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Nonlinear dependence in cryptocurrency markets

Pedro Chaim, Márcio P. Laurini*

Department of Economics – FEA-RP USP, Brazil

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ABSTRACT

We are interested in describing the returns and volatility dynamics of major cryptocurrencies. Very high volatility, large abrupt price swings, and apparent long memory in volatility are documented features of such assets. We estimate a multivariate stochastic volatility model with discontinuous jumps to mean returns and volatility. This formulation allows us to extract a time-varying shared average volatility and to account for possible large outliers. Nine cryptocurrencies with roughly three years of daily price observations are considered in the sample. Our results point to two high volatility periods in 2017 and early 2018. Qualitatively, the permanent volatility component seems driven by major market developments, as well as the level of popular interest in cryptocurrencies. Transitory mean jumps become larger and more frequent starting from early 2017, further suggesting shifts in cryptocurrencies return dynamics. Calibrated simulation exercises suggest the long memory dependence features of cryptocurrencies are well reproduced by stationary models with jump components.

1. Introduction

We are interested in exploring the joint dynamics of major cryptocurrencies. Recent attention devoted to statistical analysis of cryptocurrency markets has identified preeminent features, such as very high volatility, large return outliers, and increased temporal dependency of volatility. Another important aspect is the presence of long memory dependence structures in this asset class. In this work we use a multivariate stochastic volatility model with jumps in the mean and variance that reproduces the empirical patterns observed in the mean and conditional volatility of these series and is also able to generate the long memory behavior, reproducing all the essential aspects observed in this market.

Several recent empirical studies focus on the dynamics of volatility of cryptocurrencies returns. Generalized Autoregressive Conditional Heterosk-edasticity (GARCH) models are a popular approach. Dyhrberg (2016) and Dyhrberg (2016) estimates asymmetric threshold GARCH models to Bitcoin and argues the asset has hedging properties against stocks in the Financial Times Stock Exchange. Katsiampa (2017) compares several GARCH specifications to Bitcoin and finds an AR-CGARCH, which incorporates a long-run volatility component, to give best fit. Robustness of the results presented by Katsiampa (2017) and Dyhrberg (2016) are questioned by Charles and Darné (2018) and Baur, Dimpfl, and Kuck (2018), respectively. Baur and Dimpfl (2018) consider Bitcoin and five other cryptocurrencies in their analysis, an integrated GARCH model is shown to have best fit (in terms of likelihood criteria) for four cryptocurrencies; they also perform unconditional and conditional Value at Risk coverage tests. Return and volatility spillovers among cryptocurrencies are studied in Koutmos (2018) and Katsiampa (2018), and tail risk measures in Gkillas and Katsiampa (2018)

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* Corresponding author: Av. dos Bandeirantes 3900, 14040-905 Ribeirão Preto, SP, Brazil.
E-mail address: laurini@fearp.usp.br (M.P. Laurini).

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and Troster, Tiwari, Shahbaz, and Macedo (2018). The presence of possible jumps and regime changes was studied by Chaim and Laurini (2018), Fry (2018) and Ardia, Bluteau, and Rüede (2018). Long range dependence in cryptocurrency volatility is a well-documented feature. Bariviera, Basgall, Hasperué, and Naiouf (2017) calculate Hurst exponents for Bitcoin returns volatility and argue there is evidence of self-similarity. Lahmiri, Bekiros, and Salvi (2018) provide evidence of long-range dependence in seven Bitcoin markets using a fractionally integrated GARCH model under several specifications of error distribution. As discussed by Diebold and Inoue (2001) and Leung Lai and Xing (2006), long range dependence characteristics can appear due to structural breaks affecting parameters of processes driving conditional volatility and returns. Charfeddine and Maouchi (2018) employ several tests to argue cryptocurrency volatility has indeed true long memory rather than level shifts. The relationship between long memory and market inefficiency for cryptocurrencies was studied in Urquhart (2016) and Cheah, Mishra, Parhi, and Zhang (2018). Other aspects related to the presence of long memory in this market can be found in Phillip, Chan, and Peiris (2018), Al-Yahyaee, Mensi, and Yoon (2018), Jiang, Nie, and Ruan (2018), Alvarez-Ramirez, Rodriguez, and Ibarra-Valdez (2018) and Zargar and Kumar (2019).

In this work we show that the presence of level shifts in the structure of volatility, generated by jump processes, can reproduce the long memory behavior observed in these assets. A key aspect of data generating processes employed in this work is that series are constructed by mixing short memory processes with time varying conditional volatility, as opposed to true long memory processes. This is relevant in terms of volatility predictability, portfolio allocation, and risk management, since different underlying persistence properties are implied. Our sample consists of nine major cryptocurrencies with roughly three years of daily price observations, ranging from August 2015 to October 2018. Descriptive statistics in Section 2 illustrate cryptocurrency characteristics such as an overall very high level of unconditional volatility, the presence of large outliers, and mostly right-skewed, leptokurtic distributions of returns. Contemporaneous correlation between daily returns are positive and volatilities display signs of long-range temporal dependence, reproducing all the stylized empirical facts observed for these assets. Since optimal allocation applications and risk measures depend on model choices, the selected specification should account for peculiarities of cryptocurrencies. The multivariate stochastic volatility model introduced by Laurini, Mauad, and Auibe (2016) is suitable to deal with those characteristics. Conditional volatility is decomposed in an individual, mean-reverting, component; and a common permanent component which varies discontinuously when subject to a jump. Additionally, the level of returns is also subject to common jumps, but with contemporaneous effect only. This model can be viewed as a regime shift model in which both the number of regimes and transition probabilities are endogenously determined. Due to the presence of several latent variables model estimation is performed through a mixed Markov Chain Monte Carlo sampling procedure. The results obtained by the estimation of the model indicate that common time varying average daily volatility remains stable around 1.83% from 2015 until March 2017, when it jumps to a high level of about 3.87% daily volatility. This high volatility episode lasts until June. From June to November, average volatility rescinds to about 2.73%, still above pre-2017 levels. From late November to January 2018, amid great popular interest, volatility in cryptocurrency markets again increases. There is a steep reduction in volatility towards the end of our sample. This implied path of volatility is similar to what one finds if the model is estimated for each cryptocurrency individually. Unconditional probability of transitory mean return jumps is estimated 0.35, and those jumps become larger and more frequent starting from 2017. Employing estimated posterior means as calibration values for data generating processes, we present some Monte Carlo experiments showing that the multivariate stochastic volatility models with jumps in the mean and variance can generate the long memory patterns observed in these assets. The results obtained in our analysis indicate the importance of the presence of jumps and changes of level as components of conditional mean and volatility in econometric modelling of cryptocurrency market. The remainder of this paper is structured as follows. Section 2 presents the data and selected descriptive statistics. Section 3 describes the model and estimation procedure. Section 4 presents and discusses the main results. Section 5 concludes.

2. Data

Table 1 ranks cryptocurrencies in descending order of market capitalization. As of May 6, 2015, the entire cryptocurrency market consisted of about 400$ billion. Bitcoin concentrates 36% of the market, and Ethereum is second in market share, with 17%. Cryptocurrencies with over 1$ billion market capitalization, displayed in Table 1, accounted for about 88% of the whole market. Data availability on Ethereum limits our sample. We consider cryptocurrencies which surpassed 1$ billion in market capitalization over the cryptocurrency boom of 2017, and have roughly three years of daily price observations. These criteria leave us with nine cryptocurrencies: Bitcoin, Ethereum, Ripple, Litecoin, Stellar, Dash, Monero, NEM, and Verge[1]; with 1174 daily price observations between August 16, 2015 to October 31, 2018.

Descriptive statistics of daily log returns are displayed in Table 2. Returns have positive mean over the considered sample, which mostly cover periods of cryptocurrency expansion. Standard deviations are high relative to more traditional financial assets, as is characteristic of this particular asset class. Verge is the most volatile cryptocurrency, with daily returns varying, on average, 16.5%; and Bitcoin, the most consolidated digital currency, has daily returns varying, on average, 4%. Except for Bitcoin, cryptocurrencies display right skewed returns, with mean larger than median. Daily returns are leptokurtic, as is expected from financial assets, and some cryptocurrencies have very large outlier realizations, which increase kurtosis coefficients. Fig. 1 plots daily log returns of the nine cryptocurrencies in our sample, it is clear some of the assets, such as Ripple, Litecoin, Monero, and NEM, present very large

[1] Data was collected from (coinmetrics.io/data-downloads) (access on October 31, 2018). Ethereum limits our sample size, with earliest daily price observation dating August 07, 2015. We exclude the first eight observations due the destabilizing impact of extreme Ethereum return realizations in those days have on empirical estimates. Daily closing prices are computed at 00:00.

Table 1
Cryptocurrency markets.

