

draft

## Exploratory analysis

```
##      ID          AGE        GENDER    DRIVING_EXPERIENCE
##  Min.   : 101   16-25:2016 female:5010  0-9y  :3530
##  1st Qu.:249639 26-39:3063  male  :4990  10-19y:3299
##  Median :501777 40-64:2931           20-29y:2119
##  Mean   :500522 65+   :1990           30y+   :1052
##  3rd Qu.:753975
##  Max.   :999976
##
##      EDUCATION     CREDIT_SCORE VEHICLE_OWNERSHIP VEHICLE_YEAR
##  high school:4157   Min.   :0.05336  Min.   :0.000  after 2015 :3033
##  none       :1915   1st Qu.:0.41719  1st Qu.:0.000  before 2015:6967
##  university :3928   Median :0.52503  Median :1.000
##                  Mean   :0.51581  Mean   :0.697
##                  3rd Qu.:0.61831  3rd Qu.:1.000
##                  Max.   :0.96082  Max.   :1.000
##                  NA's    :982
##
##      MARRIED CHILDREN ANNUAL_MILEAGE VEHICLE_TYPE
##  Min.   :0.0000  Min.   :0.0000  Min.   : 2000  sedan    :9523
##  1st Qu.:0.0000  1st Qu.:0.0000  1st Qu.:10000 sports car: 477
##  Median :0.0000  Median :1.0000  Median :12000
##  Mean   :0.4982  Mean   :0.6888  Mean   :11697
##  3rd Qu.:1.0000  3rd Qu.:1.0000  3rd Qu.:14000
##  Max.   :1.0000  Max.   :1.0000  Max.   :22000
##  NA's    :957
##
##      SPEEDING_VIOLATIONS PAST_ACCIDENTS OUTCOME CLAIMS
##  Min.   : 0.000  Min.   : 0.000  Min.   :0.0000  Min.   : 0
##  1st Qu.: 0.000  1st Qu.: 0.000  1st Qu.:0.0000  1st Qu.: 0
##  Median : 0.000  Median : 0.000  Median :0.0000  Median : 0
##  Mean   : 1.483  Mean   : 1.056  Mean   :0.3133  Mean   : 2329
##  3rd Qu.: 2.000  3rd Qu.: 2.000  3rd Qu.:1.0000  3rd Qu.: 1748
##  Max.   :22.000  Max.   :15.000  Max.   :1.0000  Max.   :116328
##
```

splitting data into train and test/validation

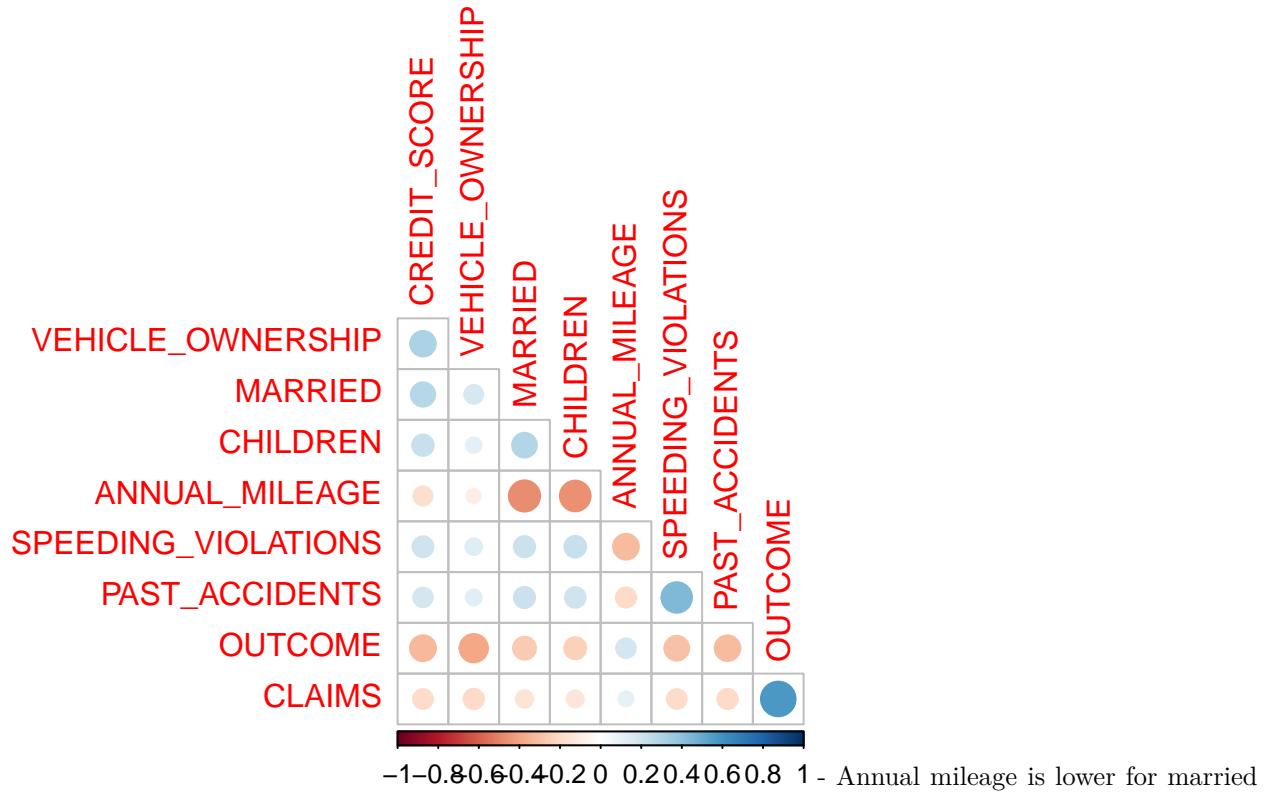
```
set.seed(67)
train_indices <- sample(1:10000,8000)
train_data <- data[train_indices, ]
test_data <- data[-train_indices, ]
```

```
train_data_x <- train_data %>% dplyr::select(-OUTCOME, -CLAIMS)
test_data_x <- test_data %>% dplyr::select(-OUTCOME, -CLAIMS)
```

## correlation matrix of numeric covariates

```
num_cols <- data %>% select(where(is.numeric))

num_cols <- num_cols[!is.na(num_cols$CREDIT_SCORE) &
                     !is.na(num_cols$ANNUAL_MILEAGE),]
num_cols <- num_cols %>% select(-ID)
corrplot(cor(num_cols), type = "lower", diag = F)
```



-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 - Annual mileage is lower for married ppl and ppl with children - past accidents has high corr with speeding violations ofc - vehicle owners are less likely to file claims?

## investigating missing values

```
print(paste("number of rows with missing annual mileage",
            sum(is.na(train_data$ANNUAL_MILEAGE)), "/8000"))
```

```
## [1] "number of rows with missing annual mileage 773 /8000"
```

```

print(paste("number of rows with missing creditscore",
           sum(is.na(train_data$CREDIT_SCORE)), "/8000"))

## [1] "number of rows with missing creditscore 804 /8000"

print(paste("number of rows missing both annual mileage and credit",
           sum(is.na(train_data$ANNUAL_MILEAGE) & is.na(train_data$CREDIT_SCORE)), "/8000"))

## [1] "number of rows missing both annual mileage and credit 76 /8000"

print(paste("percentage of ppl of claim for nonempty annual mileage rows",
           mean(train_data[!is.na(train_data$ANNUAL_MILEAGE),"OUTCOME", drop = T])))

## [1] "percentage of ppl of claim for nonempty annual mileage rows 0.31451501314515"

print(paste("percentage of ppl of claim for empty annual mileage rows",
           mean(train_data[is.na(train_data$ANNUAL_MILEAGE),"OUTCOME", drop = T])))

## [1] "percentage of ppl of claim for empty annual mileage rows 0.33764553686934"

print(paste("percentage of ppl of claim for nonempty creditscore rows",
           mean(train_data[!is.na(train_data$CREDIT_SCORE),"OUTCOME", drop = T])))

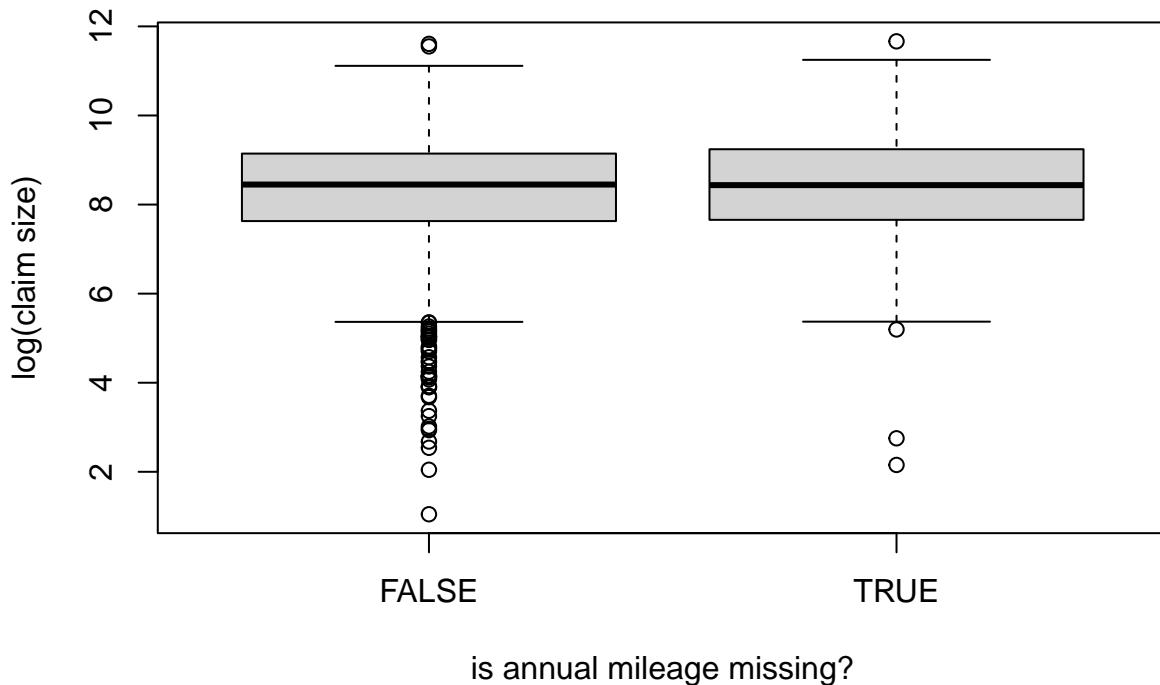
## [1] "percentage of ppl of claim for nonempty creditscore rows 0.317259588660367"

print(paste("percentage of ppl of claim for empty creditscore rows",
           mean(train_data[is.na(train_data$CREDIT_SCORE),"OUTCOME", drop = T])))

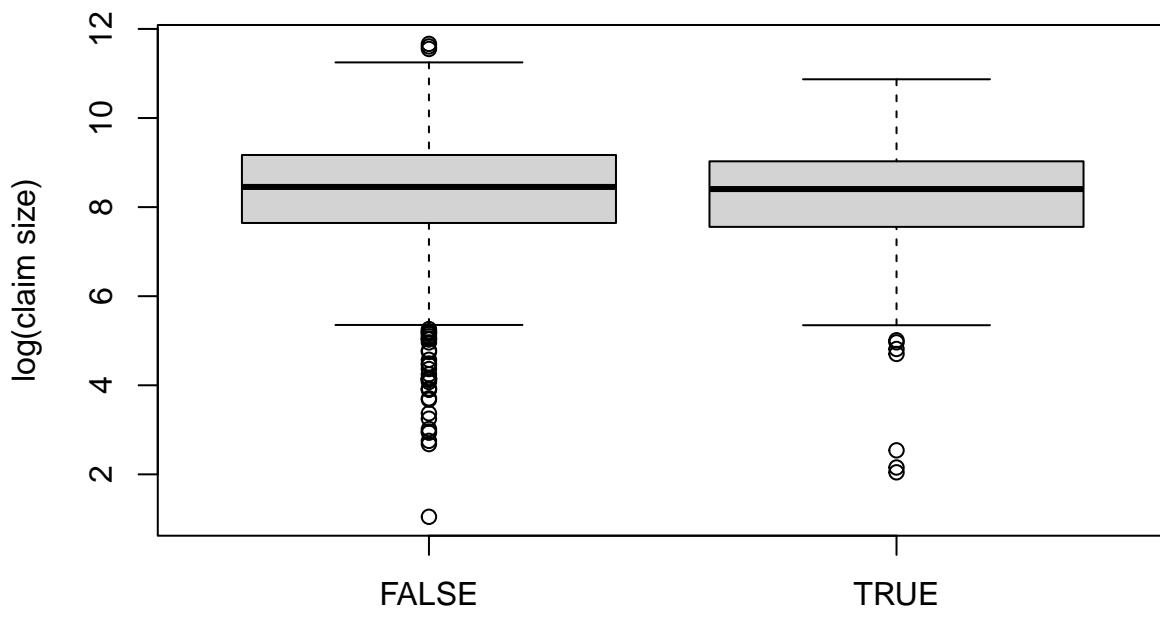
## [1] "percentage of ppl of claim for empty creditscore rows 0.312189054726368"

boxplot(log(train_data[train_data$CLAIMS>0,"CLAIMS", drop = T]) ~ is.na(train_data[train_data$CLAIMS>0,
                           xlab = "is annual mileage missing?", ylab = "log(claim size)"])

```



```
boxplot(log(train_data[train_data$CLAIMS>0,"CLAIMS", drop = T]) ~ is.na(train_data[train_data$CLAIMS>0,
  xlab = "is creditscore missing?", ylab = "log(claim size)"])
```



