degree of an extension

Let E/F be a field extension. Denote by

$$[E : F]$$

the dimension of E as an F-vector space. It may be infinite.

If $E/F/k$ is a tower of extensions, then

$$[E : k] = [E : F][F : k].$$

It follows from contructing a k-basis for E from those for E/F and F/k.

For an extension E/F and an element $\alpha \in E$, denote by

$$F[\alpha]$$

the subring generated by α.

$$F(\alpha)$$

denotes the field of fractions of $F(\alpha)$. If α is algebraic, then we have

$$F[\alpha] = F(\alpha).$$

If α is algebraic, with minimal polynomial $P(X)$, then

$$\deg P(X) = [F(\alpha) : F].$$

Question

Show that $\mathbb{Q}(\sqrt{2}+\sqrt{3})$ is an extension of $\mathbb{Q}(\sqrt{6})$? What is the degree?