# Homomorphisms

# Definition

Let $G, G'$ be monoids. If a map

$$f\colon G \longrightarrow G'$$

satisfies

$$f(e) = e'$$

and

$$f(xy) = f(x)f(y)$$

for all $x, y \in G$, then we say $f$ is a homomorphism.

# Definition: an alternate form

Let $G, G'$ be monoids. If a map

$$f\colon G \longrightarrow G'$$

preserves all finite products, then we say $f$ is a homomorphism.

# Some terminology

A homomorphisms from $G$ to $G'$ is

- an isomorphism if it is invertible,
- an endomorphism if $G = G'$,
- an automorphism if it is invertible and $G = G'$,
- an embedding if it is injective.

Let $G, G'$ be group and $f\colon G \to G'$ be a homomorphism.

### Proposition

*For $g \in G$, we have $f(g^{-1}) = f(g)^{-1}$.*

### Proof.

It suffices to observe $f(g^{-1})f(g) = f(g^{-1}g) = f(e) = e'$. $\square$