

Opposite category

Recall that a category  $\mathcal{C}$  consists of  $\text{ob}(\mathcal{C})$  and  $\text{mor}(\mathcal{C})$ . For each morphism  $\phi$ , we have objects  $s(\phi)$  and  $t(\phi)$ , the source and the target. For the moment, let us denote them by  $s_{\mathcal{C}}(\phi)$  and  $t_{\mathcal{C}}(\phi)$ , while the composition by  $\circ_{\mathcal{C}}$ .

Let us form the opposite category of  $\mathcal{C}$ , denoted by  $\mathcal{C}^{\text{op}}$ , with

$$\text{ob}(\mathcal{C}^{\text{op}}) = \text{ob}(\mathcal{C}), \quad \text{mor}(\mathcal{C}^{\text{op}}) = \text{mor}(\mathcal{C})$$

and for every  $\phi \in \text{mor}(\mathcal{C}^{\text{op}})$

$$s_{\mathcal{C}^{\text{op}}}(\phi) = t_{\mathcal{C}}(\phi)$$

$$t_{\mathcal{C}^{\text{op}}}(\phi) = s_{\mathcal{C}}(\phi).$$

The composition in  $\mathcal{C}^{\text{op}}$  satisfies

$$\phi \circ_{\mathcal{C}} \psi = \psi \circ_{\mathcal{C}^{\text{op}}} \phi.$$

## Question

Compare it with the notion of the opposite group. Is it a special case of an opposite category?