Homs, sums, and products

Let A be a ring and consider two A-modules M_1 and M_2. For any A-module N, there is a map

$$\pi\colon \operatorname{Hom}_A(N, M_1 \times M_2) \longrightarrow \operatorname{Hom}_A(N, M_1) \times \operatorname{Hom}_A(N, M_2)$$

which is component-wise obtained by composition with the projection maps $M_1 \times M_2 \to M_i$, $i = 1, 2$. By the universal property of the product, π is a bijection.

In fact, the same principle applies to an arbitrary product. The map

$$\pi\colon \mathrm{Hom}_A(N, \prod_{i\in I} M_i) \to \prod_{i\in I} \mathrm{Hom}_A(N, M_i)$$

is a bijection.

Dually, the natural map

$$\iota\colon \prod_{i\in I} \mathrm{Hom}_A(M_i, N) \longrightarrow \mathrm{Hom}_A(\bigoplus_{i\in I} M_i, N)$$

is a bijection.

Question

Suppose that each M_i is given an isomorphism $M_i \simeq A$. Interpret the bijection

$$\iota\colon \prod_{i \in I} \mathrm{Hom}_A(M_i, N) \longrightarrow \mathrm{Hom}_A(\bigoplus_{i \in I} M_i, N)$$

in terms of the universal property of free modules and obtain

$$I = \bigcup_{i \in I} \{i\}.$$