

Noetherian property of modules over principal rings

Let A be a principal entire ring, and M be a finitely generated module over A .

Theorem

Any submodule of M is also finitely generated.

One can prove this using the fact that any submodule of a free module of finite rank over A is also free of finite rank.

proof

Since M is finitely generated, there is a surjective map

$$\pi: F \twoheadrightarrow M$$

of A -modules, where F is free of finite rank over A . For a submodule $N \subset M$, $\pi^{-1}(N)$ is again free of finite rank. Therefore, we get a surjection

$$\pi^{-1}(N) \rightarrow N$$

and N is also finitely generated.

Question

Assume that A is a principal entire ring. Show that any finitely generated A -module is finitely presented.