Subgroups

Let G be a group.

Definition

A subset $H \subset G$ is a subgroup if

1. $e \in H$,
2. $x, y \in H \Rightarrow xy \in H$,
3. $x \in H \Rightarrow x^{-1} \in H$.

Immediate from the definition is that the set of subgroups is closed under arbitrary intersection.

Generation of a subgroup

If $S \subset G$ is a subset, then the intersection of all subgroups of G containing S is again a subgroup of G, say H.

We say that the group H is generated by S.

An element of H is of the form

$$x_1^{k_1} \cdots x_r^{k_r}$$

with $x_i \in S$ and $k_i \in \mathbb{Z}$ for $i = 1, 2, \cdots, r$.

Cyclic groups

A group is cyclic if it can be generated by one element.

Commutator subgroup

Let G be a group. For $x, y \in G$, their commutator is defined to be

$$[x, y] = xyx^{-1}y^{-1}.$$

The commutator subgroup is defined to be the subgroup generated by all commutators. It is often denoted by $[G, G]$.