Rank	Name	Symbol	Market Cap.	% Mkt. Cap.	Price
1	**Bitcoin**	**BTC**	**$165,947,375,277**	**36.07%**	**$9,751.33**
2	**Ethereum**	**ETH**	**$79,954,945,093**	**17.38%**	**$805.52**
3	**Ripple**	**XRP**	**$34,964,625,007**	**7.60%**	**$0.89**
4	Bitcoin Cash	BCH	$30,167,524,727	6.56%	$1,762.90
5	EOS	EOS	$14,461,111,605	3.14%	$17.26
6	**Litecoin**	**LTC**	**$9,893,568,748**	**2.15%**	**$175.41**
7	Cardano	ADA	$9,180,172,357	2.00%	$0.35
8	**Stellar**	**XLM**	**$7,819,698,649**	**1.70%**	**$0.42**
9	IOTA	MIOTA	$6,597,460,680	1.43%	$2.37
10	TRON	TRX	$5,531,270,046	1.20%	$0.08
11	NEO	NEO	$5,510,690,419	1.20%	$84.78
12	**Dash**	**DASH**	**$4,030,203,916**	**0.88%**	**$500.33**
13	**Monero**	**XMR**	**$3,824,282,820**	**0.83%**	**$238.98**
14	**NEM**	**XEM**	**$3,747,774,940**	**0.81%**	**$0.42**
15	Vechain	VEN	$2,646,691,078	0.58%	$5.03
16	Ethereum Classic	ETC	$2,464,362,518	0.54%	$24.27
17	Tether	USDT	$2,260,646,694	0.49%	$1.00
18	Qtum	QTUM	$2,011,712,156	0.44%	$22.71
19	OmiseGO	OMG	$1,750,481,140	0.38%	$17.15
20	ICON	ICX	$1,653,755,856	0.36%	$4.27
21	Binance Coin	BNB	$1,647,292,567	0.36%	$14.44
22	Lisk	LSK	$1,391,739,656	0.30%	$13.18
23	Bitcoin Gold	BTG	$1,359,545,417	0.30%	$80.04
24	Bytecoin	BCN	$1,325,028,027	0.29%	$0.01
25	Zcash	ZEC	$1,164,652,238	0.25%	$302.14
26	Nano	NANO	$1,148,466,740	0.25%	$8.62
27	**Verge**	**XVG**	**$1,138,578,906**	**0.25%**	**$0.08**
28	Zilliqa	ZIL	$1,039,166,283	0.23%	$0.14
29	Aeternity	AE	$1,021,907,254	0.22%	$4.39
30	Ontology	ONT	$1,000,404,503	0.22%	$8.88
	Total		$460,085,546,696		

Note: Table ranks cryptocurrencies in terms of market capitalization, as of 2018-05-06. Assets included in our sample are highlighted in bold. Market capitalization data collected fromhttps://coinmarketcap.com/historical/20180506/.

Table 2
Descriptive statistics of daily returns.

	Mean	Std.	Min	2.5q	50q	97.5q	Max	Skew.	Kurt.
Bitcoin	0.271	3.921	−20.208	−8.586	0.279	8.578	22.351	−0.154	7.832
Ethereum	0.400	6.618	−31.984	−12.826	−0.090	15.538	28.629	0.184	6.135
Ripple	0.341	7.688	−60.171	−11.820	−0.311	17.242	101.096	2.974	39.656
Litecoin	0.217	5.757	−39.105	−10.650	0.000	11.817	51.845	1.327	16.474
Stellar	0.392	8.441	−33.342	−14.453	−0.301	18.855	70.404	2.100	18.105
Dash	0.333	5.936	−24.343	−11.499	−0.121	14.101	38.310	0.840	8.170
Monero	0.442	7.138	−29.173	−13.223	0.000	15.623	56.767	1.012	10.331
NEM	0.562	9.203	−43.083	−14.276	0.000	20.152	106.849	2.206	23.276
Verge	0.565	16.538	−69.315	−31.366	0.000	38.621	95.572	0.641	7.649

Note: table presents descriptive statistics of daily log returns of the nine cryptocurrencies in our sample (which goes from 2015-08-16 to 2018-10-31). We report returns in percent points for better presentation. First column displays the mean. Second column shows standard deviations. Third and seventh columns show smallest and largest observations, respectively. Fourth through sixth columns show 2.5%, 50%, and 97.5% quantiles, respectively. Seventh column reports skewness coefficients, and the eighth column presents (raw) kurtosis coefficients.

outliers — possibly due to formative events in those particular markets.

Table 3 shows correlation between contemporaneous daily returns of digital currencies in our sample. Coefficients are all positive; correlation is overall highest with Bitcoin, and smallest with Verge. This is some preliminary evidence of cryptocurrency returns displaying contemporaneous comovements.

Table 4 presents selected descriptive statistics and dependence measures of returns volatility, measured by the absolute returns. Column 7 reports p-values of Augmented Dickey Fuller (ADF) unit root tests, and column 8 reports p-values of Kwiatkowski, Phillips, Schmidt, and Shin (1992) (KPSS) stationarity tests. ADF's null hypotheses of non-stationarity are uniformly rejected at 1% confidence level. Conversely, KPSS's null hypotheses of stationarity are also all rejected at 1%. Estimates of the fractional difference parameter d are between 0.29 and 0.40, which falls in the stationary range, but far from what we would expect from short memory processes. Contrasting test indications regarding stationarity, as well as estimates of fractional difference parameters, reinforce the established

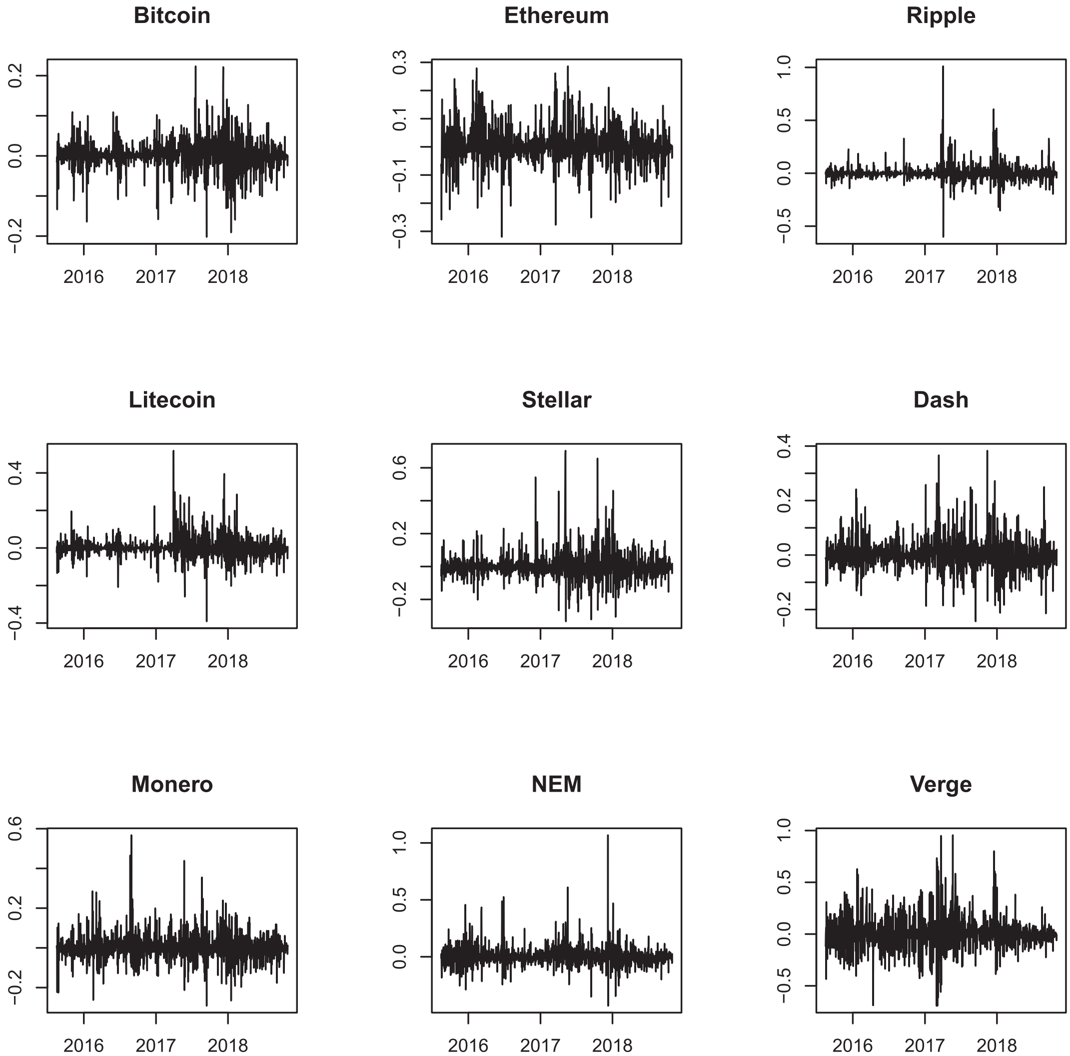


Fig. 1. Daily Returns. Note: figure plots daily log returns of the nine cryptocurrencies in our sample, from 2015-08-16 to 2018-10-31.

Table 3
Correlation between contemporaneous returns.

