product

The category of (left) modules over a ring A has product.

product

Let M_i be a family of A-modules parametrized by a set I and its typical element $i \in I$. Then, the cartesian product of sets

$$\prod_{i \in I} M_i$$

together with component-wise addition is an abelian group. Moreover, the diagonal action of A,

$$a \cdot (m_i)_{i \in I} = (am_i)_{i \in I},$$

makes it a module over A.

The universal property of the product is with respect to the projection maps

$$\pi_j \colon \prod_{i \in I} M_i \to M_j$$

for $j \in I$. If M' is another module and has maps $f_j \colon M' \to M_j$, then it uniquely factors through $f \colon M' \to \prod_{i \in I} M_i$ with $\pi_j \circ f = f_j$.

Question

The category of finite abelian groups has finite products; product parametrized by a finite set I. Does it have an arbitrary product?