

finite generation

Let E/k be a field extension. We say that E is finitely generated over k if there is a finite family

$$\alpha_1, \dots, \alpha_n \in E$$

such that

$$E = k(\alpha_1, \dots, \alpha_n).$$

Proposition

If E is a finite extension of k , then E is finitely generated.

Proof.

By assumption, there is a basis $\alpha_1, \dots, \alpha_n \in E$ as a k -vector space.

Then, $E = k(\alpha_1, \dots, \alpha_n)$.



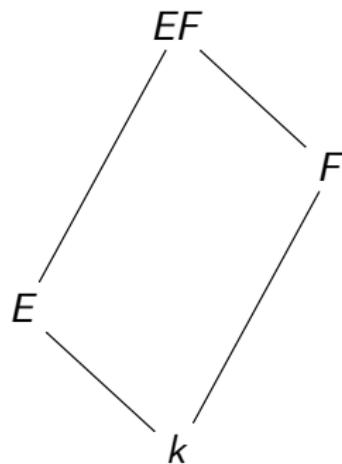
Compositum and finite generation

If $E = k(\alpha_1, \dots, \alpha_n)$ and F/k is an extension, then

$$EF = F(\alpha_1, \dots, \alpha_n).$$

We have the implication

$$E/k \text{ is finitely generated} \Rightarrow EF/F \text{ is finitely generated.}$$



Question

Find a counter-example to the implication

$$EF/F \text{ is finitely generated} \Rightarrow E/k \text{ is finitely generated.}$$