- Dont want to get rid of ~10% of the data, would be good to impute

Investigating if emptiness of the creditscore is independent to other variables i.e if creditscore is intentionally left empty - would expect younger ppl to have no credit score, so cells may be intentionally empty

```

print("TESTING FOR INDEPENDENCE OF CREDIT_SCORE EMPTINESS")

## [1] "TESTING FOR INDEPENDENCE OF CREDIT_SCORE EMPTINESS"

# Define the target variable: a binary indicator for missing credit scores
credit_score_missing <- is.na(train_data$CREDIT_SCORE)

# Get a list of all predictor variables to test, excluding ID and CREDIT_SCORE itself
predictors_to_test <- setdiff(names(train_data), c("ID", "CREDIT_SCORE"))

# Loop through each predictor and perform the appropriate test
for (var in predictors_to_test) {

  # Ignore any columns with no variation
  if (length(unique(train_data[[var]])) < 2) next

  # --- Test for association with CONTINUOUS variables ---
  if (is.numeric(train_data[[var]])) {
    # We use a t-test to see if the mean of the numeric variable is
    # different between the 'missing' and 'present' groups.
    test_result <- t.test(train_data[[var]] ~ credit_score_missing)
    p_value <- test_result$p.value
    cat(sprintf("Variable: %-20s | Test: T-test           | P-value: %.4f\n", var, p_value))

    # --- Test for association with CATEGORICAL variables ---
  } else if (is.factor(train_data[[var]]) || is.character(train_data[[var]])) {
    # We use a Chi-squared test for independence between two categorical variables.
    # We add 'simulate.p.value = TRUE' to handle cases with low expected counts.
    test_result <- chisq.test(table(train_data[[var]], credit_score_missing), simulate.p.value = TRUE)
    p_value <- test_result$p.value
    cat(sprintf("Variable: %-20s | Test: Chi-squared     | P-value: %.4f\n", var, p_value))
  }
}

## Variable: AGE                  | Test: Chi-squared   | P-value: 0.1544
## Variable: GENDER                | Test: Chi-squared   | P-value: 0.9025
## Variable: DRIVING_EXPERIENCE   | Test: Chi-squared   | P-value: 0.1494
## Variable: EDUCATION              | Test: Chi-squared   | P-value: 0.5927
## Variable: VEHICLE_OWNERSHIP    | Test: T-test        | P-value: 0.1828
## Variable: VEHICLE_YEAR          | Test: Chi-squared   | P-value: 0.0395
## Variable: MARRIED                | Test: T-test        | P-value: 0.4997
## Variable: CHILDREN                | Test: T-test        | P-value: 0.4530
## Variable: ANNUAL_MILEAGE         | Test: T-test        | P-value: 0.6424
## Variable: VEHICLE_TYPE            | Test: Chi-squared   | P-value: 0.4308
## Variable: SPEEDING_VIOLATIONS   | Test: T-test        | P-value: 0.7981
## Variable: PAST_ACCIDENTS          | Test: T-test        | P-value: 0.9907
## Variable: OUTCOME                 | Test: T-test        | P-value: 0.7688
## Variable: CLAIMS                  | Test: T-test        | P-value: 0.1725

```

- pvalues very high, so likely that annual\_mileage is left empty independently of other variables for all except vehicle year

```

plot_data <- train_data %>%
  # Create a clear factor for whether the credit score is missing
  mutate(Credit_Score_Status = factor(ifelse(is.na(CREDIT_SCORE), "Empty", "Not Empty"))) %>%
  # Count the occurrences of each vehicle year for each status
  count(VEHICLE_YEAR, Credit_Score_Status) %>%
  # Group by the status so we can calculate proportions within each group
  group_by(Credit_Score_Status) %>%
  # Calculate the proportion
  mutate(Proportion = n / sum(n))

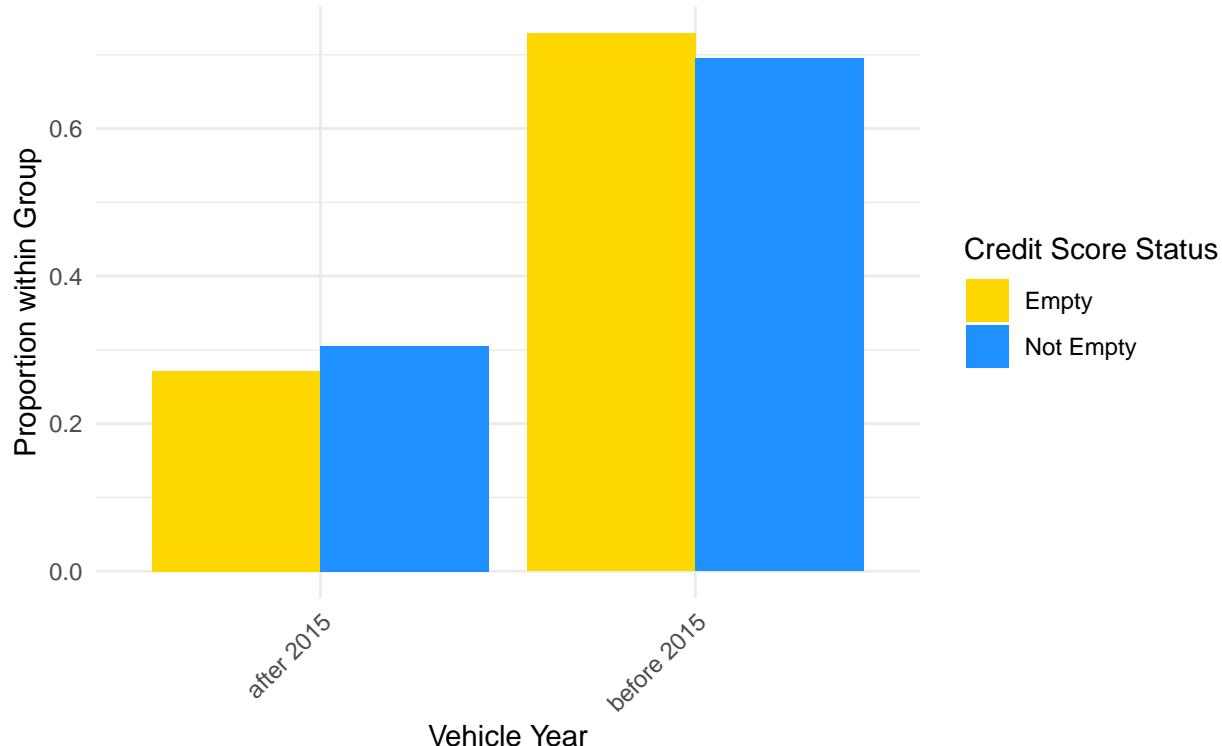
# --- 4. Create the Side-by-Side Bar Plot ---

ggplot(plot_data, aes(x = VEHICLE_YEAR, y = Proportion, fill = Credit_Score_Status)) +
  # geom_bar with stat="identity" uses the y-value directly.
  # position="dodge" places the bars next to each other.
  geom_bar(stat = "identity", position = "dodge") +
  labs(
    title = "Vehicle Year Distribution by Credit Score Availability",
    subtitle = "Comparing proportions for policies with empty vs. non-empty credit scores",
    x = "Vehicle Year",
    y = "Proportion within Group",
    fill = "Credit Score Status"
  ) +
  # Set the colors as requested
  scale_fill_manual(values = c("Empty" = "gold", "Not Empty" = "dodgerblue")) +
  # Improve readability
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))

```

## Vehicle Year Distribution by Credit Score Availability

Comparing proportions for policies with empty vs. non-empty credit scores



with no particular reason to take the conventional significance level of 0.05, conclude that the vehicle year is still independent to the emptiness of credit scores

- suggests that ppl with no credit score are not necessarily younger, so we assume that missing values are missing unintentionally

```
print("TESTING FOR INDEPENDENCE OF ANNUAL_MILEAGE EMPTINESS")  
  
## [1] "TESTING FOR INDEPENDENCE OF ANNUAL_MILEAGE EMPTINESS"  
  
AM_missing <- is.na(train_data$ANNUAL_MILEAGE)  
  
# Get a list of all predictor variables to test, excluding ID and CREDIT_SCORE itself  
predictors_to_test <- setdiff(names(train_data), c("ID", "ANNUAL_MILEAGE"))  
  
# Loop through each predictor and perform the appropriate test  
for (var in predictors_to_test) {  
  
  # Ignore any columns with no variation  
  if (length(unique(train_data[[var]])) < 2) next  
  
  # --- Test for association with CONTINUOUS variables ---  
  if (is.numeric(train_data[[var]])) {  
    # We use a t-test to see if the mean of the numeric variable is  
    # different between the 'missing' and 'present' groups.  
    test_result <- t.test(train_data[[var]] ~ AM_missing)
```

```

    p_value <- test_result$p.value
    cat(sprintf("Variable: %-20s | Test: T-test           | P-value: %.4f\n", var, p_value))

    # --- Test for association with CATEGORICAL variables ---
} else if (is.factor(train_data[[var]]) || is.character(train_data[[var]])) {
    # We use a Chi-squared test for independence between two categorical variables.
    # We add 'simulate.p.value = TRUE' to handle cases with low expected counts.
    test_result <- chisq.test(table(train_data[[var]]), AM_missing), simulate.p.value = TRUE)
    p_value <- test_result$p.value
    cat(sprintf("Variable: %-20s | Test: Chi-squared | P-value: %.4f\n", var, p_value))
}
}

## Variable: AGE           | Test: Chi-squared | P-value: 0.4203
## Variable: GENDER        | Test: Chi-squared | P-value: 0.8166
## Variable: DRIVING_EXPERIENCE | Test: Chi-squared | P-value: 0.5192
## Variable: EDUCATION      | Test: Chi-squared | P-value: 0.6092
## Variable: CREDIT_SCORE    | Test: T-test     | P-value: 0.3480
## Variable: VEHICLE_OWNERSHIP | Test: T-test     | P-value: 0.5202
## Variable: VEHICLE_YEAR     | Test: Chi-squared | P-value: 0.2689
## Variable: MARRIED         | Test: T-test     | P-value: 0.8048
## Variable: CHILDREN        | Test: T-test     | P-value: 0.4170
## Variable: VEHICLE_TYPE     | Test: Chi-squared | P-value: 0.4728
## Variable: SPEEDING_VIOLATIONS | Test: T-test     | P-value: 0.5506
## Variable: PAST_ACCIDENTS    | Test: T-test     | P-value: 0.0999
## Variable: OUTCOME          | Test: T-test     | P-value: 0.1960
## Variable: CLAIMS            | Test: T-test     | P-value: 0.1095

```

- pvalues very high, so likely that annual\_mileage is left empty independently of other variables
- assume from here on that missing values are missing unintentionally

## imputing missing values

```

# 1. Select all numeric columns from train and test sets
train_numeric <- train_data_x %>% select(where(is.numeric))
test_numeric <- test_data_x %>% select(where(is.numeric))

# 2. Build recipe: Impute ONLY ANNUAL_MILEAGE and CREDIT_SCORE, using all numeric predictors
rec <- recipe(~ ., data = train_numeric) %>%
  step_impute_knn(c("ANNUAL_MILEAGE", "CREDIT_SCORE"))

# 3. Prep the recipe (fit on training data)
rec_prep <- prep(rec, training = train_numeric)

# 4. Impute the training and test sets
train_imputed <- bake(rec_prep, new_data = train_numeric)
test_imputed <- bake(rec_prep, new_data = test_numeric)

# The result: train_imputed and test_imputed contain all numeric columns,
# with only ANNUAL_MILEAGE and CREDIT_SCORE imputed, using relationships with all numeric variables.

```

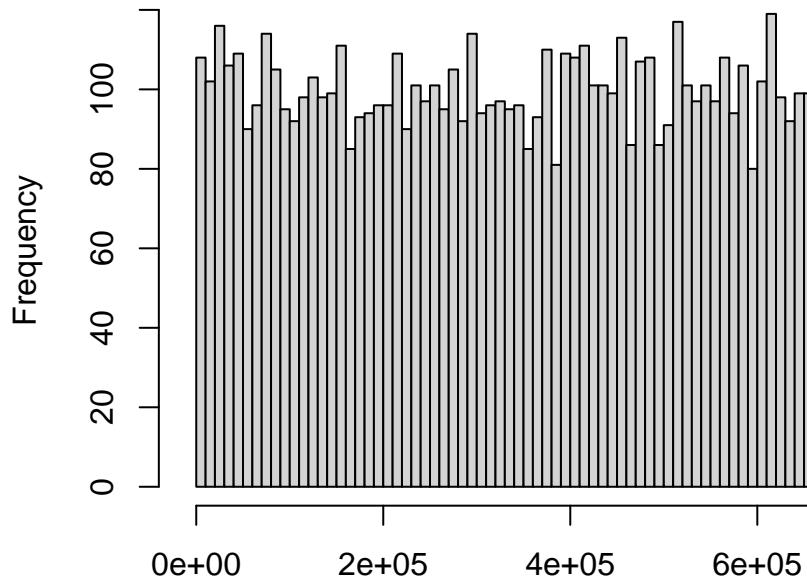
```
train_data_imputed <- train_data
train_data_imputed$ANNUAL_MILEAGE <- train_imputed$ANNUAL_MILEAGE
train_data_imputed$CREDIT_SCORE <- train_imputed$CREDIT_SCORE

test_data_imputed <- test_data
test_data_imputed$ANNUAL_MILEAGE <- test_imputed$ANNUAL_MILEAGE
test_data_imputed$CREDIT_SCORE <- test_imputed$CREDIT_SCORE
```

- used PCA between numeric variables to impute missing values
- wasnt possible to use the same model to also impute test data
- used knn imputation instead
- doesnt account for categorical variable values!

