Modelling inter-subject functional variability

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Context

- A major goal of human neuroscience is to understand
  - the structure,
  - function, and
  - inter-subject variability of the human brain
- We will focus on **inter-subject functional variability**

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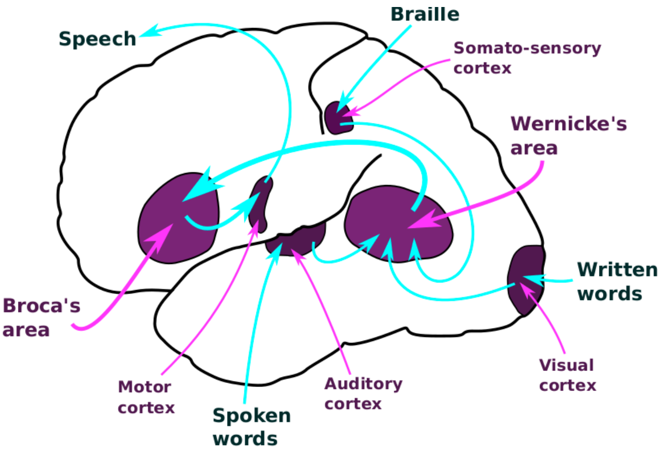
Introduction

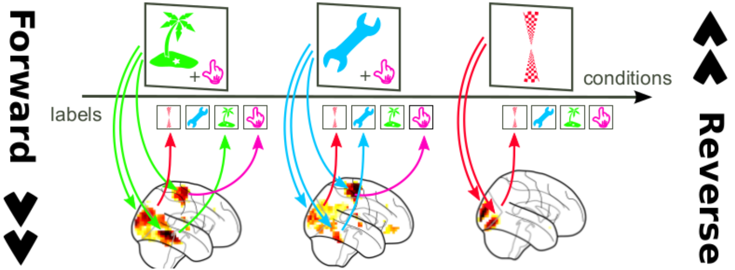
Brain function regions and networks

Part of the language network



(Picture is courtesy of Gael Varoquaux)

Mapping cognitive circuits in the brain



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- **Reverse inference / brain-decoding** [Dehaene 98; Cox 03] predicts the experimental condition from brain signals

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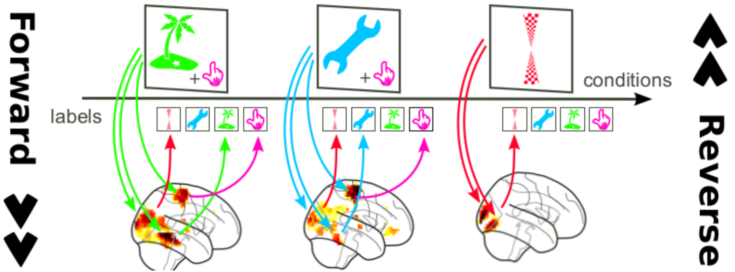

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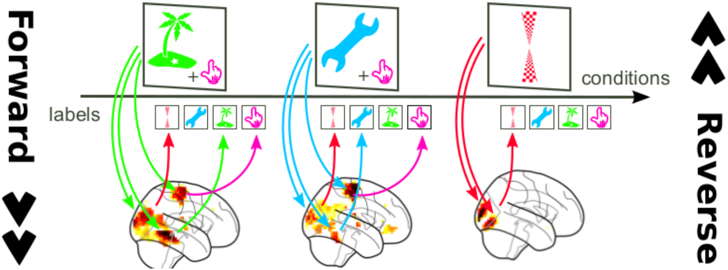

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A zoom on brain-decoding

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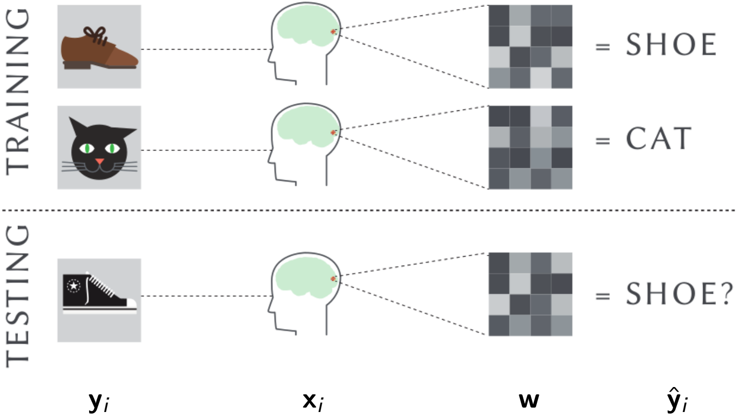

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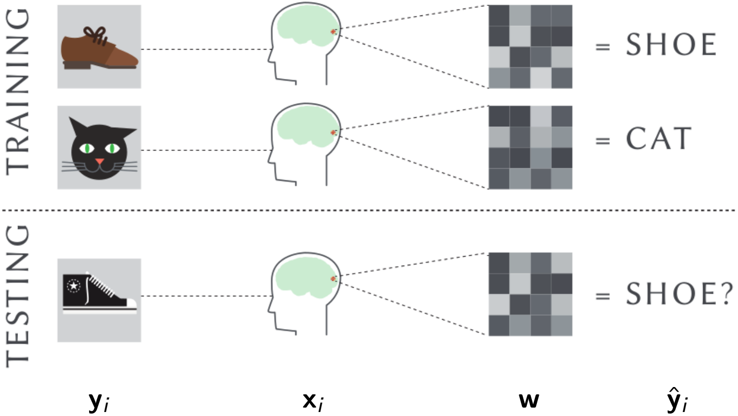
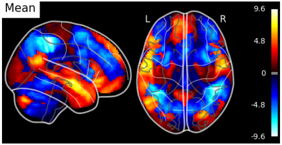


