

free abelian groups

Definition

An abelian group A is free if it is isomorphic to the direct sum of copies of \mathbb{Z} .

Basis

Let A be a free abelian group. A family of elements

$$x_i \in A, \quad i \in I$$

parametrized by a set I is a basis if

$$\begin{aligned}\epsilon_i: \mathbb{Z} &\rightarrow A \\ 1 &\mapsto x_i\end{aligned}$$

induces an isomorphism

$$\bigoplus_{i \in I} \mathbb{Z} \xrightarrow{\sim} A$$

via the universal property of direct sums.

Question

Is $\{(a, b) \in \mathbb{Z}^2 : 2 \mid a + b\}$ an abelian group? Is it free?