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## BENCHMARKING SOLVERS FOR TV- $\ell_1$ PENALIZED LOGISTIC AND LEAST SQUARES REGRESSION: APPLICATION TO BRAIN DATA

Learning predictive models from brain imaging data, as in decoding cognitive states from fMRI (functional Magnetic Resonance Imaging), is typically an ill-posed problem as it entails estimating many more parameters than available sample points. This estimation problem thus requires regularization. Total variation regularization, combined with sparse models, has been shown to yield good predictive performance, as well as stable and interpretable maps. However, the corresponding optimization problem is very challenging. Here we explore a wide variety of solvers and exhibit their convergence properties on fMRI data. Our findings show that care must be taken in solving TV- $\ell_1$  estimation in brain imaging. We highlight the successful strategies.

### PROBLEM STATEMENT

$$\text{Minimize } E(w) := \mathcal{L}(X, y, w) + \alpha (\rho \|w\|_1 + (1 - \rho) TV(w)) \quad (1)$$

For  $w \in \mathbb{R}^p$

where :

- $\mathcal{L}(X, y, w)$  is the **loss** term = the loss incurred by using the **brain map**  $w \in \mathbb{R}^p$  to **predict** a sample  $y \in \mathbb{R}^n$  of  $n$  **responses** from a sample  $X \in \mathbb{R}^{n \times p}$  of  $n$  corresponding **brain images**.
- $X$  is commonly called the **design matrix** whilst  $y$  is the **response variate**.
- $\mathcal{L}(X, y, w) \equiv \frac{1}{2} \|y - Xw\|_2^2$  for linear regression, etc.
- $n \ll p$  for brain data (**high-dimensional problem**)  $\Rightarrow$  need for regularization
- Typically,  $n \sim 10 - 10^2$  brain images and  $p \sim 10^4 - 10^6$  voxels
- $\alpha (\rho \|w\|_1 + (1 - \rho) TV(w))$  is the **regularization** (aka **penalty**) term (Michel et al. (2011); Baldassarre et al. (2012); Gramfort et al. (2013)) :
  - Encodes “**sparsity + spatial structure**” prior on the optimal brain map  $\hat{w}$ .
  - $\alpha \geq 0$  : overall amount of regularization (tradeoff between data and regularization).
  - $\|w\|_1$  :  $\ell_1$ -norm of  $w$  defined by  $\|w\|_1 := \sum_{j \in \text{voxels}} |w_j|$ .
  - $TV(w)$  : the **isotropic Total-variation** of  $w$  defined by
 
$$TV(w) := \|\nabla(w)\|_{21} := \sum_{j \in \text{voxels}} \sqrt{(\nabla^x w)_j^2 + (\nabla^y w)_j^2 + (\nabla^z w)_j^2},$$
 and  $\nabla := [\nabla^x, \nabla^y, \nabla^z]^T \in \mathbb{R}^{3p \times p}$  is the 3D discrete spatial gradient operator.
  - $\rho \in [0, 1]$  : also called the  **$\ell_1$ -ratio** controls the tradeoff between **sparsity** (enforced by the minimizing  $\ell_1(w)$ ) and **spatial structure** (enforced by minimizing  $TV(w)$ ).

### NEED FOR FAST SOLVERS

Problem (1) is a **high-dimensional non-smooth convex optimization problem** and calls for novel optimization techniques.

- The structuring power of TV- $\ell_1$  only comes into effect for well optimized solutions of problem (1).
- Lack of fast solver and explicit control of tolerance for problem (1) can lead to brain maps and conclusions that reflect properties of the solver more than of the TV- $\ell_1$  solution !

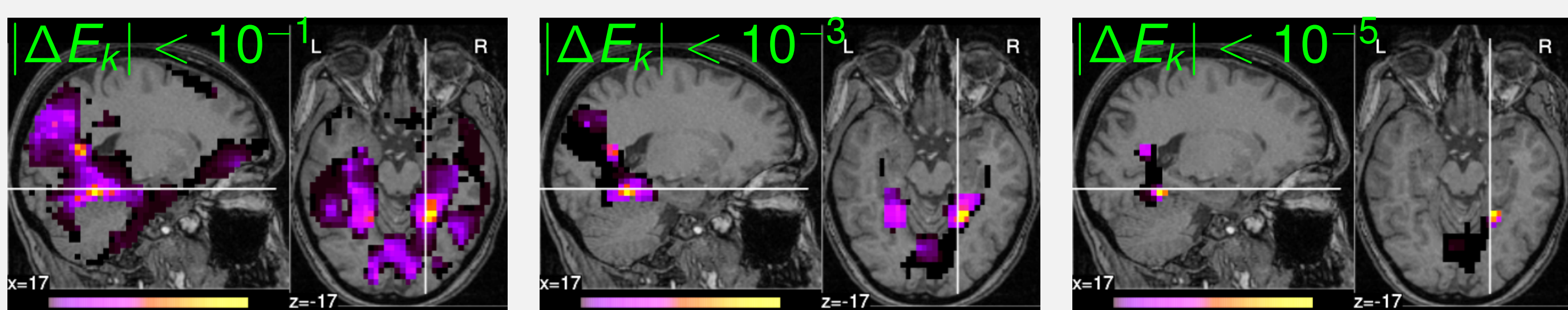


FIGURE : Optimal TV- $\ell_1$  brain maps for the face-house discrimination task on the Haxby et al. (2001) dataset, for various levels of tolerance. Dohmatob et al. (2014)