- This is **supervised machine-learning**
- We don't just want good predictions, we want **regions**

Variability in both location and magnitude of activations

- Story vs Math language contrast of HCP dataset [van Essen '12]



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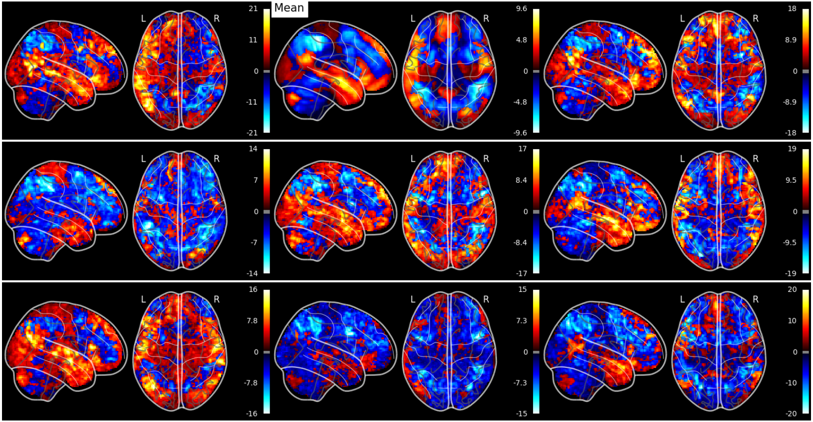

Variability in both location and magnitude of activations

- Inter-subject functional variability $\neq$ noise!
  - Usually (incorrectly) discarded in standard analysis
  - Is predictive of behavioral differences between individuals
- Cannot be corrected via **spatial normalization**, etc.
  - E.g spatial normalization cannot correct for differences in activation magnitude
- Driven by genetic and behavioral inter-individual differences
- Functional diseases can be seen as extremes of this variation

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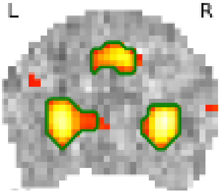
Mapping the brain with structured multi-variate models

What we mean by “structured”

Definition:

- Localized activation patterns – **sparsity**
- Clusters of active voxels – **smoothness**



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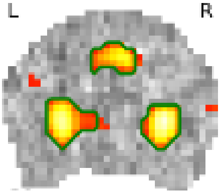
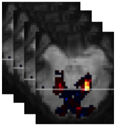


- Such a model is much more **interpretable** (i.e small number of **regions**) than classical methods like SVM, Ridge regression, Lasso
- Performs **model-estimation** and **feature-selection** jointly
- Fights the **curse-of-dimensionality**, via dimensionality reduction.

Generalized linear models with structured penalties

$\mathbb{E}[\mathbf{y}|\mathbf{x}_i] = f\Big($



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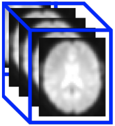

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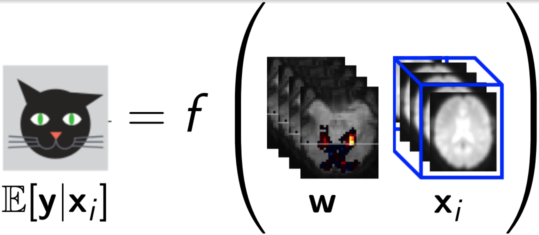

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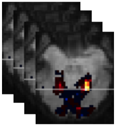

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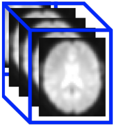

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- $f =$ "logit" in **classification**
- $f =$ "id" in **regression**
- Optimization problem:

$$\min_{\mathbf{w}\in\mathbb{R}^p} \underbrace{\frac{1}{n}\sum_{i=1}^{n}\ell(y_i, f(\langle \mathbf{w}, \mathbf{x}_i\rangle))}_{\text{data / loss term}} + \underbrace{\alpha\mathcal{P}(\mathbf{w})}_{\text{penalty}}$$

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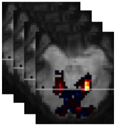

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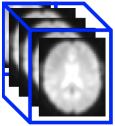

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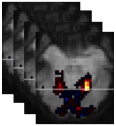
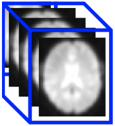
$\mathbb{E}[\mathbf{y}|\mathbf{x}_i] = f\Big($   $\Big)$

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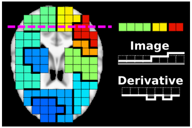

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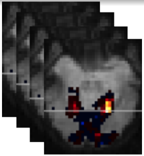

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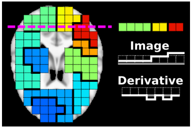

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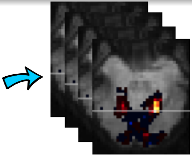

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- Bayesian interpretation

$$\underbrace{P(\mathbf{w}|\mathbf{x}_i, y_i)}_{\text{posterior}} \propto \underbrace{P(y_i|\mathbf{x}_i, \mathbf{w})}_{\text{likelihood}}\underbrace{P(\mathbf{w})}_{\text{prior}} \propto \exp(-\ell(y_i, f(\langle \mathbf{w}, \mathbf{x}_i\rangle)))\exp(-\alpha\mathcal{P}(\mathbf{w}))$$

