# Modelling inter-subject functional variability

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# Context

- A major goal of human neuroscience is to understand
  - the structure,
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  - inter-subject variability of the human brain
- We will focus on **inter-subject functional variability**

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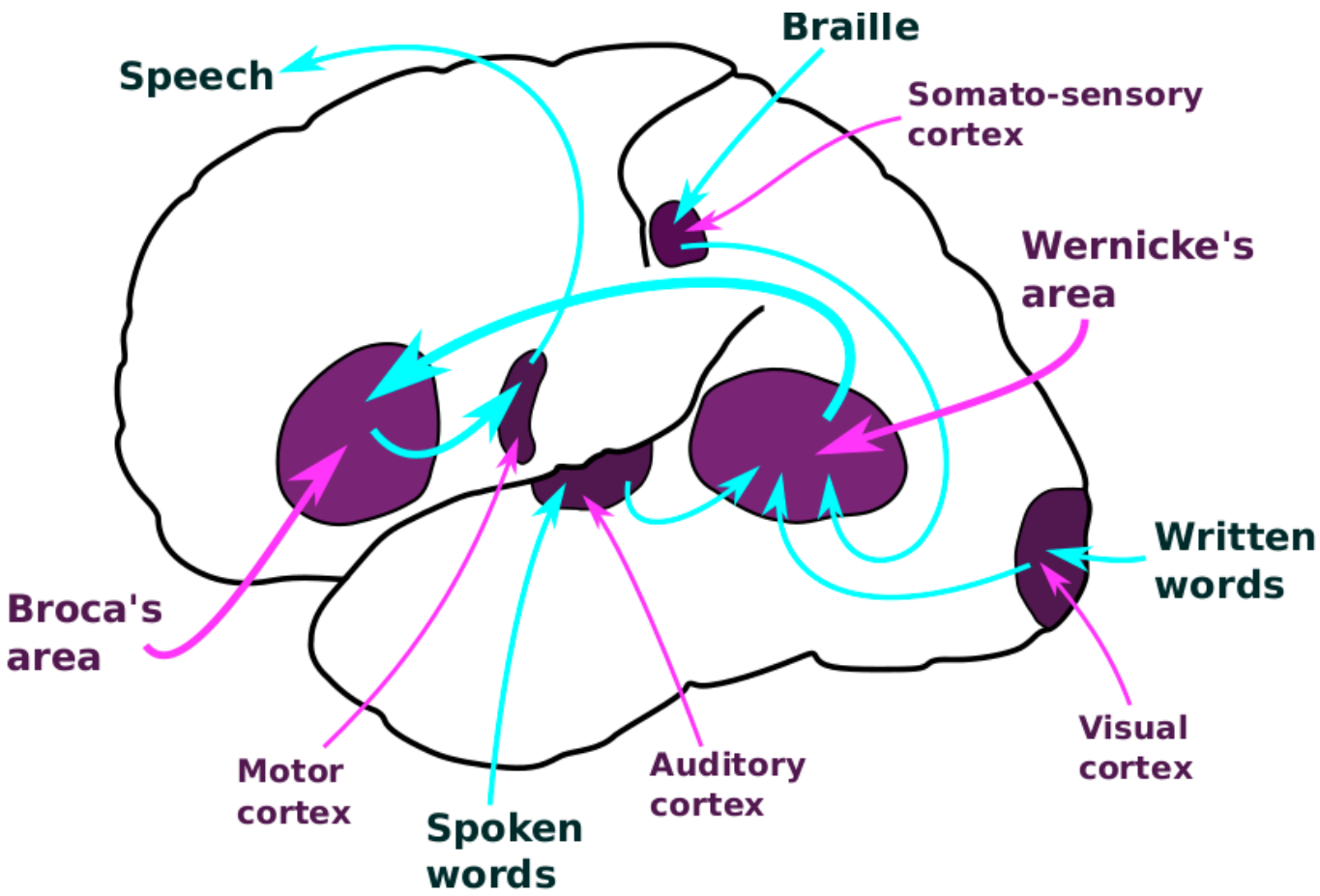
# Introduction

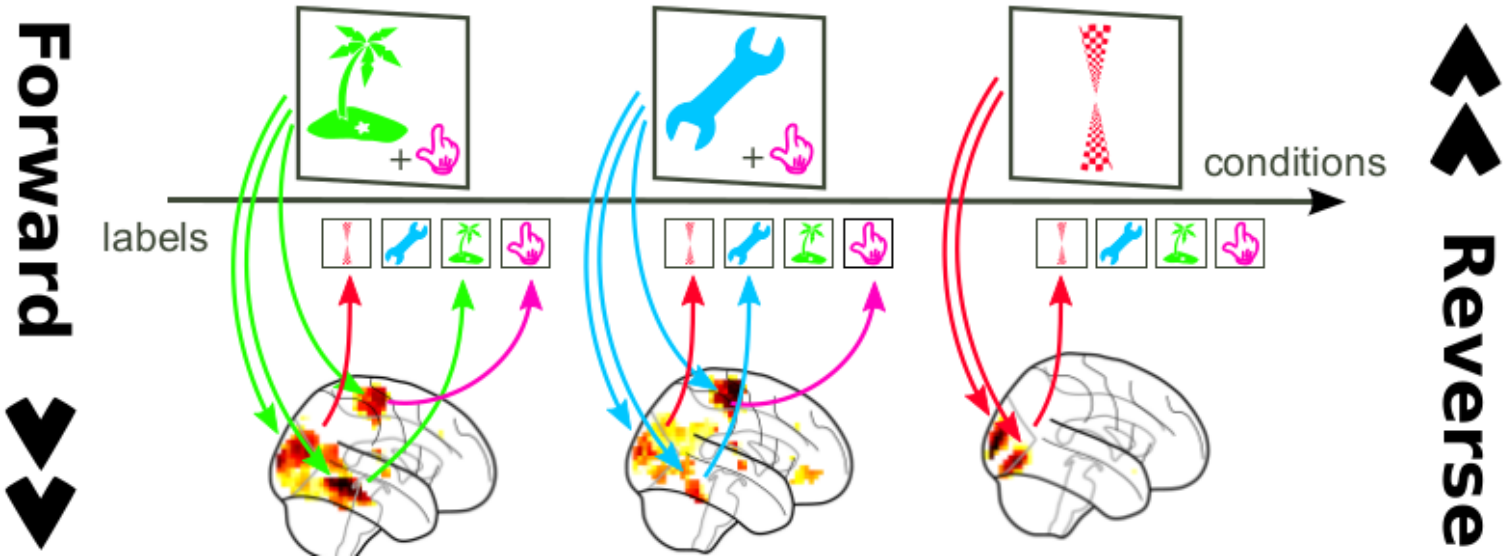
# Brain function regions and networks

## Part of the language network




*(Picture is courtesy of Gael Varoquaux)*

# Mapping cognitive circuits in the brain

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- **Reverse inference / brain-decoding** [Dehaene 98; Cox 03] predicts the experimental condition from brain signals

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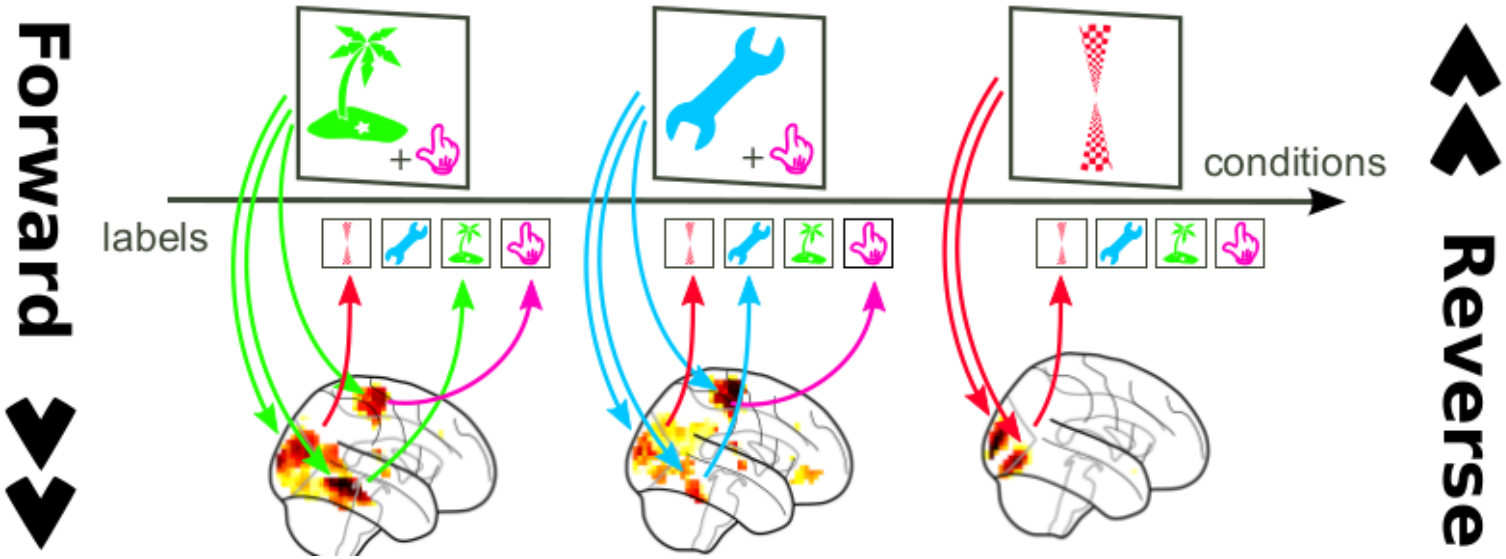


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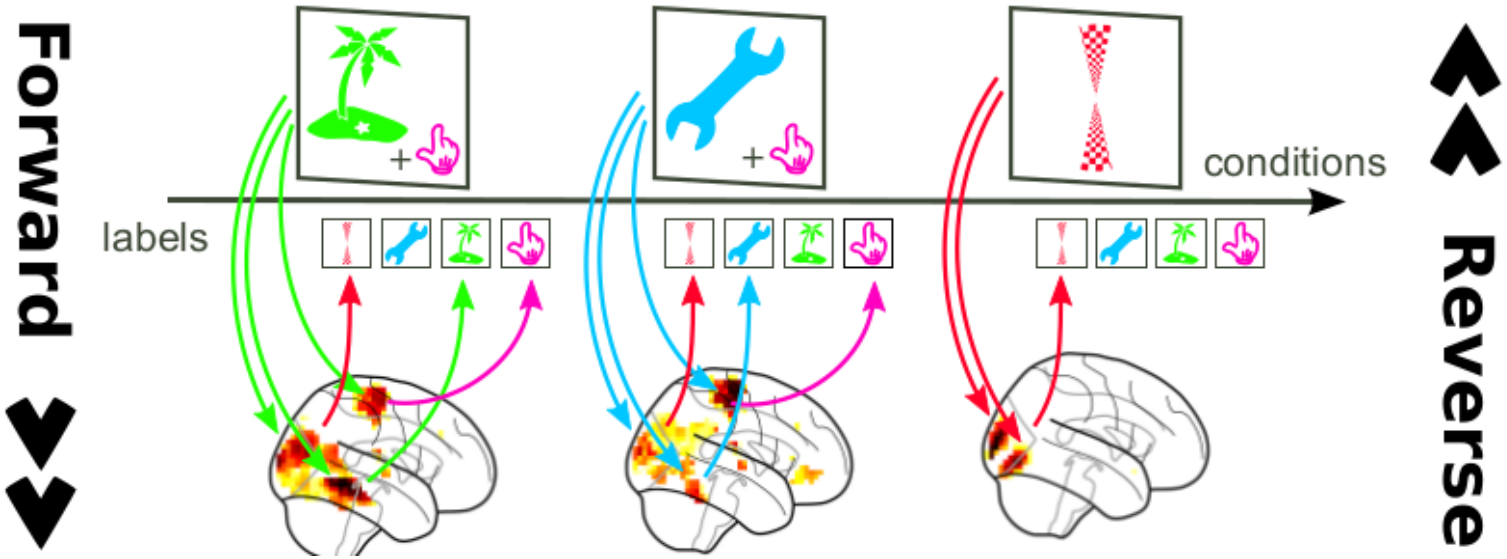


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# A zoom on brain-decoding

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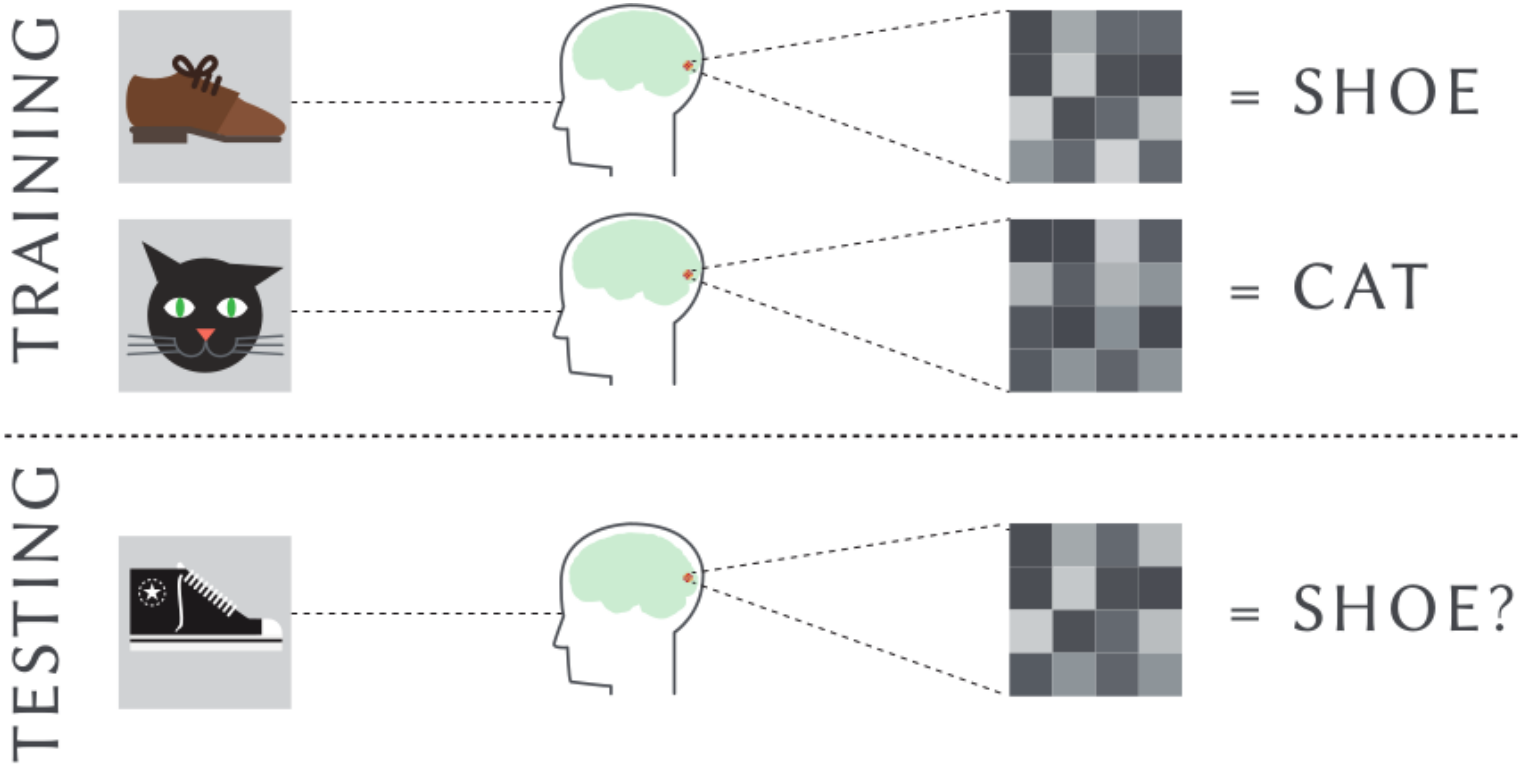


