Compositum of fields

Let E, F be field extensions of a fixed field k. If both E and F are contained in some field L, then denote by EF the smallest subfield of L containing both E and F. We call it the compositum of E and F.

If E, F are not embedded in a common field, then the compositum is undefined.

Let $E = k(t)$ and $F = k(s)$. If we regard it as a subfield of $k(t, s)$, then

$$EF = k(t, s).$$

If $E' = k(t)$ and $F' = k(t^2)$, then $F' \subset E'$ and

$$E'F' = E' = k(t).$$

In particular,

$$EF \not\simeq E'F'.$$

Note that $E \simeq E'$ and $F \simeq F'$ as fields.

Question

Compute the degree the compositum of the following pairs of fields. Every field is regarded as a subfield of $\mathbb{C}$.

$$E_1 = \mathbb{Q}(2^{\frac{1}{5}}), \qquad F_1 = \mathbb{Q}(e^{\frac{2\pi i}{5}})$$

$$E_2 = \mathbb{Q}(e^{\frac{2\pi i}{5}} + e^{\frac{-2\pi i}{5}}), \qquad F_2 = \mathbb{Q}(\sqrt{5})$$

$$E_3 = \mathbb{Q}(e^{\frac{2\pi i}{5}} + e^{\frac{-2\pi i}{5}}), \qquad F_3 = \mathbb{Q}(\sqrt{-5})$$

$$E_4 = \mathbb{Q}(2^{\frac{1}{5}}), \qquad F_4 = \mathbb{Q}(\sqrt{3})$$