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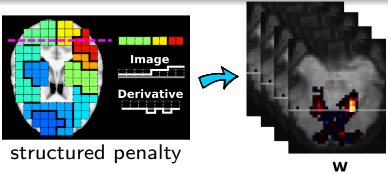

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- Sparse-Variation [Eickenberg '15]

Some notes

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- GraphNet ("Dirichlet energy") is weaker, but easier to optimize (smooth convex optimization problem)

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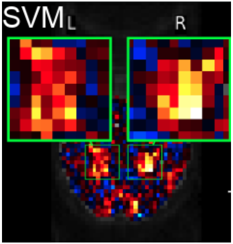
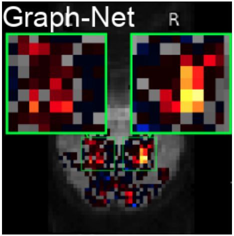
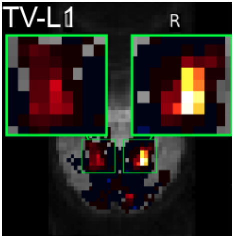
Spatial penalties $\implies$ more interpretable brain maps







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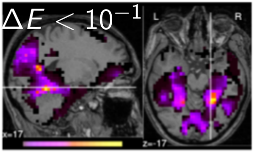

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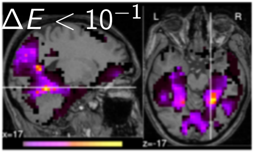

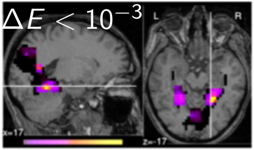

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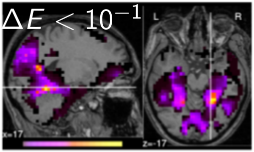

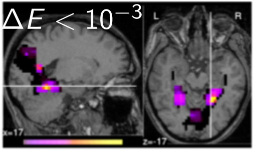

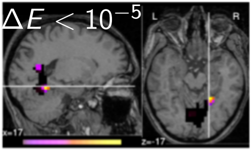

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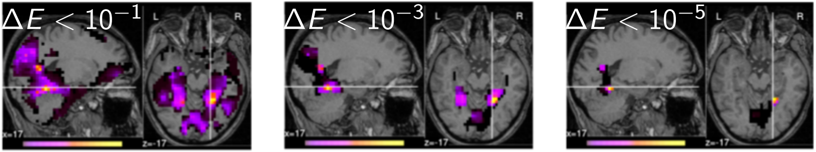

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  - **high-dimensional non-smooth ill-conditioned** problem

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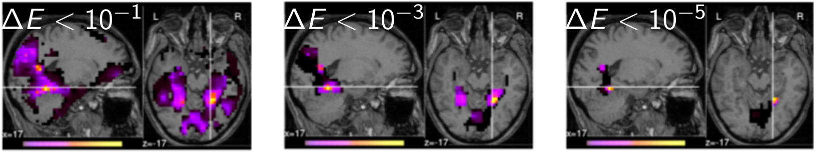

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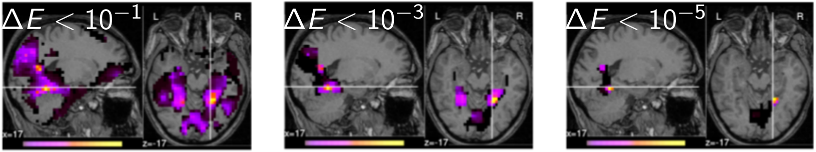

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Our contributions

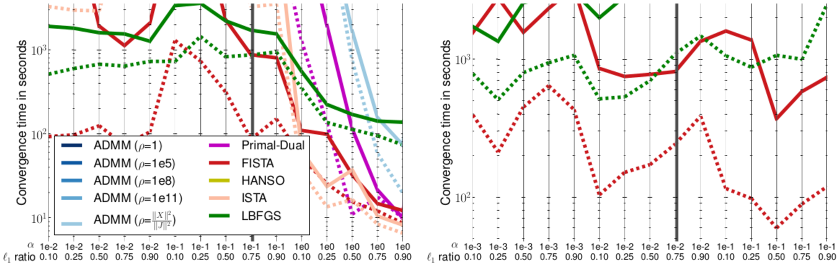
Faster, better, stronger!

We propose a combination of **algorithmic** and **implementation** improvements that make these models usable out-of-the-box

Looking for the ideal solver

[Dohmatob '14, '15 (PRNI); Varoquaux '15 (Gretsi)]

- Solver speed sensitive to hyper-parameter
- Retained strategy is nested FISTA [Beck '09] algorithm



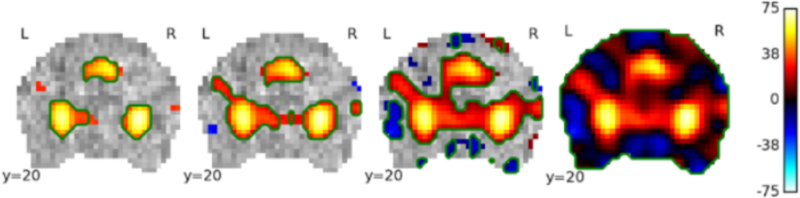
Benchmarks on "mixed-gambles" task [Jimura '12]