$\mathbf{y}_i$ $\mathbf{x}_i$ $\mathbf{w}$ $\hat{\mathbf{y}}_i$

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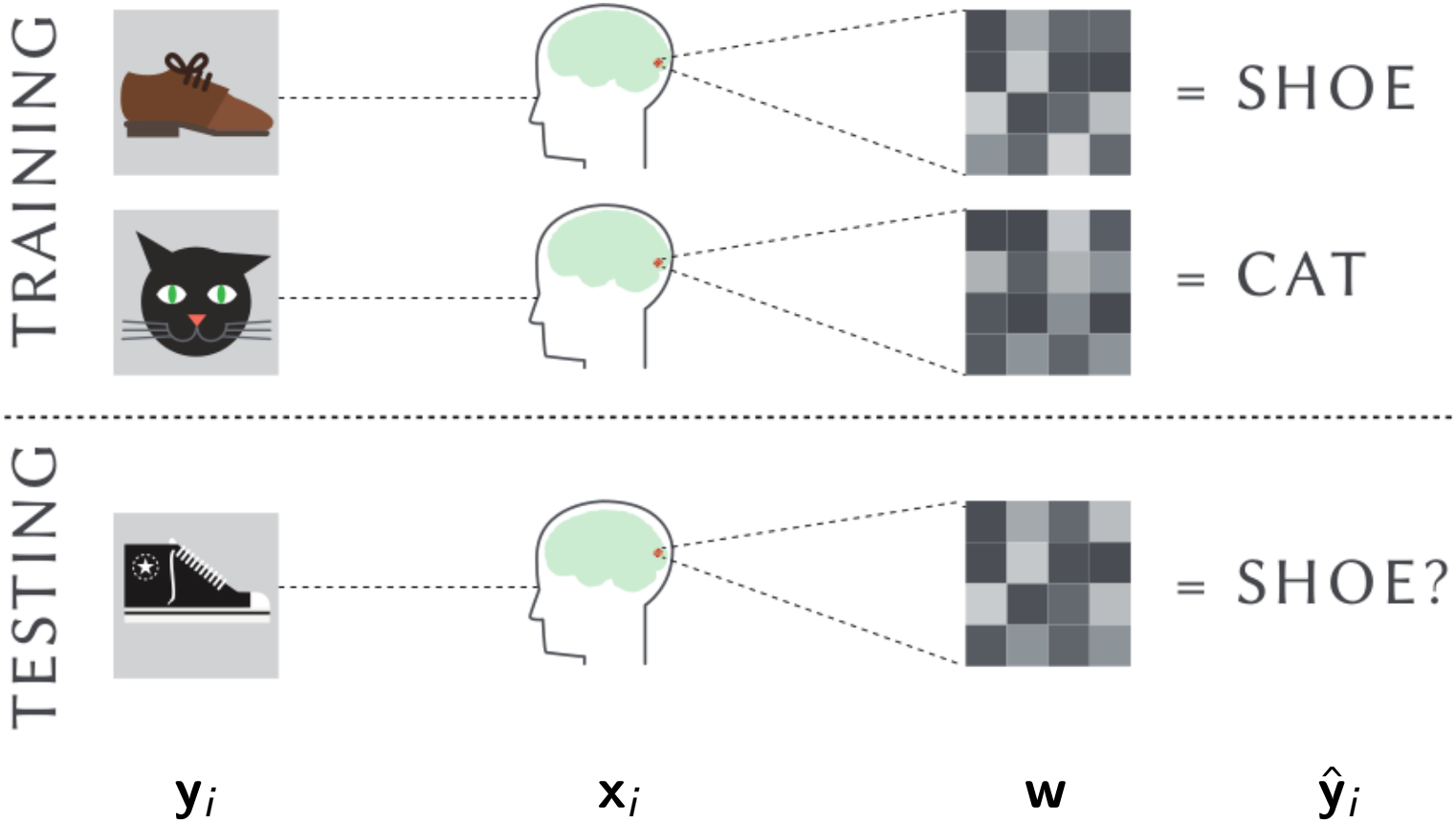
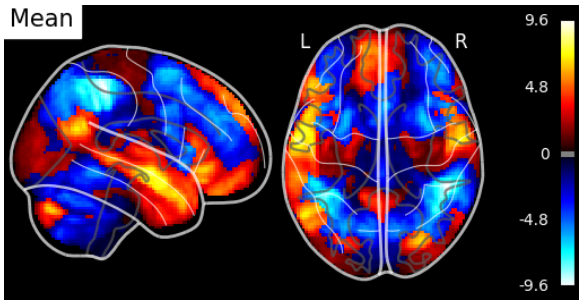



- This is **supervised machine-learning**
- We don't just want good predictions, we want **regions**

# Variability in both location and magnitude of activations

- Story vs Math language contrast of HCP dataset [van Essen '12]



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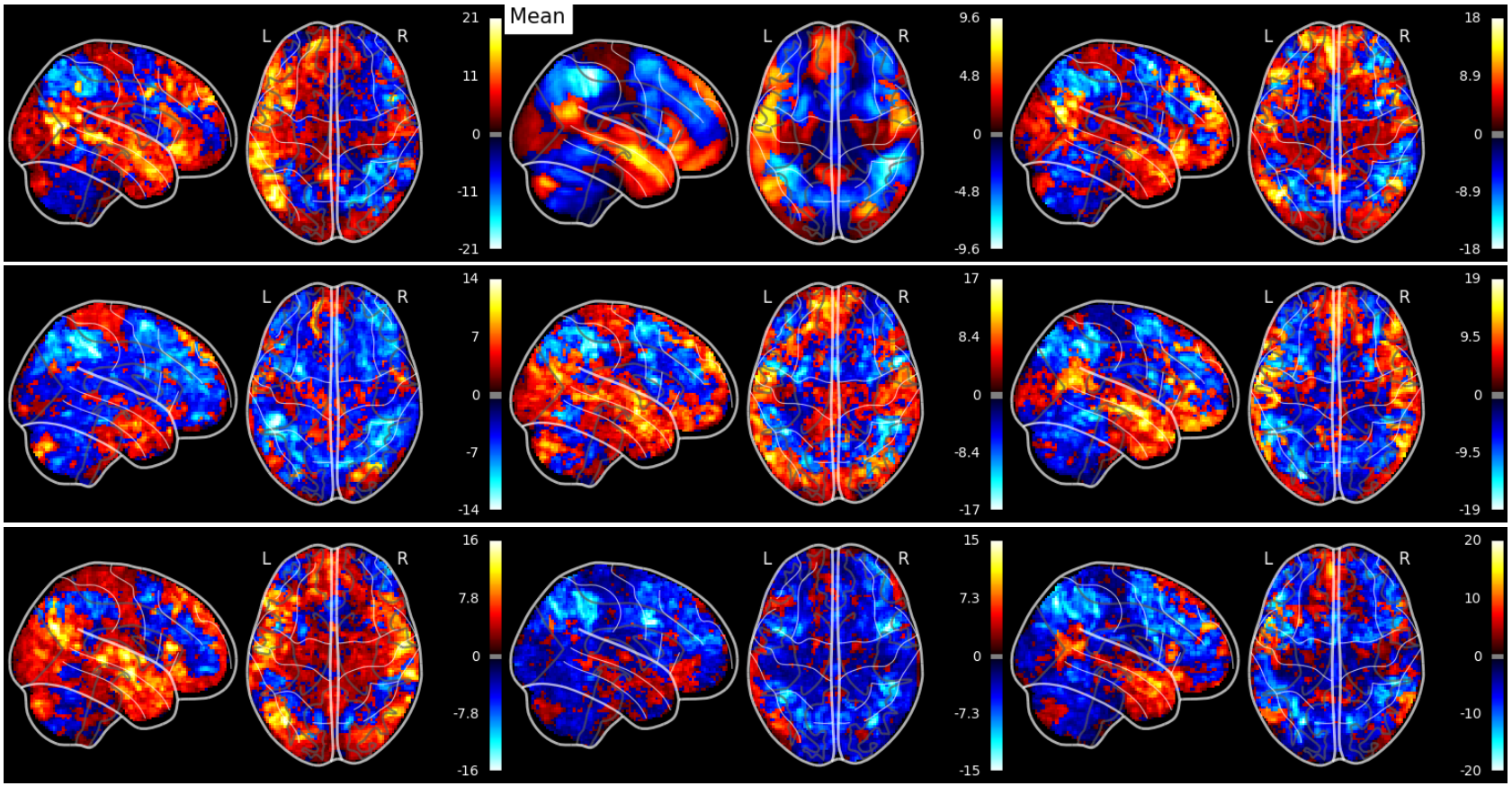

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- Inter-subject functional variability $\neq$ noise!
  - Usually (incorrectly) discarded in standard analysis
  - Is predictive of behavioral differences between individuals
- Cannot be corrected via **spatial normalization**, etc.
  - E.g spatial normalization cannot correct for differences in activation magnitude
- Driven by genetic and behavioral inter-individual differences
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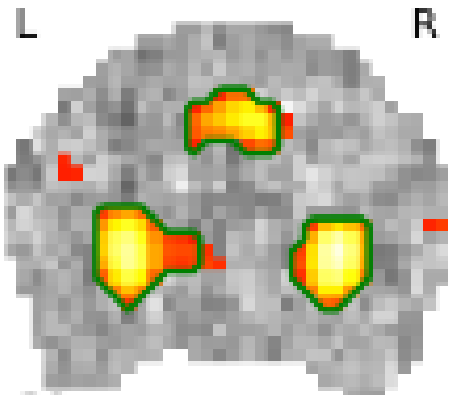
# Mapping the brain with structured multi-variate models

# What we mean by "structured"

**Definition:**

- Localized activation patterns – **sparsity**
- Clusters of active voxels – **smoothness**



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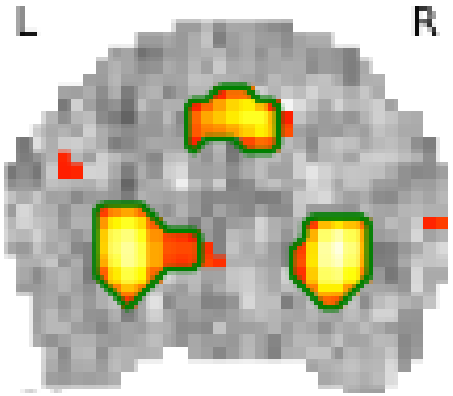


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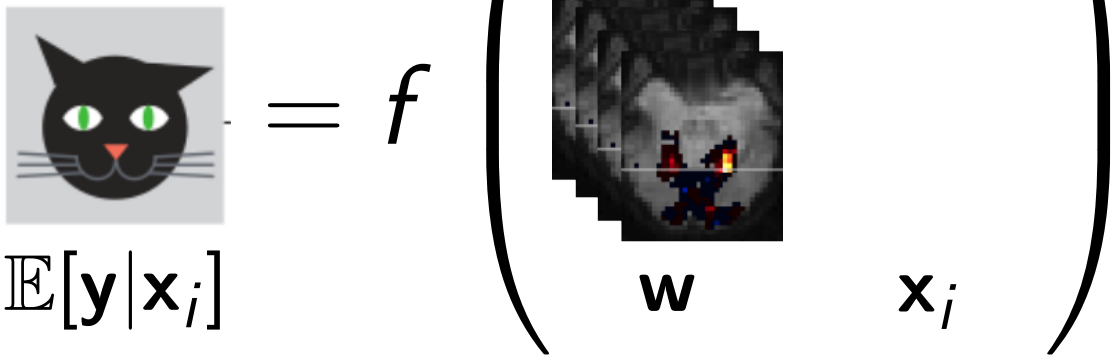
- Such a model is much more **interpretable** (i.e small number of **regions**) than classical methods like SVM, Ridge regression, Lasso
- Performs **model-estimation** and **feature-selection** jointly
- Fights the **curse-of-dimensionality**, via dimensionality reduction.

# Generalized linear models with structured penalties




- **Samples** $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^p$
  - # samples $n \sim 10^3$
  - # **features** $p \sim 10^6$ voxels

# Generalized linear models with structured penalties

$$\mathbb{E}[\mathbf{y}|\mathbf{x}_i] = f\left( \mathbf{w} \quad \mathbf{x}_i \right)$$

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- $f =$ "logit" in **classification**
- $f =$ "id" in **regression**

- Optimization problem:

$$\min_{\mathbf{w} \in \mathbb{R}^p} \underbrace{\frac{1}{n} \sum_{i=1}^{n} \ell(y_i, f(\langle \mathbf{w}, \mathbf{x}_i \rangle))}_{\text{data / loss term}} + \underbrace{\alpha \mathcal{P}(\mathbf{w})}_{\text{penalty}}$$

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- $\ell(y_i, f(\mathbf{x}_i^T \mathbf{w})) = \begin{cases} \frac{1}{2}(y_i - \langle \mathbf{w}, \mathbf{x}_i \rangle)^2, & \text{in regression,} \\ \log(1 + \exp(-y_i \langle \mathbf{w}, \mathbf{x}_i \rangle)), & \text{in classif. (OvR)} \end{cases}$

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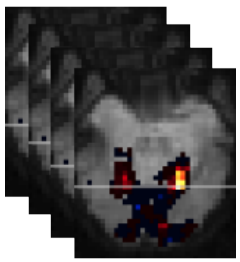
$\mathbb{E}[\mathbf{y}|\mathbf{x}_i] = f\Big($  $\mathbf{w} \qquad \mathbf{x}_i \Big)$

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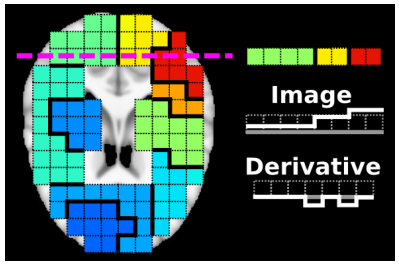
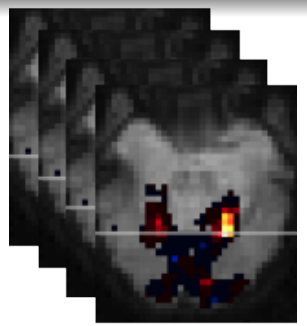



structured penalty



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$$\mathcal{P}(\mathbf{w}) = \begin{cases} \sum_{j\in[\![p]\!]} \rho|\mathbf{w}_j| + \frac{1}{2}(1-\rho)\|(\nabla \mathbf{w})_j\|_2^2, & \text{GraphNet}, \\ \sum_{j\in[\![p]\!]} \rho|\mathbf{w}_j| + (1-\rho)\|(\nabla \mathbf{w})_j\|_2, & \text{isotropic TV-L1}, \\ \sum_{j\in[\![p]\!]} (\rho^2|\mathbf{w}_j|^2 + (1-\rho)^2\|(\nabla \mathbf{w})_j\|_2^2)^{1/2}, & \text{Sparse Variation}, \\ \vdots \end{cases}$$

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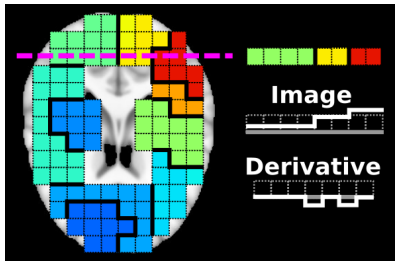


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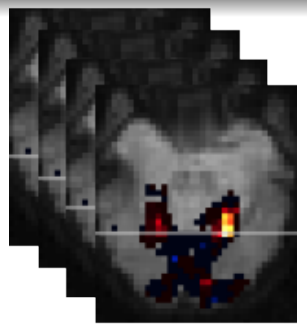

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- Bayesian interpretation

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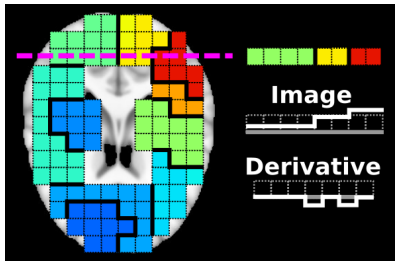


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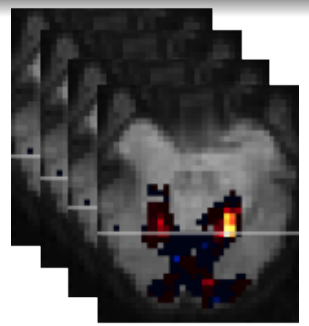

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**Some notes**

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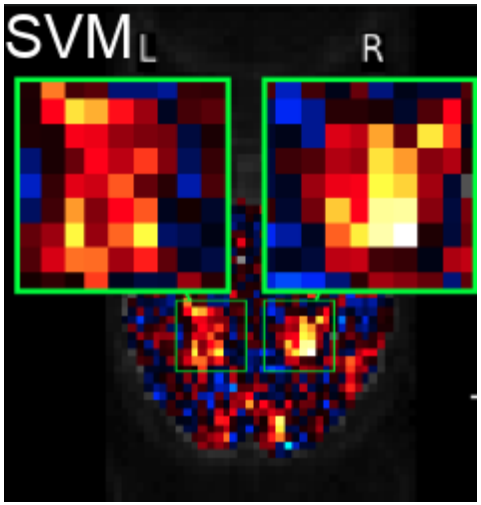