	Bitcoin	Ethereum	Ripple	Litecoin	Stellar	Dash	Monero	NEM	Verge
Bitcoin	1.000	0.376	0.277	0.591	0.340	0.453	0.465	0.366	0.238
Ethereum	0.376	1.000	0.250	0.371	0.269	0.386	0.376	0.274	0.155
Ripple	0.277	0.250	1.000	0.336	0.539	0.220	0.277	0.280	0.115
Litecoin	0.591	0.371	0.336	1.000	0.355	0.419	0.406	0.369	0.147
Stellar	0.340	0.269	0.539	0.355	1.000	0.276	0.369	0.370	0.158
Dash	0.453	0.386	0.220	0.419	0.276	1.000	0.463	0.320	0.210
Monero	0.465	0.376	0.277	0.406	0.369	0.463	1.000	0.288	0.186
NEM	0.366	0.274	0.280	0.369	0.370	0.320	0.288	1.000	0.206
Verge	0.238	0.155	0.115	0.147	0.158	0.210	0.186	0.206	1.000

Note: table reports Pearson correlation coefficients between contemporaneous daily log-returns of cryptocurrencies in our sample.

notion that returns volatility of cryptocurrencies display long range dependence characteristics. As discussed by Diebold and Inoue (2001) and Leung Lai and Xing (2006), long range dependence characteristics can appear due to structural breaks affecting parameters driving returns and volatility dynamics.

Table 4
Descriptive statistics of daily volatility (absolute returns).

	Mean	Std.	2.5q	50q	97.5q	d-GPH	ADF	KPSS
Bitcoin	2.538	2.985	0.057	1.382	10.865	0.395	0.010	0.010
Ethereum	4.542	4.821	0.116	2.945	18.387	0.330	0.010	0.010
Ripple	4.123	6.491	0.103	2.103	19.893	0.321	0.010	0.010
Litecoin	3.458	4.609	0.080	1.812	15.460	0.392	0.010	0.010
Stellar	5.189	6.665	0.142	3.205	22.018	0.296	0.010	0.010
Dash	4.085	4.314	0.112	2.839	15.730	0.366	0.010	0.010
Monero	4.872	5.213	0.146	3.273	17.900	0.357	0.010	0.010
NEM	5.867	7.102	0.214	3.863	22.317	0.206	0.010	0.010
Verge	10.827	12.510	0.221	6.846	43.559	0.297	0.010	0.010

Note: table displays selected descriptive statistics of daily log returns volatility, as measured by the absolute returns, of the nine cryptocurrencies in our sample (columns one through five), Geweke and Porter-Hudak (1983) long memory parameter *d* estimates (sixth column), p-values of unit root ADF tests (seventh column), and p-values of stationarity KPSS tests (eight column); p-values of 0.01 mean lower than 0.01. Volatility is reported in percent points for better presentation.

3. Model

In this section we describe the multivariate stochastic volatility proposed by Laurini et al. (2016), which features common jumps to the mean and volatility of the return process. The model is given by the following hierarchical representation:

$$y_{i,t} = \exp\left(\frac{h_{i,t}}{2} + \frac{s_i^v \mu_t}{2}\right)\varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(\gamma_{i,t}, 1) \tag{1}$$

$$h_{i,t} = \phi_i h_{i,t-1} + \sigma_i^h z_{i,t}^h, \quad z_{i,t}^h \sim N(0, 1), \tag{2}$$

$$\mu_t = \mu_{t-1} + \delta_t^v \sigma^v z_t^v, \quad z_t^v \sim N(0, 1), \tag{3}$$

$$\gamma_{i,t} = s_i^m \delta_t^m \sigma^m z_t^m, \quad z_t^m \sim N(0, 1), \tag{4}$$

$$\delta_t^v \sim Bernoulli(p_v), \quad \delta_t^m \sim Bernoulli(p_m), \tag{5}$$

$$h_{(i,j),t} = s_i^v s_j^v \mu_t, \tag{6}$$

where $y_{i,t}$ is the log return of the i-th cryptocurrency in time t. The conditional variance in Eq. (1) is decomposed into an idiosyncratic transitory component $h_{i,t}$, which follows a zero-mean first-order autoregressive process with persistence ϕ_i, and error standard deviation σ_i^h; and a permanent component μ_t, scaled by a factor loading s_i^v. The μ_t component can be interpreted as a common (global) variance factor subjected to jumps modelled by a compound binomial process, similar to the one employed by Qu and Perron (2013). Additionally, returns $y_{i,t}$ are subjected to a temporary jump in mean, $\gamma_{i,t}$—which is incorporated through the mean of the error process $\varepsilon_{i,t}$. Note that volatility jumps have permanent effects, while mean jumps only affect the return process contemporaneously. Incorporating jumps to the return process is a way to considering large outliers. The permanent component μ_t is subjected to a Gaussian iid disturbance with zero mean and standard deviation σ^v if the realization of the Bernoulli random variable δ_t^v (with parameter p_v) is one. The Bernoulli process captures the occurrence of jumps in common volatility. If this process realization is zero, the process maintains the previous value, and thus the common volatility level is constant. If a jump occurs, the change is given by the Gaussian innovation with standard deviation σ^v. This formulation allows to interpret this component as the mean of the latent volatility process, which is only altered by jumps with permanent effect. The transitory effects are given by the mean reverting process $h_{i,t}$. Individual asset sensibility to the permanent volatility component is given by the scaling factor s_i^v. Factor models of stochastic volatility have known identification issues related to observationally equivalent combinations of parameter values and arbitrary factor rotation schemes (Geweke & Zhou, 1996). In order to circumvent these issues, we follow a traditional approach (Aguilar & West, 2000; Chib, Nardari, & Shephard, 2006; Han, 2005; Lopes & Carvalho, 2007; Zhou, Nakajima, & West, 2014) and set the first loading s_1^v (related to Bitcoin) to 1.

A second source of common discontinuous jumps affects return processes. These jumps capture events that jointly impact returns in the system. Differently from jumps to global average volatility, which are permanent, jumps to return processes have contemporaneous effects only. If the realization of Bernoulli random variable δ_t^m, with parameter p_m, is 1, then $\varepsilon_{i,t}$ is sampled from a normal distribution with mean $\gamma_{i,t}$ and standard deviation 1 (a jump occurs); if δ_t^m comes up 0, then $\varepsilon_{i,t}$ is sampled from a standard normal distribution (no jump occurs). Note the model assumes distinct Bernoulli processes driving mean and variance jumps, thus jumps to returns and volatility are not correlated and can occur either simultaneously or independently. Individual sensibility of each asset to a jump in mean described by the scaling factor s_i^m. Here we also set s_1^m to one, as previous comments on identification apply. The intensity of jumps associated to s_1^m is given by the parameter σ_v. The size of the jump at time t, z_t^m, follows a standard normal distribution.

Covariance between two individual assets i and j at time t is given by the coefficient $h_{(i,t),y}$, which depends on the scaling factors of sensibility to volatility jumps, and the permanent volatility component μ_t. In this model, covariance between assets is given by the exposition to the common variance component factor μ_t, since idiosyncratic transitory component $h_{i,t}$ are assumed to be independent between assets. Due to the presence of several latent components, the estimation model (1)–(6) is based on a Bayesian mechanism using a mixed Markov Chain Monte Carlo procedure proposed by Laurini et al. (2016). Jump processes δ_t^m and δ_t^v are sampled using the data augmenting scheme proposed by Albert and Chib (1993). A jump occurs if the realization of an auxiliary independent, uniformly distributed variable, exceeds some threshold—which is determined by unconditional jump probabilities p_m and p_v. Remaining latent processes are sampled though a mixture of Gibbs and Metropolis steps using the Slice Sampler of Neal (2003). This setup allows to avoid, for example, the linearization step of Qu and Perron (2013), or the mixture of Gaussians used by Kim et al (1998). Estimation was carried out with 20,000 iterations of the sampling algorithm. In order to reduce the influence of initial values, the first 5,000 draws were discarded[2].

4. Results

Here we present and discuss parameter estimates of model (1)–(6) and comment on the implied path of latent variables. Then posterior means are employed as calibrated values for simulation exercises.

4.1. Posterior estimates

Posterior descriptive statistics of parameters common to all assets in the double jump model (1)–(6) are presented in Table 5. Standard deviation of permanent volatility jumps σ^v is estimated between 0.617 and 1.337 with 95% probability, and the unconditional probability such jumps, p_v has posterior mean of 0.010, which implies an average of about 3.6 jumps per year. Return jumps are more frequent, with unconditional probability p_m estimated around 0.358, and standard deviation σ^m between 0.110 and 0.127 with 95% probability.

