

duality of finite abelian groups

If A, B are two abelian groups, then the set

$$\text{Hom}(A, B)$$

of homomorphisms from A to B is an abelian group.

pairing

There is a natural bilinear map

$$A \times \operatorname{Hom}(A, B) \rightarrow B$$

given by evaluation. More generally, one can consider a bilinear map

$$A \times B \rightarrow C$$

where A, B, C are abelian groups.

Definition

Such a pairing is called perfect if $A \xrightarrow{\sim} \operatorname{Hom}(B, C)$.

Let A be a finite abelian group. Then,

$$A \times \operatorname{Hom}(A, \mathbb{Q}/\mathbb{Z}) \rightarrow \mathbb{Q}/\mathbb{Z}$$

is a perfect pairing.

Sometimes, the group $\operatorname{Hom}(A, \mathbb{Q}/\mathbb{Z})$ is written as A^\vee , and called the Pontryagin dual of A .

Question

Let A be a cyclic group of order n . Is there an isomorphism $A \rightarrow \text{Hom}(A, \mathbb{Q}/\mathbb{Z})$?