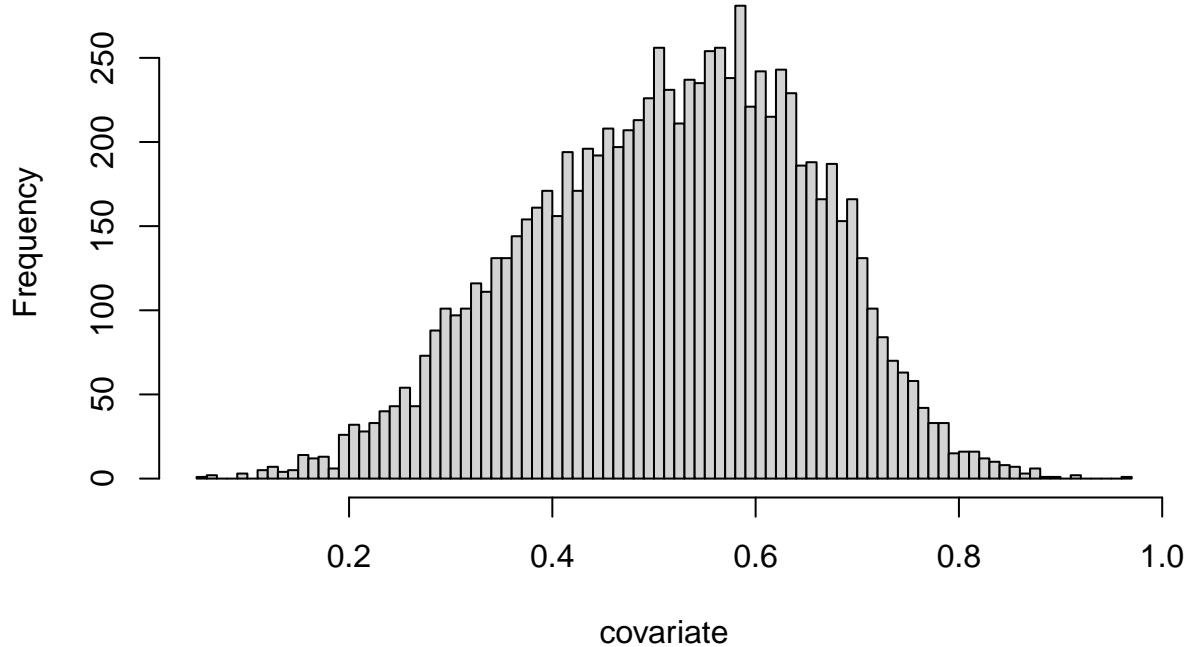
histogram of covariates

**Histogram of ID**

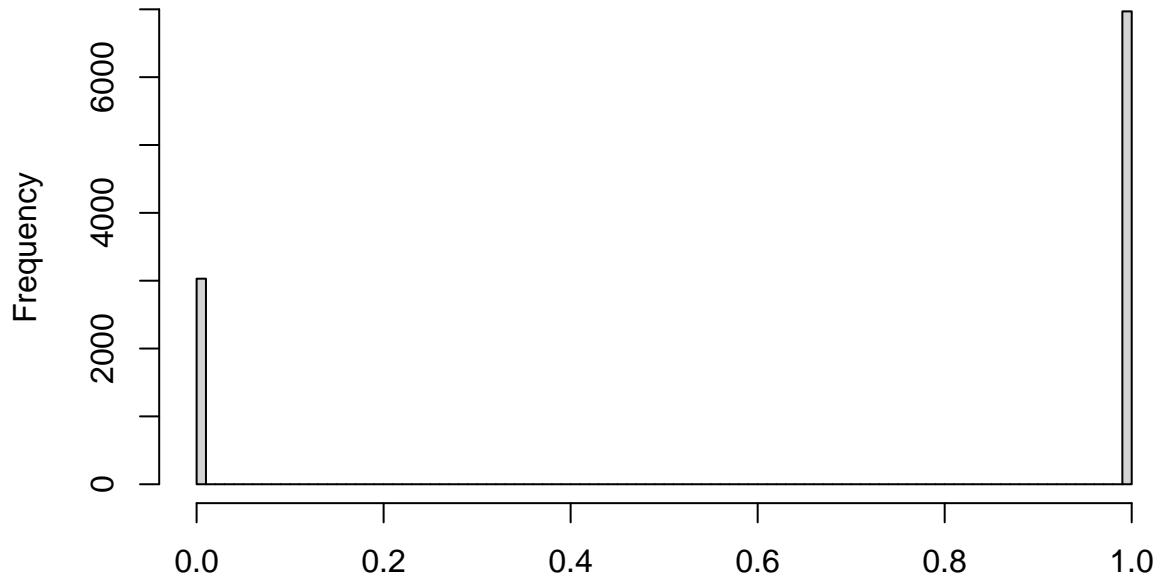


- suppressed as it makes the knitted document untidy

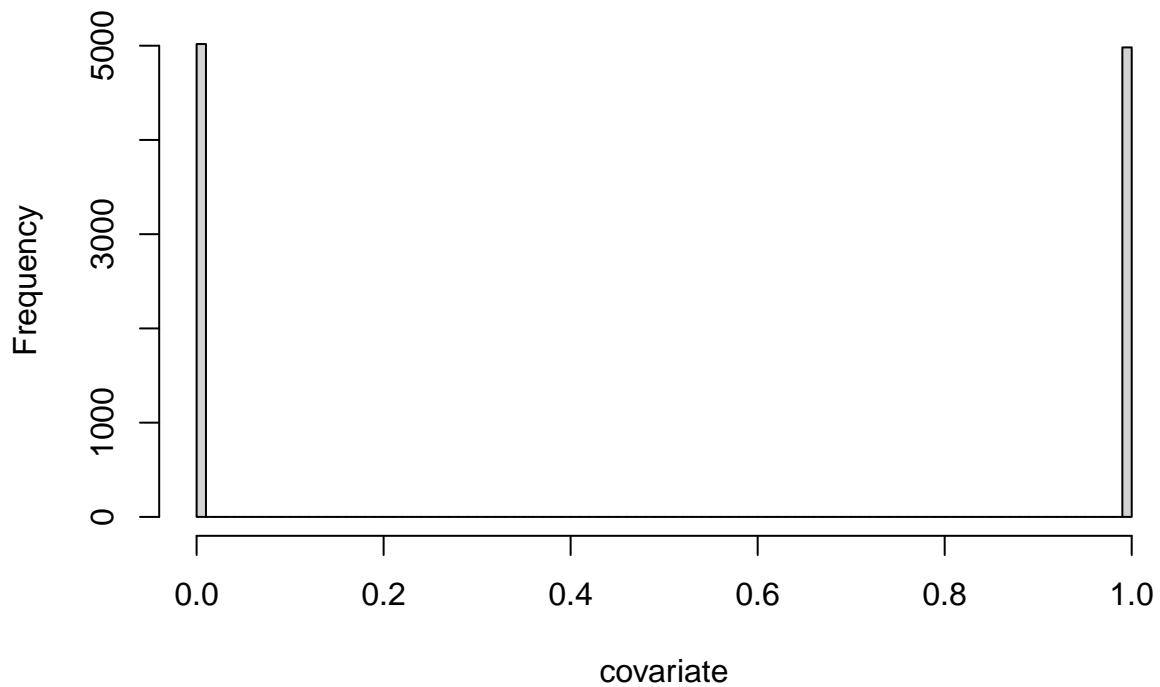
**Histogram of CREDIT\_SCORE**



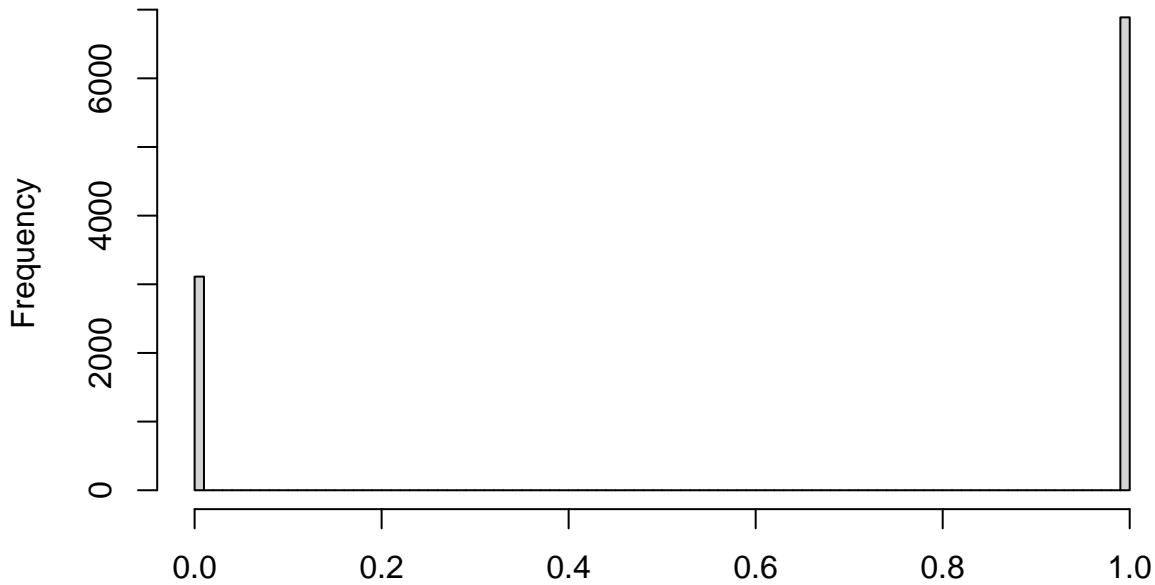
**Histogram of VEHICLE\_OWNERSHIP**



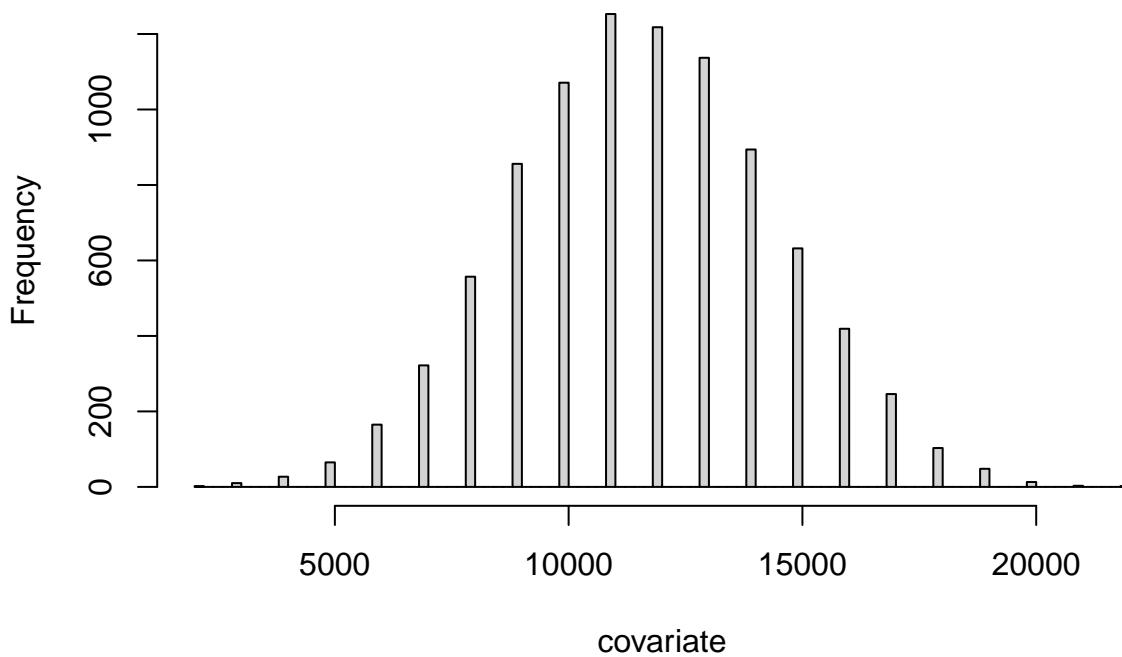
**Histogram of MARRIED**



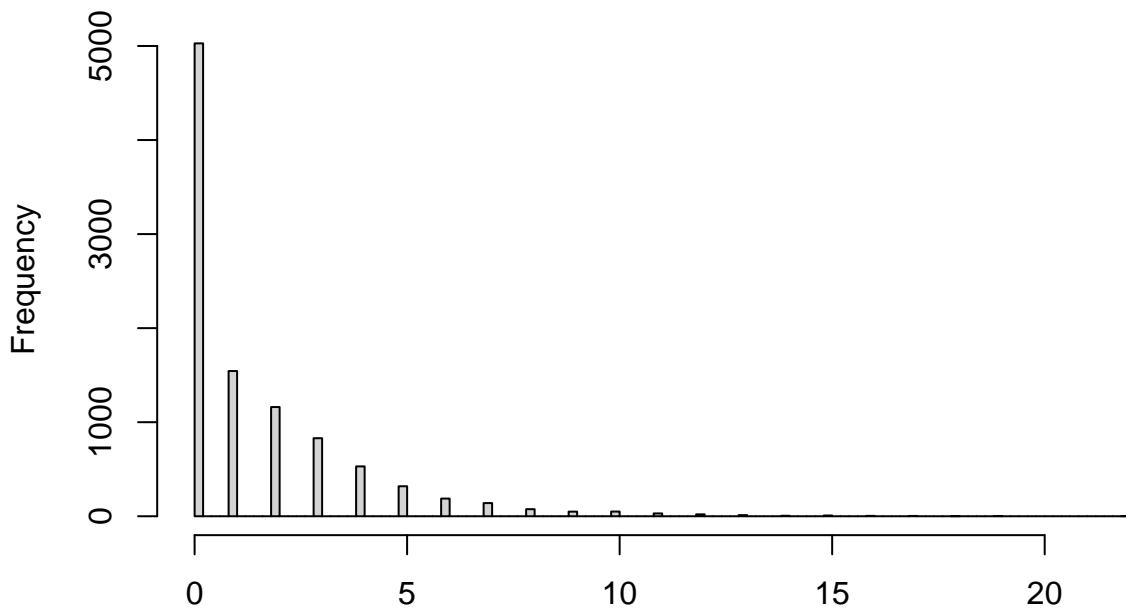
**Histogram of CHILDREN**



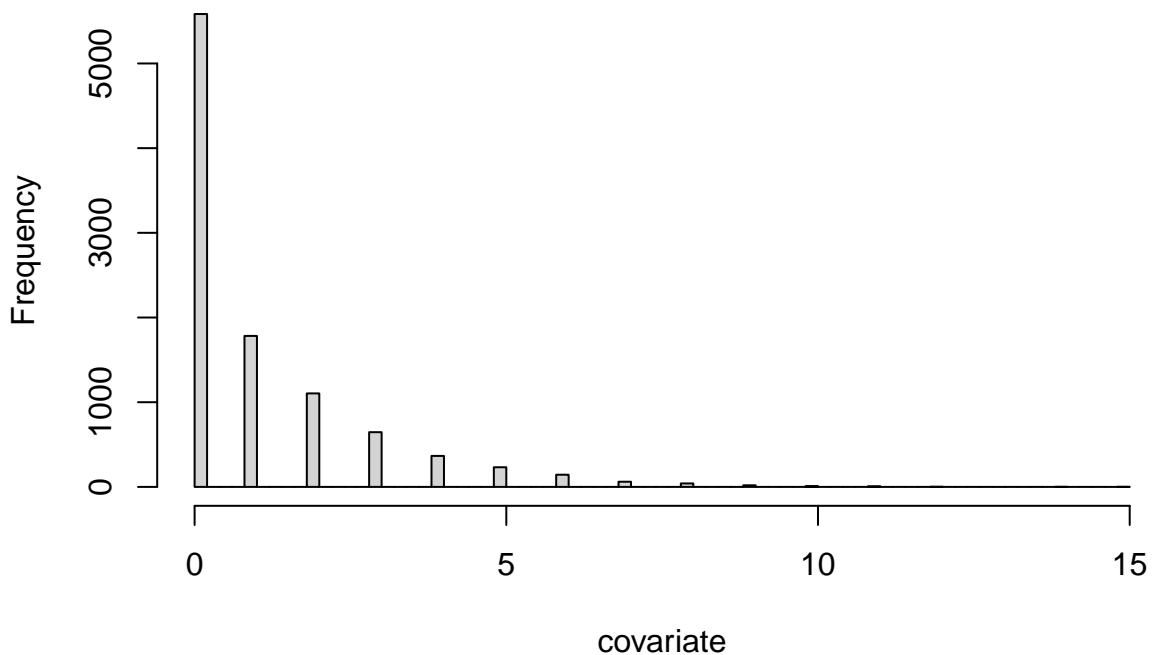
**Histogram of ANNUAL\_MILEAGE**



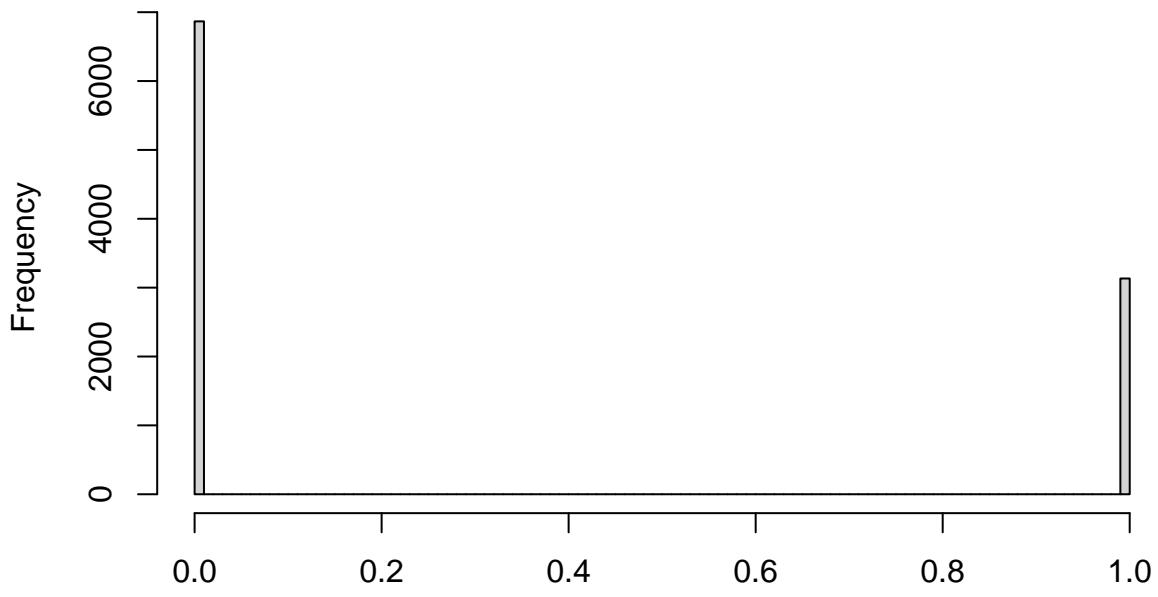
**Histogram of SPEEDING\_VIOLATIONS**



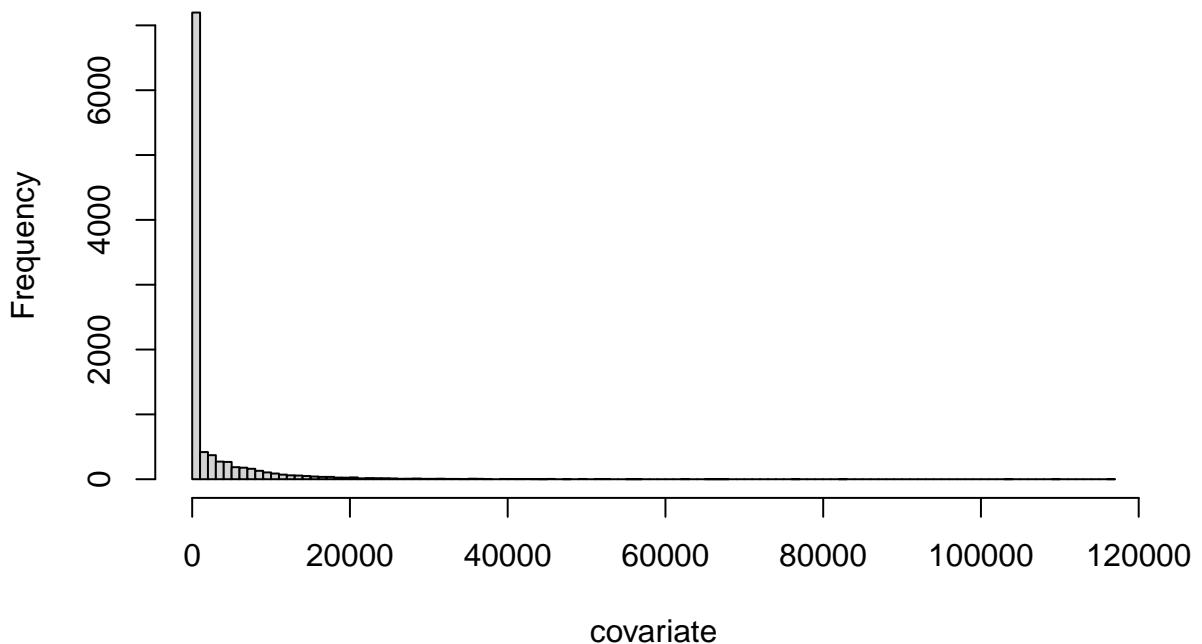
**Histogram of PAST\_ACCIDENTS**  
covariate



## Histogram of OUTCOME



## covariate Histogram of CLAIMS



```
summary(train_data)
```

```
##      ID        AGE      GENDER    DRIVING_EXPERIENCE
##  Min.   : 125   16-25:1611  female:3993  0-9y  :2822
##  1st Qu.:249025 26-39:2459   male  :4007   10-19y:2641
```

```

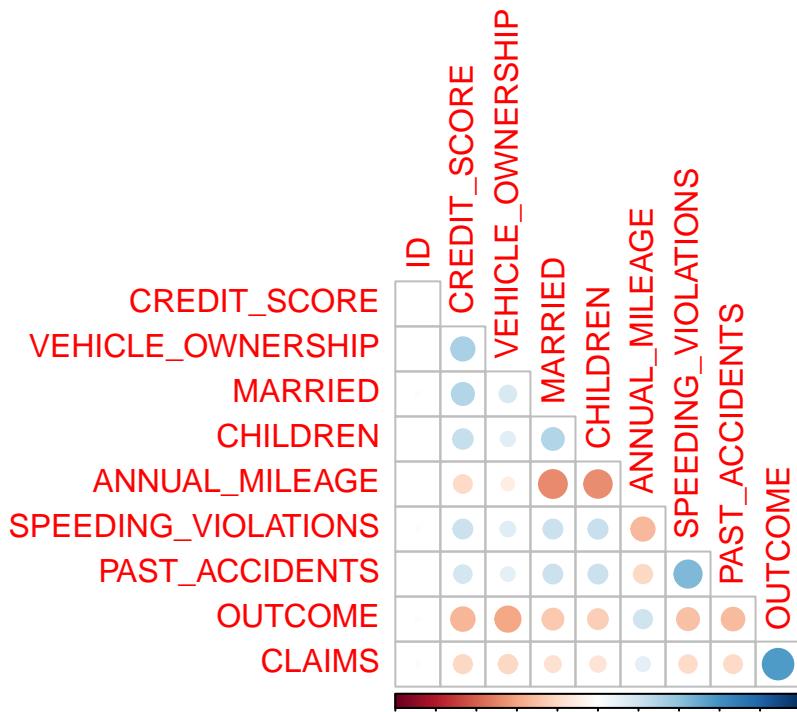
## Median :500664 40-64:2339          20-29y:1688
## Mean   :499893 65+   :1591          30y+   : 849
## 3rd Qu.:753723
## Max.   :999976
##
##           EDUCATION      CREDIT_SCORE      VEHICLE_OWNERSHIP      VEHICLE_YEAR
## high school:3335    Min.   :0.05336    Min.   :0.000    after 2015 :2416
## none       :1524     1st Qu.:0.41637    1st Qu.:0.000    before 2015:5584
## university :3141    Median :0.52456    Median :1.000
##                   Mean   :0.51528    Mean   :0.695
##                   3rd Qu.:0.61700    3rd Qu.:1.000
##                   Max.   :0.96082    Max.   :1.000
##                   NA's    :804
##
##           MARRIED      CHILDREN      ANNUAL_MILEAGE      VEHICLE_TYPE
## Min.   :0.0000    Min.   :0.0000    Min.   : 3000    sedan    :7623
## 1st Qu.:0.0000    1st Qu.:0.0000    1st Qu.:10000   sports car: 377
## Median :0.0000    Median :1.0000    Median :12000
## Mean   :0.4951    Mean   :0.6871    Mean   :11723
## 3rd Qu.:1.0000    3rd Qu.:1.0000    3rd Qu.:14000
## Max.   :1.0000    Max.   :1.0000    Max.   :22000
##                   NA's    :773
##
##           SPEEDING_VIOLATIONS PAST_ACCIDENTS      OUTCOME      CLAIMS
## Min.   : 0.000    Min.   : 0.000    Min.   :0.0000    Min.   :    0
## 1st Qu.: 0.000    1st Qu.: 0.000    1st Qu.:0.0000    1st Qu.:    0
## Median : 0.000    Median : 0.000    Median :0.0000    Median :    0
## Mean   : 1.484    Mean   : 1.055    Mean   :0.3167    Mean   : 2360
## 3rd Qu.: 2.000    3rd Qu.: 2.000    3rd Qu.:1.0000    3rd Qu.: 1838
## Max.   :22.000    Max.   :15.000    Max.   :1.0000    Max.   :116328
##

```

```

num_cols = train_data_imputed %>% select_if(is.numeric)
cor_matrix = cor(num_cols)
corrplot(cor_matrix, type = "lower", diag = F)

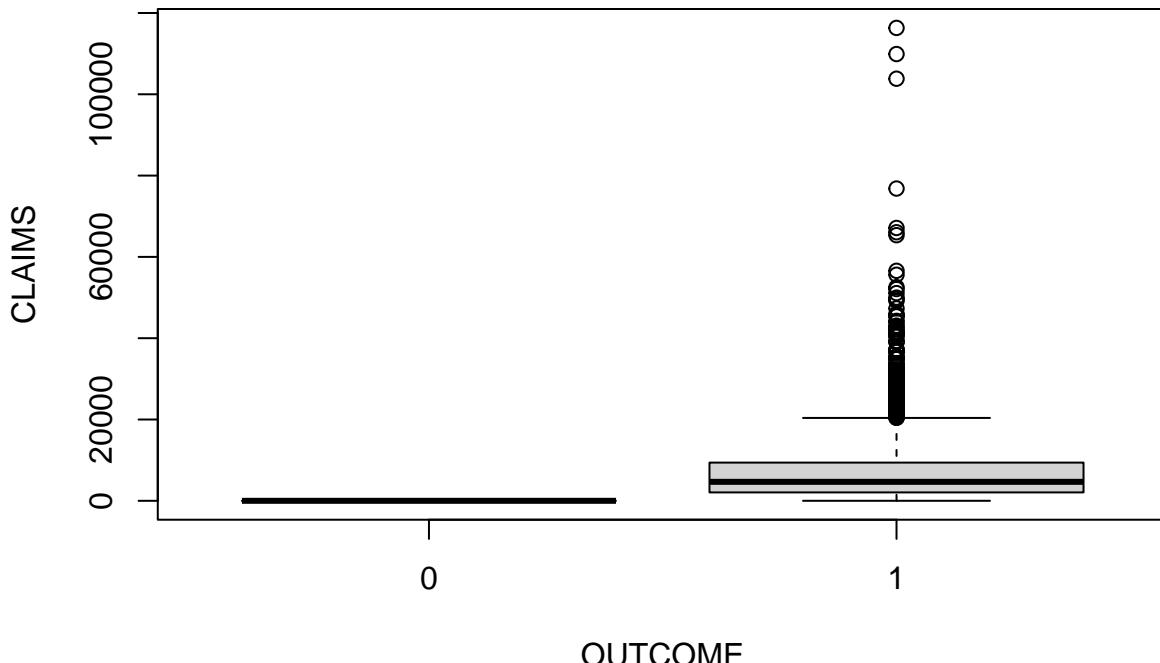
```



-1 -0.8 -0.6 -0.4 -0.2 0 0.2 0.4 0.6 0.8 1 - looks like annual mileage has a negative correlation with being married / having children - ppl with speeding violations are much more likely to have past accidents - surprisingly non vehicle owners have a higher correlation with making claims

## investigation of claim sizes

**Boxplot of claim sizes by outcome**



```
## [1] 0
## [1] 0
```

- Looks like there are no cases where outcome = 1 and there is a claimsize of 0
- suggests that claims sizes are always nonzero and zero claims should be accounted for within the counts distribution

