Categories

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A category $\mathcal{C}$ consists of the following data

1. A collection of objects, ob($\mathcal{C}$)
2. A collection of morphisms, mor($\mathcal{C}$)
3. For each morphism ϕ, there is the 'source' $s(\phi)$, an object, and the 'target' $t(\phi)$.
4. A law of 'composition'; whenever $s(\phi) = t(\psi)$ for two morphisms ϕ and ψ, there is a third morphism $\phi \circ \psi$ satisfying

$$s(\phi \circ \psi) = s(\psi)$$
$$t(\phi \circ \psi) = t(\phi)$$

subject to the two conditions we explain now.

Composition is associative

For any composable tripe of morphisms ϕ_1, ϕ_2, ϕ_3, that is to say $t(\phi_3) = s(\phi_2)$ and $t(\phi_2) = s(\phi_1)$, we have

$$\phi_1 \circ (\phi_2 \circ \phi_3) = (\phi_1 \circ \phi_2) \circ \phi_3.$$

Each object has an identity morphism

For each object A, there is an identity morphism 1_A with $s(\phi) = t(\phi) = A$ so that

$$1_A \circ \phi = \phi$$

for any morphism ϕ with $t(\phi) = A$ and

$$\phi \circ 1_A = \phi$$

for any morphism ϕ with $s(\phi) = A$.

Question

For an object A in a category, we have an identity morphism 1_A. Is it unique?