Rings

Definition

Let A be a set with two laws of composition; multiplication and addition. We say A is a ring if

1. $(A, +)$ is a commutative group.
2. $(A, \cdot)$ is a monoid.
3. Multiplication is distributive; $(x + y)z = xz + yz$ and $z(x + y) = zx + zy$ for all $x, y, z \in A$.

Notation

Additive identity is written as 0.
Multiplicative identity is written 1.

From the definition of a ring A, we have

$$0x = (0 + 0)x = 0x + 0x$$

so $0x = 0$ for all x.

Convention

We allow $1 = 0$. In this case, $A = \{0\}$.

Indeed, if $x \in A$, then $x = 1x = 0x = 0$.

Let M be an abelian group. Show that the set of endomorphisms, group homomorphisms from M into M, is a ring.