Product

Let $\mathcal{C}$ be a category. Let X, Y be two objects in $\mathcal{C}$. Consider the functor

$$F\colon \mathcal{C}^{\mathrm{op}} \to \mathbf{Set}$$

sending A to $\mathrm{Mor}_{\mathcal{C}}(A, X) \times \mathrm{Mor}_{\mathcal{C}}(A, Y)$.

Suppose that F is representable; $F \xrightarrow{\simeq} h_Z$ for some Z. Then, $1_Z \in h_Z(Z)$ corresponds to a pair of morphisms $Z \to X$ and $Z \to Y$. Yoneda's lemma tells us that it satisfies the universal property;

$$\begin{tikzcd}
& Z \arrow[dl] \arrow[dr] & \\
X & & Y \\
& A \arrow[ul] \arrow[ur] \arrow[uu, dashed] &
\end{tikzcd}$$

In other words, Z is the 'product' of X and Y.

Question

Can you interpret disjoint union in a categorical way? The result is called ‘coproduct’.