$\mathbb{Z}$ as an initial object

initial objects

In a category $\mathcal{C}$, an object c is called initial if

$$\mathrm{Mor}_{\mathcal{C}}(c, x) = \{*\}$$

for any object $x \in \mathrm{ob}(\mathcal{C})$.

Let A be a ring. There is a unique ring homomorphism

$$i : \mathbb{Z} \to A.$$

Indeed, such a homomorphism must satisfy $i(1) = 1$. It extends uniquely to $\mathbb{Z}$ as a ring homomorphism. In other words, $\mathbb{Z}$ is the initial object in the category of rings.

Question

Does the category of rings have a terminal object?