Table 6 summarizes posterior descriptive statistics of parameters in model (1)–(6) that are particular to each individual asset *i*: loadings associated to volatility and returns, s_i^v and s_i^m; persistence parameter of autoregressive transitory volatility ϕ_i, and standard deviation of idiosyncratic volatility innovations σ_i^h. Ethereum and Bitcoin display higher persistence of transitory volatility h_t^i, suggesting these preeminent cryptocurrencies have (relatively) more autonomous volatility dynamics. Credibility intervals of persistence parameters ϕ_i are between 0.656 and 0.957, indicating some persistence of idiosyncratic volatility processes but no clear suggestion of nonstationarity. Volatility loadings s_i^v measure the exposure of each asset to the global time varying average volatility μ_t. Because μ_t is expressed in terms of log-volatility, smaller s_i^v means an increase in μ_t (in that it becomes less negative) will contribute less to the conditional volatility of asset *i*. That said, loadings s_i^v are all less than one, which is expected since Bitcoin is the less volatile cryptocurrency. Mean return jumps have amplified impact on altcoins other than Ripple; it is, their loadings s_i^m are above 1. It is the combination of jump probability p_m, standard deviation σ^m, and exposure s_i^m that determine the impact of mean return jumps on individual conditional volatility.

Fig. 2 plots the common permanent volatility component μ_t in panel (a), and the probability of a permanent volatility jump in panel (b). From the beginning of our sample on August 16, 2015, until March 2017, log-variance component μ_t remains stable at about -8, which implies $\exp(-8/2) = 1.83\%$ daily return volatility in the long run. On March 9 2017, μ_t jumps to -6.7, implying 3.50%. This (first) high volatility regime is sustained until June 1st, when daily volatility falls to about 2.87%, still above pre-2017 levels. By the end of November 2017, amid soaring prices and great popular interest, daily average cryptocurrency volatility again rises to high levels—where it remains until mid-January 2018. From January 2018 through the end of our sample volatility steadily falls. This common long run average volatility path is consistent with what is found when one estimates the jump model for Bitcoin alone (see Chaim & Laurini (2018)).

In Appendix A we compare the common time varying average component μ_t just estimated with the univariate version of the model. The main difference from model (1)–(6) is that now the permanent component is no longer a common factor among all assets, and as the model is estimated individually for each asset all loading's parameters are fixed in the unit value. One can notice a spike in volatility during 2017 and sharp volatility reduction towards the end of the sample are features shared by most cryptocurrencies.

Jumps in mean returns are very frequent, as we can see in Fig. 3. Unconditional probability p_m is estimated around 0.35 (see Table 5). This relatively high probability of joint return jumps suggests strong interconnection between cryptocurrencies, as already hinted by contemporaneous correlations in Table 3. Return jumps seem of larger magnitude and to happen more often from early 2017 onward. This phenomenon is evident from jumps $\gamma_{1,t}$ in panel (a) and probabilities of return jumps in panel (b), both from Fig. 3[3]. The year of 2017 was a period of high returns and volatility for cryptocurrency markets. Indeed, we observe the global volatility component μ_t increases steeply in early 2017, then again by the end of the year, to then fall from early 2018 towards the end of our sample. This change in cryptocurrency volatility dynamics could be due to increased popular attention. Measures such as Google Trends Index point to a peak in popular interest in cryptocurrencies during the year of 2017. See Kristoufek (2013), Kristoufek (2015), Yelowitz and Wilsonelowitz and Wilson (2015), for examples of discussions on the relation between Bitcoin and Google Trends Index.

In order to illustrate some fit properties of the model, we take absolute returns as proxy of true volatility and compare accuracy measures against standard univariate stochastic volatility (SV) models. To estimate SV models, we employ the algorithm of Kastner

[2] Programs are available upon request.

[3] Remember these are return jumps assuming loading $s_i^m = 1$. If the loading is not equal to one, the jump is scaled by the loading value.

Table 5
Posterior distributions common parameters.

	Mean	Std.	2.5q	50q	97.5q	Skew.	Kurt
σ^v	0.910	0.186	0.617	0.885	1.337	0.636	3.245
σ^m	0.118	0.004	0.110	0.118	0.127	0.057	3.019
p_v	0.010	0.004	0.004	0.010	0.017	0.390	3.017
p_m	0.358	0.024	0.312	0.359	0.398	−0.286	2.339

Note: table summarizes posterior descriptive statistics of parameters of the multivariate jump model (1)–(6) common to all assets. The standard deviation of permanent volatility innovations σ^v has prior $IG(1.5, 1.25)$, the standard deviation of jumps to contemporaneous mean returns σ^m has prior $IG(0.5, 2.5)$, unconditional probabilities of jumps to volatility and mean, p_v and p_m, have prior $B(1, 40)$. Sampling was carried out with 20,000 repetitions, and statistics calculated after a burn-in period of 5000 draws.

and Frühwirth-Schnatter (2014) to draw a single chain of 30,000 sample draws. From Table 7 one can see the multivariate double jump model has smaller mean errors and mean squared errors, but larger root mean squared errors for six out of nine cryptocurrencies. Interestingly, the double jump model seems to, on average, underestimate volatility. One could argue that from the point of view of risk management it is undesirable to underestimate volatility. Still mean errors of the multivariate jump model are considerably smaller in magnitude. This evidence suggests model (1)–(6) has fit properties comparable to traditional models such as standard SV. It is difficult to compare the adjustment results obtained in our estimations with other results found in the literature, since due to the large temporal variations in the empirical standards of cryptocurrencies between the several samples studied in these works.

4.2. Simulation evidence for the presence of long memory processes

As discussed in Section 1, there is a long literature reporting the presence of long memory effects in digital currency financial data. Long range dependence in cryptocurrency volatility is a well-documented feature. Bariviera et al. (2017) calculate Hurst exponents for Bitcoin returns volatility and argue there is evidence of self-similarity. Lahmiri et al. (2018) provide evidence of long-range dependence in seven Bitcoin markets using a fractionally integrated GARCH model under several specifications of error distribution. The possible presence of long memory is an important effect since this may indicate the presence of market inefficiency for these assets (e.g., Kristoufek (2018)), having important consequences for the predictability of these assets and in portfolio allocation procedures and risk management. However, these possible long-memory effects may be spurious statistical effects generated by the presence of structural breaks, changes in parameters and regimes, and thus an effect generated by the aggregation of several short-memory processes, as discussed by Diebold and Inoue (2001) and Leung Lai and Xing (2006). Charfeddine and Maouchi (2018) employ several tests to argue cryptocurrency volatility has indeed true long memory rather than level shifts. Perron and Qu (2010) showed S&P500 volatility dependence features are better described by a stationary model with level shifts (for example the stochastic volatility model with permanent jumps to volatility of Qu & Perron (2013)) rather than a true long memory process. Charfeddine and Maouchi (2018) employ several spectral-based estimators to argue that regarding major cryptocurrencies, volatility indeed display long memory characteristics. Our exercise follows Perron and Qu (2010) argumentation in showing stationary models augmented with levels shifts are often the source of long memory features in financial asset data, and that this could be that case for cryptocurrencies. To verify if the structure of jumps in the mean and volatility given by the model presented in Section 3 can generate these long memory effects, we perform Monte Carlo simulation. We employ posterior (mean) estimates to calibrate simulations exercises to verify whether the model considered is able to produce the increased temporal persistence characteristic of cryptocurrency volatility, and identify which features of those models are responsible for apparent long memory. We used four different specifications in the Monte Carlo experiments to study the effect of different jump components on the long memory parameter. The first specification consists of simulating a Gaussian iid process, which serves as a reference for a short memory process with no jumps in the mean or variance. The next specification assumes jumps only in the common volatility component, and in this specification we assume that the returns have zero mean with only jumps in the common variance component. The third specification assumes that the common component of volatility is constant in time ($\mu_t = \mu_{t-1}$), and we only have the jump process in the mean given by Eq. (4). The process μ_t assumes the value at time 1 estimated by the empirical model. The fourth specification assumes the complete model with jumps in the mean and variance structure. As mentioned, we use the posterior mean of the parameters estimated in the model applied to the real data as parameters in these different specifications. For these different specifications, we generate 2,000 simulated samples, for sample sizes of 200, 500 and 1000 observations, and estimated the long memory parameter d using the GPH estimator for two different measures of conditional variance, the squared returns and the log of the squared returns. The squared returns are a measure closer to the estimation of GARCH models, which assume that the squared returns follows an ARMA process for short memory or ARFIMA for long memory, and the log squared returns is a proxy linked to the stochastic volatility model, since usual forms of estimation for this model use a discretization based on the log squared returns as the dependent variable in a state space model. Table 8 reports the GPH estimates for the squared returns, and the Table 9 for the log squared returns. Specification details are presented in Appendix.

In general, the results presented in Tables 8 and 9 are quite clear to indicate the effectiveness of jump components in generating long memory patterns for the measures of conditional variance studied. As expected, the iid process without jumps in the mean or variance presents estimated values for the coefficient d close to zero, with a proportion of rejections of the null hypothesis of short

Table 6
Posterior distributions individual parameters.