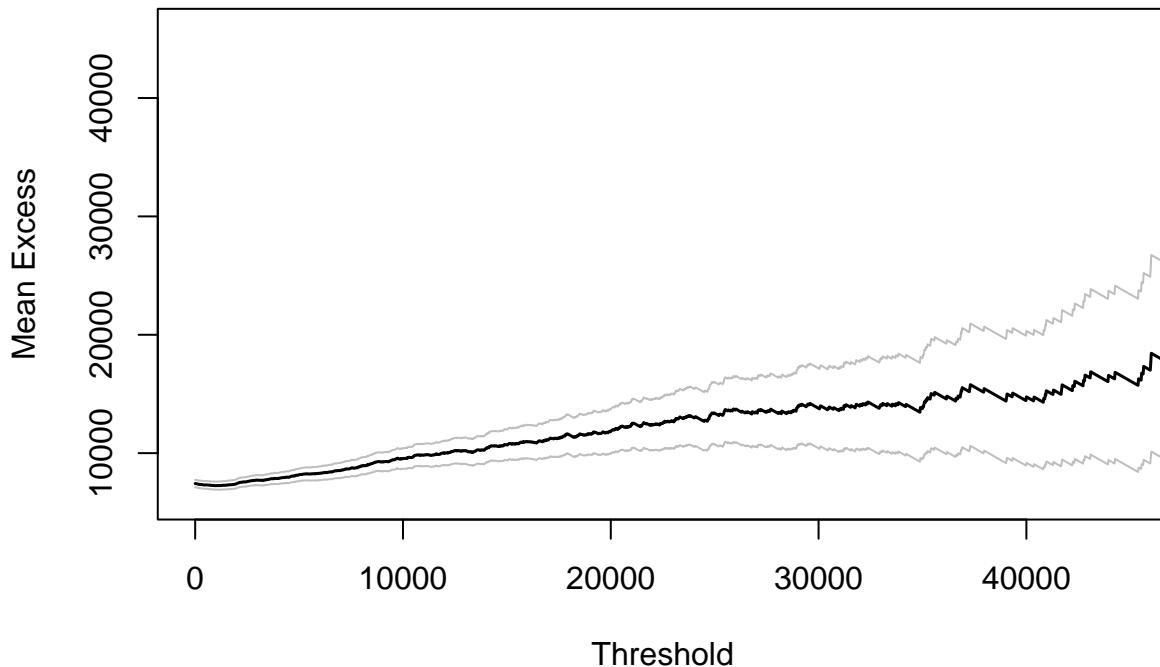
## Extreme value analysis of claimsizes

```
nonzerosizes <- data[data$OUTCOME>0, "CLAIMS", drop = T]
quantile(nonzerosizes, c(0.95,0.97, 0.99, 0.995))
```

```
##      95%      97%      99%   99.5%
## 23303.06 29080.47 42786.68 51570.25
```

```
# mean excess plot
mrlplot(nonzerosizes, xlim = c(0,45000))
```

**Mean Residual Life Plot**



- Mean excess function is rising, clearly a heavy tailed distribution

```
# Fitting GEV distribution
block_size <- 50
blocks <- ceiling(seq_along(nonzerosizes) / block_size)
GEVdf <- data.frame(nonzerosizes, blocks)
```

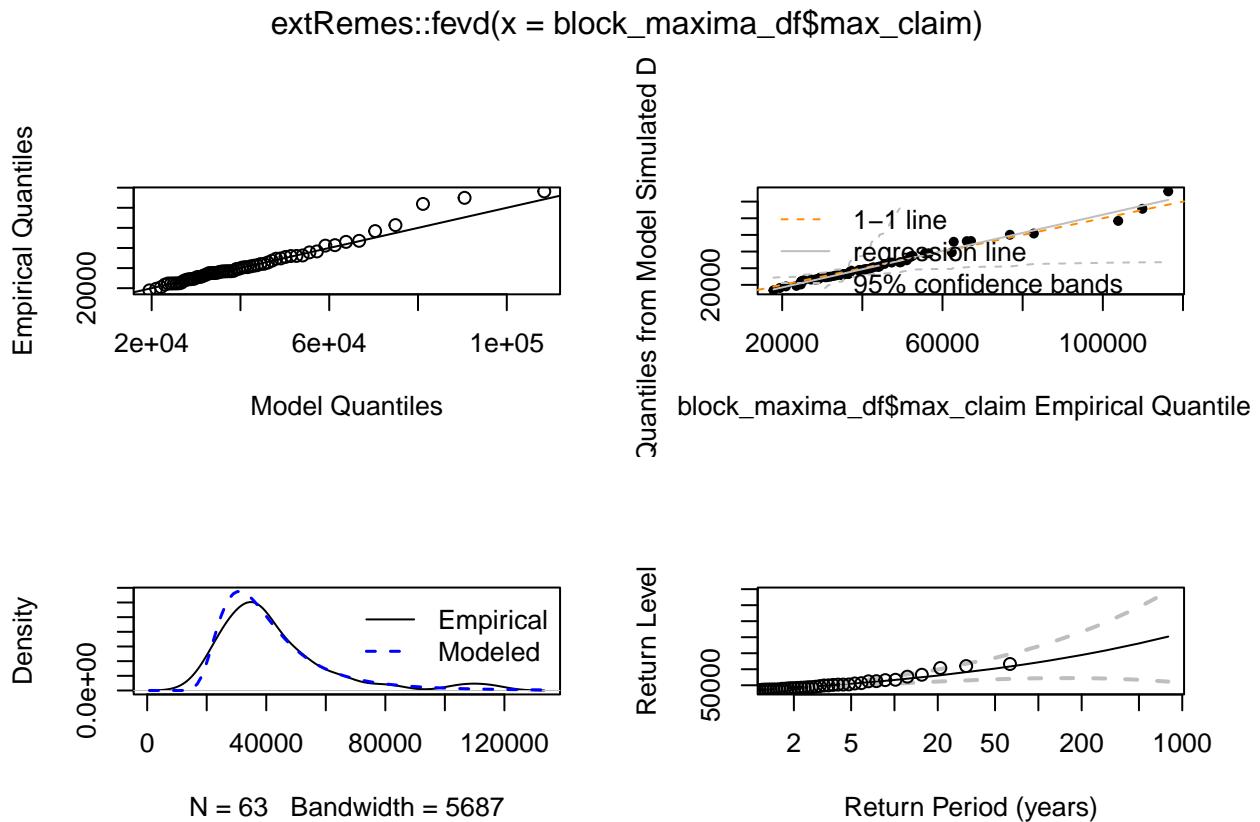
```

block_maxima_df <- GEVdf %>%
  group_by(blocks) %>%
  summarise(
    max_claim = max(nonzerosizes)
  )
gev_fit <- extRemes::fevd(block_maxima_df$max_claim)
summary(gev_fit)

##
## extRemes::fevd(x = block_maxima_df$max_claim)
##
## [1] "Estimation Method used: MLE"
##
##
## Negative Log-Likelihood Value: 695.5335
##
##
## Estimated parameters:
##   location      scale      shape
## 3.312419e+04 1.112776e+04 2.191021e-01
##
## Standard Error Estimates:
##   location      scale      shape
## 1591.904652 1247.447031  0.101009
##
## Estimated parameter covariance matrix.
##   location      scale      shape
## location 2534160.42263 1.131005e+06 -50.31878400
## scale    1131004.89478 1.556124e+06 -6.45730616
## shape     -50.31878 -6.457306e+00  0.01020283
##
## AIC = 1397.067
##
## BIC = 1403.496

plot(gev_fit)

```



```
# Hypothesis test with H0: shape = 0
# Assuming a normal distribution of the shape parameter estimator
shape.estimate <- 0.219102
shape.stderr <- 0.101009

print(shape.estimate/shape.stderr)

## [1] 2.169133
```

- clearly, the shape parameter is significantly different from 0, so we can reject the null hypothesis of an exponential tail. It is quite obvious that we have a frechet type distribution of the nonzero claim sizes
- Suggests use of frechet family
- Could use gamma or lognormal from the exponential family

## lognormal model

```
# lognormal
# lm is equivalent to fitting glm(., gaussian(link = identity))
lognormal_claimsize_df <- train_data_imputed[train_data_imputed$OUTCOME > 0,] %>%
  select(-OUTCOME)
names(train_data_imputed)
```

```
## [1] "ID"           "AGE"          "GENDER"
```

```

## [4] "DRIVING_EXPERIENCE" "EDUCATION" "CREDIT_SCORE"
## [7] "VEHICLE_OWNERSHIP" "VEHICLE_YEAR" "MARRIED"
## [10] "CHILDREN" "ANNUAL_MILEAGE" "VEHICLE_TYPE"
## [13] "SPEEDING_VIOLATIONS" "PAST_ACCIDENTS" "OUTCOME"
## [16] "CLAIMS"

lognormal_claimsize_df$CLAIMS <- log(lognormal_claimsize_df$CLAIMS)

# base model no interactions
claimsize.0.lognormal <- lm(CLAIMS ~.,
                             data = lognormal_claimsize_df)

names(train_data_x)

## [1] "ID"                 "AGE"                "GENDER"
## [4] "DRIVING_EXPERIENCE" "EDUCATION"          "CREDIT_SCORE"
## [7] "VEHICLE_OWNERSHIP"  "VEHICLE_YEAR"        "MARRIED"
## [10] "CHILDREN"          "ANNUAL_MILEAGE"    "VEHICLE_TYPE"
## [13] "SPEEDING_VIOLATIONS" "PAST_ACCIDENTS"

# model with interactions
claimsize.1.lognormal <- lm(CLAIMS ~(AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION +
                                         CREDIT_SCORE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +
                                         MARRIED + CHILDREN + ANNUAL_MILEAGE +
                                         VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2,
                                         data = lognormal_claimsize_df)

par(mfrow = c(2,2))
print('-----FULL MODEL NO INTERACTION TERMS-----')

## [1] "-----FULL MODEL NO INTERACTION TERMS-----"

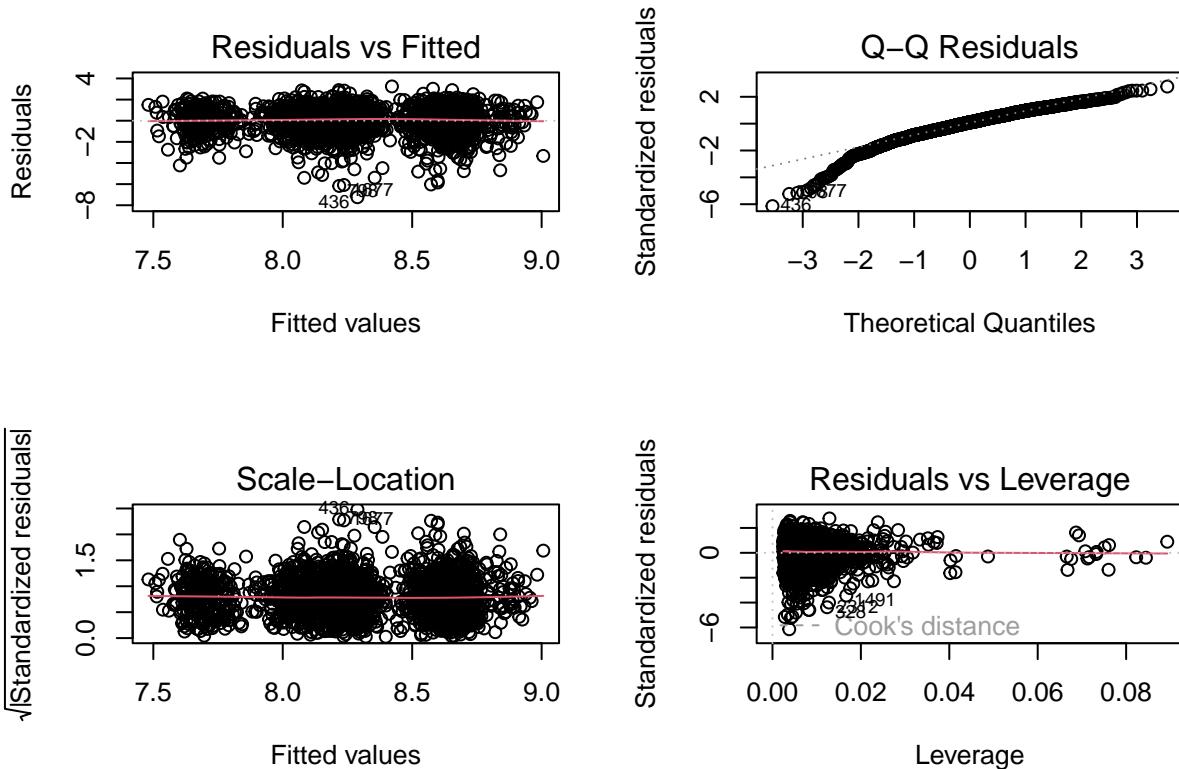
summary(claimsize.0.lognormal);plot(claimsize.0.lognormal)

## 
## Call:
## lm(formula = CLAIMS ~ ., data = lognormal_claimsize_df)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -7.2413 -0.6461  0.1103  0.7844  3.2424 
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept) 8.069e+00 2.061e-01 39.150 < 2e-16 ***
## ID          -2.414e-08 8.025e-08 -0.301 0.76355    
## AGE26-39    2.899e-03 7.042e-02  0.041 0.96716    
## AGE40-64    1.707e-02 8.819e-02  0.194 0.84657    
## AGE65+     -9.327e-02 1.220e-01 -0.764 0.44470    
## GENDERmale  3.992e-01 4.998e-02  7.986 2.10e-15 *** 
## 
```

```

## DRIVING_EXPERIENCE10-19y -5.834e-01 8.632e-02 -6.758 1.73e-11 ***
## DRIVING_EXPERIENCE20-29y -5.004e-01 1.748e-01 -2.863 0.00424 **
## DRIVING_EXPERIENCE30y+ -1.203e-01 3.363e-01 -0.358 0.72070
## EDUCATIONnone -5.531e-03 5.795e-02 -0.095 0.92397
## EDUCATIONuniversity -3.734e-02 6.044e-02 -0.618 0.53678
## CREDIT_SCORE 2.333e-01 2.222e-01 1.050 0.29384
## VEHICLE_OWNERSHIP 5.863e-02 5.046e-02 1.162 0.24542
## VEHICLE_YEARbefore 2015 -1.502e-02 8.035e-02 -0.187 0.85168
## MARRIED -6.217e-02 5.984e-02 -1.039 0.29891
## CHILDREN -1.120e-02 5.505e-02 -0.203 0.83886
## ANNUAL_MILEAGE 8.497e-06 1.066e-05 0.797 0.42545
## VEHICLE_TYPEsports car 2.300e-01 1.089e-01 2.112 0.03477 *
## SPEEDING_VIOLATIONS 5.875e-03 2.546e-02 0.231 0.81754
## PAST_ACCIDENTS 2.089e-02 3.748e-02 0.557 0.57736
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.181 on 2514 degrees of freedom
## Multiple R-squared: 0.06563, Adjusted R-squared: 0.05856
## F-statistic: 9.293 on 19 and 2514 DF, p-value: < 2.2e-16

