

arithmetic with modules and ideals

If I is a left-ideal of A , then it is a left module over A . Several operations for ideals apply to modules as well.

If $M', M'' \subset M$ are two submodules of a given left ideal M , then

$$M' \cap M''$$

is again a submodule. The submodule generated by M' and M'' is denoted by

$$M' + M'' \subset M.$$

For an ideal I and a left module M ,

$$IM = \{am : a \in I, m \in M\}$$

is again a left module. The multiplication is associative in that if $J \subset A$ is another ideal, then we have

$$(IJ)M = I(JM).$$

It is distributive in that

$$I(M' + M'') = IM' + IM''.$$

for two submodules $M', M'' \subset M$.

Question

Let $A = \mathbb{Z}$ and $M = \mathbb{Q}/\mathbb{Z}$. Determine IM for a non-zero ideal $I \subset \mathbb{Z}$.