

Natural transformations

Definition

Let \mathcal{C} and \mathcal{D} be categories and $F, G: \mathcal{C} \rightarrow \mathcal{D}$ be functors. A natural transformation η from F to G is the data of

$$\eta(A): F(A) \rightarrow G(A),$$

a morphism in \mathcal{D} , associated to every object $A \in \text{ob}(\mathcal{C})$. They should satisfy the commutativity of

$$\begin{array}{ccc} F(A) & \xrightarrow{\eta(A)} & G(A) \\ F(\phi) \downarrow & & \downarrow G(\phi) \\ F(B) & \xrightarrow{\eta(B)} & G(B) \end{array}$$

for every $\phi \in \text{Mor}_{\mathcal{C}}(A, B)$.

Functor category

$\text{Fun}(\mathcal{C}, \mathcal{D})$ together with natural transformations form a category.
Isomorphisms in this category will be called a natural isomorphism.

Question

When is a natural transformation an isomorphism?