Smallness

Reorganizing data

Recall that a category $\mathcal{C}$ consists

1. A collection of objects, $\mathrm{ob}(\mathcal{C})$
2. A collection of morphisms, $\mathrm{mor}(\mathcal{C})$

together with a law of composition. We may reorganize them as follows. For each pair (A, B) of objects, one may form the collection

$$Mor(A, B)$$

of all morphisms from A to B. For every triple (A, B, C) we have a map

$$Mor(A, B) \times Mor(B, C) \to Mor(A, C).$$

Smallness

In a broad sense, if $\mathcal{C}$ is a category, neither $\mathrm{ob}(\mathcal{C})$ nor $\mathrm{mor}(\mathcal{C})$ is required to be a set.

Definition

A category $\mathcal{C}$ is locally small if $Mor(A, B)$ is a set for every pair (A, B) of objects in $\mathcal{C}$. A locally small category is small if $ob(\mathcal{C})$ is a set.

Examples

Sets together with functions form a category

$$\mathbf{Set},$$

and monoids together with monoid homomorphisms form a category

$$\mathbf{Mon}$$

and so on. These are locally small but not small.

Convention

We will most of the time deal with locally small categories. So a category will mean a locally small category, and we will say a large category to mean a category which may not be locally small.

Remark

We will avoid what it means to be a collection.

Question

Do sets and bijections form a category? What about sets and injections?