Normal subgroups and homomorphisms

Let G be a group equipped with a subgroup $H \subset G$.

Definition

H is called a normal subgroup of G if any left coset of H is also a right coset.

Proposition

If G' is another group and $f\colon G \to G'$ is a homomorphism, then $\ker(f)$ is a normal subgroup.

Let G be a group equipped with a normal subgroup $H \subset G$. Let G/H be the set of left cosets of H. Since H is normal, it is equal to the set of right cosets.

Proposition

Under the operation

$$(aH, bH) \mapsto (ab)H$$

the quotient G/H becomes a group.

In summary, kernels are normal subgroups and every normal subgroup is a kernel for some homomorphism.