two-sided ideals and homomorphisms

Let A, B be rings. A homomorphism from A to B is a map

$$\phi \colon A \to B$$

such that they are monoid homomorphisms in two ways; for all $x, y \in A$,

$$\phi(x + y) = \phi(x) + \phi(y),$$
$$\phi(0) = 0$$

and

$$\phi(xy) = \phi(x)\phi(y),$$
$$\phi(1) = 1.$$

Let A be a ring. Given a two-sided ideal $\mathfrak{a} \subset A$, the set

$$A/\mathfrak{a}$$

of additive cosets of $\mathfrak{a}$ is again a ring. It is called a factor ring. It is equipped with a natural map $A \to A/\mathfrak{a}$.

If B is another ring and $\phi\colon A \to B$ is a ring homomorphism, then its kernel

$$\ker(\phi) := \{a \in A \colon \phi(a) = 0\}$$
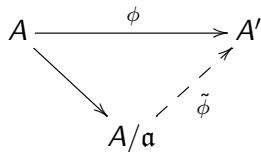
is a two-sided ideal. We have

$$\ker(A \to A/\mathfrak{a}) = \mathfrak{a}.$$

So kernels of ring homomorphisms are precisely the two-sided ideals.

Universal property

If $\phi\colon A \to A'$ is a ring homomorphism with $\phi(\mathfrak{a}) = 0$, then there exists a unique $\tilde{\phi}$ making the diagram below commutative:-

Can you interpret it as a universal property?

Can you interpret $\mathbb{C}$ as a factor ring of $\mathbb{R}[x]$?