Characteristic

Let A be a ring. The kernel of the unique homomorphism

$$i\colon \mathbb{Z} \to A.$$

is of the form $n\mathbb{Z}$, where $n \geq 0$ is an integer. Such n is unique.

Definition

The integer n is called the characteristic of A.

Proposition

Let A be a division ring. Then, its characteristic is either 0 or a prime.

Proof.

Suppose that the chacteristic n is a positive integer which is composite. Write $n = ab$, $n > a, b > 1$. The definition of characteristics imply that there is an inclusion $\mathbb{Z}/n\mathbb{Z} \to A$. Since $ab = 0$ but $a, b \neq 0$, this contradicts the assumption that A is a division ring. $\square$

Question

If A and B are rings, how the characteristics of A, B, and $A \times B$ are related?