

# Natural transformations

## Definition

Let  $\mathcal{C}$  and  $\mathcal{D}$  be categories and  $F, G: \mathcal{C} \rightarrow \mathcal{D}$  be functors. A natural transformation  $\eta$  from  $F$  to  $G$  is the data of

$$\eta(A): F(A) \rightarrow G(A),$$

a morphism in  $\mathcal{D}$ , associated to every object  $A \in \text{ob}(\mathcal{C})$ . They should satisfy the commutativity of

$$\begin{array}{ccc} F(A) & \xrightarrow{\eta(A)} & G(A) \\ F(\phi) \downarrow & & \downarrow G(\phi) \\ F(B) & \xrightarrow{\eta(B)} & G(B) \end{array}$$

for every  $\phi \in \text{Mor}_{\mathcal{C}}(A, B)$ .

## Functor category

$\text{Fun}(\mathcal{C}, \mathcal{D})$  together with natural transformations form a category.  
Isomorphisms in this category will be called a natural isomorphism.

## Question

When is a natural transformation an isomorphism?