direct sums

definition

Let I be a set paremetrizing abelian groups A_i for $i \in I$. We define the direct sum

$$\bigoplus_{i \in I} A_i$$

to be the set of finite formal sums

$$\sum_{k=1}^{n} x_{i_k}$$

where n is a non-negative integer, $i_k \in I$, and $x_{i_k} \in A_{i_k}$.

the universal property

We have maps

$$\epsilon_i \colon A_i \to \bigoplus_{i \in I} A_i$$

for each $i \in I$. Here, the target is universal in the following sense. If there is an abelian group X together with a homomorphism

$$f_i \colon A_i \to X$$

for every i, then there exists a unique homomorphism

$$\phi \colon \bigoplus_{i \in I} A_i \to X$$

such that $\phi \circ \epsilon_i = f_i$ for all $i \in I$.

the universal property as a diagram

$$
\begin{tikzcd}
 & \bigoplus_{i \in I} A_i \arrow[dd, dashed, "\phi"] \\
A_i \arrow[ur, "\epsilon_i"] \arrow[dr, "f_i"'] & \\
 & X
\end{tikzcd}
$$

a bigger diagram

$$\begin{tikzcd}
 & & \bigoplus_{i\in I} A_i \arrow[d, dashed, "\phi"'] \arrow[ddr, dashed, "\psi"] & \\
A_i \arrow[urr, "\epsilon_i"] \arrow[rr, "f_i"] \arrow[drrr, "g_i"'] & & X \arrow[dr, "h"] & \\
 & & & Y
\end{tikzcd}$$

Question

Let A_i be a family of sets parametrized by $i \in I$. Can you interpret the diagram of sets

$$\begin{tikzcd}
 & \prod_{i \in I} A_i \arrow[dl] \\
A_i & \\
 & X \arrow[ul] \arrow[uu, dashed]
\end{tikzcd}$$

as a universal property of the cartesian product?