Functors

Definition

Let $\mathcal{C}$ and $\mathcal{D}$ be categories. A covariant functor F from $\mathcal{C}$ to $\mathcal{D}$ consists of the following data:

1. for every object $A \in \mathrm{ob}\,(\mathcal{C})$, an object $F(A) \in \mathrm{ob}\,(\mathcal{D})$
2. for every morphism $\phi \in \mathrm{Mor}_{\mathcal{C}}(A, B)$, a morphisms $F(\phi) \in \mathrm{Mor}_{\mathcal{D}}(F(A), F(B))$

such that $F(1_A) = 1_{F(A)}$ and $F(\phi \circ \psi) = F(\phi) \circ F(\psi)$.

Such a functor is written $F\colon \mathcal{C} \to \mathcal{D}$.

The collection of all such functors is written $\mathrm{Fun}(\mathcal{C}, \mathcal{D})$.

A diagram

$$
\begin{tikzcd}
A \arrow[r, mapsto] \arrow[d, "\phi"'] & F(A) \arrow[r, equal] \arrow[d, "F(\phi)"] & F(A) \arrow[dd, "F(\psi\circ\phi)"] \\
B \arrow[r, mapsto] \arrow[d, "\psi"'] & F(B) \arrow[d, "F(\psi)"] & \\
C \arrow[r, mapsto] & F(C) \arrow[r, equal] & F(C)
\end{tikzcd}
$$

Definition

A contravariant functor F from $\mathcal{C}$ to $\mathcal{D}$ is by definition a functor

$$F\colon \mathcal{C}^{\mathsf{op}} \to \mathcal{D}.$$

Convention

We will adopt the convention that a functor is covariant.

Question

Do we have $\mathsf{Fun}(\mathcal{C}^{\mathsf{op}}, \mathcal{D}) = \mathsf{Fun}(\mathcal{C}, \mathcal{D}^{\mathsf{op}})$?