More speed via univariate feature-screening

[Dohmatob '15 (PRNI)]

- t_k := kth percentile of the vector $|\mathbf{X}^T\mathbf{y}| := (|\mathbf{x}_1^T\mathbf{y}|, \ldots, |\mathbf{x}_p^T\mathbf{y}|)$.
- Discard jth voxel if $|\mathbf{x}_j^T\mathbf{y}| < t_k$

$k = 10\%$ $k = 20\%$ $k = 50\%$ $k = 100\%$



Mixed gambling

More speed via univariate feature-screening

[Dohmatob '15 (PRNI)]

- t_k := kth percentile of the vector $|\mathbf{X}^T\mathbf{y}| := (|\mathbf{x}_1^T\mathbf{y}|, \ldots, |\mathbf{x}_p^T\mathbf{y}|)$.
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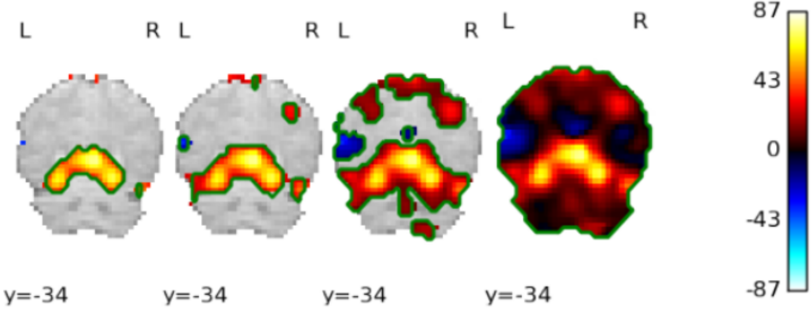

Visual recognition

More speed via univariate feature-screening

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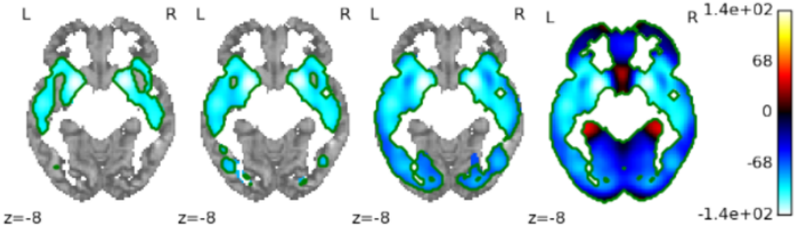
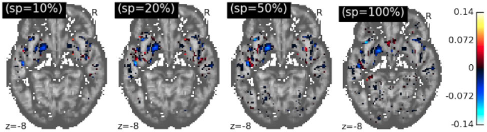


Age prediction from gray-matter maps

More speed via univariate feature-screening: results

- Age prediction



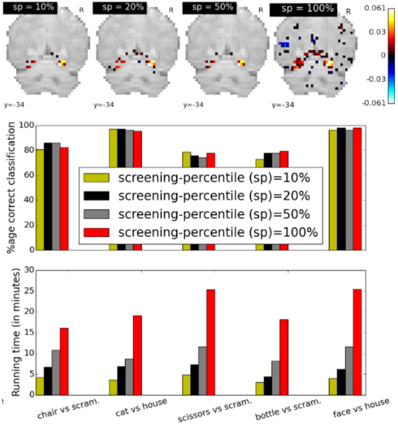
p	100%	50%	20%	10%
MSE	8.37	9.10	9.23	9.19

[Dohmatob '15 (PRNI)]

- Solve on **subset of features**
- Reduced **training time**

More speed via univariate feature-screening: results

- Visual object recognition



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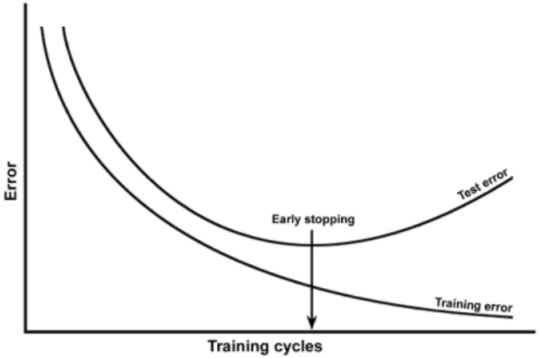
Early-stopping

- Stop optimization if accuracy on validation data stops improving [Dohmatob '15 (PRNI)]



- old idea (e.g [Bottou '07])
- saves training time
- implicit regularization
- helps against overfitting
- it's a compromise
  - it doesn't destroy accuracy
  - but may lead to sub-optimal brain maps

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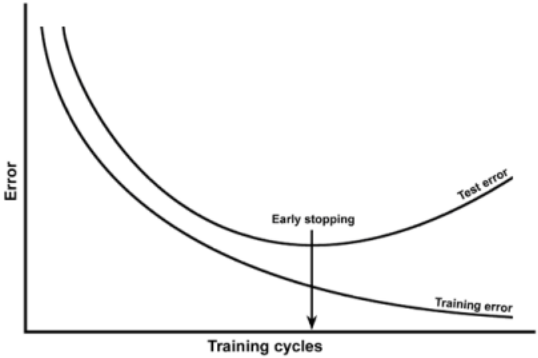

