

Examples of group action

A group is almost indistinguishable from its actions. A group is often defined as the way it acts on certain set. Conversely, groups often carry natural actions.

Example

Let G be a group. Then, the multiplication

$$G \times G \rightarrow G$$

is an action of G , viewed as a group, on G , viewed as a set.

Example

Let G be a group equipped with a subgroup. Then, the multiplication map

$$H \times G \rightarrow G$$

is an action of H , viewed as a group, on G , viewed as a set.

Example

Let G be a group. The map

$$\begin{aligned} G \times G &\rightarrow G \\ (x, y) &\mapsto xyx^{-1} \end{aligned}$$

is an action. This is called the conjugation action. In the other direction we get

$$(x, y) \mapsto y^{-1}xy$$

which is a right action of G on itself.

Example

Let V be a vector space, and

$$G = \mathrm{GL}(V)$$

be the group of invertible linear transformations of V into itself.
Then, application of the linear transformation

$$\begin{aligned} G \times V &\rightarrow V \\ (g, v) &\mapsto gv \end{aligned}$$

induces an action of G on V .

Example

Let G be a group and H its subgroup. The set

$$G/H$$

of left cosets of H is a G -set under the law

$$(x, yH) \mapsto xyH.$$

Similarly, G acts on the right of the set $H \backslash G$ of all right H -cosets.