

Monads

Definition

Let \mathcal{C} be a category. A monad is a functor

$$F: \mathcal{C} \rightarrow \mathcal{C}$$

together with natural transformations

$$m: F \circ F \rightarrow F$$

$$e: 1_{\mathcal{C}} \rightarrow F$$

satisfying $m \cdot mF = m \cdot Fm$ and $m \cdot eF = 1_F = m \cdot Fe$.

Two diagrams

The equations can be written as commutativity.

$$\begin{array}{ccc} FFF & \xrightarrow{Fm} & FF \\ mF \downarrow & & \downarrow m \\ FF & \xrightarrow{m} & F \end{array}$$

$$\begin{array}{ccccc} F & \xrightarrow{eF} & FF & \xleftarrow{Fe} & F \\ & \searrow 1_F & \downarrow m & \swarrow 1_F & \\ & & F & & \end{array}$$

Example

The power set functor $P: \mathbf{Set} \rightarrow \mathbf{Set}$ where m is union, and e is singleton.

Example

If R is right adjoint to L , $R \circ L$ is a monad; called the adjunction monad.

Question

Let $R: \mathbf{Gp} \rightarrow \mathbf{Set}$ be the forgetful functor, sending a group to its underlying set. Its left adjoint is the free group functor. Can you describe the adjunction monad?