Maximality and uniqueness

Recall:-

Theorem
Let G be a finite group and p be a prime divisor of $|G|$. Let P be a fixed p-Sylow subgroup. Any p-subgroup H is contained in a conjugate of P.

We note that this implies the uniqueness:-

Theorem
Two p-Sylow subgroups of G are conjugate to each other.

Proof.
Take H to be a p-Sylow subgroup. $\qquad\qquad\qquad\qquad\qquad\square$