		Mean	Std.	2.5q	50q	97.5q	Skew.	Kurt
Bitcoin	s_1^v	1.000	0.000	1.000	1.000	1.000	0.000	0.000
	s_1^m	1.000	0.000	1.000	1.000	1.000	0.000	0.000
	ϕ_1	0.904	0.031	0.837	0.906	0.957	−0.433	3.139
	σ_1^h	0.306	0.091	0.147	0.305	0.488	0.221	2.682
Ethereum	s_2^v	0.840	0.037	0.761	0.841	0.905	−0.225	2.585
	s_2^m	1.132	0.032	1.071	1.132	1.192	−0.014	2.921
	ϕ_2	0.916	0.023	0.867	0.918	0.957	−0.421	3.224
	σ_2^h	0.324	0.081	0.188	0.319	0.496	0.386	2.728
Ripple	s_3^v	0.917	0.028	0.856	0.917	0.974	−0.097	3.159
	s_3^m	0.993	0.033	0.927	0.993	1.059	−0.001	2.957
	ϕ_3	0.770	0.038	0.694	0.771	0.842	−0.167	3.383
	σ_3^h	1.124	0.178	0.787	1.120	1.481	0.135	3.127
Litecoin	s_4^v	0.990	0.032	0.929	0.989	1.056	0.135	2.911
	s_4^m	1.090	0.031	1.028	1.090	1.152	0.024	3.117
	ϕ_4	0.766	0.052	0.656	0.768	0.860	−0.337	3.127
	σ_4^h	1.120	0.225	0.709	1.096	1.605	0.387	2.958
Stellar	s_5^v	0.818	0.028	0.770	0.815	0.879	0.440	2.889
	s_5^m	1.161	0.043	1.077	1.161	1.247	0.089	2.926
	ϕ_5	0.753	0.042	0.661	0.756	0.827	−0.442	3.094
	σ_5^h	0.771	0.134	0.544	0.756	1.076	0.538	3.483
Dash	s_6^v	0.865	0.026	0.823	0.862	0.920	0.492	2.630
	s_6^m	1.112	0.036	1.041	1.112	1.183	−0.037	2.971
	ϕ_6	0.785	0.046	0.681	0.790	0.864	−0.526	3.452
	σ_6^h	0.487	0.109	0.293	0.481	0.722	0.324	2.916
Monero	s_7^v	0.817	0.028	0.772	0.814	0.879	0.570	2.924
	s_7^m	1.192	0.039	1.116	1.192	1.269	0.015	2.913
	ϕ_7	0.867	0.029	0.804	0.869	0.916	−0.447	3.109
	σ_7^h	0.375	0.081	0.235	0.365	0.558	0.503	3.128
NEM	s_8^v	0.782	0.028	0.733	0.781	0.835	0.128	2.374
	s_8^m	1.267	0.040	1.192	1.265	1.348	0.138	2.988
	ϕ_8	0.814	0.042	0.721	0.819	0.886	−0.522	3.278
	σ_8^h	0.600	0.133	0.389	0.584	0.884	0.522	2.899
Verge	s_9^v	0.618	0.025	0.569	0.620	0.662	−0.175	3.149
	s_9^m	1.486	0.054	1.378	1.487	1.592	−0.047	3.000
	ϕ_9	0.828	0.043	0.736	0.831	0.905	−0.346	2.866
	σ_9^h	0.930	0.278	0.498	0.898	1.634	0.768	3.588

Note: table summarizes posterior descriptive statistics of parameters of the multivariate jump model (1)–(6) common to all assets. The standard deviation of permanent volatility innovations σ^v has prior $IG(1.5, 1.25)$, the standard deviation of jumps to contemporaneous mean returns σ^m has prior $IG(0.5, 2.5)$, unconditional probabilities of jumps to volatility and mean, p_v and p_m, have prior $B(1, 40)$. Sampling was carried out with 20,000 repetitions, and statistics calculated after a burn-in period of 5000 draws.

memory (d parameter equals zero) close to the expected level of 5%, confirming the good properties of the GPH estimator in the validity of null hypothesis. In all other specifications with the inclusion of jump components, the results are quite different. We can observe that both the individual inclusion of jump components in the mean or variance structures, as well as the inclusion of jump components in these two components at the same time, are able to generate long memory effects in the generated series. We can observe that for specifications with jumps the estimated, coefficient d is close to the values found in the long memory estimates for cryptocurrencies (approximately ranging from 0.1 to 0.28 in our experiments), with a proportion of violations well above the expected 5% proportion of rejections, with this proportion increasing with the number of observations. These results are very important, since they show that different natures of jumps can induce the presence of long memory in the process of conditional variance. Jumps in the common component of volatility generate level changes in volatility, which are persistent and affect all series

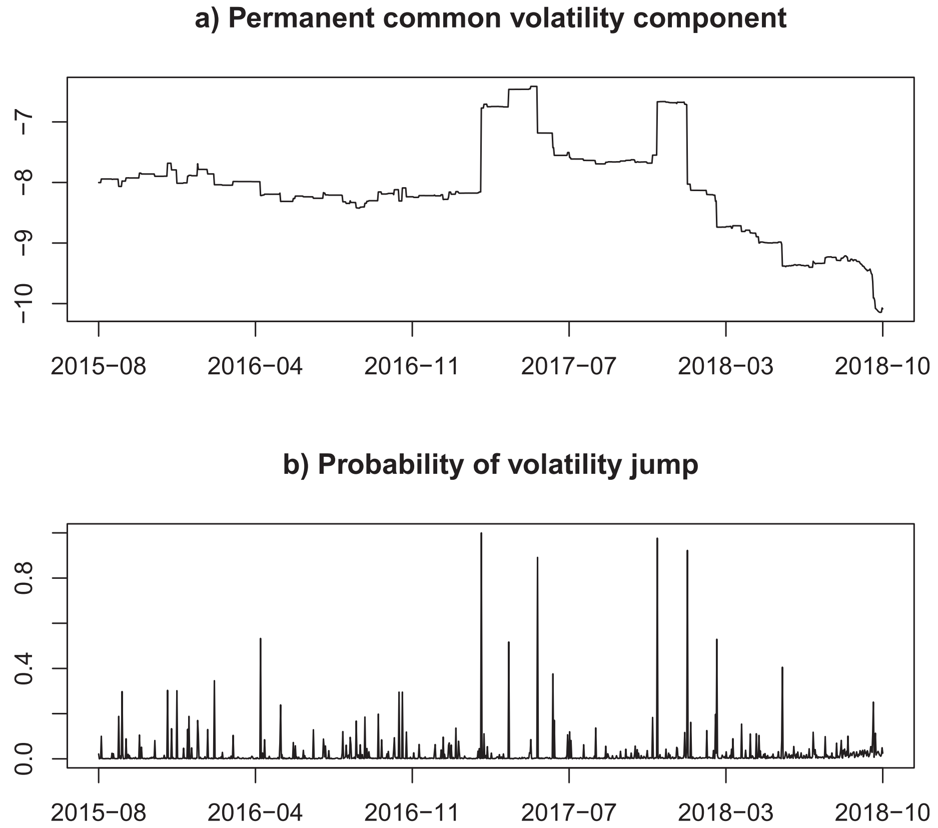


Fig. 2. Volatility component and probability of jumps. Note: figure presents the permanent volatility component μ_t in panel (a) and the probability of a volatility jump in panel (b).

in the process, inducing a direct effect on the persistence of conditional volatility. This effect may be related to the influence of level changes and regime switching for the generation of long memory effects. But note that even a non-persistent process, such as the jump structure in the mean used in this work, can generate long memory effects for the conditional variance. This effect can be related to observations of long memory generated by the aggregation of distinct stochastic processes of short memory or zero memory. The combination of these two types of jumps also generates the long memory effects observed in financial assets, especially the cryptocurrency market studied in this work. Note that the results are robust to the choice of the squared returns or log squared returns and the sample size used.

4.3. Sub-sample analysis

When looking at Fig. 3 we can see that there is an apparent change in the process of jumps in the mean between the period until approximately the end of 2016 and the period after 2017. In this final period, we can note a significantly increase in the number of jumps in mean, which correspond to the common events directly affecting the returns on all assets. To verify if there are relevant changes in the parameters of the model in these two periods, we estimate the model in two sub-samples. The first one runs from August 2015 until December 2016, and the second from January 2017 until October 2018. For space reasons we focus our analysis on the parameters related to the jumps in the mean, but the other results for this sub-sample analysis are available with the authors. The procedures of estimation and structure of priors are identical to those used in the estimation with the full sample. The Fig. 4 shows the estimated posterior distribution for the two sub-samples for the parameter p_m, which is the unconditional probability of jumps in mean. We can note that there is a large difference between these two estimates. In the first sub-sample the parameter p_m is concentrated near zero, indicating that the common jumps in the returns have low probability, showing that until 2017 the chance of common events (jumps) in the returns is almost null. After 2017 this situation changes in a relevant way. We can observe that in the second sub-sample the posterior distribution of the parameter p_m concentrates around 0.52, indicating that there is now a relevant dynamic of direct transmission of shocks in returns for these assets. The conditional jump probability in the mean for the two sub-periods confirms this interpretation, as shown in Fig. 5. For the first sub-sample there is no day with jump probability greater than 0.3, while for the second sub-sample there is a relevant proportion of days with jump probability close to 1. This result indicates a