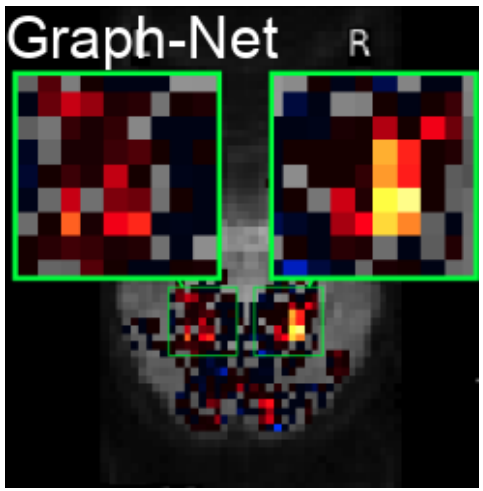


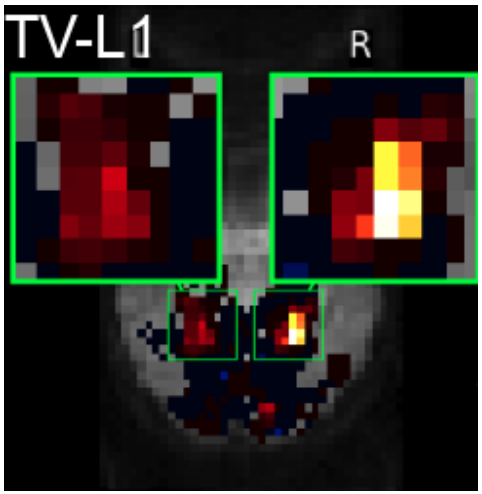

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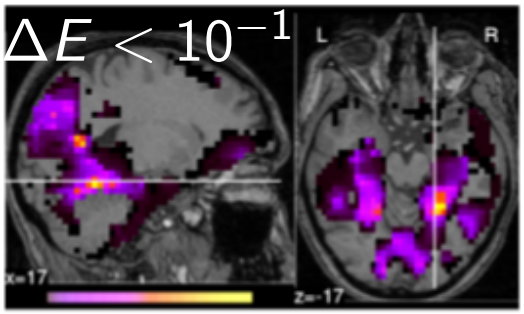

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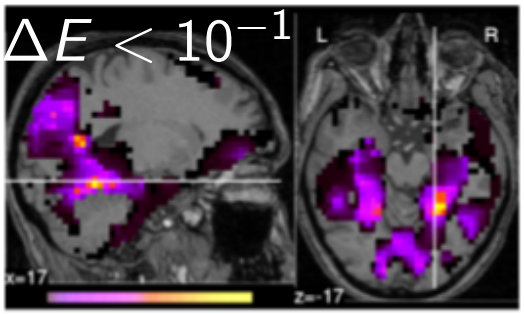


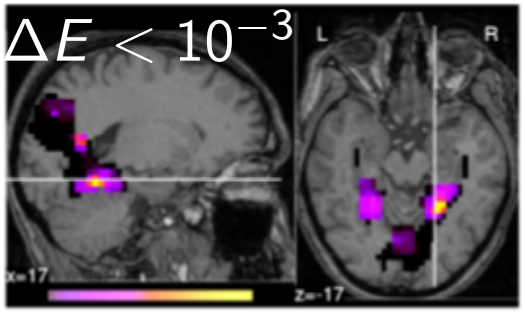

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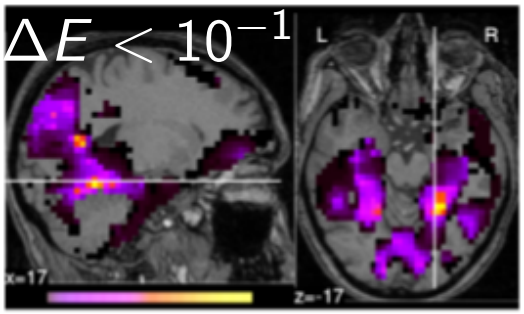


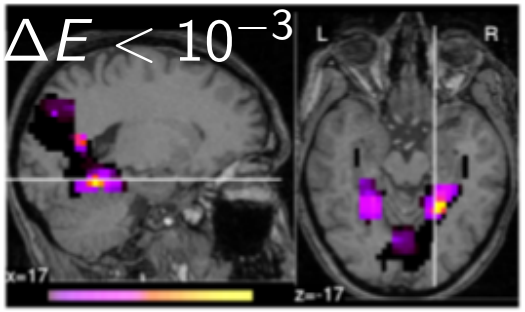


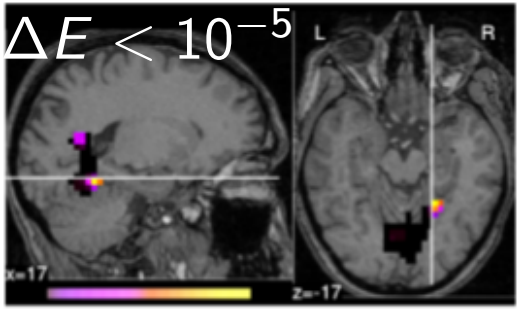


- Structured penalties $\Longrightarrow$ **more interpretable models**

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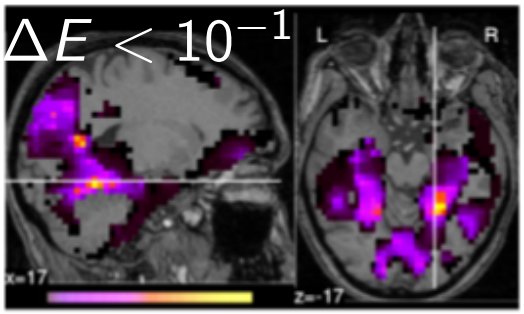


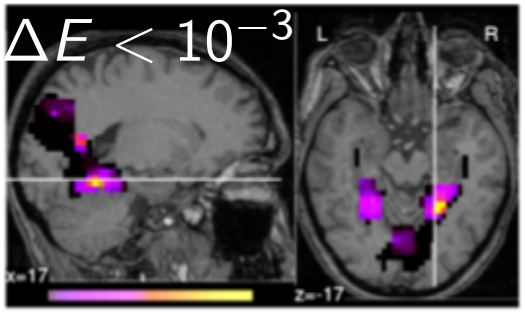


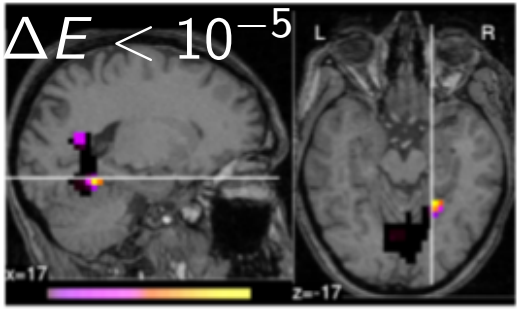


- Structured penalties $\Longrightarrow$ **more interpretable models**
- Corresponding optim. problem is much **harder** (than SVM, etc.)
  - **high-dimensional non-smooth ill-conditioned** problem

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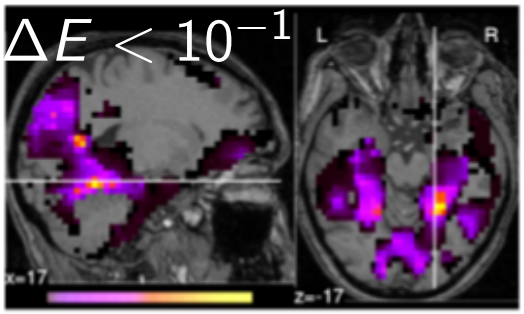


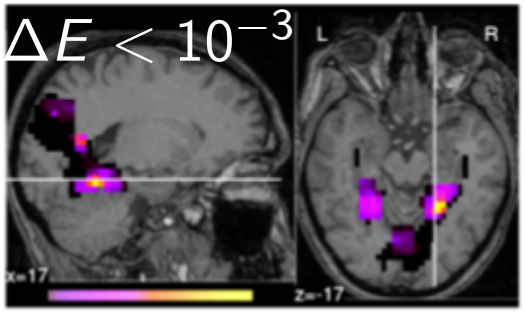


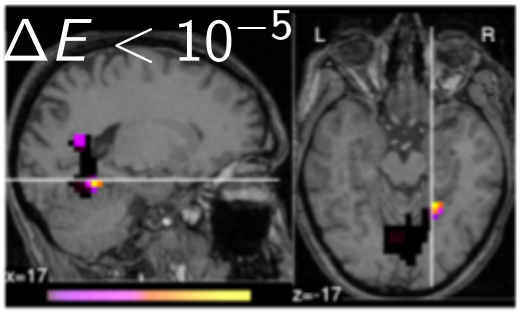


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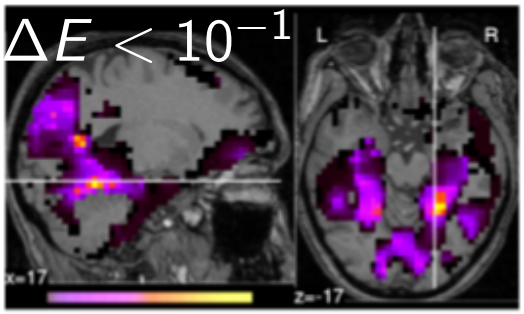


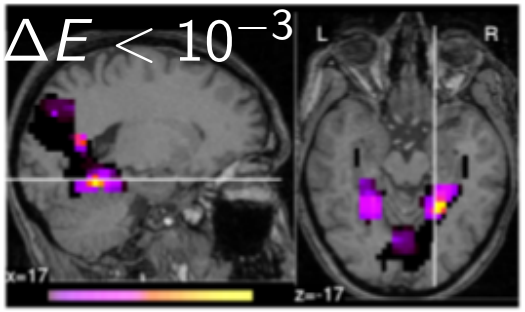


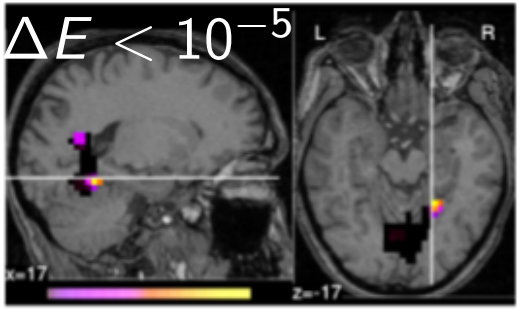


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# Our contributions

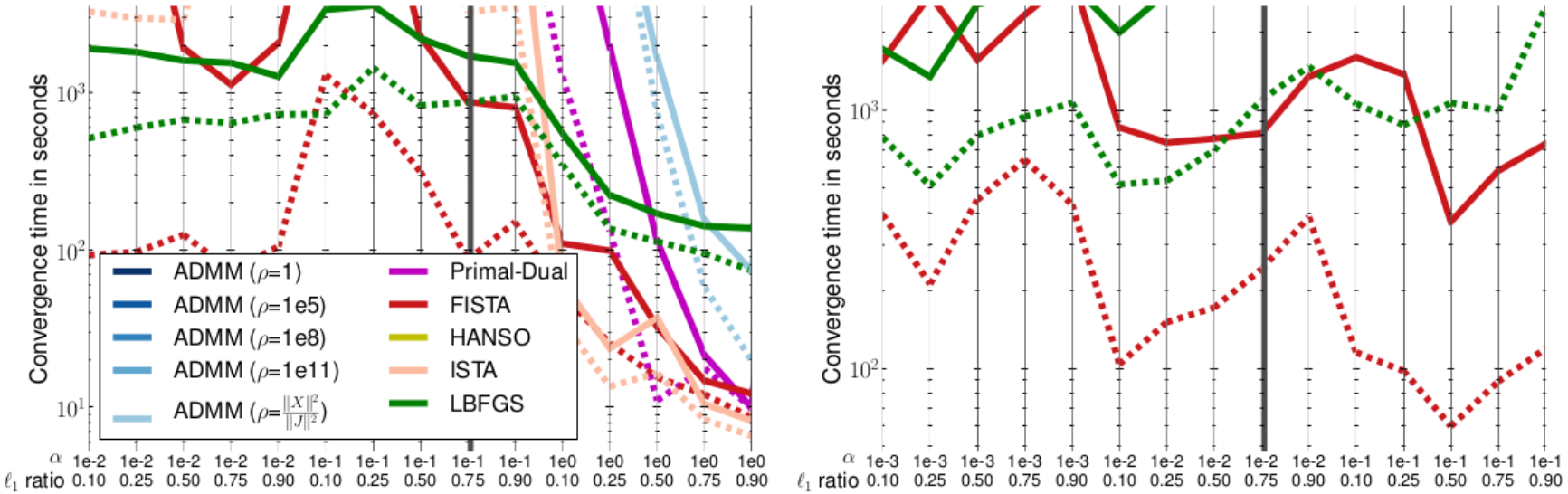
**Faster, better, stronger!**

We propose a combination of **algorithmic** and **implementation** improvements that make these models usable out-of-the-box

# Looking for the ideal solver

[Dohmatob '14, '15 (PRNI); Varoquaux '15 (Gretsi)]

- Solver speed sensitive to hyper-parameter
- Retained strategy is nested FISTA [Beck '09] algorithm



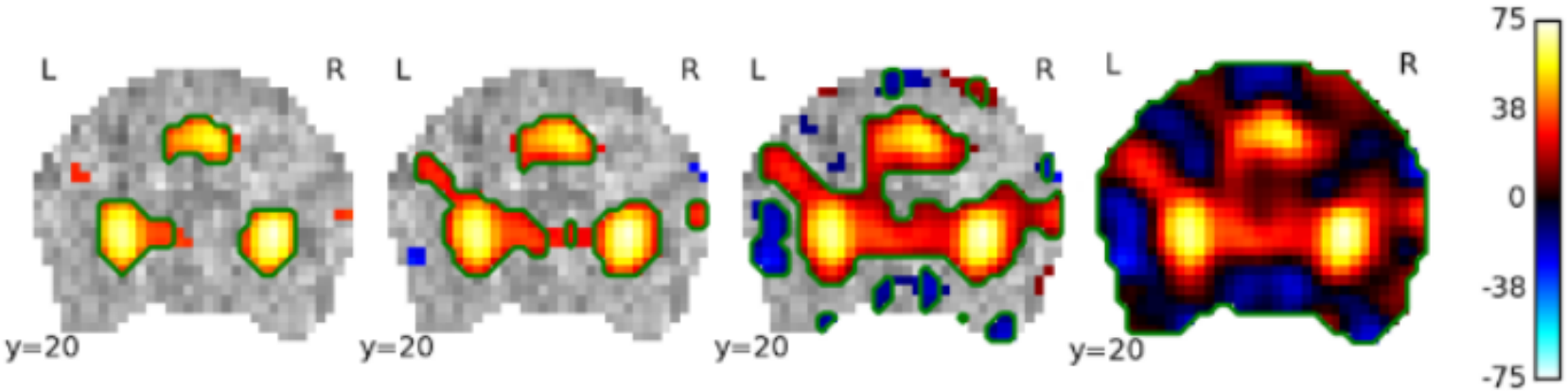

Benchmarks on "mixed-gambles" task [Jimura '12]

# More speed via univariate feature-screening

[Dohmatob '15 (PRNI)]

- $t_k :=$ $k$th percentile of the vector $|\mathbf{X}^T\mathbf{y}| := (|\mathbf{x}_1^T\mathbf{y}|, \ldots, |\mathbf{x}_p^T\mathbf{y}|)$.
- Discard $j$th voxel if $|\mathbf{x}_j^T\mathbf{y}| < t_k$