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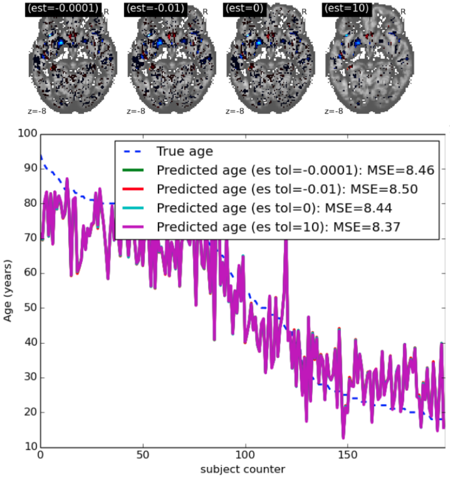

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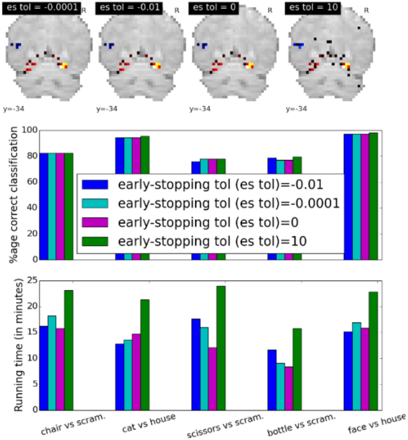

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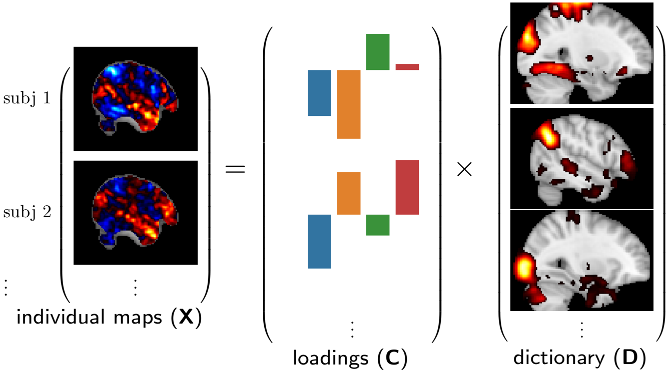
Section wrap-up

- Building on prior work, we have developed enhanced structured penalties for multi-variate brain-decoding
- Such penalties lead to more interpretable brain maps (a small number of smooth spatially localized regions)
- Focus on practical usability (fast model training)
- Our contributions are available as part of **Nilearn** toolkit.

Modelling inter-subject variability via dictionary-learning

Learn latent model for inter-subject variability

- **Goal:** Learn a latent model of inter-subject functional variability



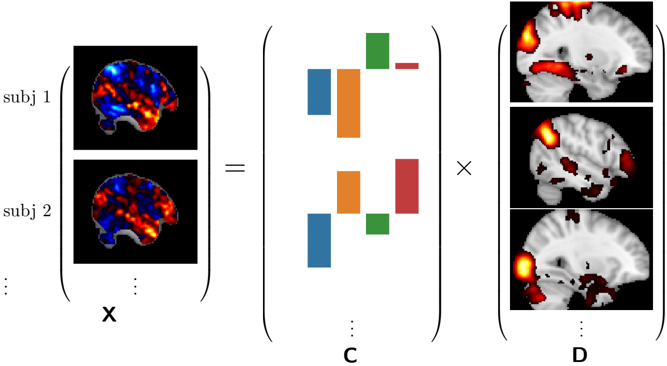
- Each **cognitive map** $\mathbf{x}_i$ with p voxels gets **encoded** over a **dictionary** $\mathbf{D}$ as k **loading coefficients** $\mathbf{c}_i$, with $k \ll p$

The challenge

[Dohmatob '16 (NIPS)]

- Sparsity: spatially localized atoms
- Smooth regions: each atom = interpretable blobs
- Scalable / online: model should trainable online

Introducing the proposed model [Dohmatob '16 (NIPS)]



$$\min_{\mathbf{D}\in\mathbb{R}^{p\times k}} \left(\lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} \min_{\mathbf{c}_t \in \mathbb{R}^k} \frac{1}{2}\|\mathbf{x}_t - \mathbf{D}\mathbf{c}_t\|_2^2 + \frac{1}{2}\alpha\|\mathbf{c}_t\|_2^2 \right)$$

subject to $\mathbf{d}^1, \ldots, \mathbf{d}^k \in \mathcal{K}$ [Mairal '09']

- $\mathcal{K} \subseteq \mathbb{R}^p$ is an ℓ_1 ball

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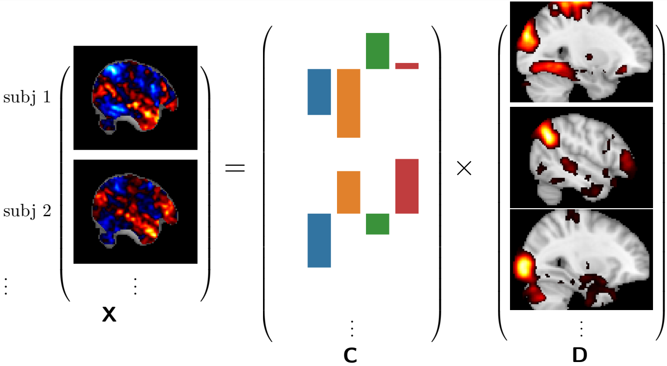

