

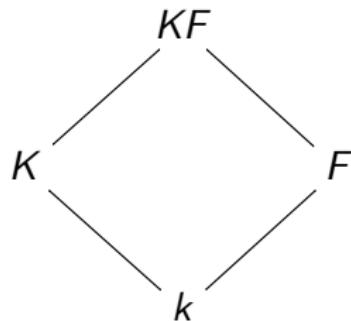
lifting normal extensions

## Theorem

*A normal extension remains normal under lifting, compositum, and intersection.*

## lifting

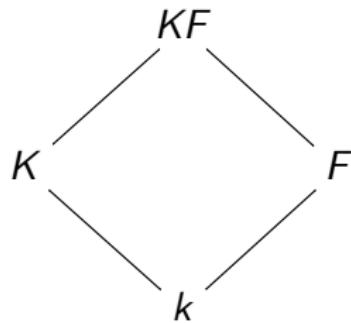
Consider the diagram of fields:



If  $K/k$  is normal, it is the splitting field of a family of the polynomials, say  $(f_i)_{i \in I}$ . Then,  $KF/F$  is the splitting field of the same family  $(f_i)_{i \in I}$  of polynomials.

## compositum

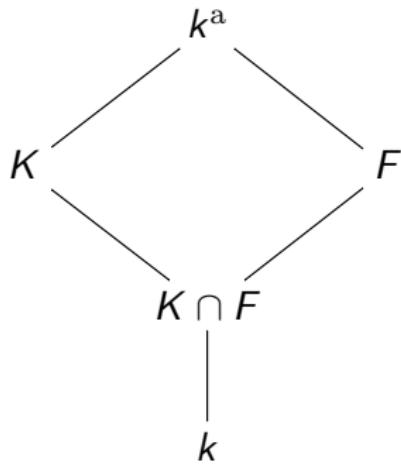
Consider the diagram of fields:



If  $K/k$  and  $F/k$  are normal, then they are the splitting field of the families of the polynomials, say  $(f_i)_{i \in I}$  and  $(g_j)_{j \in J}$ , respectively. Then,  $KF/F$  is the splitting field of the composite family  $(f_i)_{i \in I} \cup (g_j)_{j \in J}$  of polynomials.

## intersection

Consider the diagram of fields:



Any embedding  $K \rightarrow k^a$  has image in  $K$ . Similarly, any embedding  $F \rightarrow k^a$  has image in  $F$ . Since an embedding  $K \cap F \rightarrow k^a$  can be extended to an embedding of either  $F$  or  $K$ , its image is contained in both  $F$  and  $K$ .

## Question

In the diagram

$$E \downarrow F \downarrow k$$

with  $E/k$  normal, show that  $E/F$  is normal.