

duality of finite abelian groups

If  $A, B$  are two abelian groups, then the set

$$\text{Hom}(A, B)$$

of homomorphisms from  $A$  to  $B$  is an abelian group.

## pairing

There is a natural bilinear map

$$A \times \text{Hom}(A, B) \rightarrow B$$

given by evaluation. More generally, one can consider a bilinear map

$$A \times B \rightarrow C$$

where  $A, B, C$  are abelian groups.

### Definition

Such a pairing is called perfect if  $A \xrightarrow{\sim} \text{Hom}(B, C)$ .

Let  $A$  be a finite abelian group. Then,

$$A \times \text{Hom}(A, \mathbb{Q}/\mathbb{Z}) \rightarrow \mathbb{Q}/\mathbb{Z}$$

is a perfect pairing.

Sometimes, the group  $\text{Hom}(A, \mathbb{Q}/\mathbb{Z})$  is written as  $A^\vee$ , and called the Pontryagin dual of  $A$ .

## Question

Let  $A$  be a cyclic group of order  $n$ . Is there an isomorphism  $A \rightarrow \text{Hom}(A, \mathbb{Q}/\mathbb{Z})$ ?