Groups

We say an element $x \in S$ in a monoid is invertible if there exists $y \in S$ such that $xy = yx = e$.

Definition

A group is a monoid in which every element is invertible.

A remark

We could have required for invertibility that there exist y_1, y_2 such that $xy_1 = y_2x = e$. Since it follows that $y_1 = y_2$, there is no need to worry about 'left inverse' and 'right inverse' being different.