$k = 10\%$ $k = 20\%$ $k = 50\%$ $k = 100\%$




Mixed gambling

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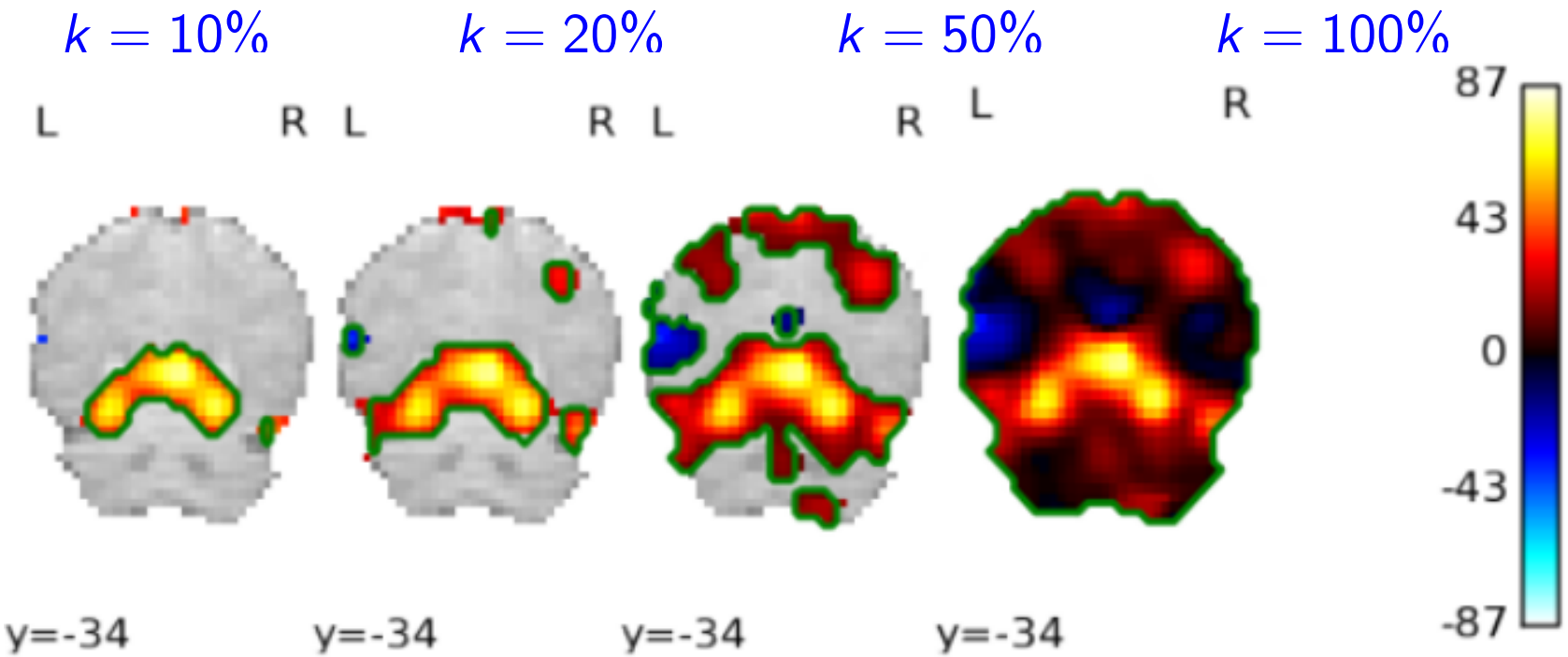


Visual recognition

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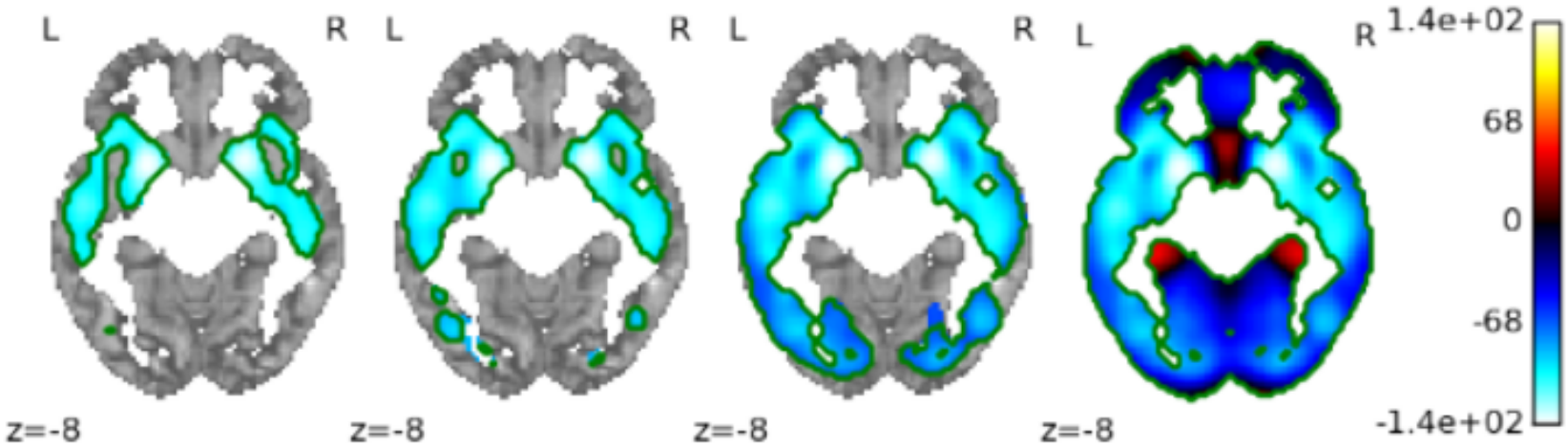
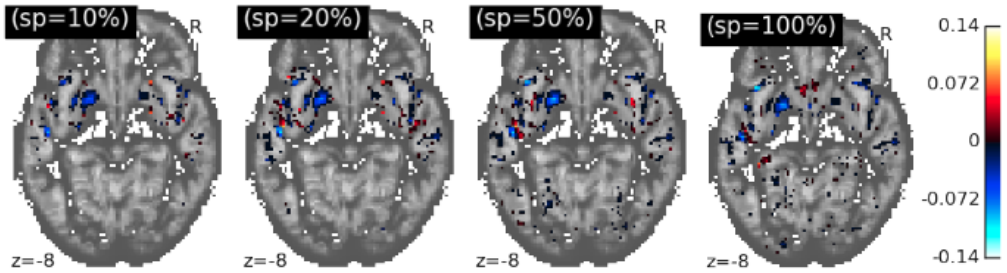



Age prediction from gray-matter maps

# More speed via univariate feature-screening: results

- Age prediction



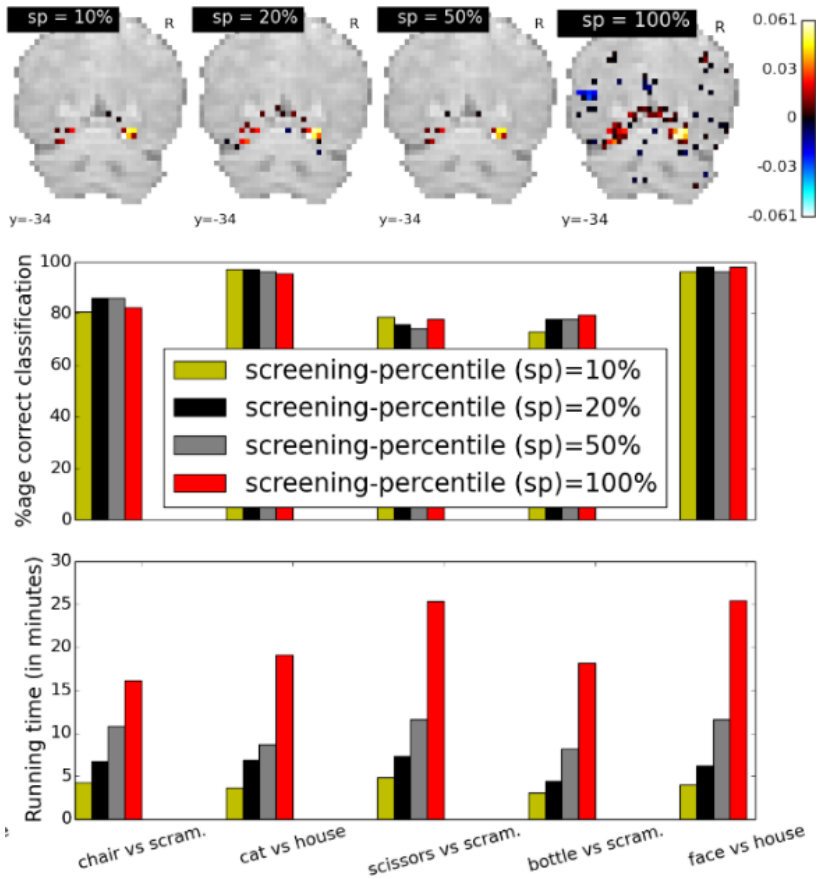

| $p$ | 100% | 50% | 20% | 10% |
|---|---|---|---|---|
| MSE | 8.37 | 9.10 | 9.23 | 9.19 |

[Dohmatob '15 (PRNI)]

- Solve on **subset of features**
- Reduced **training time**

# More speed via univariate feature-screening: results

- Visual object recognition

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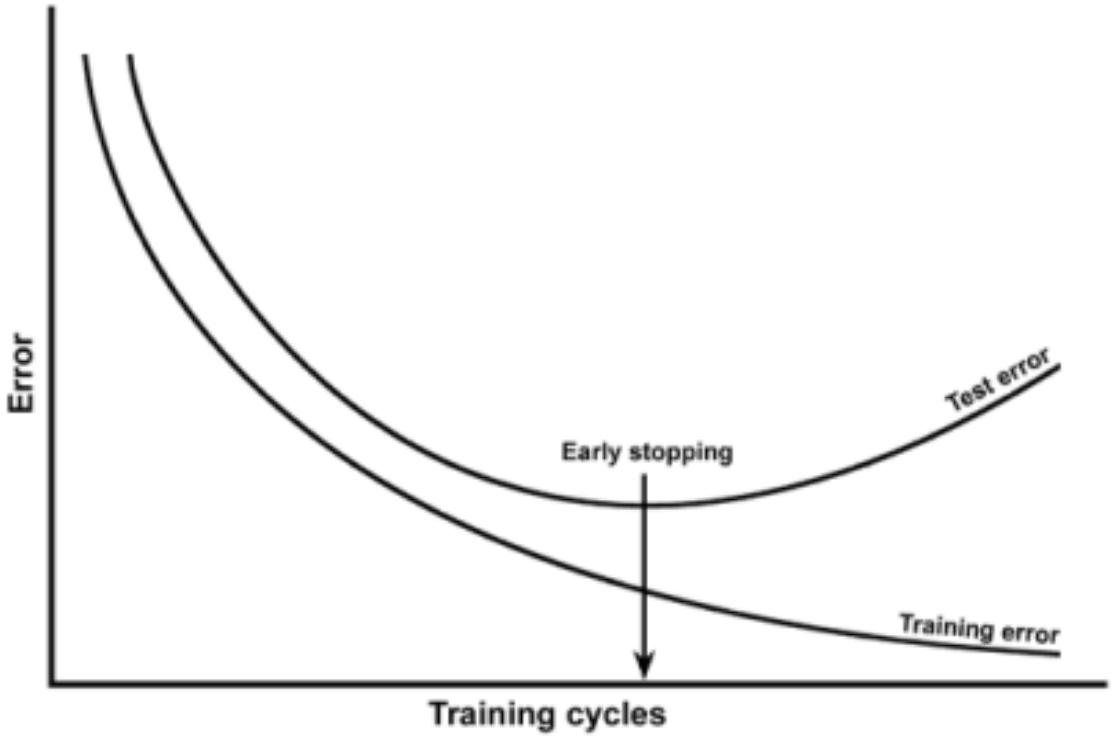
# Early-stopping

- Stop optimization if accuracy on validation data stops improving [Dohmatob '15 (PRNI)]




- old idea (e.g [Bottou '07])
- saves training time
- implicit regularization
- helps against overfitting
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  - but may lead to sub-optimal brain maps

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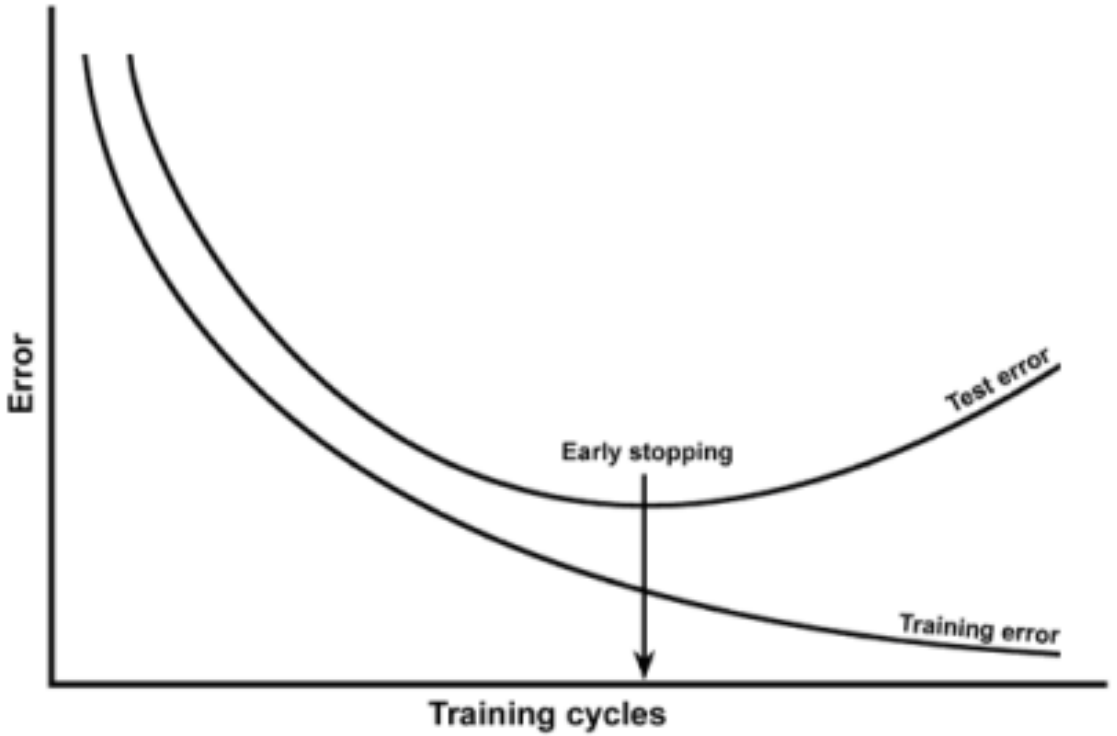


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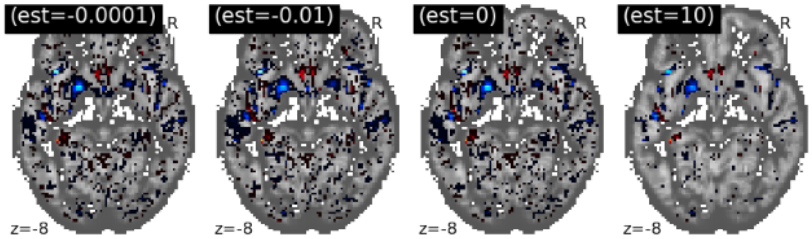


