conjugates

Let E/k be an algebraic extension. Fix an algebraic closure k^{a}/k.

Definition

Let

$$\sigma\colon E \to k^{\mathrm{a}}$$

be an embedding of E over k. Then,

$$\sigma(E) \subset k^{\mathrm{a}}$$

is called a conjugate of E in k^{a}. If $\alpha \in E$, then $\sigma(\alpha)$ is called a conjugate of α in k^{a}.

If K^{a}/k is any extension that is algebraically closed, then one can talk about conjugates of an algebraic extension E/k in K^{a}.

Let $k^{\mathrm{a}} \subset K^{\mathrm{a}}$ be the subfield generated by the elements that are algebraic over k. Then, k^{a} is an algebraic closure of k. If

$$\sigma \colon E \to K^{\mathrm{a}}$$

is an embedding over k, then we have $\sigma(E) \subset k^{\mathrm{a}}$.

Question

Find the number of distinct conjugates of the following extensions.

1. $\mathbb{C}/\mathbb{R}$
2. $\mathbb{Q}(\sqrt{-1})/\mathbb{Q}$
3. $\mathbb{Q}(3^{\frac{1}{3}})/\mathbb{Q}$
4. $\mathbb{Q}(\alpha)/\mathbb{Q}$, $\alpha^3 + 3\alpha + 1 = 0$