

generation of ideals

Let  $A$  be a ring. Given two left ideals  $\mathfrak{a}, \mathfrak{b} \subset A$ ,

$$\mathfrak{a} \cap \mathfrak{b}$$

is also a left ideal.

For any subset  $X \subset A$ , the family of left ideals of  $A$  containing  $X$  is non-empty, since  $A$  is always a member. The intersection of them is an ideal; the left ideal generated by  $X$ .

One can similarly define right and two-sided ideals generated by  $X$ .

Let  $A$  be a ring. Given two left ideals  $\mathfrak{a}, \mathfrak{b} \subset A$ , the left ideal generated by  $\mathfrak{a}$  and  $\mathfrak{b}$  is equal to

$$\mathfrak{a} + \mathfrak{b} = \{a + b : a \in \mathfrak{a}, b \in \mathfrak{b}\}.$$

## Question

In  $\mathbb{Z}$ , compute  $2\mathbb{Z} + 5\mathbb{Z}$ . For positive integers  $n, m$ , when is  $n\mathbb{Z} + m\mathbb{Z}$  equal to  $\mathbb{Z}$ ?