

direct sums

## definition

Let  $I$  be a set parametrizing abelian groups  $A_i$  for  $i \in I$ . We define the direct sum

$$\bigoplus_{i \in I} A_i$$

to be the set of finite formal sums

$$\sum_{k=1}^n x_{i_k}$$

where  $n$  is a non-negative integer,  $i_k \in I$ , and  $x_{i_k} \in A_{i_k}$ .

## the universal property

We have maps

$$\epsilon_i: A_i \rightarrow \bigoplus_{i \in I} A_i$$

for each  $i \in I$ . Here, the target is universal in the following sense. If there is an abelian group  $X$  together with a homomorphism

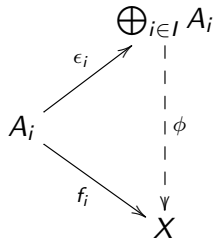
$$f_i: A_i \rightarrow X$$

for every  $i$ , then there exists a unique homomorphism

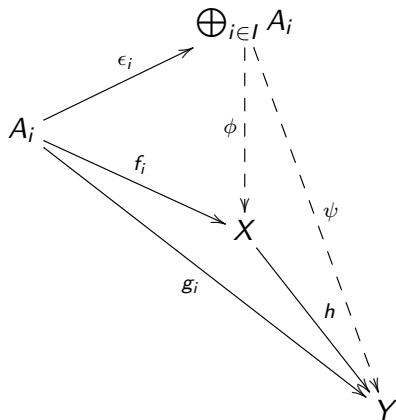
$$\phi: \bigoplus_{i \in I} A_i \rightarrow X$$

such that  $\phi \circ \epsilon_i = f_i$  for all  $i \in I$ .

the universal property as a diagram

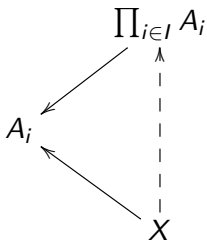


a bigger diagram



## Question

Let  $A_i$  be a family of sets parametrized by  $i \in I$ . Can you interpret the diagram of sets



as a universal property of the cartesian product?