

finitely generated algebraic extensions

Let  $E/k$  be a finitely generated field extension of the form

$$E = k(\alpha_1, \dots, \alpha_n).$$

### Proposition

*If each  $\alpha_i$  is algebraic over  $k$ , then  $E/k$  is finite. In particular, it is algebraic.*

### Proof.

Let  $E_0 = k$  and for  $m \geq 1$   $E_m = k(\alpha_1, \alpha_2, \dots, \alpha_m)$ . Then, for each  $i = 0, 1, \dots, m-1$ ,  $E_{i+1} = E_i(\alpha_{i+1})$  is finite. Thus, dimension of  $E/k$  is at most

$$\prod_{i=0}^{m-1} [E_{i+1}/E_i]$$

which is finite.



## Question

Find an algebraic extension which is not finite.