$$\frac{dN}{dt} = r \cdot N \cdot (1 - N/K) \tag{1}$$

$$\int_{N_0}^{N} \frac{dN'}{N' \cdot (1 - N'/K)} = \int_0^t r \cdot dt' \tag{2}$$

$$x = \frac{N}{K} \Rightarrow \frac{dx}{dN} = 1/K \Rightarrow K \cdot dx = dN \tag{3}$$

$$\int_{N_0/K}^{N/K} \frac{K \cdot dN'}{N' \cdot (1 - N'/K)} = \int_0^t r \cdot dt' \tag{4}$$

$$\int_{N_0/K}^{N/K} \frac{dx}{x \cdot (1 - x)} = \int_0^t r \cdot dt' \tag{5}$$

$$\int \frac{dx}{x \cdot (1 - x)} = \int \frac{((1 - x) + x)dx}{x \cdot (1 - x)} = \int \frac{dx}{x} + \int \frac{dx}{(1 - x)} \tag{6}$$

$$\int \frac{dx}{(1 - x)} = \int \frac{-du}{u} \; with \; 1 - x = u; \frac{du}{dx} = -1 \tag{7}$$

$$\int \frac{dx}{x \cdot (1 - x)} = ln(x) - ln(1 - x) = ln(\frac{x}{1 - x}) \tag{8}$$

$$\int_{N_0/K}^{N/K} \frac{dx}{x \cdot (1 - x)} = \int_0^t r \cdot dt' \Rightarrow [ln(\frac{x}{1 - x})]_{N_0/K}^{N/K} = r \cdot t \tag{9}$$

$$ln(\frac{N/K}{1 - N/K}) - ln(\frac{N_0/K}{1 - N_0/K}) = r \cdot t \tag{10}$$

$$ln(\frac{N/K \cdot (1 - N_0/K)}{(1 - N/K) \cdot N_0/K}) = r \cdot t \tag{11}$$

$$\Rightarrow \frac{N}{K} \cdot (1 - N_0/K) = N_0/K \cdot (1 - N/K) \cdot e^{r \cdot t} = N_0/K \cdot e^{r \cdot t} - N/K \cdot N_0/K \cdot e^{r \cdot t} \tag{12}$$

$$N/K \cdot [(1 - N_0/K) + N_0/K \cdot e^{r \cdot t}] = N_0/K \cdot e^{r \cdot t} \tag{13}$$

$$N/K = N_0/K \cdot e^{r \cdot t}/[(1 - N_0/K) + N_0/K \cdot e^{r \cdot t}] \tag{14}$$

Divide right-hand side's numerator and denominator by $N_0/K \cdot e^{r \cdot t}$:

$$N/K = \frac{1}{(1 - N_0/K) \cdot K/N_0 \cdot e^{-r \cdot t} + 1}] = \frac{1}{(K/N_0 - 1) \cdot e^{-r \cdot t} + 1} \tag{15}$$

$$\Rightarrow N = \frac{K}{1 + (K/N_0 - 1) \cdot e^{-r \cdot t}} \tag{16}$$