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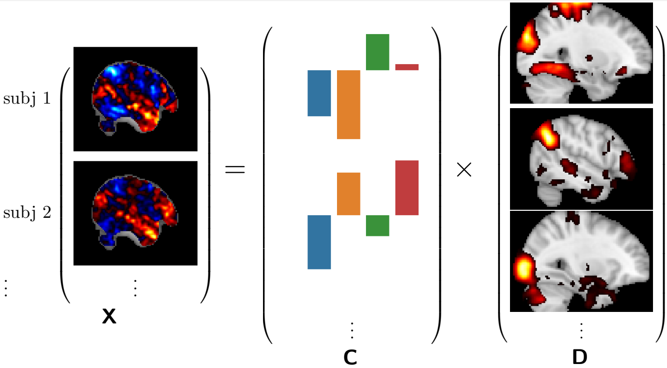

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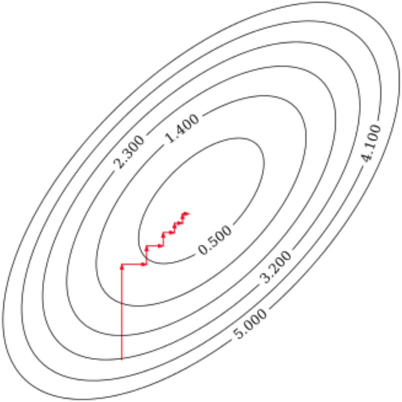
Reminder on coordinate-descent (CD)

- Optimize w.r.t a variable, and then w.r.t to another, and so on ...



The proposed algorithm

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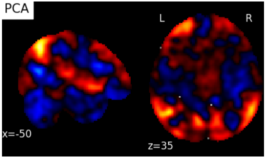
Experimental results on HCP fMRI data: qualitative



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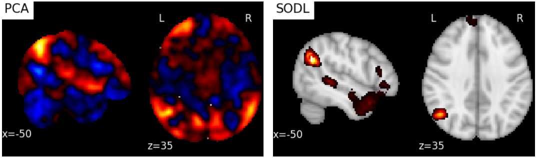

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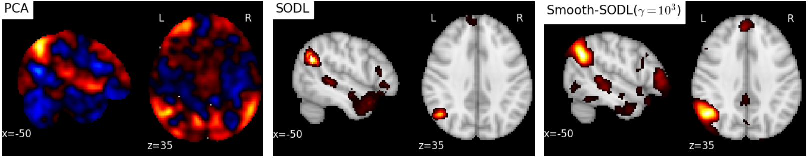

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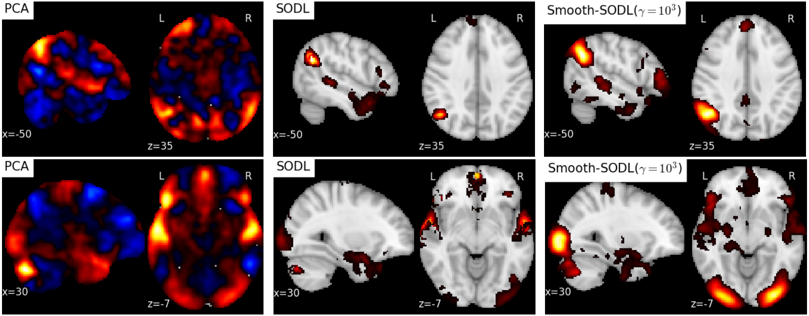

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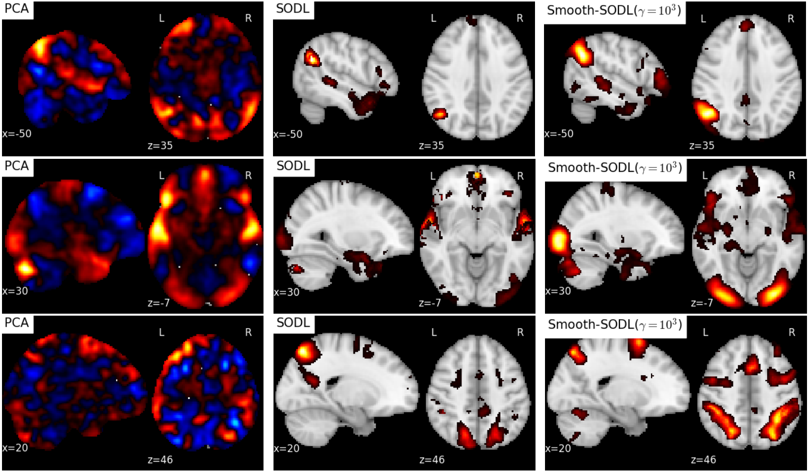

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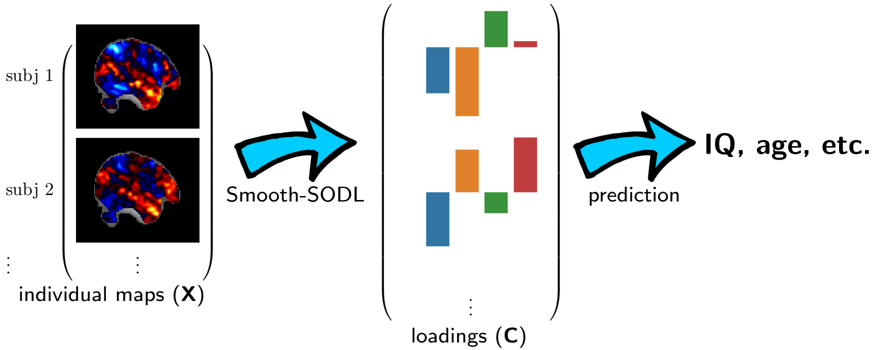
Learned latent dimensions capture inter-subject variability