```



```
print('-----WHITE TEST-----')
```

```
## [1] "-----WHITE TEST-----"
```

```
white_test(claimsizelognormal)
```

```
## White's test results
```

```

## Null hypothesis: Homoskedasticity of the residuals
## Alternative hypothesis: Heteroskedasticity of the residuals
## Test Statistic: 0.45
## P-value: 0.797812

print('-----VIF-----')

## [1] "-----VIF-----"

vif(claimsize.0.lognormal, type = "predictor")

## GVIFs computed for predictors

##          GVIF Df GVIF^(1/(2*Df)) Interacts With
## ID        1.005306  1      1.002650      --
## AGE       3.033224  3      1.203143      --
## GENDER    1.102305  1      1.049907      --
## DRIVING_EXPERIENCE 4.569123  3      1.288167      --
## EDUCATION 1.284516  2      1.064596      --
## CREDIT_SCORE 1.491735  1      1.221366      --
## VEHICLE_OWNERSHIP 1.135285  1      1.065498      --
## VEHICLE_YEAR   1.068256  1      1.033565      --
## MARRIED     1.365148  1      1.168396      --
## CHILDREN    1.372683  1      1.171616      --
## ANNUAL_MILEAGE 1.594910  1      1.262897      --
## VEHICLE_TYPE  1.009676  1      1.004826      --
## SPEEDING_VIOLATIONS 2.000328  1      1.414329      --
## PAST_ACCIDENTS 1.657310  1      1.287366      --
##
## ID          AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_OWNERSHIP, VEH...
## AGE         ID, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_OWNERSHIP, VEH...
## GENDER      ID, AGE, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_OWNERSHIP, VEH...
## DRIVING_EXPERIENCE ID, AGE, GENDER, EDUCATION, CREDIT_SCORE, VEHICLE_OWNERSHIP, VEH...
## EDUCATION   ID, AGE, GENDER, DRIVING_EXPERIENCE, CREDIT_SCORE, VEHICLE_OWNERSHIP, VEH...
## CREDIT_SCORE ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, VEHICLE_OWNERSHIP, VEH...
## VEHICLE_OWNERSHIP ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEH...
## VEHICLE_YEAR   ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_O...
## MARRIED     ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_OWN...
## CHILDREN    ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_OWN...
## ANNUAL_MILEAGE ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_O...
## VEHICLE_TYPE  ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_O...
## SPEEDING_VIOLATIONS ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEH...
## PAST_ACCIDENTS ID, AGE, GENDER, DRIVING_EXPERIENCE, EDUCATION, CREDIT_SCORE, VEHICLE_O...

```

- good fit around the middle, but the fit near the tails suffer
- doesn't seem heteroskedastic, linear model is appropriate in that regards
- introducing interaction terms increases adj Rsq
- not much multicollinearity between variables w no interaction terms

## Variable selection

- only investigate adjr2 due to the size of the predictorspace

```
# forward selection
# not running because it takes too long to knit
print('-----FORWARD-----')
lognormal_fwdselection <- ols_step_forward_adj_r2(claimsize.1.lognormal)
lognormal_fwdselection

# backward selection
print('-----backward-----')

## [1] "-----backward-----"

print('adjr2')

## [1] "adjr2"

lognormal_bwdselection <- stats::step(claimsize.1.lognormal, direction = "backward", trace = 0)
summary(lognormal_bwdselection)

##
## Call:
## lm(formula = CLAIMS ~ GENDER + DRIVING_EXPERIENCE + EDUCATION +
##     VEHICLE_OWNERSHIP + VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE +
##     VEHICLE_TYPE + SPEEDING_VIOLATIONS + GENDER:CHILDREN + GENDER:SPEEDING_VIOLATIONS +
##     EDUCATION:ANNUAL_MILEAGE + VEHICLE_OWNERSHIP:VEHICLE_YEAR +
##     VEHICLE_YEAR:MARRIED + CHILDREN:ANNUAL_MILEAGE + ANNUAL_MILEAGE:SPEEDING_VIOLATIONS,
##     data = lognormal_claimsize_df)
##
## Residuals:
##      Min      1Q      Median      3Q      Max 
## -7.2824 -0.6446   0.1127   0.7990   3.1821 
##
## Coefficients:
## (Intercept)          Estimate Std. Error t value
## GENDERmale           8.985e+00  2.733e-01  32.875
## DRIVING_EXPERIENCE10-19y -5.679e-01  6.927e-02 -8.198
## DRIVING_EXPERIENCE20-29y -4.499e-01  1.547e-01 -2.908
## DRIVING_EXPERIENCE30y+  -9.899e-02  3.287e-01 -0.301
## EDUCATIONnone         -6.055e-01  2.683e-01 -2.257
## EDUCATIONuniversity    1.865e-02  2.595e-01  0.072
## VEHICLE_OWNERSHIP      -2.356e-01  1.502e-01 -1.569
## VEHICLE_YEARbefore 2015 -7.289e-02  1.182e-01 -0.617
## MARRIED                1.485e-01  1.539e-01  0.965
## CHILDREN               -9.208e-01  2.632e-01 -3.499
## ANNUAL_MILEAGE        -4.430e-05  1.762e-05 -2.515
## VEHICLE_TYPEsports car  2.277e-01  1.082e-01  2.105
## SPEEDING_VIOLATIONS   -1.705e-01  8.576e-02 -1.988
```

```

## GENDERmale:CHILDREN           1.633e-01  9.654e-02  1.691
## GENDERmale:SPEEDING_VIOLATIONS 7.766e-02  5.128e-02  1.514
## EDUCATIONnone:ANNUAL_MILEAGE    4.567e-05  2.056e-05  2.222
## EDUCATIONuniversity:ANNUAL_MILEAGE -3.995e-06  2.047e-05 -0.195
## VEHICLE_OWNERSHIP:VEHICLE_YEARbefore 2015 3.365e-01  1.578e-01  2.132
## VEHICLE_YEARbefore 2015:MARRIED      -2.333e-01  1.608e-01 -1.452
## CHILDREN:ANNUAL_MILEAGE          6.365e-05  1.962e-05  3.243
## ANNUAL_MILEAGE:SPEEDING_VIOLATIONS 1.140e-05  6.538e-06  1.743
##
## Pr(>|t|)
## (Intercept) < 2e-16 ***
## GENDERmale 7.08e-05 ***
## DRIVING_EXPERIENCE10-19y 3.86e-16 ***
## DRIVING_EXPERIENCE20-29y 0.003664 **
## DRIVING_EXPERIENCE30y+ 0.763314
## EDUCATIONnone 0.024092 *
## EDUCATIONuniversity 0.942719
## VEHICLE_OWNERSHIP 0.116844
## VEHICLE_YEARbefore 2015 0.537372
## MARRIED 0.334749
## CHILDREN 0.000476 ***
## ANNUAL_MILEAGE 0.011963 *
## VEHICLE_TYPEsports_car 0.035401 *
## SPEEDING_VIOLATIONS 0.046959 *
## GENDERmale:CHILDREN 0.090965 .
## GENDERmale:SPEEDING_VIOLATIONS 0.130076
## EDUCATIONnone:ANNUAL_MILEAGE 0.026385 *
## EDUCATIONuniversity:ANNUAL_MILEAGE 0.845321
## VEHICLE_OWNERSHIP:VEHICLE_YEARbefore 2015 0.033100 *
## VEHICLE_YEARbefore 2015:MARRIED 0.146762
## CHILDREN:ANNUAL_MILEAGE 0.001198 **
## ANNUAL_MILEAGE:SPEEDING_VIOLATIONS 0.081382 .
##
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.175 on 2512 degrees of freedom
## Multiple R-squared:  0.07705,   Adjusted R-squared:  0.06934
## F-statistic: 9.986 on 21 and 2512 DF,  p-value: < 2.2e-16

```

- improvement in adjusted R squared from full model
- backward selection models have more variables but higher adjRsq
- Select out of backward selection models for this
- all seem to have same adj R sq and AIC, choose one with the lowest number of variables, i.e the model chosen by prioritising AIC

```

# backward stepwise
lognormal_fwdselection<- stats::step(claimsiz.1.lognormal, direction = "forward", trace = 0)
summary(lognormal_fwdselection)

```

- backward selected model is chosen as the reduced model due to the increased adjusted R squared and parsimony

```
claimsize.2.lognormal <- lognormal_bwdselection
```

## Significance test

```
# between full model with all interaction terms and backward stepwise model
anova(claimsize.2.lognormal, claimsize.1.lognormal)

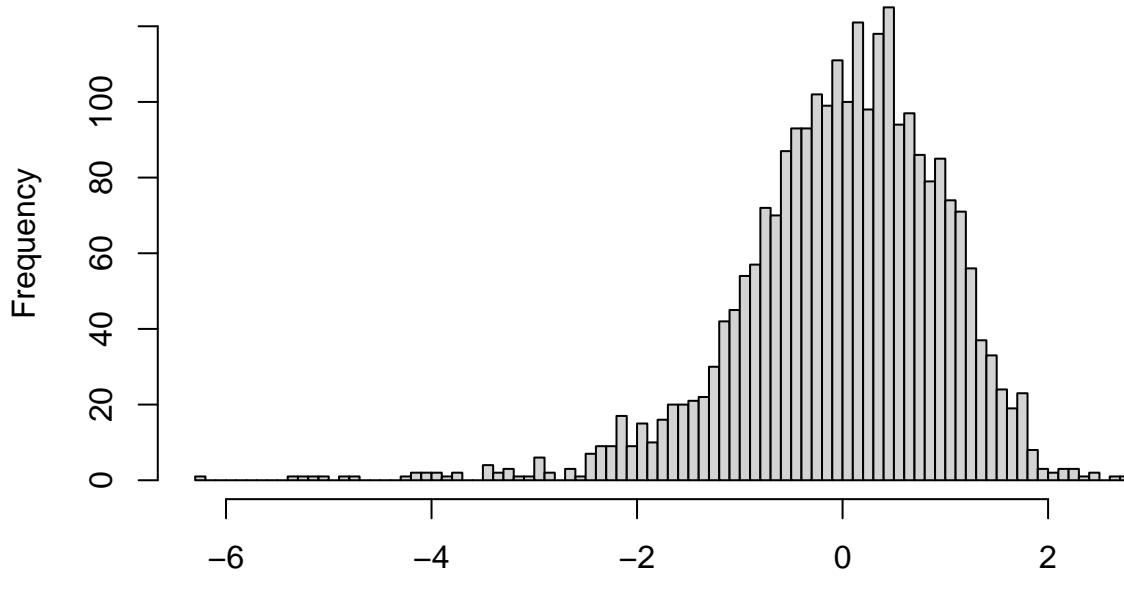
## Analysis of Variance Table
##
## Model 1: CLAIMS ~ GENDER + DRIVING_EXPERIENCE + EDUCATION + VEHICLE_OWNERSHIP +
##           VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE + VEHICLE_TYPE +
##           SPEEDING_VIOLATIONS + GENDER:CHILDREN + GENDER:SPEEDING_VIOLATIONS +
##           EDUCATION:ANNUAL_MILEAGE + VEHICLE_OWNERSHIP:VEHICLE_YEAR +
##           VEHICLE_YEAR:MARRIED + CHILDREN:ANNUAL_MILEAGE + ANNUAL_MILEAGE:SPEEDING_VIOLATIONS
## Model 2: CLAIMS ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION + CREDIT_SCORE +
##           VEHICLE_OWNERSHIP + VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE +
##           VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2
##   Res.Df   RSS Df Sum of Sq    F Pr(>F)
## 1    2512 3465.6
## 2    2379 3338.8 133     126.87 0.6797 0.9979
```

- Clearly, the dropped variables were not that significant

## Residual analysis

```
# plot of type 1 standardised residuals
hist(summary(claimsize.2.lognormal)$res / summary(claimsize.2.lognormal)$sigma, breaks = 100,
      main = "histogram of standardised residuals", xlab = "standardised residuals")
```

## histogram of standardised residuals



standardised residuals - looks left  
skewed, suggests havent taken account of patterns in the data, possibly due to poor treatment of tail

## Gamma model

- consider common interaction terms and interaction terms suggested by correlation analysis

