unique factorization

Let A be an entire ring. For a non-zero element $a \in A$, we consider its factorization

$$a = u \prod_{i=1}^{n} p_i$$

where p_i is irreducible and $u \in A^{\times}$. Such a factorization is called a factorization of a into irreducibles.

Two factorizations

$$a = u \prod_{i=1}^{n} p_i$$

and

$$a = v \prod_{j=1}^{m} q_j$$

are called equivalent if $(p_1), (p_2), \cdots, (p_n)$ is equal to $(q_1), \cdots, (q_m)$ with multiplicities. This amounts to saying that $n = m$ and there exists some permutation σ and units $r_0, \cdots, r_n \in A^\times$, such that

$$u = r_0 v$$

and

$$p_i = r_i q_{\sigma i}$$

for $i = 1, \cdots, n$.

Definition

A is a factorial ring if it is entire and every element has a unique factorization into irreducible elements.

An example

For a field k, $\mathbb{Z}$ and $k[x]$ are both factorial.

two non-examples

Examples of non-factorial rings include $\mathbb{Z}[\sqrt{-5}]$ and $k[x^2, x^3]$ for a field k.

Another non-example

$k[x, y, z]/(z^2 - xy)$ is not factorial.

Question

Find two factorizations of $x^6 \in k[x^2, x^3]$.