

Free abelian groups as an adjunction

Let S be a set, A an abelian group. Then, we have a natural bijection

$$\text{Mor}_{\mathbf{Ab}}(\mathbb{Z}[S], A) \xrightarrow{\sim} \text{Mor}_{\mathbf{Set}}(S, u(A))$$

where $\mathbb{Z}[S]$ denotes the free abelian group on S and $u(A)$ denotes the set underlying A . This is an example of an adjunction. One can write:-

$$\mathbb{Z}[-] \dashv u$$

Question

Consider $(-)^{\text{ab}}: \mathbf{Gp} \rightarrow \mathbf{Ab}$ be the functor which sends a group G to its abelianization G^{ab} . On the other hand, one has $u: \mathbf{Ab} \rightarrow \mathbf{Gp}$ which 'forgets' the group being abelian. Show that one has an adjunction of the form:-

$$\text{Mor}_{\mathbf{Ab}}(G^{\text{ab}}, A) \xrightarrow{\sim} \text{Mor}_{\mathbf{Gp}}(G, u(A))$$