r + (T_{RR} - T_{RL}) \cdot \frac{t_r}{2r_e}$$

$$I_z \cdot \dot{r} = l_f \cdot \left[-C_{\alpha f}\left(\beta + \frac{r l_f}{v_x} - \delta\right)\right] - l_r \cdot \left[-C_{\alpha r}\left(\beta - \frac{r l_r}{v_x}\right)\right] + (T_{RR} - T_{RL}) \cdot \frac{t_r}{2r_e}$$

$$I_z \cdot \dot{r} = \left[-\frac{l_f^2 C_{\alpha f} + l_r^2 C_{\alpha r}}{v_x}\right] \cdot r + \left[-(l_f C_{\alpha f} - l_r C_{\alpha r})\right] \cdot \beta + \left[l_f C_{\alpha f}\right] \cdot \delta + (T_{RR} - T_{RL}) \cdot \frac{t_r}{2r_e}$$

$$\dot{r} = \left[-\frac{l_f^2 C_{\alpha f} + l_r^2 C_{\alpha r}}{I_z v_x}\right] \cdot r + \left[-\frac{l_f C_{\alpha f} - l_r C_{\alpha r}}{I_z}\right] \cdot \beta + \left[\frac{l_f C_{\alpha f}}{I_z}\right] \cdot \delta + \left[-\frac{t_r}{2 I_z r_e}\right] \cdot T_{RL} + \left[\frac{t_r}{2 I_z r_e}\right] \cdot T_{RR}$$

3.2. 횡방향 운동

$$m(v_x \cdot \dot{\beta} + v_x \cdot r) = F_{yf} + F_{yr}$$

$$\dot{\beta} = \frac{F_{yf} + F_{yr}}{m v_x} - r$$

$$\dot{\beta} = \frac{1}{m v_x}\left[-C_{\alpha f}\left(\beta + \frac{r l_f}{v_x} - \delta\right) - C_{\alpha r}\left(\beta - \frac{r l_r}{v_x}\right)\right] - r$$

$$\dot{\beta} = \left[-\frac{C_{\alpha f} + C_{\alpha r}}{m v_x}\right] \cdot \beta + \left[-\frac{l_f C_{\alpha f} - l_r C_{\alpha r}}{m v_x^2}\right] \cdot r + \left[\frac{C_{\alpha f}}{m v_x}\right] \cdot \delta - r$$

$$\dot{\beta} = \left[-\frac{l_f C_{\alpha f} - l_r C_{\alpha r}}{m v_x^2} - 1\right] \cdot r + \left[-\frac{C_{\alpha f} + C_{\alpha r}}{m v_x}\right] \cdot \beta + \left[\frac{C_{\alpha f}}{m v_x}\right] \cdot \delta$$

3.3. 종방향 운동

$$m\dot{v}_x = F_{x,RL} + F_{x,RR} - F_{aero} - R_{roll}$$

$$F_{aero}(v_x) \approx F_{aero}(v_{x0}) + \dot{F}_{aero}(v_{x0}) \cdot (v_x - v_{x0})$$

$$F_{aero}(v_x) \approx \frac{1}{2}\rho A_f C_d v_{x0}^2 + \rho A_f C_d v_{x0} \cdot (v_x - v_{x0})$$

$$F_{aero}(v_x) \approx (\rho A_f C_d v_{x0}) \cdot v_x - \frac{1}{2}\rho A_f C_d v_{x0}^2$$

$$R_{roll} = fmg$$

$$\Sigma F_{resistance} = F_{aero} + R_{roll}$$

$$m\dot{v}_x = \frac{T_{RL}}{r_e} + \frac{T_{RR}}{r_e} - \left[(\rho A_f C_d v_{x0}) \cdot v_x - \frac{1}{2}\rho A_f C_d v_{x0}^2\right] - fmg$$

$$\dot{v}_x = \left[-\frac{\rho A_f C_d v_{x0}}{m}\right] \cdot v_x + \left[\frac{1}{mr_e}\right] \cdot T_{RL} + \left[\frac{1}{mr_e}\right] \cdot T_{RR} + \left[\frac{\rho A_f C_d v_{x0}^2}{2m} - fg\right]$$

4. 상태 공간 모델

$$x = \begin{bmatrix} r \\ \beta \\ v_x \end{bmatrix}, \qquad u = \begin{bmatrix} T_{RL} \\ T_{RR} \end{bmatrix}, \qquad d = [\delta]$$

4.1 시스템 행렬 A

	r	β	v_x
$\dot{r}$	$-\frac{l_f^2 C_{\alpha f} + l_r^2 C_{\alpha r}}{I_z v_x}$	$-\frac{l_f C_{\alpha f} - l_r C_{\alpha r}}{I_z}$	0
$\dot{\beta}$	$-\frac{l_f C_{\alpha f} - l_r C_{\alpha r}}{m v_x^2} - 1$	$-\frac{C_{\alpha f} + C_{\alpha r}}{m v_x}$	0
$\dot{v}_x$	0	0	$-\frac{\rho A_f C_d v_{x0}}{m}$

4.2 제어 입력 행렬 B

	T_{RL}	T_{RR}
$\dot{r}$	$-\frac{t_r}{2 I_z r_e}$	$\frac{t_r}{2 I_z r_e}$
$\dot{\beta}$	0	0
$\dot{v}_x$	$\frac{1}{m r_e}$	$\frac{1}{m r_e}$

4.3 외란 입력 행렬 D

	δ
$\dot{r}$	$\frac{l_f C_{\alpha f}}{I_z}$
$\dot{\beta}$	$\frac{C_{\alpha f}}{m v_x}$
$\dot{v}_x$	0

4.4 상수 행렬 E

	Val
$\dot{r}$	0
$\dot{\beta}$	0
$\dot{v}_x$	$\frac{\rho A_f C_d v_{x0}^2}{2m} - fg$