

Yoneda's Lemma

Let \mathcal{C} be a category, $C \in \text{ob}(\mathcal{C})$ an object. Recall that

$$h_C: \mathcal{C}^{\text{op}} \rightarrow \mathbf{Set}$$

is the functor; $h_C(-) = \text{Mor}_{\mathcal{C}}(-, C)$.

Lemma (Yoneda's Lemma)

If $F: \mathcal{C}^{\text{op}} \rightarrow \mathbf{Set}$ is a functor, we have a canonical bijection

$$\text{Mor}_{\text{Fun}(\mathcal{C}^{\text{op}}, \mathbf{Set})}(h_C, F) \xrightarrow{\sim} F(C).$$

Corollary

As a special case, we have a canonical bijection

$$\mathrm{Mor}_{\mathrm{Fun}(\mathcal{C}^{\mathrm{op}}, \mathbf{Set})}(h_C, h_D) \xrightarrow{\sim} \mathrm{Mor}_{\mathcal{C}}(C, D).$$

In words, the corollary says: any natural transformation from h_C to h_D is induced by a morphism from C to D .

Question

Can you formulate Yoneda's lemma involving functors of the form $\text{Mor}(C, -)$?