homomorphisms

Let M and M' be two (left) modules over a ring A. Then, an additive map

$$f \colon M \to M'$$

is a homomorphism between A-modules if

$$f(am) = af(m)$$

for all $a \in A$ and all $m \in M$.

For a homomorphism $f\colon M \to M'$, one has its kernel

$$\ker(f) = \{m \in M\colon f(m) = 0\}$$

and image

$$\mathrm{im}(f) = \{m' \in M'\colon m' = f(m) \text{ for some } m \in M\}.$$

Also, one has the cokernel

$$M \to \mathrm{coker}(f) = M'/\mathrm{im}(f).$$

A submodule of M is a subset $M_0 \subset M$ such that

1. M_0 is an abelian group, and
2. $AM_0 \subset M_0$.

Then, one can form the factor module and has the reduction map

$$M \to M' = M/M_0$$

whose kernel of M_0.

In other words, submodules are characterized as kernels of homomorphisms.

An isomorphism between two A-modules M and M' is a homomorphism

$$f\colon M \to M'$$

which has an inverse; another homomorphism

$$g\colon M' \to M$$

such that $f \circ g = 1_{M'}$ and $g \circ f = 1_M$.

Question

Characterize the isomorphisms as homomorphisms that are both injective and surjective.