torsion-part as a direct summand

Let A be a principal entire ring, and M be a finitely generated module over A. We want to show:

Theorem

There is a free submodule $F \subset M$ with $M = F \oplus M_{tor}$.

proof

Consider the exact sequence

$$0 \to M_{\text{tor}} \to M \xrightarrow{\pi} M/M_{\text{tor}} \to 0.$$

Note that M/M_{tor} is finitely generated and torsion-free, which implies that it is free. Thus, there is a section of $\pi\colon M \to M/M_{\text{tor}}$, say $f\colon M/M_{\text{tor}} \to M$. Then, the image of f is free. It implies that

$$\text{im}(f) \cap M_{\text{tor}} = 0.$$

That is to say, we have the desired decomposition

$$M \simeq M_{\text{tor}} \oplus \text{im}(f).$$

Question

Show that the choice of the section f cannot be made natural.