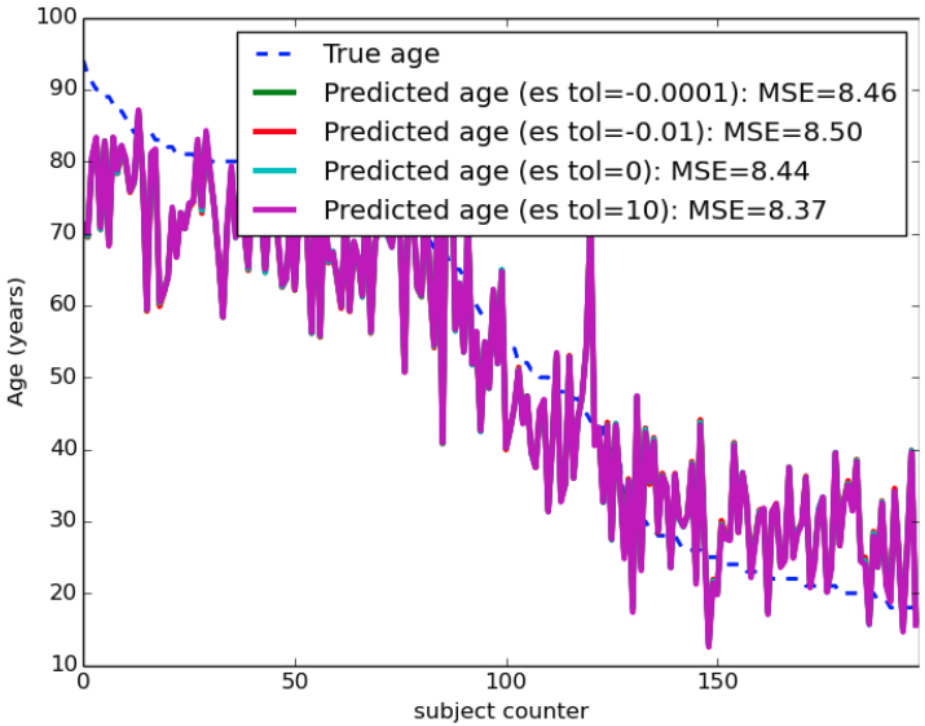


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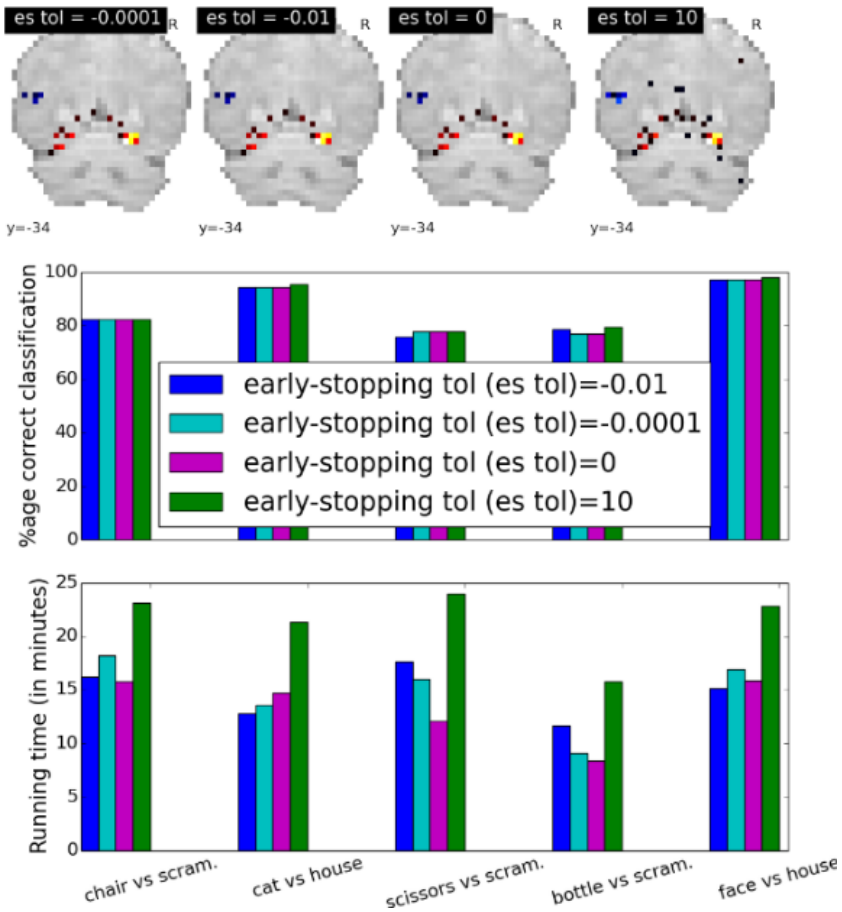


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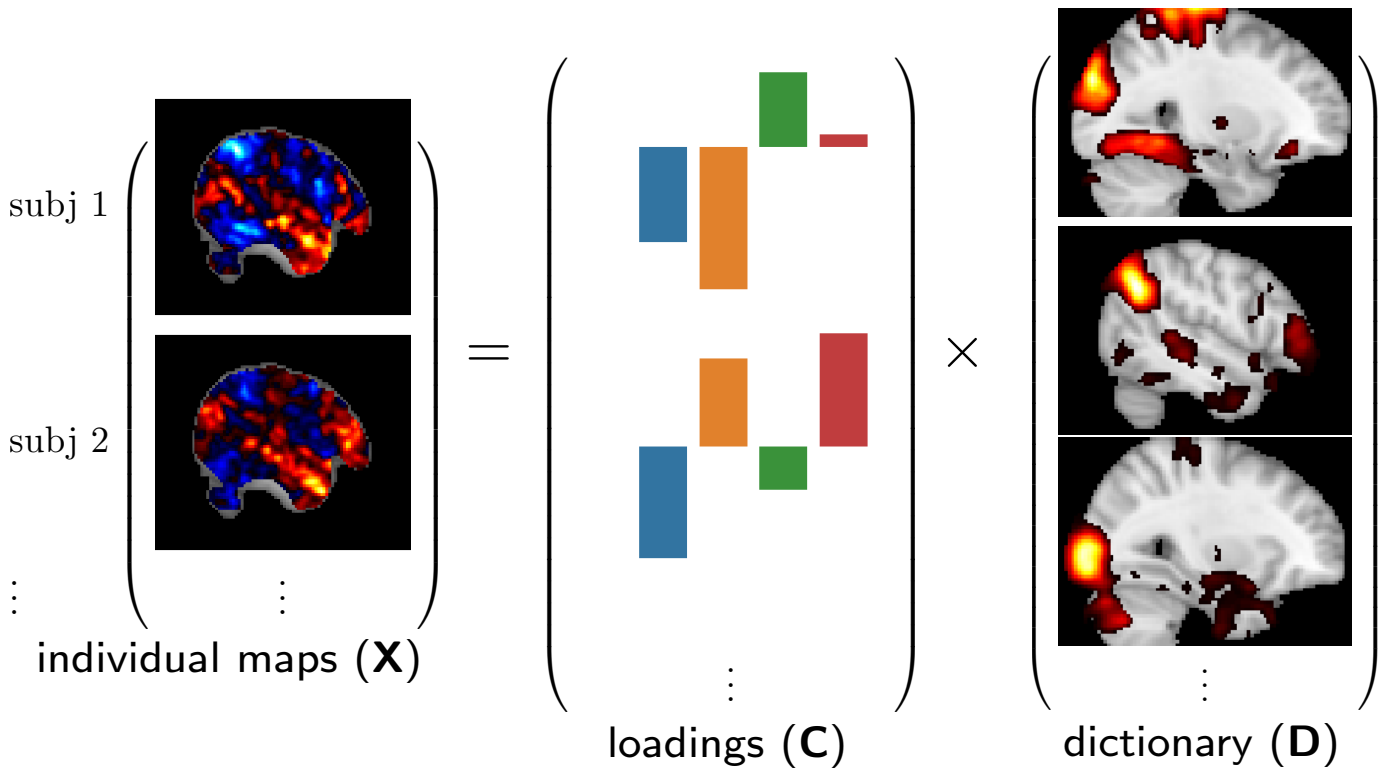
# Section wrap-up

- Building on prior work, we have developed enhanced structured penalties for multi-variate brain-decoding
- Such penalties lead to more interpretable brain maps (a small number of smooth spatially localized regions)
- Focus on practical usability (fast model training)
- Our contributions are available as part of **Nilearn** toolkit.

# Modelling inter-subject variability via dictionary-learning

# Learn latent model for inter-subject variability

- **Goal:** Learn a latent model of inter-subject functional variability



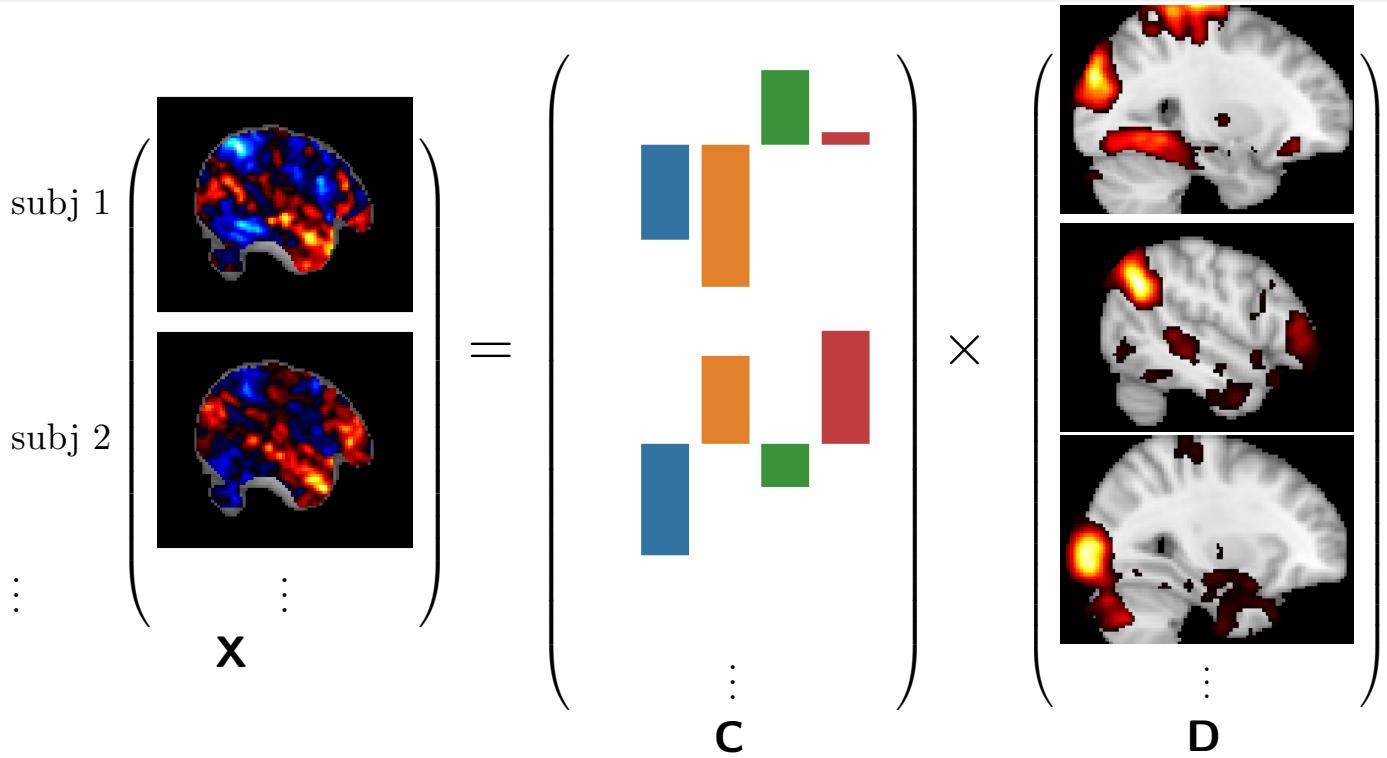

- Each **cognitive map** $\mathbf{x}_i$ with $p$ voxels gets **encoded** over a **dictionary** $\mathbf{D}$ as $k$ **loading coefficients** $\mathbf{c}_i$, with $k \ll p$

# The challenge

[Dohmatob '16 (NIPS)]

- Sparsity: spatially localized atoms
- Smooth regions: each atom = interpretable blobs
- Scalable / online: model should trainable online

# Introducing the proposed model [Dohmatob '16 (NIPS)]




$$\min_{\mathbf{D}\in\mathbb{R}^{p\times k}} \left( \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \min_{\mathbf{c}_t\in\mathbb{R}^k} \frac{1}{2}\|\mathbf{x}_t - \mathbf{D}\mathbf{c}_t\|_2^2 + \frac{1}{2}\alpha\|\mathbf{c}_t\|_2^2 \right)$$

subject to $\mathbf{d}^1, \ldots, \mathbf{d}^k \in \mathcal{K}$ [Mairal '09']

- $\mathcal{K} \subseteq \mathbb{R}^p$ is an $\ell_1$ ball

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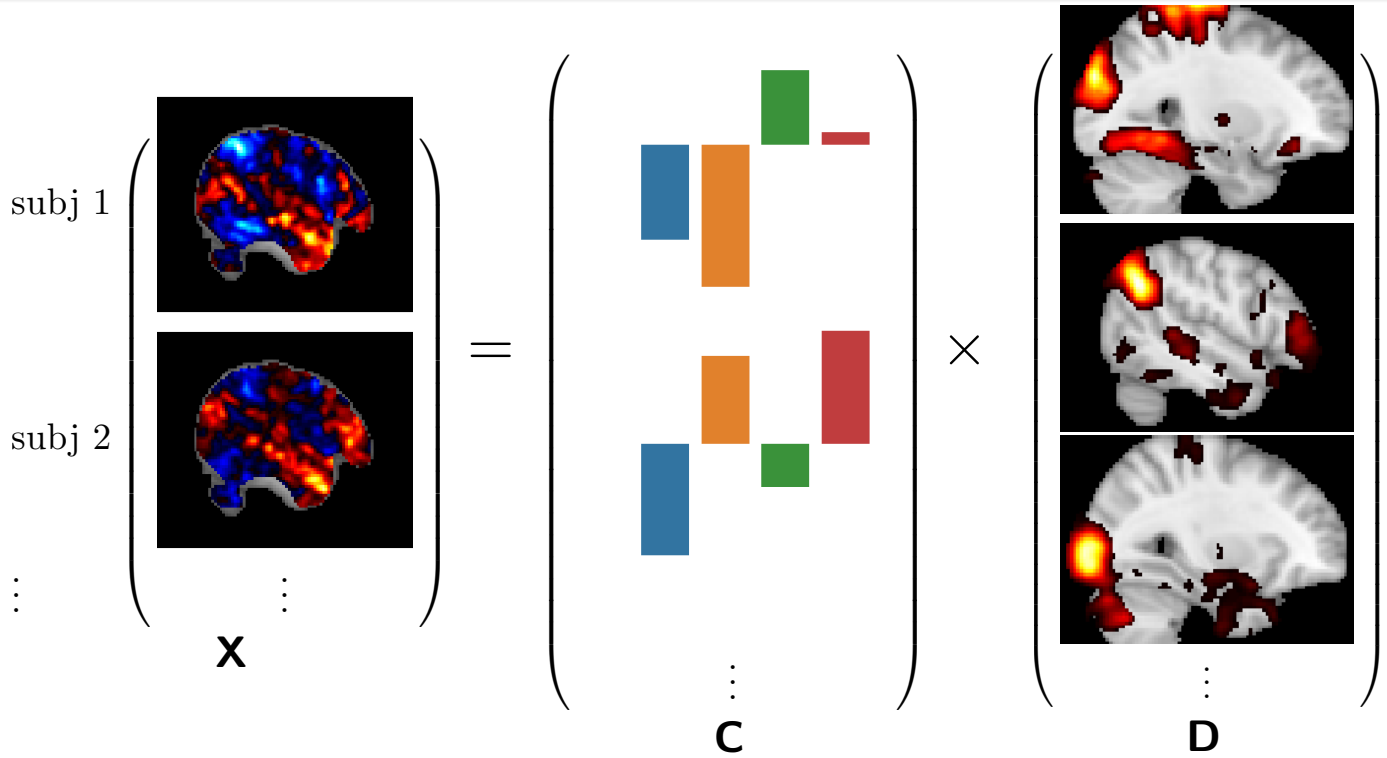


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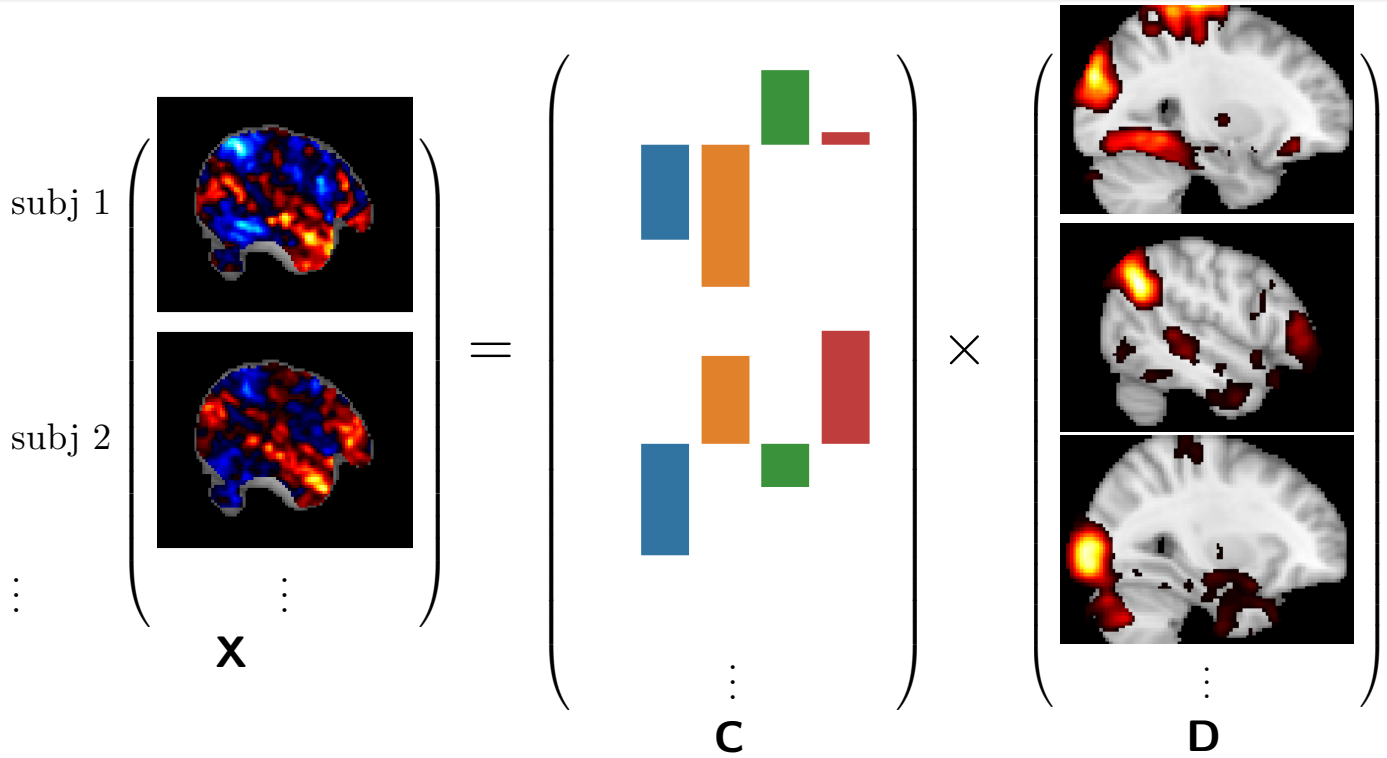