- Predicting behavior from **compressed representation** of Story vs Math contrast of language task maps [van Essen '12]



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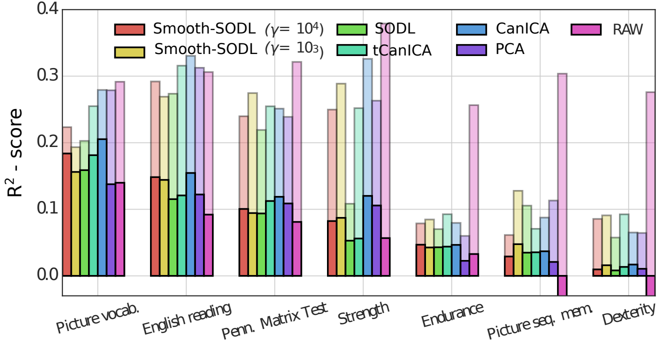
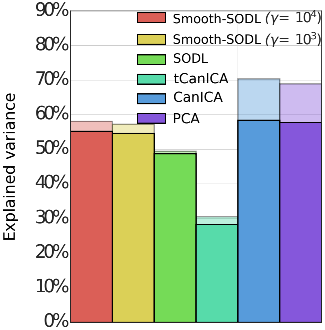


- Thick bars $\implies$ scores on **test** set; faint bars $\implies$ on **train**
- Proposed **Smooth-SODL** overfits the least (i.e generalizes best)

What's happening



- Unregularized models **overfit**
- Models thresholded post-training **underfit**

Spatial prior reduces sample-complexity

Nb. subjects	vanilla [Mairal '10]	Proposed model	gain factor
17	2%	**31%**	**13.8**
92	37%	**50%**	**1.35**
167	47%	**54%**	**1.15**
241	49%	**55%**	**1.11**

Learning-curve for "boost" in explained variance of our proposed Smooth-SODL model over the reference SODL model.

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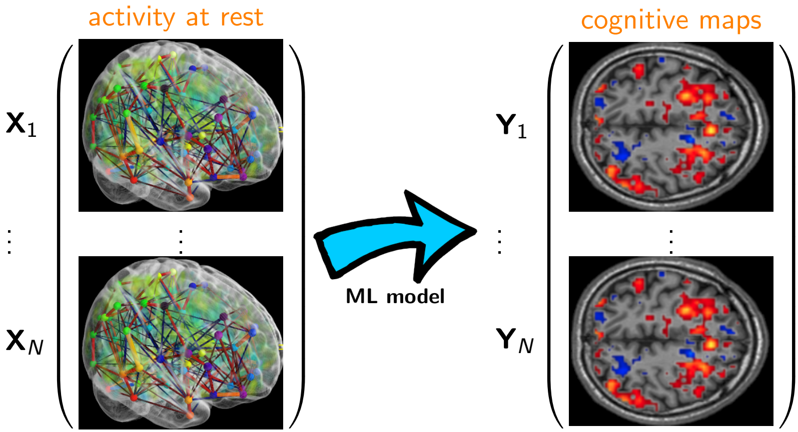
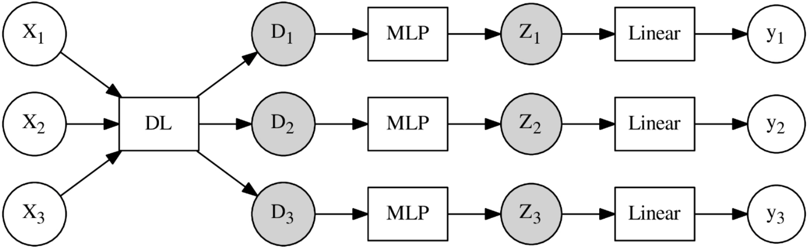
Can we predict task maps from resting-state data ?



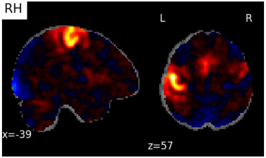
- $\mathbf{X}_s$: resting-state functional connectivity graph for subject s
- $\mathbf{Y}_s$: task-specific activation maps for subject s

Proposal: Deep semi-supervised voxel encoding



- $\mathbf{Y} \in \mathbb{R}^{p \times C}$: subject-specific GLM maps of brain activity
- $\mathbf{X} \in \mathbb{R}^{p \times T}$: resting-state fMRI data

Preliminary results: learned features



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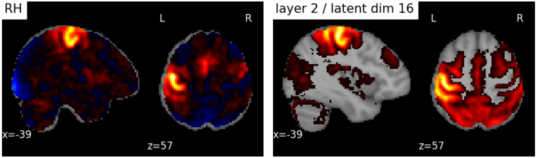

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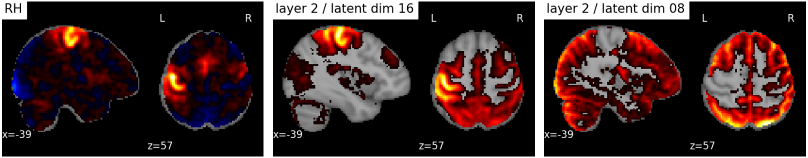

