group of homomorphisms

Given a ring A, its left modules together with homomorphisms form a category. Denote it by $\mathcal{C} = \boldsymbol{Mod}_A$.

If M and M' are two (left) modules over a ring A, then

$$\mathrm{Mor}_{\mathcal{C}}(M, M')$$

is an abelian group. The group law is point-wise addition;

$$(f + g)(m) = f(m) + g(m)$$

If A is commutative, one can also scale them. For $a \in A$ and $f \in \mathrm{Mor}_{\mathcal{C}}(M, M')$, define af by

$$(af)(m) = af(m).$$

Then, it is again a homomorphism because

$$(af)(bm) = f(abm) = f(bam) = b(f(am)) = b((af)(m)).$$

Question

A map $f\colon M \to M'$ induces a map

$$f^*\colon \mathrm{Mor}_{\mathcal{C}}(M', M'') \to \mathrm{Mor}_{\mathcal{C}}(M, M'')$$

for every A-module M''. Is it a homomorphism between abelian groups? If A is commutative, is it a module homomorphism?