

## Field extensions

Let  $E$  be a field and  $F \subset E$  be a subfield. We also say that  $E$  is an extension field of  $F$ . Often, we simply say  $E$  is an extension of  $F$ .

Either

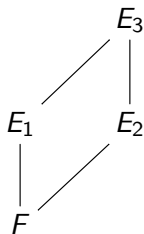
$$E/F$$

or

$$\begin{array}{c} E \\ | \\ F \end{array}$$

denotes an extension.

Diagrams like



are useful.

If  $E$  is an extension of  $F$ , then  $E$  is a vector space over  $F$ .

### Definition

A finite extension of  $F$  is an extension  $E/F$  which is finite dimensional over  $F$ .

## Question

Is  $\mathbb{C}/\mathbb{Q}$  finite? What about  $\mathbb{C}/\mathbb{R}$ ?