

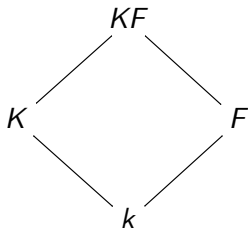
lifting normal extensions

Theorem

A normal extension remains normal under lifting, compositum, and intersection.

lifting

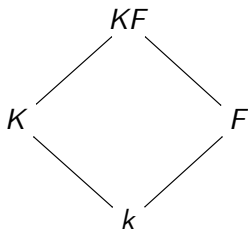
Consider the diagram of fields:



If K/k is normal, it is the splitting field of a family of the polynomials, say $(f_i)_{i \in I}$. Then, KF/F is the splitting field of the same family $(f_i)_{i \in I}$ of polynomials.

compositum

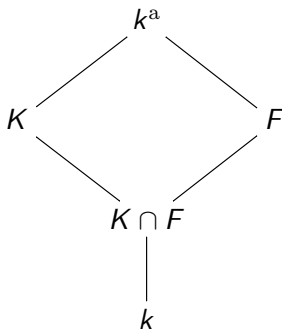
Consider the diagram of fields:



If K/k and F/k are normal, then they are the splitting field of the families of the polynomials, say $(f_i)_{i \in I}$ and $(g_j)_{j \in J}$, respectively. Then, KF/F is the splitting field of the composite family $(f_i)_{i \in I} \cup (g_j)_{j \in J}$ of polynomials.

intersection

Consider the diagram of fields:



Any embedding $K \rightarrow k^a$ has image in K . Similarly, any embedding $F \rightarrow k^a$ has image in F . Since an embedding $K \cap F \rightarrow k^a$ can be extended to an embedding of either F or K , its image is contained in both F and K .

Question

In the diagram

$$\begin{array}{c} E \\ | \\ F \\ | \\ k \end{array}$$

with E/k normal, show that E/F is normal.