

direct sums

definition

Let I be a set parametrizing abelian groups A_i for $i \in I$. We define the direct sum

$$\bigoplus_{i \in I} A_i$$

to be the set of finite formal sums

$$\sum_{k=1}^n x_{i_k}$$

where n is a non-negative integer, $i_k \in I$, and $x_{i_k} \in A_{i_k}$.

the universal property

We have maps

$$\epsilon_i: A_i \rightarrow \bigoplus_{i \in I} A_i$$

for each $i \in I$. Here, the target is universal in the following sense. If there is an abelian group X together with a homomorphism

$$f_i: A_i \rightarrow X$$

for every i , then there exists a unique homomorphism

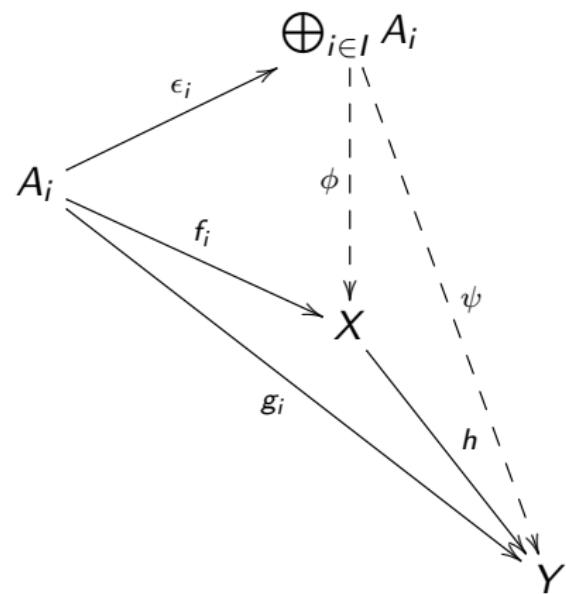
$$\phi: \bigoplus_{i \in I} A_i \rightarrow X$$

such that $\phi \circ \epsilon_i = f_i$ for all $i \in I$.

the universal property as a diagram

$$\begin{array}{ccc} & \oplus_{i \in I} A_i & \\ \epsilon_i \nearrow & | & \\ A_i & | \phi & \\ \searrow f_i & | & \\ & X & \end{array}$$

a bigger diagram



Question

Let A_i be a family of sets parametrized by $i \in I$. Can you interpret the diagram of sets

$$\begin{array}{ccc} & \prod_{i \in I} A_i & \\ \swarrow & & \uparrow \\ A_i & & | \\ & \searrow & | \\ & X & \end{array}$$

as a universal property of the cartesian product?