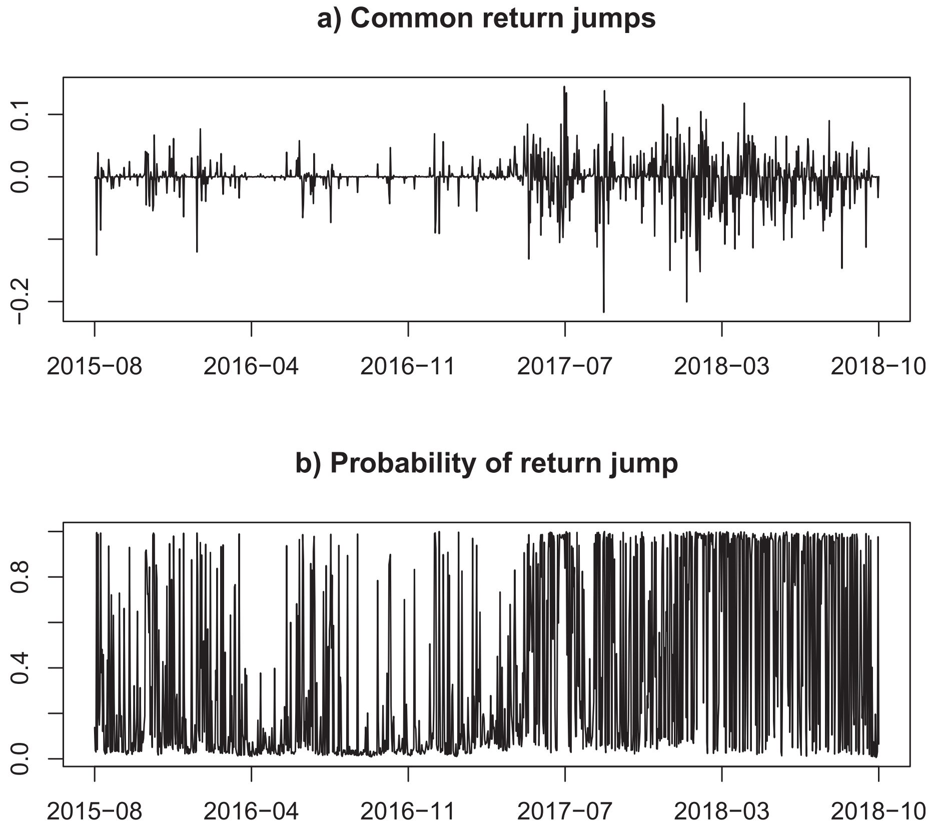


Fig. 3. Mean jumps and probability of jumps. Note: figure presents common return jumps γ_t in panel (a) and the probability of return jumps in panel (b).

Table 7
Model Fit Models.

	Mean Error			Root Mean Squared Error			Mean Absolute Error		
	Jump	SV	Δ%	Jump	SV	Δ%	Jump	SV	Δ%
BTC	−0.00453	0.0077	58.8%	0.027	0.0236	114.4%	0.0165	0.0173	95.4%
ETH	−0.00361	0.0123	29.3%	0.0407	0.0369	110.3%	0.0282	0.0288	97.9%
XRP	−0.00532	0.0107	49.7%	0.0408	0.0414	98.6%	0.0231	0.0252	91.7%
LTC	−0.00902	0.0102	88.4%	0.0358	0.0327	109.5%	0.0207	0.022	94.1%
XLM	−0.0053	0.013	40.8%	0.0441	0.0453	97.4%	0.0275	0.0315	87.3%
DASH	−0.00547	0.0109	50.2%	0.0368	0.0353	104.2%	0.0242	0.0261	92.7%
XMR	−0.00484	0.013	37.2%	0.044	0.0415	106.0%	0.0293	0.0308	95.1%
XEM	−0.00516	0.015	34.4%	0.0524	0.0518	101.2%	0.0332	0.0359	92.5%
XVG	0.00611	0.0303	20.2%	0.0805	0.0903	89.1%	0.0574	0.0664	86.4%

Note: table reports comparison of forecast accuracy measures between the multivariate double jump model (Jump) and univariate standard log-normal stochastic volatility models (SV). Columns Δ% stands for Jump model errors as proportion of SV error measures. 30000 sample draws using the algorithm of Kastner and Frühwirth-Schnatter (2014) are employed in estimating SV models.

relevant change in the cryptocurrency market from the year 2017. This result is consistent with the public interest in this market, which has grown substantially in the second sub-sample.

5. Conclusion

We investigated the joint dynamics of nine major cryptocurrencies by estimating a model with common discontinuous jumps to returns and volatility, introduced by Laurini et al. (2016). This model is based on a nonlinear structure that captures the presence of

Table 8
GPH Estimation for the Squared Returns.

Model	S. Size		Bitcoin	Ethereum	Ripple	Litecoin	Stellar	Dash	Monero	NEM	Verge
iid	200	mean d	0.0028	−0.0002	0.0003	0.0028	0.0019	−0.0008	0.0028	−0.0011	0.0006
		prop. rej.	0.058	0.065	0.0595	0.071	0.0605	0.0575	0.0545	0.062	0.057
	500	mean d	0.0013	0.0009	0.0005	−0.0015	3.4e−05	0.0002	−0.0023	0.0012	−0.0010
		prop. rej.	0.0545	0.061	0.0555	0.0495	0.062	0.045	0.058	0.0575	0.0495
	1000	mean d	−0.0011	5.97e−05	−0.0017	−0.0020	−0.0005	0.00021	0.0014	−0.0022	0.00151
		prop. rej.	0.0525	0.051	0.051	0.049	0.0515	0.059	0.0475	0.0585	0.051
variance only	200	mean d	0.183	0.198	0.169	0.175	0.162	0.142	0.174	0.181	0.208
		prop. rej.	0.397	0.431	0.423	0.438	0.362	0.274	0.362	0.401	0.518
	500	mean d	0.195	0.219	0.136	0.141	0.146	0.133	0.182	0.168	0.185
		prop. rej.	0.658	0.755	0.48	0.475	0.467	0.401	0.617	0.56	0.645
	1000	mean d	− 0.202	0.224	0.107	0.118	0.124	0.124	0.183	0.151	0.166
		prop. rej.	0.879	0.927	0.452	0.517	0.534	0.521	0.819	0.658	0.749
mean only	200	mean d	0.144	0.188	0.162	0.172	0.155	0.127	0.172	0.178	0.209
		prop. rej.	0.289	0.402	0.408	0.419	0.338	0.237	0.356	0.386	0.52
	500	mean d	0.166	0.212	0.135	0.142	0.134	0.123	0.177	0.164	0.185
		prop. rej.	0.529	0.722	0.469	0.481	0.435	0.365	0.604	0.556	0.654
	1000	mean d	0.169	0.216	0.115	0.113	0.121	0.116	0.177	0.148	0.165
		prop. rej.	0.745	0.907	0.492	0.483	0.51	0.482	0.793	0.665	0.746
full model	200	mean d	0.145	0.19	0.162	0.165	0.154	0.125	0.171	0.171	0.212
		prop. rej.	0.293	0.419	0.41	0.397	0.34	0.241	0.348	0.376	0.528
	500	mean d	0.16	0.208	0.137	0.139	0.138	0.126	0.175	0.161	0.191
		prop. rej.	0.517	0.725	0.467	0.477	0.449	0.38	0.585	0.527	0.663
	1000	mean d	0.171	0.216	0.111	0.115	0.124	0.116	0.175	0.149	0.167
		prop. rej.	0.756	0.908	0.489	0.499	0.527	0.48	0.785	0.67	0.725

Table reports average d GPH estimates (using the $m = [T^{0.5}]$ lowest frequencies) and the proportion of rejection of the $d = 0$ hypothesis with 95% confidence level, from 2000 squared returns simulations of four model specifications described in Appendix.

Table 9
GPH Estimation for the Log of Squared Returns.

