Limits

Let $\mathcal{C}$ be a category, a small category I, and the associated functor category

$$\mathcal{C}^I = \mathsf{Fun}(I, \mathcal{C}).$$

There is a diagonal $\mathcal{C} \to \mathcal{C}^I$. If the diagonal has a right adjoint, we call it the limit functor. Write it as

$$\lim \colon \mathcal{C}^I \to \mathcal{C}.$$

Then, the adjunction equation is the universal property of the limit.

Example

If I is the empty category, we get the terminal object as a limit.

Example

If I has two objects and no morphisms, we get the product.

Example

If I is a directed set, we recover the notion of inverse limit as a limit.

Question

Can you define colimits as an adjunction?