

Modules

Let A be a ring. A left A -module is an abelian group M together with a map

$$\begin{aligned} A \times M &\rightarrow M \\ (a, m) &\mapsto am \end{aligned}$$

such that

1. $ab(m) = a(bm)$
2. $a(m + m') = am + am'$

for $a, b \in A$ and $m, m' \in M$.

Similarly, one can define a right A -module of M . If A is commutative, a left module is a right module as well.

Question

Let k be a field, $n \geq 1$ an integer. Let A be the ring of all $n \times n$ matrices over k . Show that the set of column vectors of length n is a left module over A . What about the row vectors?