

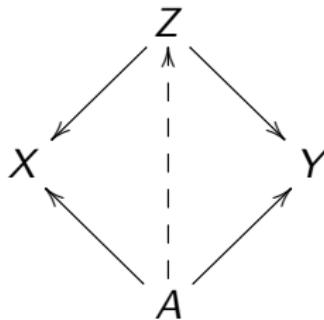
# Product

Let  $\mathcal{C}$  be a category. Let  $X, Y$  be two objects in  $\mathcal{C}$ . Consider the functor

$$F: \mathcal{C}^{\text{op}} \rightarrow \mathbf{Set}$$

sending  $A$  to  $\text{Mor}_{\mathcal{C}}(A, X) \times \text{Mor}_{\mathcal{C}}(A, Y)$ .

Suppose that  $F$  is representable;  $F \xrightarrow{\sim} h_Z$  for some  $Z$ . Then,  $1_Z \in h_Z(Z)$  corresponds to a pair of morphisms  $Z \rightarrow X$  and  $Z \rightarrow Y$ . Yoneda's lemma tells us that it satisfies the universal property;



In other words,  $Z$  is the ‘product’ of  $X$  and  $Y$ .

## Question

Can you interpret disjoint union in a categorical way? The result is called ‘coproduct’.