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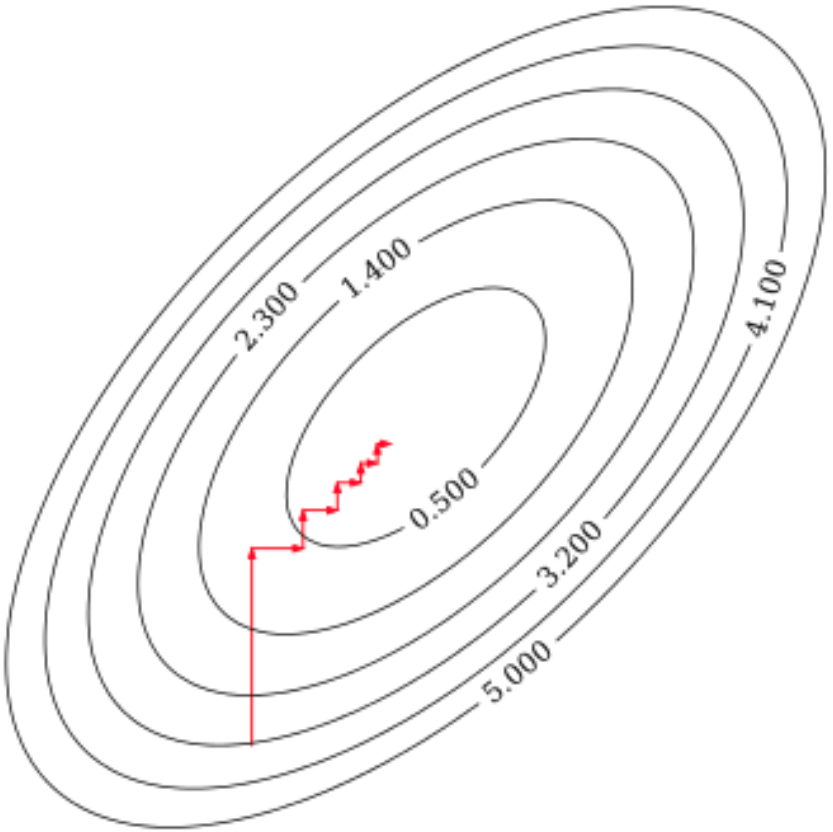
# Reminder on coordinate-descent (CD)

- Optimize w.r.t a variable, and then w.r.t to another, and so on ...



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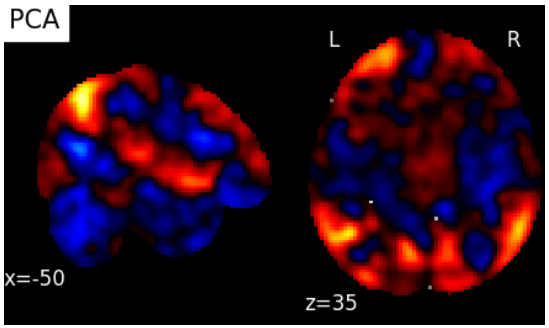
# Experimental results on HCP fMRI data: qualitative



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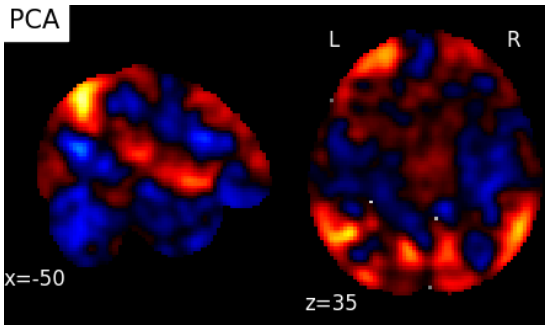


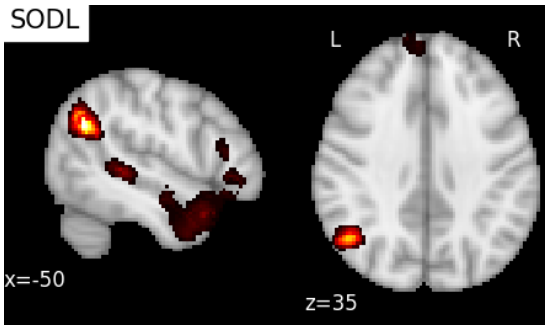

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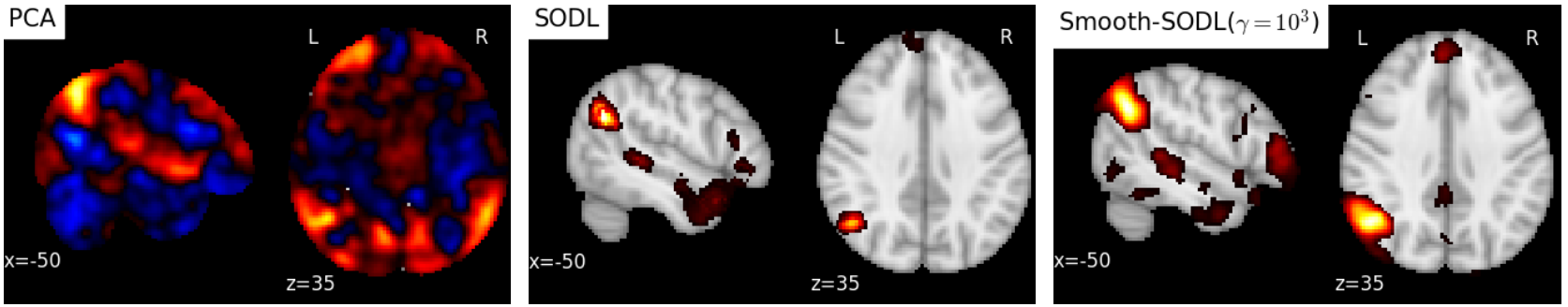


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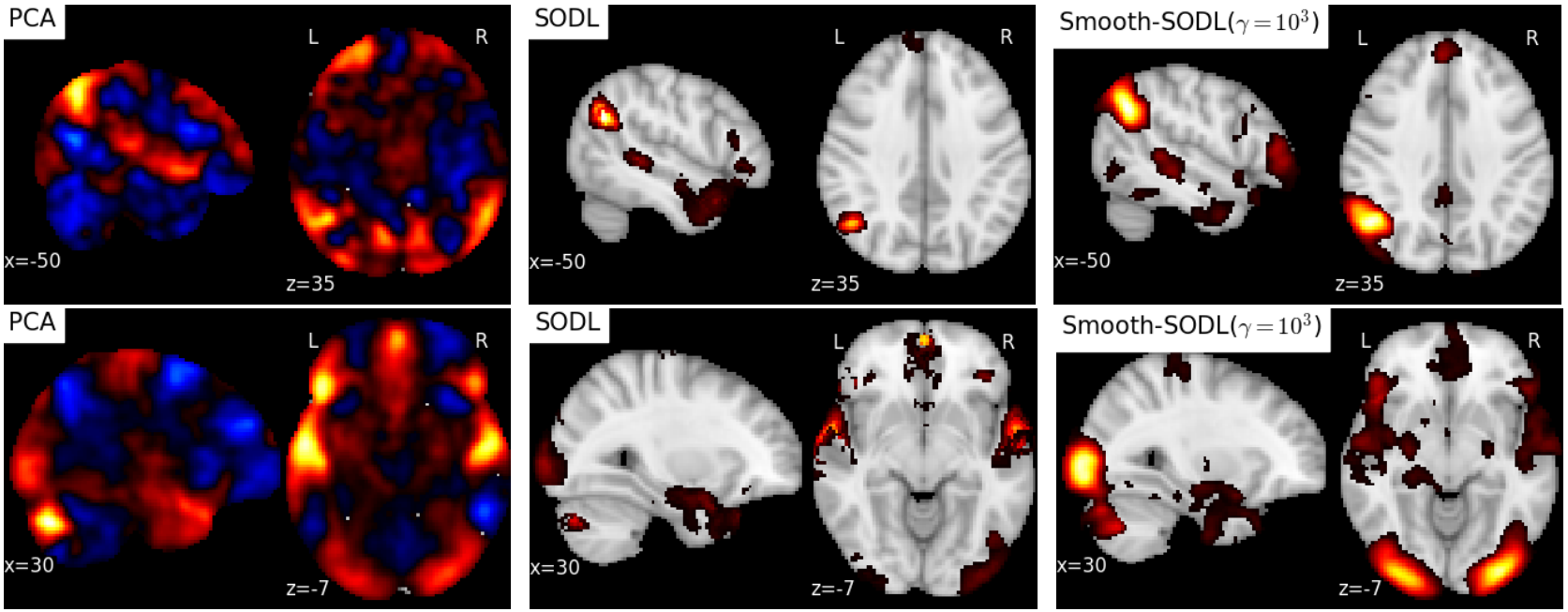


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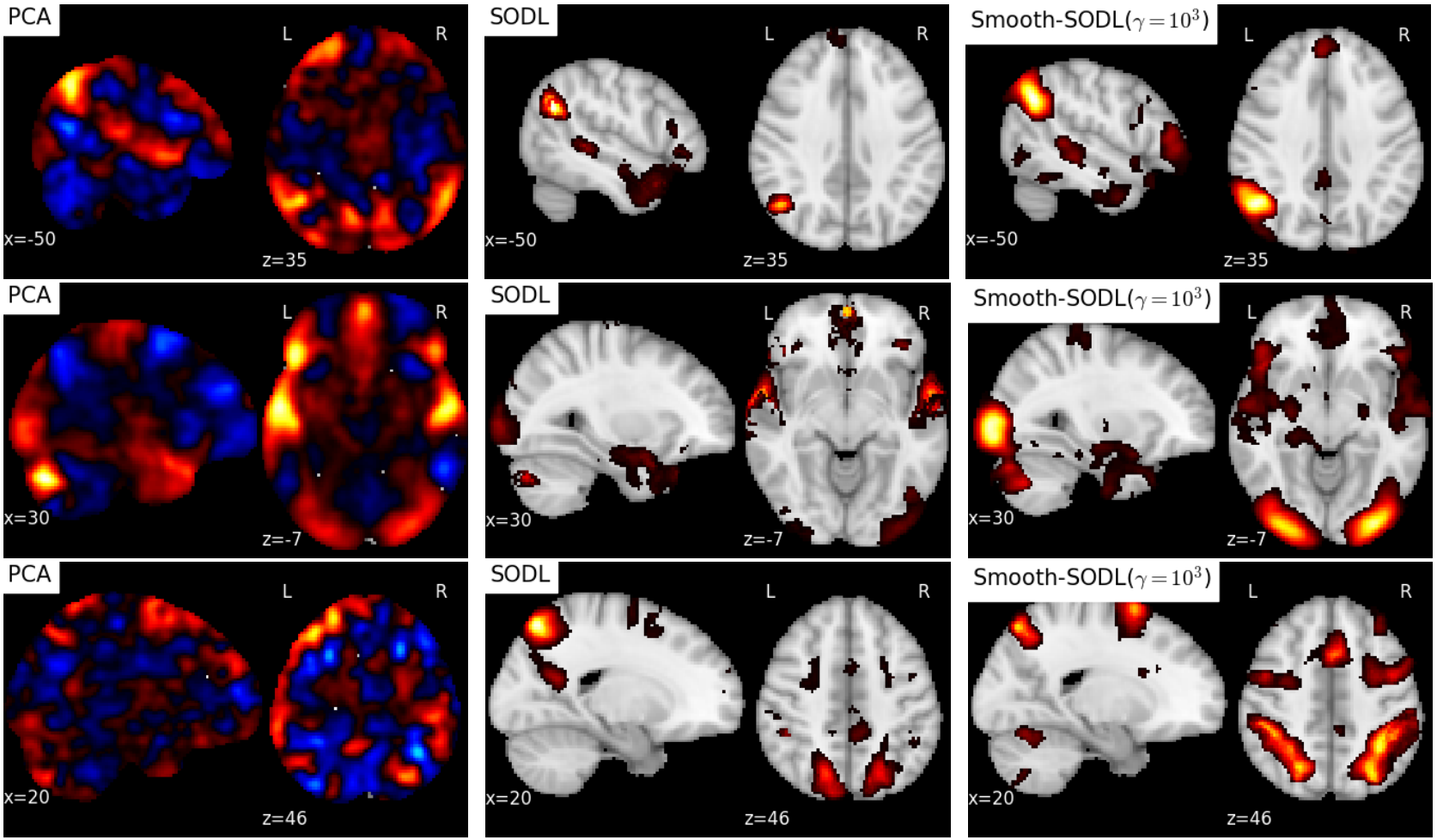
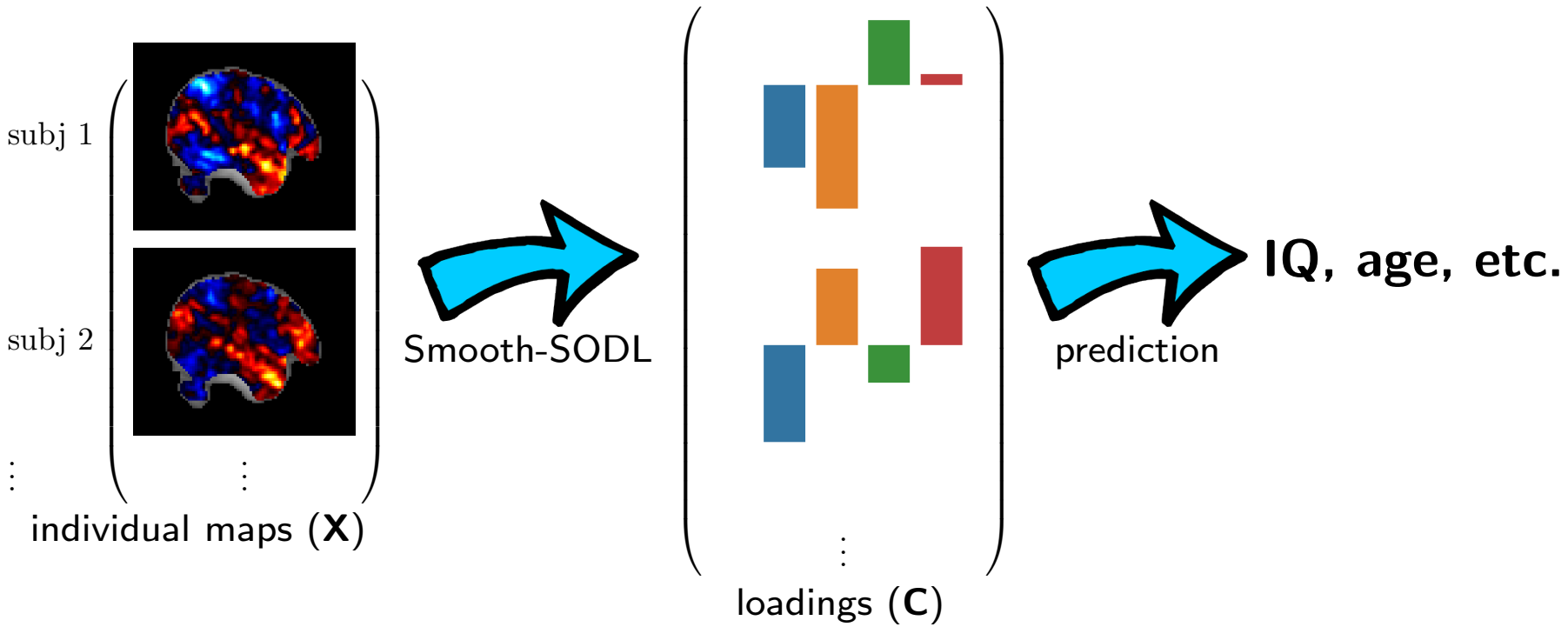
# Experimental results on HCP fMRI data: qualitative