Model	S. Size		Bitcoin	Ethereum	Ripple	Litecoin	Stellar	Dash	Monero	NEM	Verge
iid	200	mean d	0.0015	−0.0056	0.0034	−0.00012	−0.0016	−0.0009	−0.0016	0.0004	0.0012
		prop. rej.	0.0555	0.0645	0.069	0.0645	0.067	0.057	0.0545	0.055	0.0655
	500	mean d	−0.0017	−0.0004	−1.96e−05	0.0013	0.0014	−0.0003	0.0012	−0.0004	−0.0028
		prop. rej.	0.051	0.0555	0.058	0.054	0.058	0.0635	0.058	0.0535	0.0575
	1000	mean d	0.0006	0.0022	−0.0008	0.0003	7.55e−05	−5.73e−05	1.82e−05	−0.0011	−0.0011
		prop. rej.	0.062	0.058	0.054	0.0535	0.053	0.05	0.0565	0.0475	0.048
jumps in variance only	200	mean d	0.121	0.145	0.27	0.257	0.168	0.1	0.13	0.165	0.271
		prop. rej.	0.205	0.261	0.653	0.6	0.343	0.156	0.22	0.314	0.648
	500	mean d	0.133	0.169	0.239	0.231	0.156	0.0963	0.136	0.157	0.257
		prop. rej.	0.398	0.573	0.835	0.821	0.51	0.234	0.405	0.497	0.879
	1000	mean d	0.143	0.175	0.213	0.204	0.141	0.0926	0.136	0.147	0.247
		prop. rej.	0.659	0.807	0.92	0.898	0.645	0.333	0.605	0.674	0.973
jumps in mean only	200	mean d	0.0965	0.134	0.242	0.214	0.156	0.091	0.118	0.15	0.263
		prop. rej.	0.153	0.237	0.57	0.481	0.288	0.124	0.191	0.273	0.624
	500	mean d	0.105	0.156	0.215	0.195	0.143	0.0895	0.122	0.146	0.248
		prop. rej.	0.288	0.506	0.76	0.679	0.443	0.207	0.346	0.468	0.863
	1000	mean d	0.115	0.16	0.194	0.177	0.13	0.0824	0.123	0.135	0.239
		prop. rej.	0.478	0.746	0.87	0.812	0.57	0.283	0.525	0.6	0.966
jumps in mean and variance	200	mean d	0.0913	0.134	0.246	0.216	0.152	0.0893	0.113	0.151	0.263
		prop. rej.	0.148	0.227	0.58	0.485	0.289	0.141	0.179	0.271	0.642
	500	mean d	0.106	0.152	0.216	0.194	0.142	0.0855	0.12	0.144	0.254
		prop. rej.	0.289	0.497	0.76	0.69	0.447	0.188	0.336	0.453	0.88
	1000	mean d	0.114	0.161	0.198	0.179	0.13	0.0822	0.126	0.139	0.238
		prop. rej.	0.465	0.75	0.894	0.83	0.583	0.278	0.56	0.631	0.961

Table reports average d GPH estimates (using the $m = [T^{0.5}]$ lowest frequencies) and the proportion of rejection of the $d = 0$ hypothesis with 95% confidence level, from 2,000 log squared returns simulations of four model specifications described in Appendix.

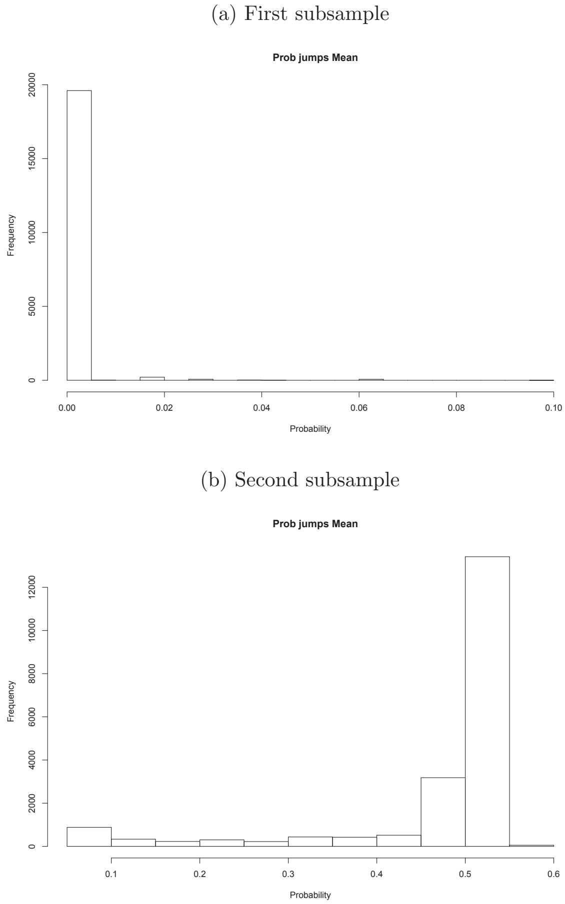
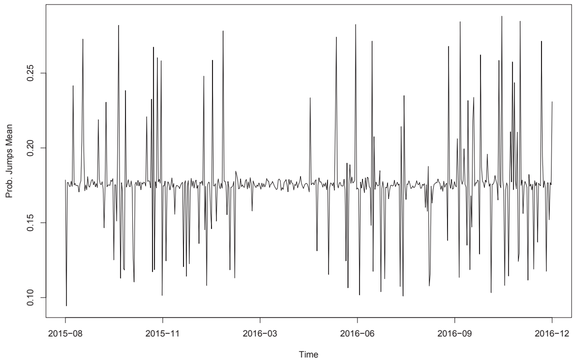


Fig. 4. Unconditional probability of jumps in mean – subsample analysis. *Note:* Unconditional Probability of Jumps in Mean. (a) First subsample August 2015 – December 2016, (b) Second subsample – January 2017 – October 2018.

common jumps (co-jumps) in the mean and conditional volatility structures, and allows to capture the most important stylized facts in the cryptocurrency market. The structure of common jumps incorporates the existence of joint movements in the cryptocurrency market, a fact that is very important for the investors of these assets. Joint variations of high magnitude reflect changes in risk levels and transmission of shocks in this market. Unlike traditional assets such as shares traded on centralized stock exchanges, cryptocurrencies are subject to additional sources of risk such as regulatory changes, forks and possibilities for fraud and price manipulation via hacking, and to large effects on prices due to the possible existence of price bubbles in these assets. In general, the occurrence of these events has effects on the whole market of cryptocurrencies, generating the common movements observed in this market. The results obtained in the estimation of the model indicate that the common variance component μ_t remains relatively stable around -8, implying 1.83% mean daily volatility from 2015-08 to 2017-01. One can notice two high volatility episodes, in early 2017 and then in early 2018, then steady reduction towards the end of our sample on October 2018. The path of common average volatility is similar to individually estimated ones. Transitory mean jumps occur, on average, once every three days; though become more frequent and of larger magnitude from 2017 onward. These are some indications of a structural change in cryptocurrency dynamics in past years. From an estimation in sub-samples, it is possible to observe that from 2017 the return dynamics presents a high proportion of common shocks in the returns, indicating a substantial increase in the dependence of the returns in this market, with a high proportion of days where common shocks affect the returns on all assets. Another important result obtained in this work is to show that the observed long memory evidences for the most important cryptocurrencies can be generated by the presence of jumps in the mean and variance of these assets. The Monte Carlo experiments performed show that the individual jumping structures in the mean and variance can generate the observed long memory patterns, and thus giving an alternative interpretation to the reported market inefficiencies for these assets. The methodology used in this work presents some important restrictions. The first restriction is that we consider all jump processes to be common among all assets, and all individual dynamics are placed in the specific factors, which are assumed to follow continuous random variables. One possibility is to introduce specific jump components to the individual assets, which would allow capturing specific large discrete variations. Although this possibility is interesting in terms of model fit, the

(a) First subsample



(b) Second subsample

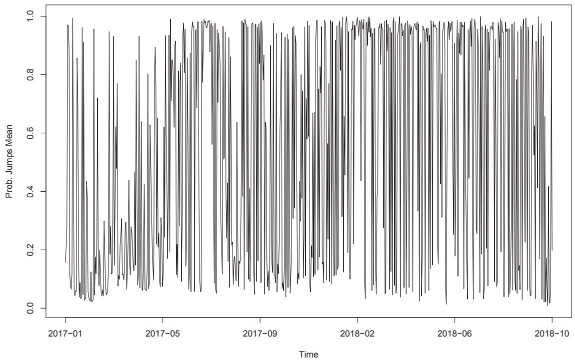


Fig. 5. Probability of jumps in mean – subsample analysis. *Note:* Probability of Jumps in Mean. First subsample August 2015 – December 2016, (b) Second subsample – January 2017 – October 2018.

number of latent parameters and processes would be quite high, which can pose problems in terms of model estimation and possible overfitting. A second interesting modification is to allow the jump process to be time dependent, in contrast to the independent Bernoulli process used in this work. In a structure of dependent jumps the presence of a jump could increase or decrease the probability of a jump at the next moment. This possibility could be implemented using a temporal dependence structure for the parameters related to the probability of jumps. This is a possibility not yet explored in this class of models. In this work our focus was to propose an identification mechanism for the effect of possible jumps on the mean and variance in the structure of returns and conditional volatilities for a relevant set of cryptocurrencies. The results indicate that these components are important in the temporal dynamics of these assets and can adequately explain the empirical characteristics of this sector, such as the presence of large price changes and persistent changes in volatility levels. We also show that this jumping mechanism can generate the observed long memory phenomena for this class of assets, and that this mechanism can be generated by common jumps in the mean, variance or in these two components simultaneously. Important aspects in terms of portfolio allocation and risk management in the presence of these jump processes still need to be explored for this asset class. Using the same class of models used in this paper, Laurini et al. (2016) show that jumps have important allocation and risk effects for companies in the oil sector. They show that a minimum variance allocation strategy based on the common component of conditional volatility has a better performance in terms of Sharpe ratio and turnover in relation to allocation strategies based on conditional volatility models of the GARCH family. They also show that the inclusion of jump components is important for the calculation of tail risk measures, such as Value at Risk. An important extension for this work would be to verify the performance of these methods for the cryptocurrency market, which presents complex challenges in terms of allocation and risk management.

