algebras

Let E be a module over a commutative ring A. If there is a bilinear map,

$$\mu\colon E \times E \to E$$

then it can be viewed as a law of composition.

Definition

E is called an A-algebra if the law of composition is associative and has a unit element.

Caution

Non-associative non-unital composition laws are also intensively studied. Examples include Lie algebras.

According to our convention, any algebra E has a unique multiplicative unit $1 \in E$. In particular, one has a map

$$A \to E$$

sending $a \mapsto a1$. This is a ring homomorphism.

Conversely, if E is a ring equipped with a ring homomorphism $A \to E$, then E becomes an A-algebra.

Question

Let $n \geq 1$ be an integer and A be a commutative ring. The set R consisting of all $n \times n$ matrices over A is an example of an A-algebra, where

$$R \times R \to R$$

is given by matrix multiplication. What is the map $A \to R$?