- Our method produces localized and smooth decompositions

# Learned latent dimensions capture inter-subject variability

- Predicting behavior from **compressed representation** of Story vs Math contrast of language task maps [van Essen '12]



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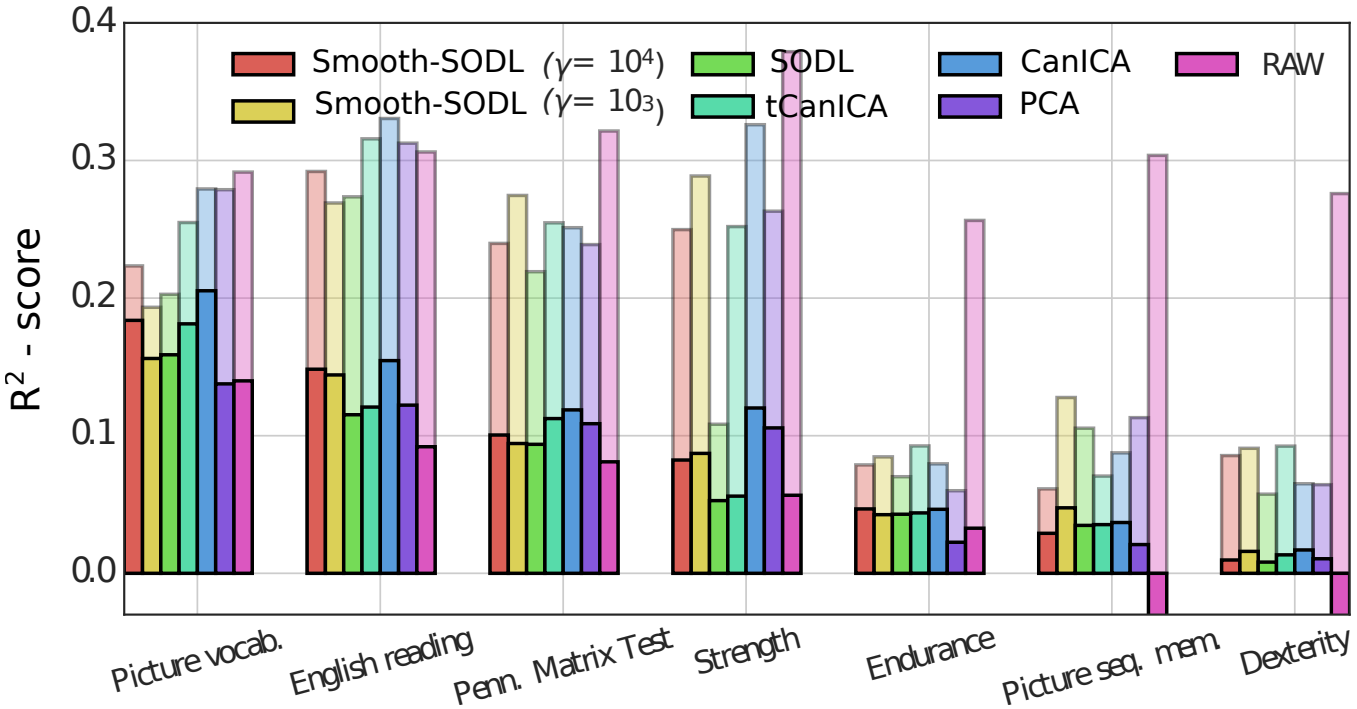
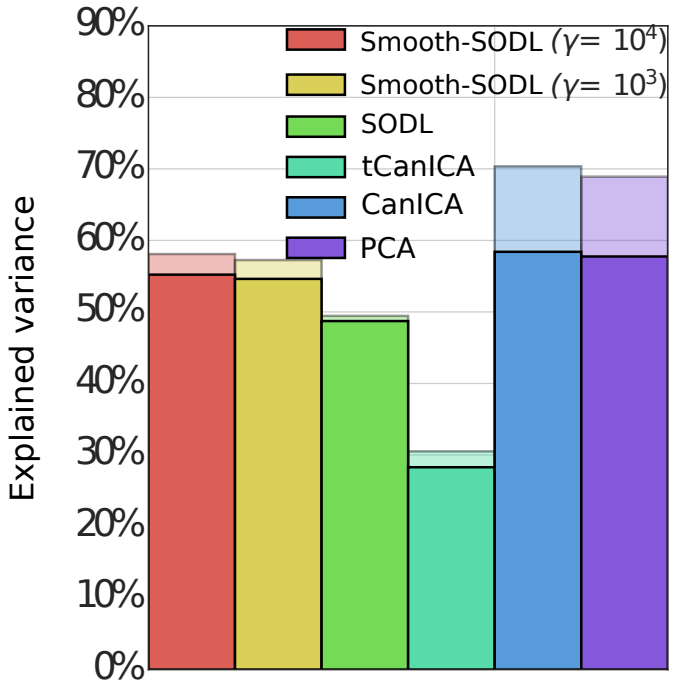



- Thick bars $\Longrightarrow$ scores on **test** set; faint bars $\Longrightarrow$ on **train**
- Proposed **Smooth-SODL** overfits the least (i.e generalizes best)

# What's happening




- Unregularized models **overfit**
- Models thresholded post-training **underfit**

# Spatial prior reduces sample-complexity

| Nb. subjects | vanilla [Mairal '10] | Proposed model | gain factor |
|---|---|---|---|
| 17 | 2% | **31%** | **13.8** |
| 92 | 37% | **50%** | **1.35** |
| 167 | 47% | **54%** | **1.15** |
| 241 | 49% | **55%** | **1.11** |

**Learning-curve** for "boost" in explained variance of our proposed Smooth-SODL model over the reference SODL model.

# Concluding remarks

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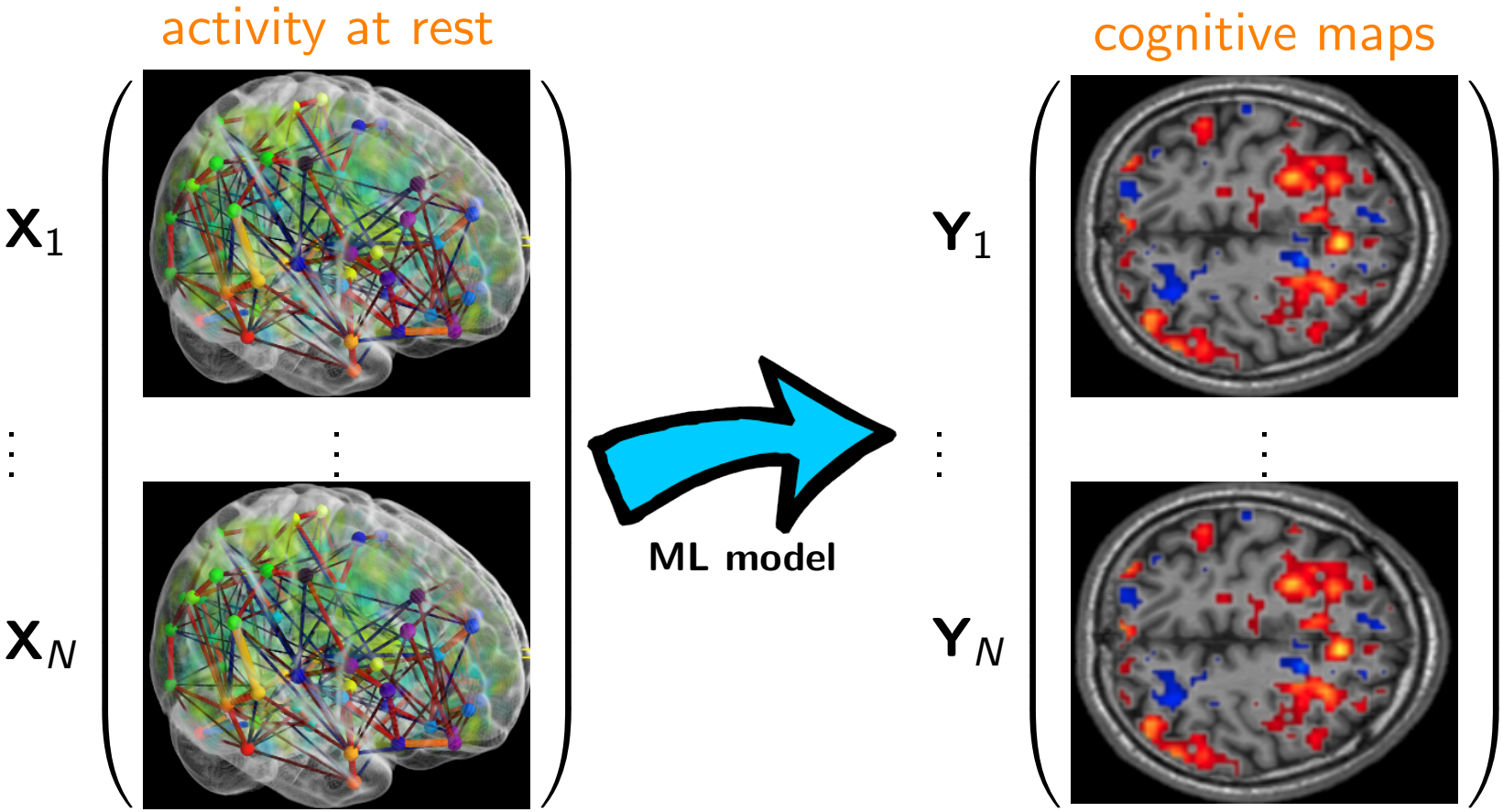
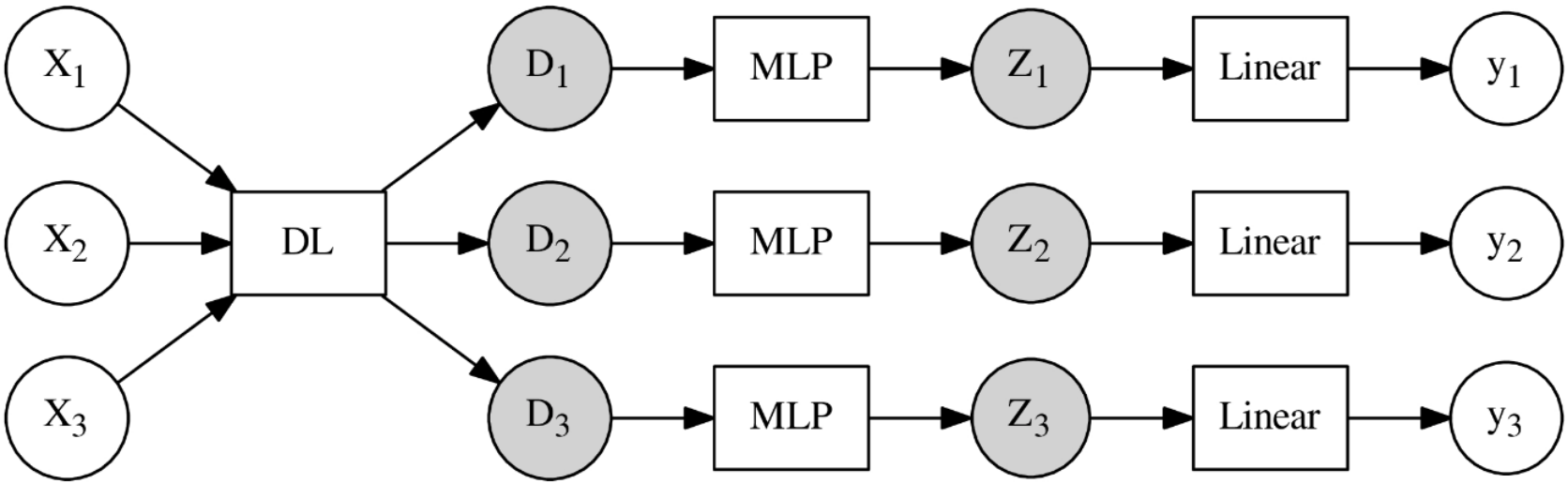
# Can we predict task maps from resting-state data ?



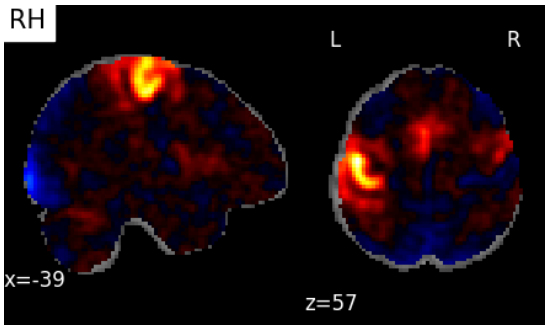

- $\mathbf{X}_s$: resting-state functional connectivity graph for subject $s$
- $\mathbf{Y}_s$: task-specific activation maps for subject $s$

# Proposal: Deep semi-supervised voxel encoding




- $\mathbf{Y} \in \mathbb{R}^{p \times C}$: subject-specific GLM maps of brain activity
- $\mathbf{X} \in \mathbb{R}^{p \times T}$: resting-state fMRI data

# Preliminary results: learned features



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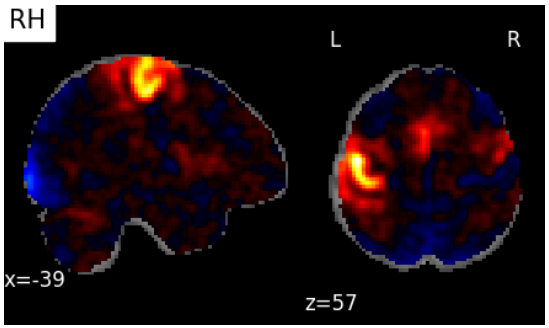


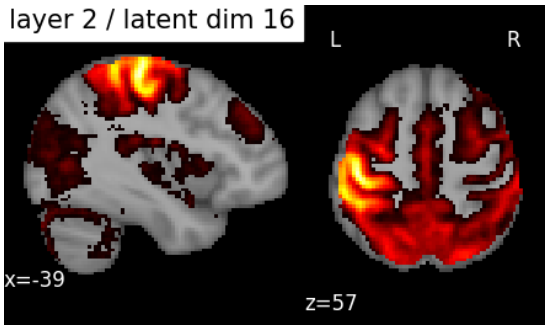

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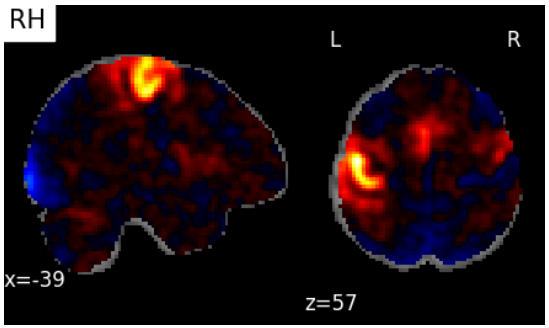


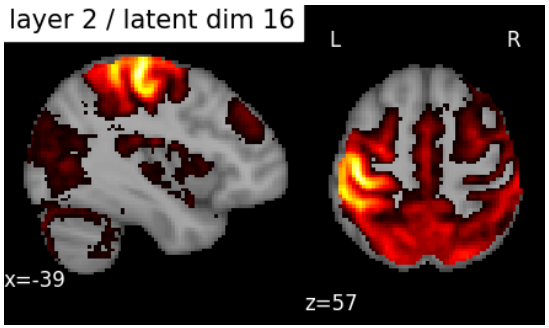


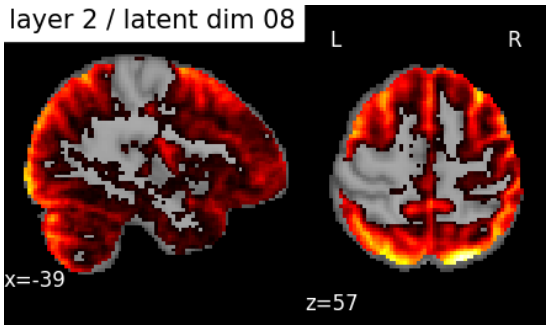

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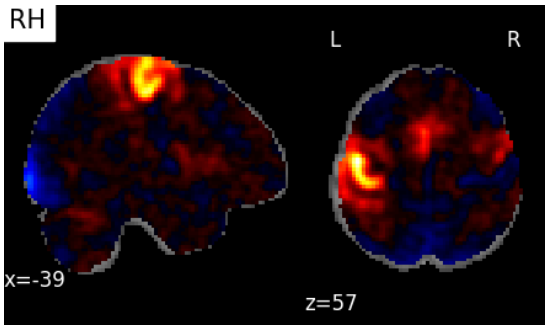