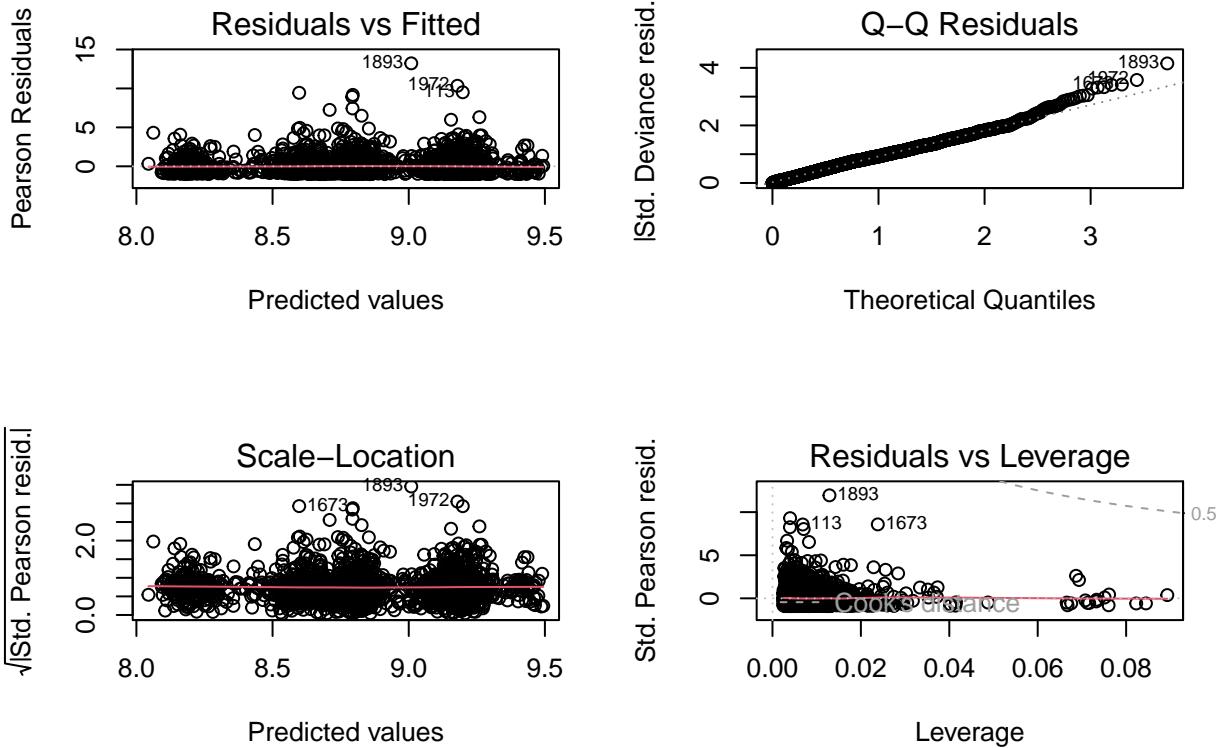
```
claimsize_df <- train_data_imputed[train_data_imputed$OUTCOME >0,] %>%  
  select(-OUTCOME)  
  
# base model no interaction  
claimsize.0.gamma <- glm(CLAIMS ~ ., family = Gamma(link = "log"),  
  data = claimsize_df)  
  
# w interactions  
claimsize.1.gamma <- glm(CLAIMS ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION +  
  CREDIT_SCORE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +  
  MARRIED + CHILDREN + ANNUAL_MILEAGE +  
  VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2, family = Gamma(link = "log"),  
  data = claimsize_df)  
  
## Warning: glm.fit: algorithm did not converge  
  
par(mfrow = c(2,2))  
print('-----FULL MODEL NO INTERACTION TERMS-----')  
  
## [1] "-----FULL MODEL NO INTERACTION TERMS-----"
```

```

summary(claimsize.0.gamma);plot(claimsize.0.gamma)

## 
## Call:
## glm(formula = CLAIMS ~ ., family = Gamma(link = "log"), data = claimsize_df)
## 
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)           8.555e+00  1.944e-01 44.012 < 2e-16 ***
## ID                  -2.713e-08  7.568e-08 -0.358 0.720044    
## AGE26-39            -7.097e-02  6.641e-02 -1.069 0.285391    
## AGE40-64            -5.188e-02  8.318e-02 -0.624 0.532879    
## AGE65+              -1.822e-01  1.151e-01 -1.583 0.113531    
## GENDERmale          4.028e-01  4.714e-02  8.544 < 2e-16 ***
## DRIVING_EXPERIENCE10-19y -5.776e-01  8.141e-02 -7.095 1.68e-12 ***
## DRIVING_EXPERIENCE20-29y -5.457e-01  1.649e-01 -3.310 0.000946 ***
## DRIVING_EXPERIENCE30y+   -1.892e-01  3.172e-01 -0.597 0.550841    
## EDUCATIONnone         -7.026e-04  5.466e-02 -0.013 0.989745    
## EDUCATIONuniversity   -1.457e-02  5.700e-02 -0.256 0.798212    
## CREDIT_SCORE          3.253e-01  2.095e-01  1.553 0.120622    
## VEHICLE_OWNERSHIP     -2.409e-03  4.759e-02 -0.051 0.959626    
## VEHICLE_YEARbefore 2015 8.060e-02  7.578e-02  1.064 0.287580    
## MARRIED              -2.411e-02  5.644e-02 -0.427 0.669320    
## CHILDREN             6.204e-03  5.192e-02  0.119 0.904892    
## ANNUAL_MILEAGE        5.129e-06  1.005e-05  0.510 0.610004    
## VEHICLE_TYPEsports car 2.353e-01  1.027e-01  2.291 0.022035 *  
## SPEEDING_VIOLATIONS   2.758e-02  2.402e-02  1.148 0.250901    
## PAST_ACCIDENTS        -2.947e-03  3.535e-02 -0.083 0.933551    
## --- 
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 
## (Dispersion parameter for Gamma family taken to be 1.241379)
## 
## Null deviance: 2963.2 on 2533 degrees of freedom
## Residual deviance: 2724.3 on 2514 degrees of freedom
## AIC: 50079
## 
## Number of Fisher Scoring iterations: 6

```



```
print('-----WHITE TEST-----')
```

```
## [1] "-----WHITE TEST-----"
```

```
white_test(claims0.gamma)
```

```
## White's test results
##
## Null hypothesis: Homoskedasticity of the residuals
## Alternative hypothesis: Heteroskedasticity of the residuals
## Test Statistic: 1
## P-value: 0.605183
```

```
print('-----VIF-----')
```

```
## [1] "-----VIF-----"
```

```
vif(claims0.gamma, type = "predictor")
```

```
## Warning in vif.lm(claims0.gamma, type = "predictor"): type = 'predictor' is available only for un
```

	GVIF	Df	GVIF <sup>(1/(2*Df))</sup>
## ID	1.005306	1	1.002650
## AGE	3.033224	3	1.203143
## GENDER	1.102305	1	1.049907

```

## DRIVING_EXPERIENCE 4.569123 3      1.288167
## EDUCATION          1.284516 2      1.064596
## CREDIT_SCORE        1.491735 1      1.221366
## VEHICLE_OWNERSHIP  1.135285 1      1.065498
## VEHICLE_YEAR        1.068256 1      1.033565
## MARRIED            1.365148 1      1.168396
## CHILDREN           1.372683 1      1.171616
## ANNUAL_MILEAGE     1.594910 1      1.262897
## VEHICLE_TYPE        1.009676 1      1.004826
## SPEEDING_VIOLATIONS 2.000328 1      1.414329
## PAST_ACCIDENTS     1.657310 1      1.287366

```

## Variable selection

```

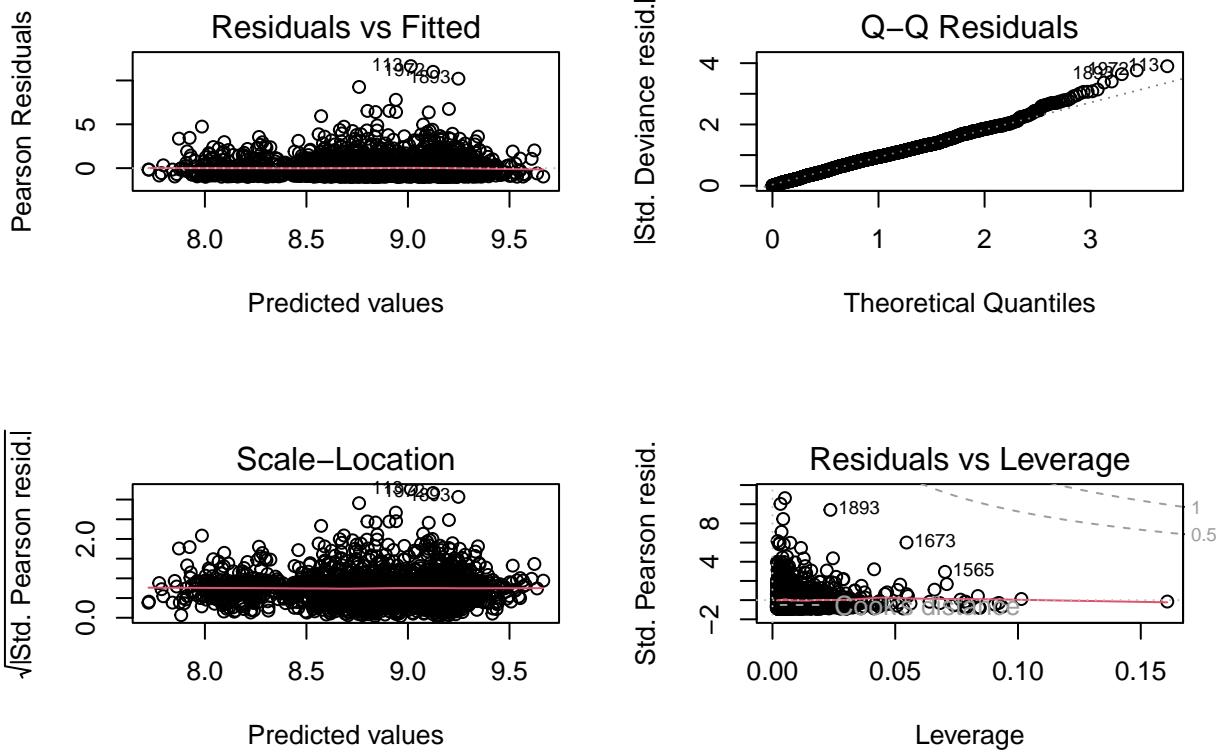
##
## Call:
## glm(formula = CLAIMS ~ GENDER + DRIVING_EXPERIENCE + CREDIT_SCORE +
##       VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE + VEHICLE_TYPE +
##       SPEEDING_VIOLATIONS + PAST_ACCIDENTS + GENDER:CHILDREN +
##       CHILDREN:ANNUAL_MILEAGE + CREDIT_SCORE:PAST_ACCIDENTS + VEHICLE_YEAR:MARRIED +
##       GENDER:SPEEDING_VIOLATIONS + CREDIT_SCORE:VEHICLE_TYPE, family = Gamma(link = "log"),
##       data = df, control = glm.control(maxit = 100))
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                8.764e+00  2.361e-01  37.126 < 2e-16 ***
## GENDERmale                  2.660e-01  6.497e-02   4.094 4.37e-05 ***
## DRIVING_EXPERIENCE10-19y    -5.953e-01  7.069e-02  -8.422 < 2e-16 ***
## DRIVING_EXPERIENCE20-29y    -5.480e-01  1.549e-01  -3.537 0.000412 ***
## DRIVING_EXPERIENCE30y+      -1.502e-01  3.088e-01  -0.486 0.626840
## CREDIT_SCORE                 4.418e-01  1.950e-01   2.266 0.023558 *
## VEHICLE_YEARbefore 2015      2.311e-01  9.350e-02   2.472 0.013510 *
## MARRIED                      2.449e-01  1.430e-01   1.713 0.086869 .
## CHILDREN                     -7.110e-01  2.434e-01  -2.921 0.003518 **
## ANNUAL_MILEAGE                -2.069e-05 1.366e-05  -1.515 0.129961
## VEHICLE_TYPEsports car       7.186e-01  3.624e-01   1.983 0.047499 *
## SPEEDING_VIOLATIONS          -4.699e-02  4.963e-02  -0.947 0.343826
## PAST_ACCIDENTS                 1.749e-01  1.134e-01   1.543 0.123067
## GENDERmale:CHILDREN           2.082e-01  8.995e-02   2.315 0.020709 *
## CHILDREN:ANNUAL_MILEAGE       4.511e-05  1.809e-05   2.494 0.012702 *
## CREDIT_SCORE:PAST_ACCIDENTS   -4.260e-01  2.275e-01  -1.872 0.061255 .
## VEHICLE_YEARbefore 2015:MARRIED -3.099e-01  1.493e-01  -2.076 0.037983 *
## GENDERmale:SPEEDING_VIOLATIONS 8.845e-02  4.833e-02   1.830 0.067357 .
## CREDIT_SCORE:VEHICLE_TYPEsports car -1.067e+00  7.690e-01  -1.387 0.165605
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for Gamma family taken to be 1.202638)
##
## Null deviance: 2963.2 on 2533 degrees of freedom
## Residual deviance: 2695.0 on 2515 degrees of freedom
## AIC: 50045
##
```

```

## Number of Fisher Scoring iterations: 6

##
## --- Generating Diagnostic Plots for Final Model ---

```



```

## (Intercept) 6401.5457825
## DRIVING_EXPERIENCE10-19y 0.5513907
## DRIVING_EXPERIENCE30y+ 0.8605605
## VEHICLE_YEARbefore 2015 1.2599878
## CHILDREN 0.4911376
## VEHICLE_TYPEsports car 2.0516241
## PAST_ACCIDENTS 1.1911398
## CHILDREN:ANNUAL_MILEAGE 1.0000451
## VEHICLE_YEARbefore 2015:MARRIED 0.7335132
## CREDIT_SCORE:VEHICLE_TYPEsports car 0.3442101

## [1] "-----"

```

	GENDERmale
DRIVING_EXPERIENCE10-19y	1.3047019
DRIVING_EXPERIENCE20-29y	0.5781134
CREDIT_SCORE	1.5555631
MARRIED	1.2775392
ANNUAL_MILEAGE	0.9999793
SPEEDING_VIOLATIONS	0.9540989
GENDERmale:CHILDREN	1.2314638
CREDIT_SCORE:PAST_ACCIDENTS	0.6531375
GENDERmale:SPEEDING_VIOLATIONS	1.0924750

```

##                                     GVIF Df GVIF^(1/(2*Df))
## GENDER                           2.160919  1    1.470006
## DRIVING_EXPERIENCE               2.832738  3    1.189509
## CREDIT_SCORE                     1.333893  1    1.154943
## VEHICLE_YEAR                     1.678581  1    1.295601
## MARRIED                          9.047066  1    3.007834
## CHILDREN                         31.138872 1    5.580221
## ANNUAL_MILEAGE                  3.038172  1    1.743035
## VEHICLE_TYPE                     12.979648 1    3.602728
## SPEEDING_VIOLATIONS             8.816592  1    2.969275
## PAST_ACCIDENTS                  17.606395 1    4.195998
## GENDER:CHILDREN                 3.670345  1    1.915814
## CHILDREN:ANNUAL_MILEAGE         24.160274 1    4.915310
## CREDIT_SCORE:PAST_ACCIDENTS     17.465245 1    4.179144
## VEHICLE_YEAR:MARRIED            9.025613  1    3.004266
## GENDER:SPEEDING_VIOLATIONS     7.373019  1    2.715330
## CREDIT_SCORE:VEHICLE_TYPE       13.055910 1    3.613296

```

- high vif because we have interaction terms now
- we opt to keep CHILDREN despite the high vif due to the principle of hierarchy, to ensure model interpretation as otherwise interpretation becomes difficult

```

# between full model and stepwise selected
anova(claimsize.2.gamma, claimsize.1.gamma, test = "LRT")

```

### likelihood ratio tests

```

## Analysis of Deviance Table
##
## Model 1: CLAIMS ~ GENDER + DRIVING_EXPERIENCE + CREDIT_SCORE + VEHICLE_YEAR +
##           MARRIED + CHILDREN + ANNUAL_MILEAGE + VEHICLE_TYPE + SPEEDING_VIOLATIONS +
##           PAST_ACCIDENTS + GENDER:CHILDREN + CHILDREN:ANNUAL_MILEAGE +
##           CREDIT_SCORE:PAST_ACCIDENTS + VEHICLE_YEAR:MARRIED + GENDER:SPEEDING_VIOLATIONS +
##           CREDIT_SCORE:VEHICLE_TYPE
## Model 2: CLAIMS ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION + CREDIT_SCORE +
##           VEHICLE_OWNERSHIP + VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE +
##           VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      2515    2695.0
## 2      2379    2560.1 136    134.9   0.7544

```

- removal of the interaction terms doesn't seem significant under H<sub>0</sub>, meaning the stepwise selected model is preferred.

