

# Modules

Let  $A$  be a ring. A left  $A$ -module is an abelian group  $M$  together with a map

$$\begin{aligned}A \times M &\rightarrow M \\(a, m) &\mapsto am\end{aligned}$$

such that

1.  $ab(m) = a(bm)$
2.  $a(m + m') = am + am'$

for  $a, b \in A$  and  $m, m' \in M$ .

Similarly, one can define a right  $A$ -module of  $M$ . If  $A$  is commutative, a left module is a right module as well.

## Question

Let  $k$  be a field,  $n \geq 1$  an integer. Let  $A$  be the ring of all  $n \times n$  matrices over  $k$ . Show that the set of column vectors of length  $n$  is a left module over  $A$ . What about the row vectors?