

arithmetic with modules and ideals

If  $I$  is a left-ideal of  $A$ , then it is a left module over  $A$ . Several operations for ideals apply to modules as well.

If  $M', M'' \subset M$  are two submodules of a given left ideal  $M$ , then

$$M' \cap M''$$

is again a submodule. The submodule generated by  $M'$  and  $M''$  is denoted by

$$M' + M'' \subset M.$$

For an ideal  $I$  and a left module  $M$ , denote by  $IM$  the left module generated by

$$\{am: a \in I, m \in M\}.$$

The multiplication is associative in that if  $J \subset A$  is another ideal, then we have

$$(IJ)M = I(JM).$$

It is distributive in that

$$I(M' + M'') = IM' + IM''.$$

for two submodules  $M', M'' \subset M$ .

## Question

Let  $A = \mathbb{Z}$  and  $M = \mathbb{Q}/\mathbb{Z}$ . Determine  $IM$  for a non-zero ideal  $I \subset \mathbb{Z}$ .