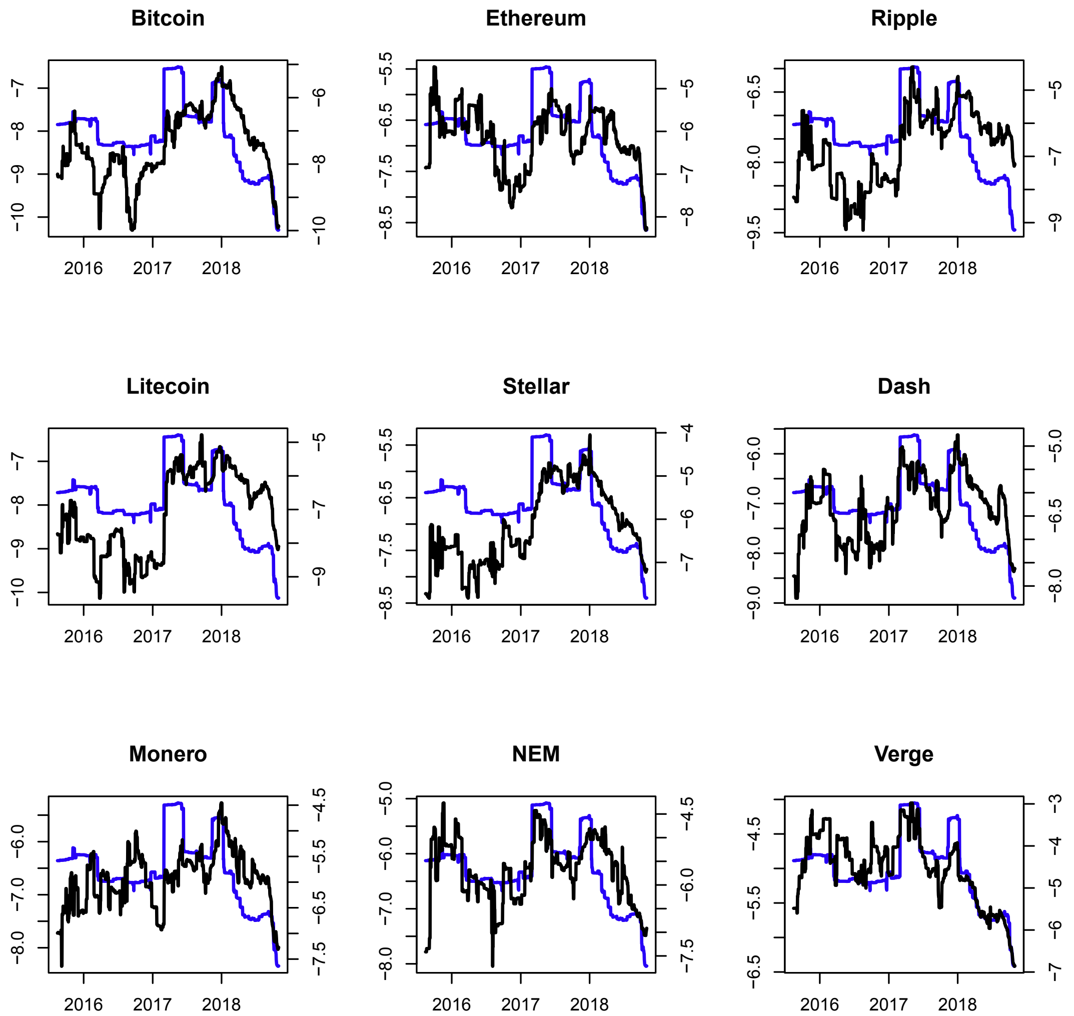


Fig. 6. Common and individual long-run volatility. *Note:* Fig. 6 plots the common time-varying average volatility μ_t estimated in Section 4 (solid blue lines) against the implied trend volatility when model (1)–(6) is estimated for each cryptocurrency individually (solid black lines).

Appendix A. Appendix

A.1. Simulation details

Here we describe the Geweke and Porter-Hudak (1983) estimator and the specification of models employed in our simulation exercises.

A.2. GPH log-periodogram d estimator

Let $I_{x,T}(\omega_j)$ be the sample periodogram of a univariate time series $\{x_t\}_{t=0}^{T}$ at Fourier frequency $\omega_j = 2\pi j/T$ $(j = 1, \ldots, [T/2])$, then the GPH estimate of d is given by the least-squares regression

$$\log(I_{x,T}(\omega_j)) = c - 2d\log(2\sin(\omega_j/w)) + \varepsilon_j, \tag{7}$$

using the $m = [T^a]$ frequencies closest to zero, $j = 1, \ldots, m$. From a central limit theorem (see Beran (2017)) we have that

$$\sqrt{m}\,(\hat{d}_{GPH} - d) \to N\left(0, \frac{\pi^2}{24}\right). \tag{8}$$

A.3. Specification of simulated models

iid model: This is the simplest normal log-normal mixture model, with uncorrelated error terms.

$$y_t = \exp(h_t/2)\varepsilon_t, \tag{9}$$

$$\varepsilon_t \sim N(0, 1), \tag{10}$$

$$h_t \sim N(0, \sigma^2). \tag{11}$$

where y_t are log-returns, h_t is an iid normal random variable with variance σ^2, and ε_t is a standard normal disturbance.

Model jumps to variance only: this is the specification of Qu and Perron (2013), in which log-volatility is disentangled into a transitory, zero-mean, autoregressive term and a discontinuously evolving permanent average.

$$y_{i,t} = \exp\left(\frac{h_{i,t}}{2} + \frac{s_i^v \mu_t}{2}\right)\varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(0, 1) \tag{12}$$

$$h_{i,t} = \phi_i h_{i,t-1} + \sigma_i^h z_{i,t}^h, \quad z_{i,t}^h \sim N(0, 1), \tag{13}$$

$$\mu_t = \mu_{t-1} + \delta_t^v \sigma^v z_t^v, \quad z_t^v \sim N(0, 1), \tag{14}$$

$$\delta_t^v \sim Bernoulli(p_v), \tag{15}$$

$$h_{(i,j),t} = s_i^v s_j^v \mu_t. \tag{16}$$

where $y_{i,t}$ are log-returns of asset i, $h_{i,t}$ transitory autoregressive log-volatility, μ_t is the common, discontinuously varying, average log-volatility, s_i^v is the exposure of asset i to μ_t, p_v is the unconditional probability of a permanent jump to volatility, and $h_{(i,j),t}$ is the implied covariance between assets i and j at time t.

Model with jumps to mean only: This model is a modification of the multivariate double jump model employed by Laurini et al. (2016), in which log-volatility follows a stationary process, and there are only common contemporaneous jumps to mean returns:

$$y_{i,t} = \exp\left(\frac{h_{i,t}}{2} + \frac{s_i^v \mu_t}{2}\right)\varepsilon_{i,t}, \quad \varepsilon_{i,t} \sim N(\gamma_{i,t}, 1) \tag{17}$$

$$h_{i,t} = \phi_i h_{i,t-1} + \sigma_i^h z_{i,t}^h, \quad z_{i,t}^h \sim N(0, 1), \tag{18}$$

$$\mu_t = \mu_{t-1}, \tag{19}$$

$$\gamma_{i,t} = s_i^m \delta_t^m \sigma^m z_t^m, \quad z_t^m \sim N(0, 1), \tag{20}$$

$$\delta_t^m \sim Bernoulli(p_m), \tag{21}$$

$$h_{(i,j),t} = s_i^v s_j^v \mu_t. \tag{22}$$

where $y_{i,t}$ are log-returns of asset i, $h_{i,t}$ transitory autoregressive log-volatility, μ_t is the common, constant, average log-volatility, $\gamma_{i,t}$ is the return jump of asset i at time t, which depends on the loading s_i^m, the standard deviation σ^m, the standard normal error term z_t^m, and the Bernoulli random variable $\gamma_{i,t}$, which has unconditional probability of success p_m.

A.4. Univariate jump models

As robustness exercise, we estimate nine univariate versions of model (1)–(6), and compare the implied permanent volatility components between multivariate and univariate specifications. In the univariate version of the jumps model, the μ_t component loses the common factor in volatility interpretation, but still represents permanent changes in the level of volatility. In this formulation all load parameters are equal to one, and there is no dynamics for the covariance included in the model. This model is a generalization of the models used in Qu and Perron (2013) and Laurini and Mauad (2014), including the jump component in mean. We focused on this section in comparing the long-term component μ_t obtained in the multivariate estimation with the values obtained in the univariate estimates. For space reasons we do not show the estimated parameters in the univariate versions and the other results, but they are available with the authors. In Fig. 6 we plot the permanent volatility component μ_t of the multivariate double jump model (1)–(6) against the time-varying volatility obtained when the same model is estimated for each cryptocurrency individually. The results obtained show that in general the permanent component estimated in the multivariate and univariate versions of the jump model has similar patterns, especially in terms of average values. As discussed in Section 4, one can notice a spike in volatility during 2017 and sharp volatility reduction from early 2018 towards the end of the sample. This feature is shared by most cryptocurrencies, indicating the global volatility component is a good representation of the volatility level in cryptocurrency markets. The μ_t component estimated by the multivariate model appears to be an aggregation of the permanent components estimated by the univariate models, showing a consistency between the results of these estimates, showing the robustness of the results obtained by this model.

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