

exact sequences

definition

Let A be a ring. A diagram of A -modules homomorphisms

$$M' \rightarrow M \rightarrow M'', \tag{1}$$

for A -modules M , M' , and M'' , is called exact, if

$$\ker(M \rightarrow M'') = \text{im}(M' \rightarrow M).$$

One can also say that the sequence (1) is exact in the middle.

A short exact sequence refers to a diagram

$$0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0 \quad (2)$$

which is exact at M' , M , and M'' . That is to say, all subsequences

$$0 \rightarrow M' \rightarrow M$$

$$M' \rightarrow M \rightarrow M''$$

$$M \rightarrow M'' \rightarrow 0$$

are exact.

One can also say that the sequence (2) is exact.

A long exact sequence refers to a diagram

$$\cdots \rightarrow M_{n-1} \rightarrow M_n \rightarrow M_{n+1} \rightarrow \cdots \quad (3)$$

which is exact at each M_i 's.

One can also say that the sequence (3) is exact.

Question

For an exact sequence

$$0 \rightarrow M' \rightarrow M \rightarrow M'' \rightarrow 0 \quad (4)$$

show that the map $M \rightarrow M''$ is the cokernel of the map $M' \rightarrow M$ in terms of the universal properties.

