Polynomial rings

Given a set X, we have the polynomial ring

$$\mathbb{Z}[X]$$

with variables in X. A typical element of $\mathbb{Z}[X]$ is of the form

$$f = \sum a_{i_1,\cdots,i_n} x_1^{i_1} \cdots x_n^{i_n}$$

where $x_1, \cdots, x_n \in X$, $i_1, \cdots, i_n \in \mathbb{Z}_{\geq 0}$, and all but finitely many $a_{i_1,\cdots,i_n}$'s are zero.

Polynomial rings can be defined in terms of their universal property. Let ***Comm*** be the category of commutative rings and ring homomorphisms. Then, we have the forgetful functor

$$u\colon \boldsymbol{Comm} \to \boldsymbol{Set}$$

which maps a (commutative) ring A to its underlying set. In the other direction, we have

$$F\colon \boldsymbol{Set} \to \boldsymbol{Comm}$$

which maps a set X to the polynomial ring

$$\mathbb{Z}[X].$$

They form an adjunction $F \dashv u$, which amounts to existence of a natural bijection

$$\mathrm{Mor}_{\boldsymbol{Comm}}(\mathbb{Z}[X], A) \xrightarrow{\sim} \mathrm{Mor}_{\boldsymbol{Set}}(X, u(A)).$$

For a given set map $t\colon X \to u(A)$, the associated ring homomorphism

$$t_*\colon \mathbb{Z}[X] \to A$$

is the evaluation map, which acts on polynomials as

$$t_*\colon \sum a_{i_1,\cdots,i_n} x_1^{i_1} \cdots x_n^{i_n} \longmapsto \sum a_{i_1,\cdots,i_n} t(x_1)^{i_1} \cdots t(x_n)^{i_n}.$$

Question

If X is a set and k is an arbitrary commutative ring, one can define polynomial rings $k[X]$ with variables in X and coefficients in k. In this context, you might want to regard $k[X]$ as a ring equipped with a homomorphism $k \to k[X]$. What is their universal property?