Normal extensions

Let F/k be an algebraic extension.

Definition

An extension F/k contained in an algebraic closure k^{a} is called normal if one of the following equivalent conditions are satisfied.

1. Every embedding of F into k^{a} over k induces an automorphism of F.
2. F is the splitting field of a family of polynomials in $k[X]$.
3. Every irreducible polynomial of $k[X]$ which has a root in F splits into linear factors in F.

Note that the normality is independent of choice of an embedding $F \hookrightarrow k^{\mathrm{a}}$.

The class of normal extensions is not distinguished. Consider the tower

$$\begin{array}{c} \mathbb{Q}(2^{\frac{1}{4}}) \\ | \\ \mathbb{Q}(2^{\frac{1}{2}}) \\ | \\ \mathbb{Q} \end{array}$$

which consists of two quadratic extensions. Every quadratic extension is normal, but the total extension $\mathbb{Q}(2^{\frac{1}{4}})/\mathbb{Q}$ is not normal.

Question

Show that every quadratic extension is normal.