### RESULTS

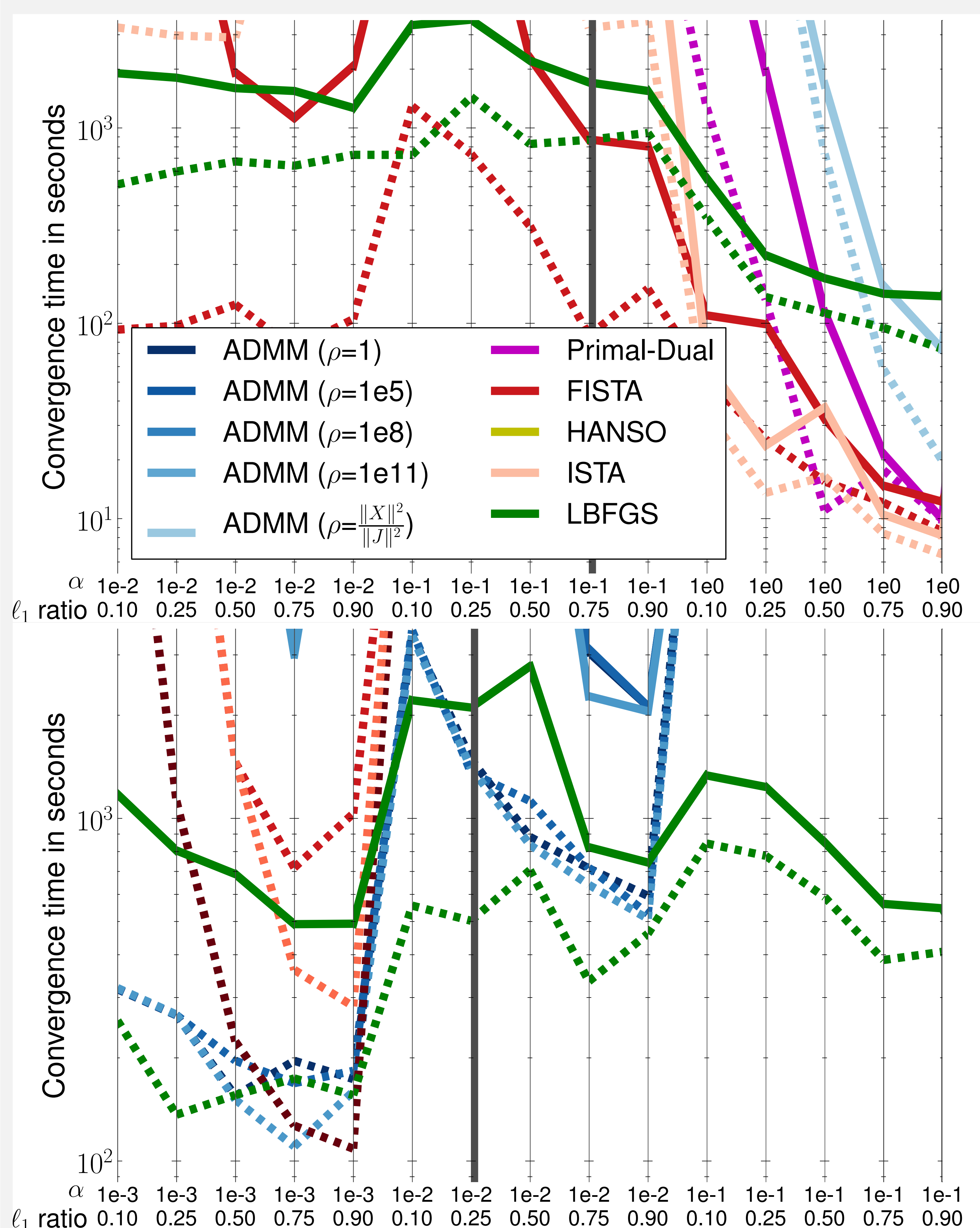


FIGURE : Benchmarks on Haxby et al. (2001) dataset. **Top** : TV- $\ell_1$  penalized least squares regression. **Bottom** : Logistic regression. See Dohmatob et al. (2014)

### CONCLUSION

- TV- $\ell_1$  penalized regression for brain imaging leads to very high-dimensional, non-smooth and very ill-conditioned optimization problems.
- We have presented a comprehensive comparison of state-of-the-art solvers (ADMM, ISTA, FISTA, HANSO, LBFGS, etc.) in these settings.
- Solvers were implemented with all known algorithmic improvements and implementation were carefully profiled and optimized.
- The implemented solvers are part of the open-source Python library *nilearn* : <http://www.github.com/nilearn/nilearn>.
- Our results outline best solvers : **monotonous FISTA** with a adaptive control of the tolerance of the TV proximal operator, in the case of squared loss ; smoothed quasi-newton (for example **LBFGS** here) based on surrogate upper-bounds of the non-smooth TV- $\ell_1$  penalty, for logistic loss.

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