

# Conjugacy classes in $S_n$

Let  $\pi(n)$  be the set of all partition of  $\{1, 2, \dots, n\}$ . Let  $\text{Conj}(S_n)$  be the set of all conjugacy classes of  $S_n$ . The map

$$S_n \rightarrow \pi(n)$$

sending  $\sigma$  to the set of its orbits induce the bijection

$$\text{Conj}(S_n) \xrightarrow{\sim} \pi(n).$$

To see this, suffices to observe how conjugation acts on the cycle decomposition of a permutation; on a cycle, the action is given by

$$\sigma[a_1 a_2 \cdots a_r] \sigma^{-1} = [\sigma(a_1) \sigma(a_2) \cdots \sigma(a_r)].$$

## Question

Find all conjugacy classes of  $S_n$  for  $2 \leq n \leq 5$ . Do the same for  $A_n$ .