

Free groups as an adjunction

Let S be a set, G a group. Let $F(S)$ be the free group on S . Then, we have a natural bijection

$$\text{Mor}_{\mathbf{Grp}}(F(S), G) \xrightarrow{\sim} \text{Mor}_{\mathbf{Set}}(S, u(G))$$

where $F(S)$ denotes the free group on S and $u(G)$ denotes the set underlying G . This is an example of an adjunction

$$F \dashv u$$

meaning that “ F is the left adjoint for u ”, or equivalently “ u is the right adjoint for F ”.

Any adjunction gives rise to a monad. In this case, we have

$$u(F(u(F(S)))) \rightarrow u(F(S))$$

which amounts to concatenation.

Question

When is a free group abelian?