General group actions

Definition

Let $\mathcal{C}$ be a category. For an object A of $\mathcal{C}$, the set of endomorphisms

$$\mathrm{Mor}_{\mathcal{C}}(A, A)$$

is a monoid under composition.

If G is a group and

$$\phi\colon G \longrightarrow \mathrm{Mor}_{\mathcal{C}}(A, A)$$

is a monoid homomorphism. We say G acts on A. This generalizes the notion of a group acting on a set.

If M is a monoid and G is a group, then any morphism

$$G \to M$$

factors through

$$M^\times \subset M$$

the submonoid consisting of invertible elements.

In particular, if G acts on A, then G always acts on A as automorphisms.

Question

If $\mathcal{C}$ is a category and A is its object, does '$G \times A$' make sense? Can you use a function

$$G \times A \to A$$

to define an action?