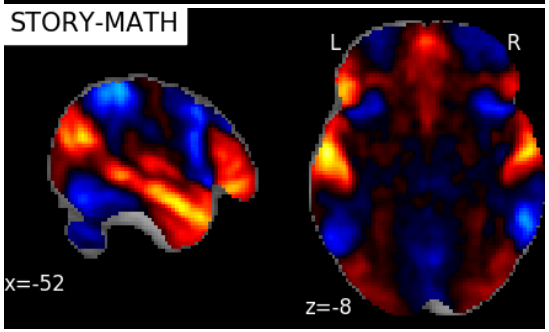


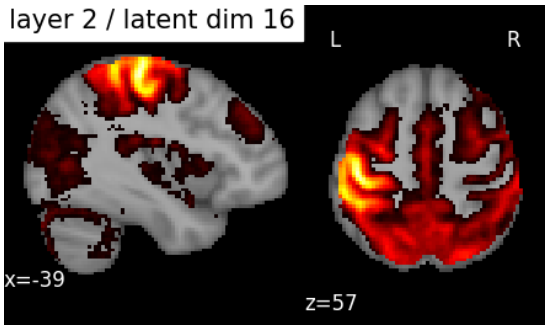


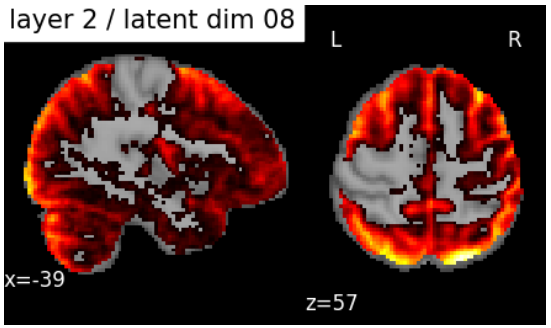

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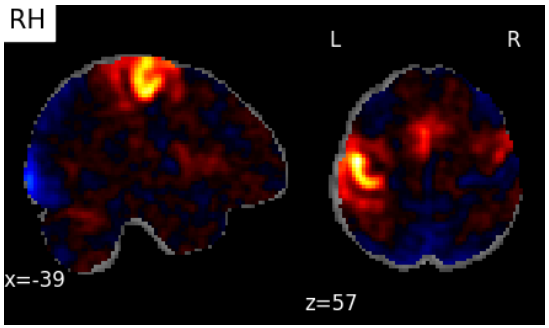


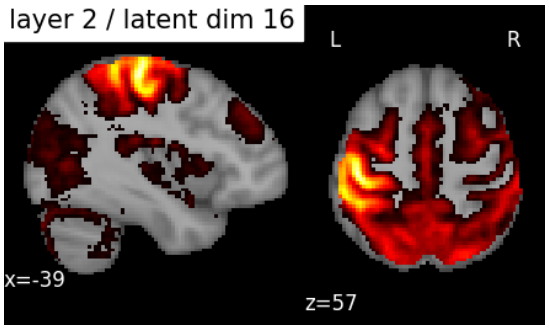


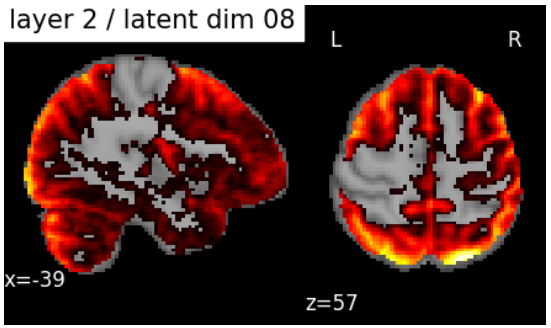


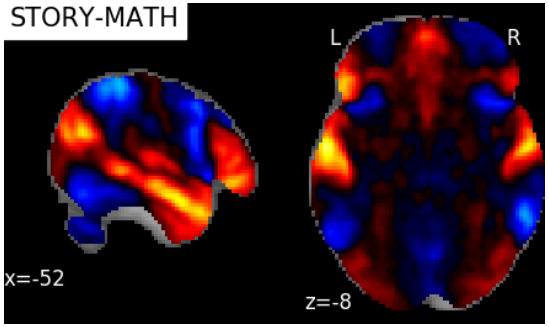


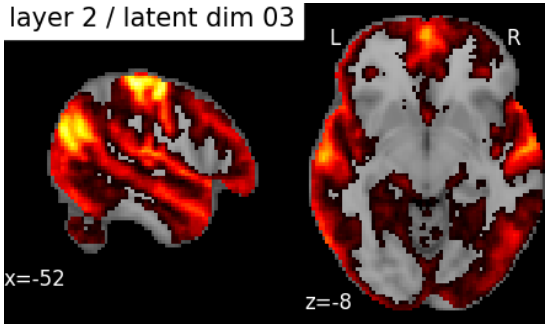

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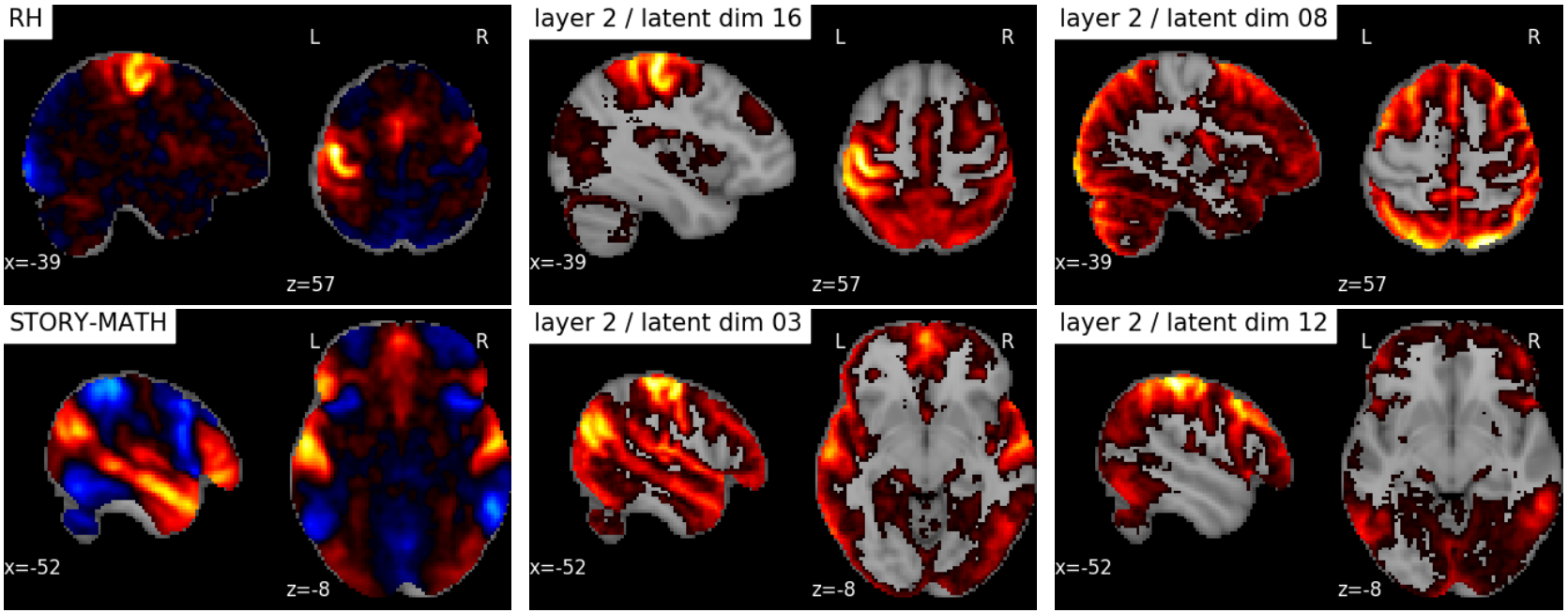

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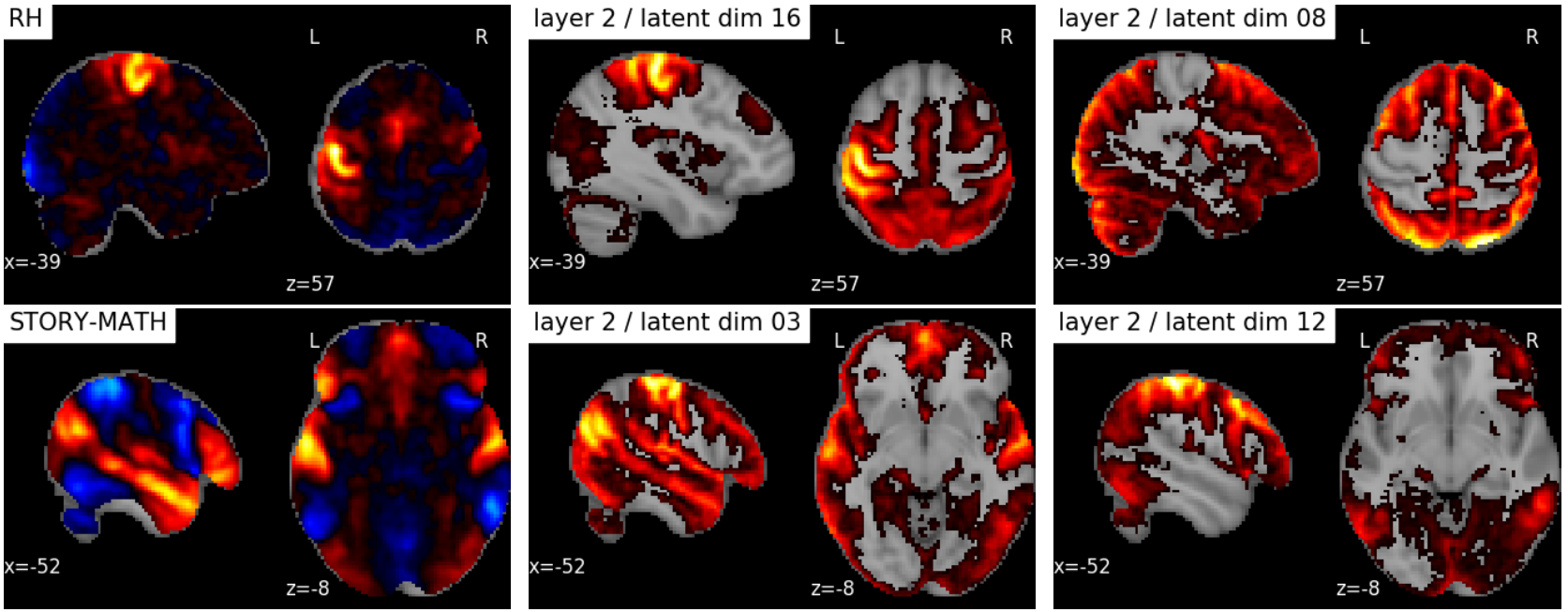
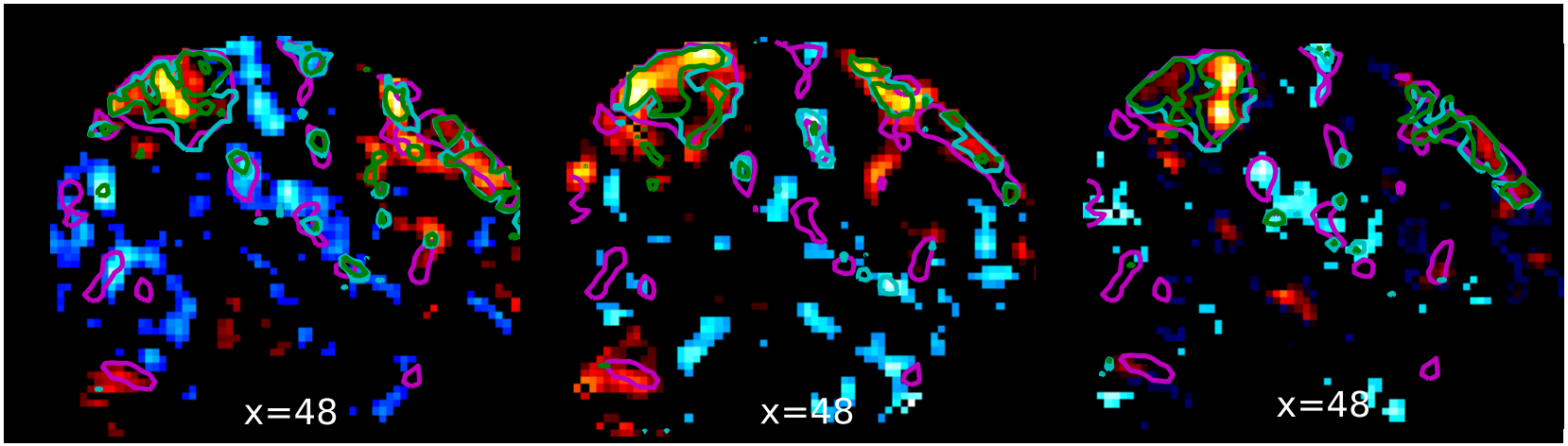



- Learned the a presentation of task activity in resting-state space!
- This is ongoing application of models developed in previous sections!

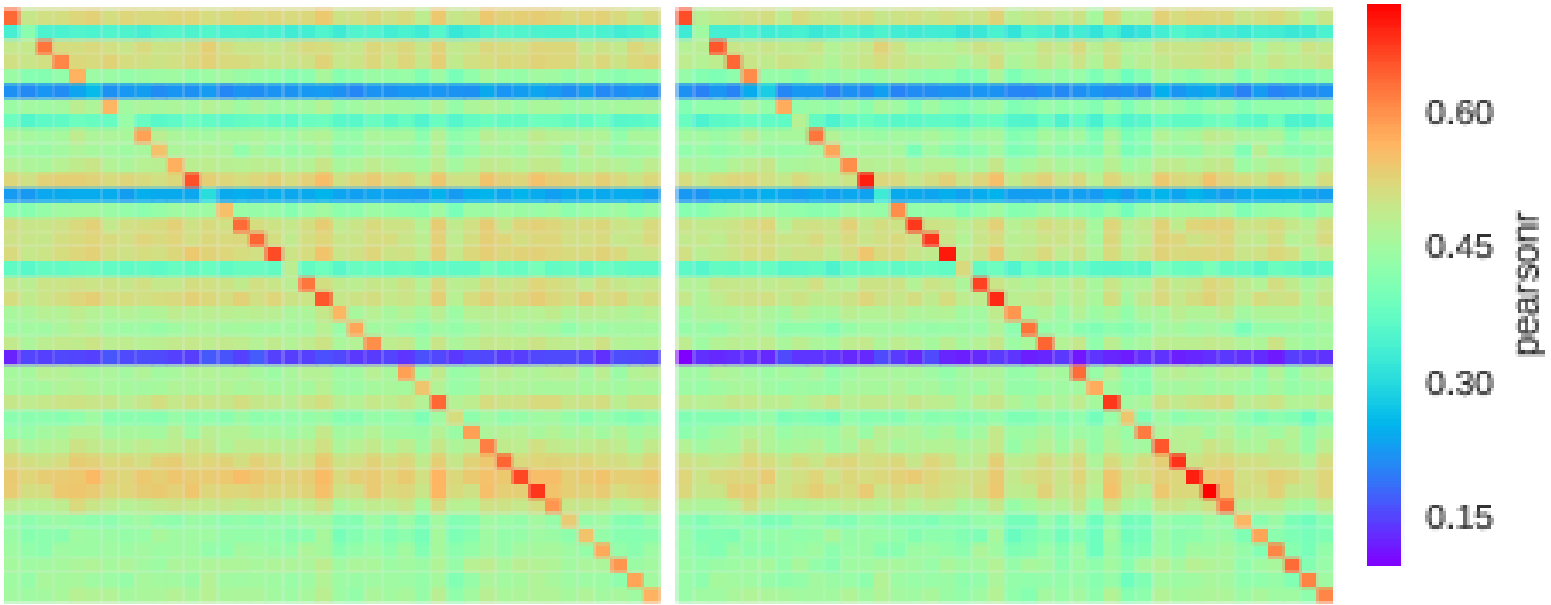
# Preliminary results: predicted individual maps




**2BK vs 0BK** contrast of the Working Memory task
[van Essen '12]

- magenta = population mean
- reference method [Tavor '16]
- proposed method
  - Prediction agrees with subject's topography more faithfully

# Preliminary results: quantitative




**Confusion matrix** for predicted versus true activation maps

# Relevant contributions I