Multiplicative subgroup of a field

Definition

Let k be a field. A multiplicative subgroup of k means a subgroup

$$U \subset k^\times$$

which is a group under multiplication.

Theorem

Let U be a finite multiplicative subgroup of a field k. Then, U is cyclic.

Proof

Note that U is a torsion abelian group. Let n be the smallest positive integer such that $u^n = 1$ for all $u \in U$. Then, $|U| \geq n$. Consider the polynomial

$$f(x) = x^n - 1$$

so every $u \in U$ is a root. Since f has at most n roots, $|U| \leq n$. We conclude that $|U| = n$ and it is cyclic.

Corollary

If k is finite, then $k^\times$ is cyclic.

Question

Let p be a prime. For any positive integer n, is there a finite field k of characteric p with $k^\times$ containing an element of order n? If yes, what is the minimal cardinality of such a field?