Representing a permutation using cycles

Cycles

Let $n \geq 1$ be an integer and consider the set $\{1, 2, \cdots, n\}$. An n-cycle refers to

$$\sigma = [a_1 \cdots a_n]$$

with $\{a_1, \cdots, a_n\} = \{1, 2, \cdots, n\}$. We interpret it as a permutation acting on letters by

$$\begin{aligned} a_1 &\mapsto a_2 \\ &\vdots \\ a_{n-1} &\mapsto a_n \\ a_n &\mapsto a_1. \end{aligned}$$

We call σ a cycle of length n.

Abuse of notation

Sometimes, 3-cycle [123] may denote somethign like

$$[123][4][5] \in S_5$$

by abuse of notation, ommiting all 1-cycles.

One can say that we implicitly use embeddings

$$\mathrm{Perm}(\{1,2,3\}) \times \mathrm{Perm}(\{4\}) \times \mathrm{Perm}(\{5\}) \hookrightarrow \mathrm{Perm}(\{1,2,3,4,5\})$$

Cycle decomposition

Let $n \geq 1$ be an integer and σ be a permutation on the set $\{1, 2, \cdots, n\}$. Then, we have the orbit decomposition

$$\{1, 2, \cdots, n\} = \coprod_{x} O_x$$

Each orbit is also called a cycle for σ. Then, the restriction

$$\sigma_x = \sigma|_{O_x}$$

is a cycle of length equal to the cardinality of O_x. The cycle decomposition refers to

$$\sigma = \prod_{x} \sigma_x.$$

Question

In the cycle decomposition of a permutation, do we need to worry about the order of multiplication?