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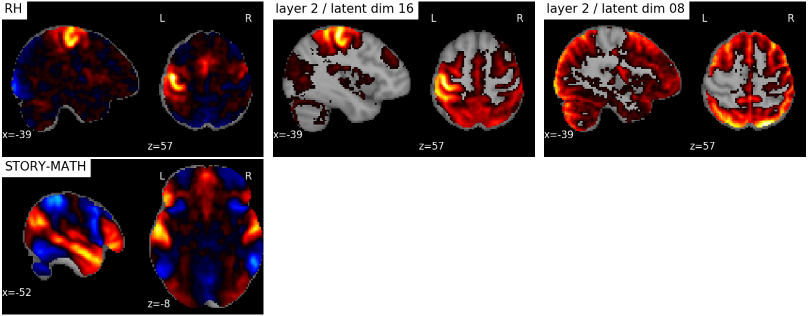

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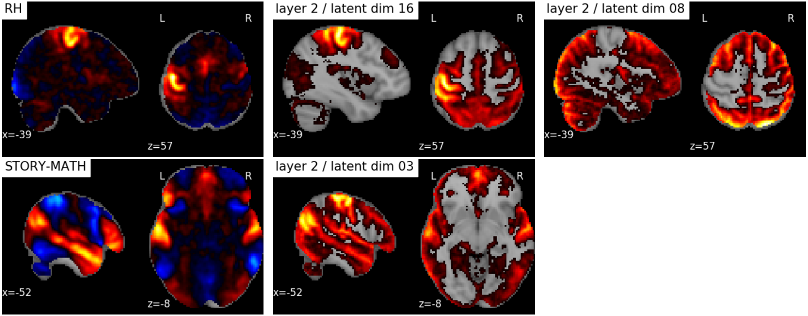

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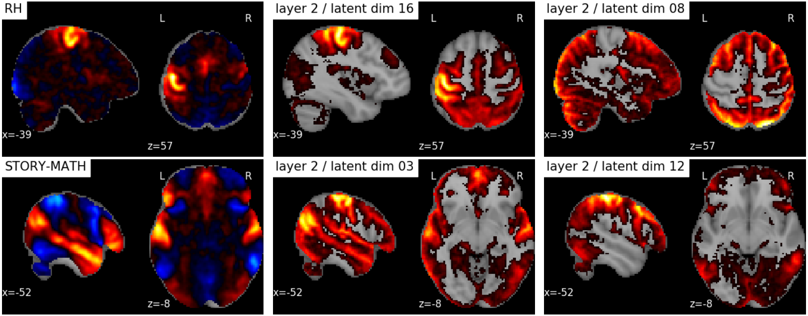

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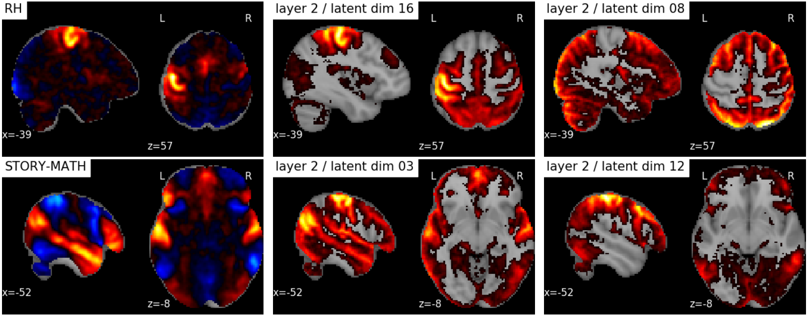
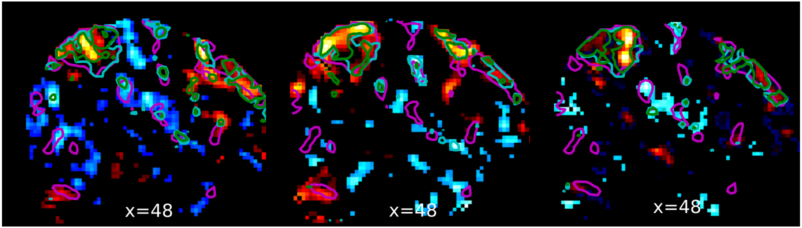


- Learned the a presentation of task activity in resting-state space!
- This is ongoing application of models developed in previous sections!

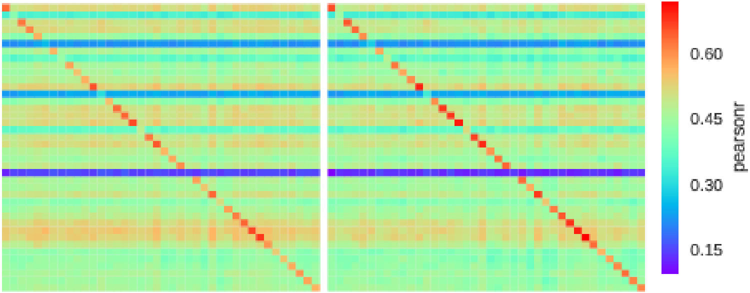
Preliminary results: predicted individual maps



2BK vs 0BK contrast of the Working Memory task
[van Essen '12]

- magenta = population mean
- reference method [Tavor '16]
- proposed method
  - Prediction agrees with subject's topography more faithfully

Preliminary results: quantitative



Confusion matrix for predicted versus true activation maps