Conjugacy classes in S_n

Let $\pi(n)$ be the set of all partition of $\{1, 2, \cdots, n\}$. Let $P(n)$ be the set of all integer partitions of n. Let $\mathrm{Conj}(S_n)$ be the set of all conjugacy classes of S_n. There is a natural map $\pi(n) \to P(n)$. The map

$$S_n \to \pi(n)$$

sending σ to the set of its orbits induce the bijection

$$\mathrm{Conj}(S_n) \xrightarrow{\sim} P(n).$$

To see this, suffices to observe how conjugation acts on the cycle decomposition of a permutation; on a cycle, the action is given by

$$\sigma[a_1 a_2 \cdots a_r]\sigma^{-1} = [\sigma(a_1)\sigma(a_2)\cdots\sigma(a_r)].$$

Question

Find all conjugacy classes of S_n for $2 \leq n \leq 5$. Do the same for A_n.