Group rings

Let $\boldsymbol{Ab}$ be the category of abelian groups. If G acts on an abelian group A, that is to say, we have a morphism

$$\phi\colon G \to \mathrm{Mor}_{\boldsymbol{Ab}}(A, A)$$

between monoids.

$\boldsymbol{Ab}$ has a forgetful functor to $\boldsymbol{Set}$, an action yields a set function

$$\begin{aligned} \mu\colon G \times A &\to A \\ (g, a) &\mapsto ga \end{aligned}$$

which satisfies the following conditions.

1. $1a = a$ for all $a \in A$.
2. $(gh)a = g(h(a))$ for all $g, h \in G$ and all $a \in A$.
3. $g(a + b) = ga + gb$ for all $g \in G$ and all $a, b \in A$.

The action $\mu\colon G \times A \to A$ can be extended as follows. If

$$x\colon \sum_{g \in G} m_g g \tag{1}$$

is a formal finite $\mathbb{Z}$-linear combination of elements from G, then we define

$$xa := \sum_{g \in G} m_g (ga). \tag{2}$$

The set $\mathbb{Z}[G]$ of all elements of the form (1) is a ring. The action (2) can be interpreted as a ring homomorphism

$$\mathbb{Z}[G] \to \mathrm{Mor}_{\boldsymbol{Ab}}(A, A)\,.$$

Question

Let G be a finite non-trivial group. Can you find a zero-divisor of $\mathbb{Z}[G]$?