Group action

Definition

Let G be a group and S a set. An action of G on S is a homomorphism

$$G \to \mathrm{Perm}(S).$$

Equivalently, an action is a map

$$G \times S \to S$$

satisfying

1. $es = s$ for all $s \in S$
2. $(xy)s = x(ys)$ for all $x, y \in G$ and all $s \in S$.

We will freely go back and forth between two points of view.

Since elements of G act from the left of those from S, such an action is often called a left action. We also say S is a (left) G-set.

Right actions are defined as a map

$$S \times G \to S$$

satisfying similar axioms.

In order to interprete a right action as a homomorphism, one my consider functions composing in the other way around; the value of a function f at an argument x is denoted by

$$xf$$

and the composition of two functions f and g are defined as

$$x(fg) = (xf)g.$$