Adjunction

Definition

Let $\mathcal{C}$ and $\mathcal{D}$ be categories. A pair of functors

$$L\colon \mathcal{C} \to \mathcal{D}$$
$$R\colon \mathcal{D} \to \mathcal{C}$$

form an adjunction if one has a natural bijection

$$\mathrm{Mor}(L(A), B) \simeq \mathrm{Mor}(A, R(B))$$

for all $A \in \mathrm{ob}(\mathcal{C})$ and $B \in \mathrm{ob}(\mathcal{D})$. We say that L is the left adjoint of R, and R is the right adjoint of L.

The naturality of the bijection means; this is an isomorphism between functors in the variables A and B.

Proposition

For a functor F, its right adjoint is unique up natural isomorphism if exists.

Question

The set of real numbers together with inequalities form a category. What is the left adjoint of the inclusion functor $\mathbb{Z} \to \mathbb{R}$?