torsion abelian groups

Definition

An abelian group A is torsion if every element $a \in A$ has a finite order.

Proposition

A finite abelian group is torsion.

However, the converse is not true. A counterexample is $\mathbb{Q}/\mathbb{Z}$.

Definition

Let p be a prime and A an abelien group. The p-primary subgroup $A(p) \subset A$ is defined to be the subset of elements killed by some power of p. The torsion subgroup A_{tor} is defined to be the subgroup of elements killed by some positive integer.

Proposition

Let A be a torsion abelian group. Then, $A(p) \to A$ induces an isomorphism

$$\bigoplus_p A(p) \xrightarrow{\sim} A_{tor}.$$

Question

What is the decomposition of $\mathbb{Q}/\mathbb{Z}$ into p-primary subgroups?