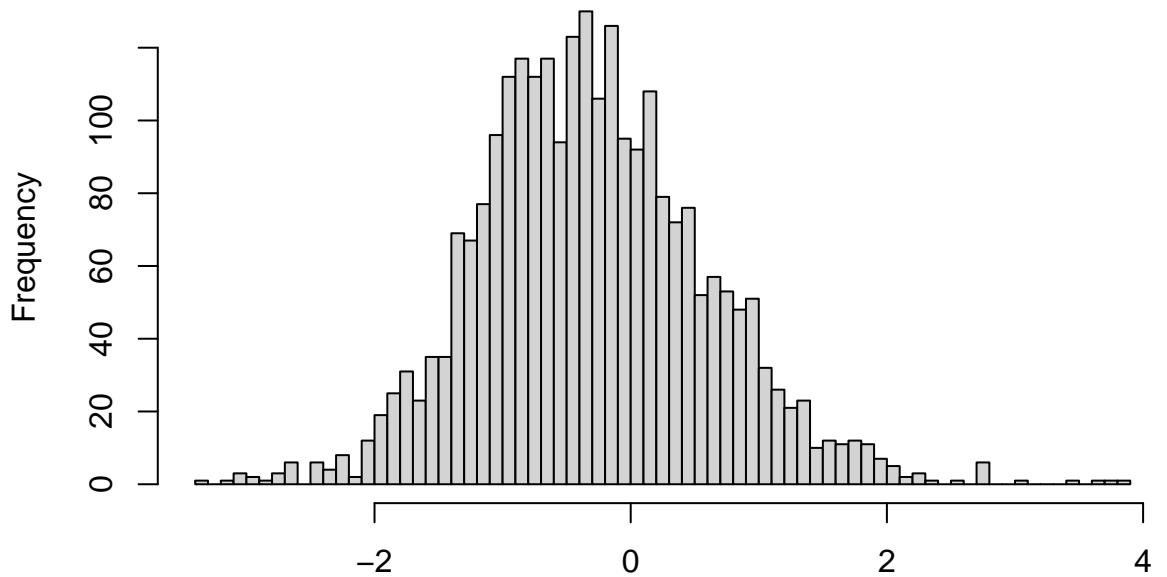
### Residual analysis

```

# histogram of type 1 standardised residuals
hist(rstandard(claimsize.2.gamma), breaks = 100)

```

## Histogram of rstandard(claims2.gamma)



rstandard(claims2.gamma)

proximately centered around mean of zero, but still some skewness

- ap-

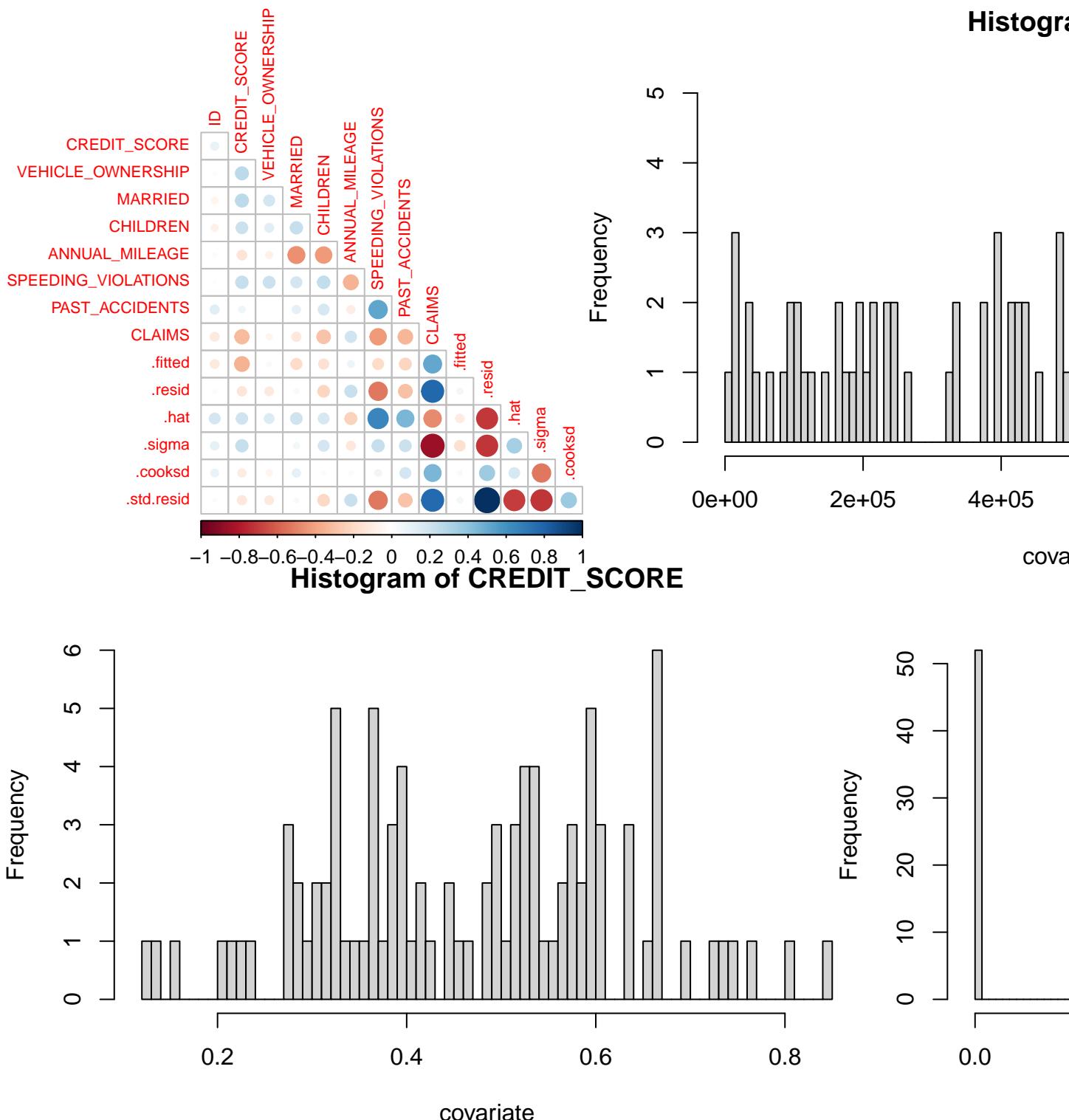
- signs of heteroskedasticity as residuals are not uniform over fitted values

identifying influential points (outliers, high leverage)

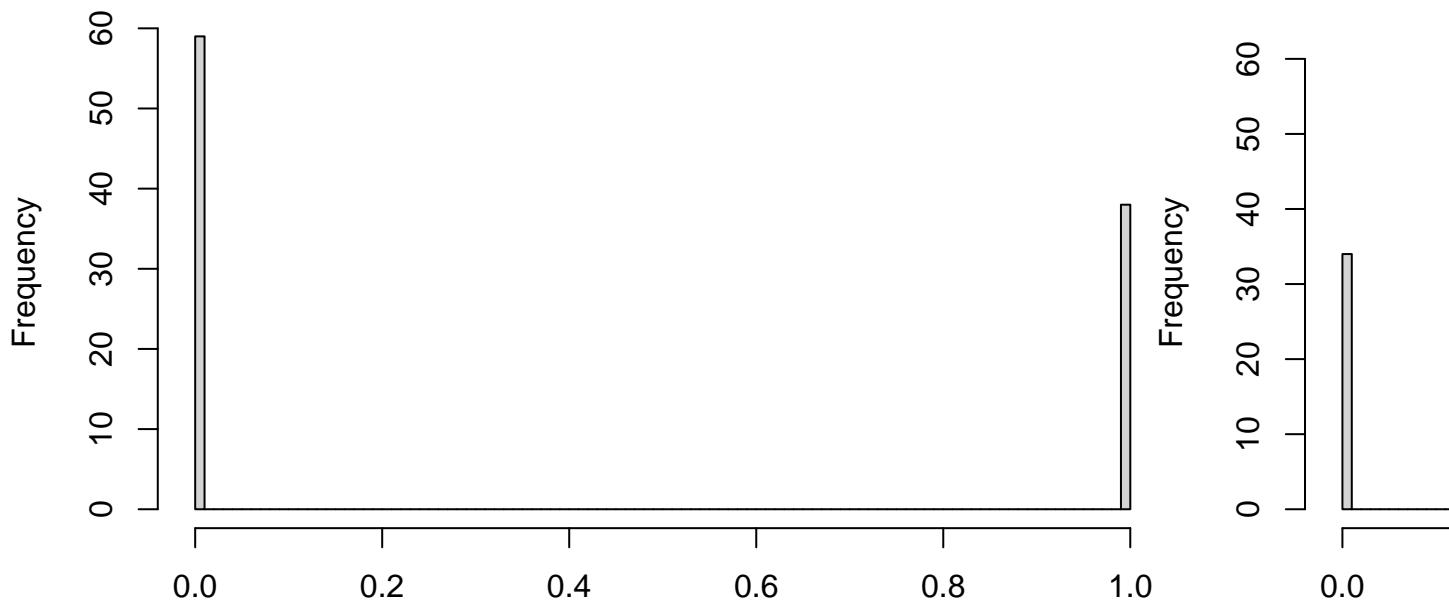
```
## [1] "cooks threshold is 0.0015785319652723"

## # A tibble: 97 x 21
##       ID AGE   GENDER DRIVING_EXPERIENCE EDUCATION CREDIT_SCORE
##   <dbl> <fct> <fct>    <fct>          <fct>        <dbl>
## 1 393748 16-25 female  0-9y      university  0.282
## 2 955572 26-39 male   10-19y     high school 0.290
## 3 943240 65+   female  30y+     university  0.596
## 4 167289 16-25 male   0-9y      none        0.337
## 5 991678 40-64 male   10-19y     high school 0.536
## 6 36193  26-39 female  10-19y     high school 0.604
## 7 959649 26-39 female  0-9y      high school 0.633
## 8 210243 16-25 male   0-9y      none        0.274
## 9 937825 40-64 female  0-9y      university  0.669
## 10 98177 26-39 male   10-19y     high school 0.459
## # i 87 more rows
## # i 15 more variables: VEHICLE_OWNERSHIP <dbl>, VEHICLE_YEAR <fct>,
## # MARRIED <dbl>, CHILDREN <dbl>, ANNUAL_MILEAGE <dbl>, VEHICLE_TYPE <fct>,
## # SPEEDING_VIOLATIONS <dbl>, PAST_ACCIDENTS <dbl>, CLAIMS <dbl>,
## # .fitted <dbl>, .resid <dbl>, .hat <dbl>, .sigma <dbl>, .cooks <dbl>,
## # .std.resid <dbl>
```

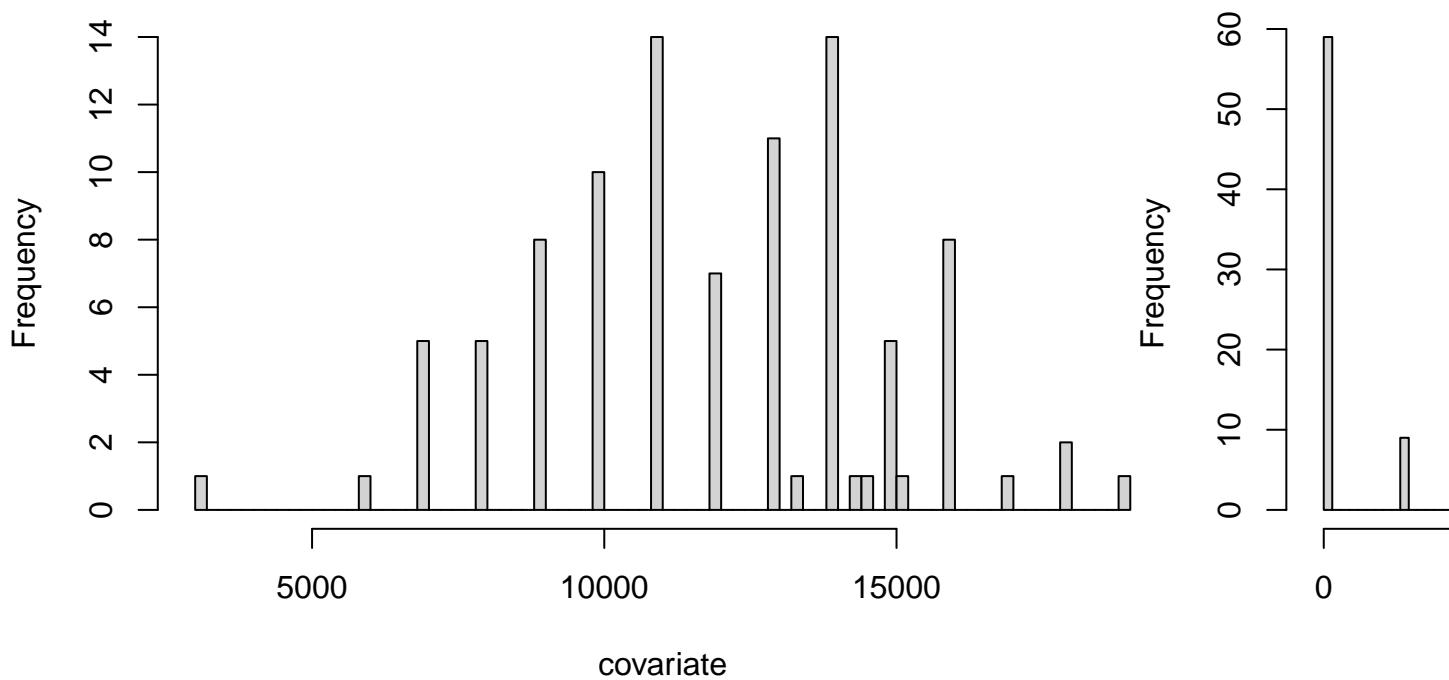
## Histogram



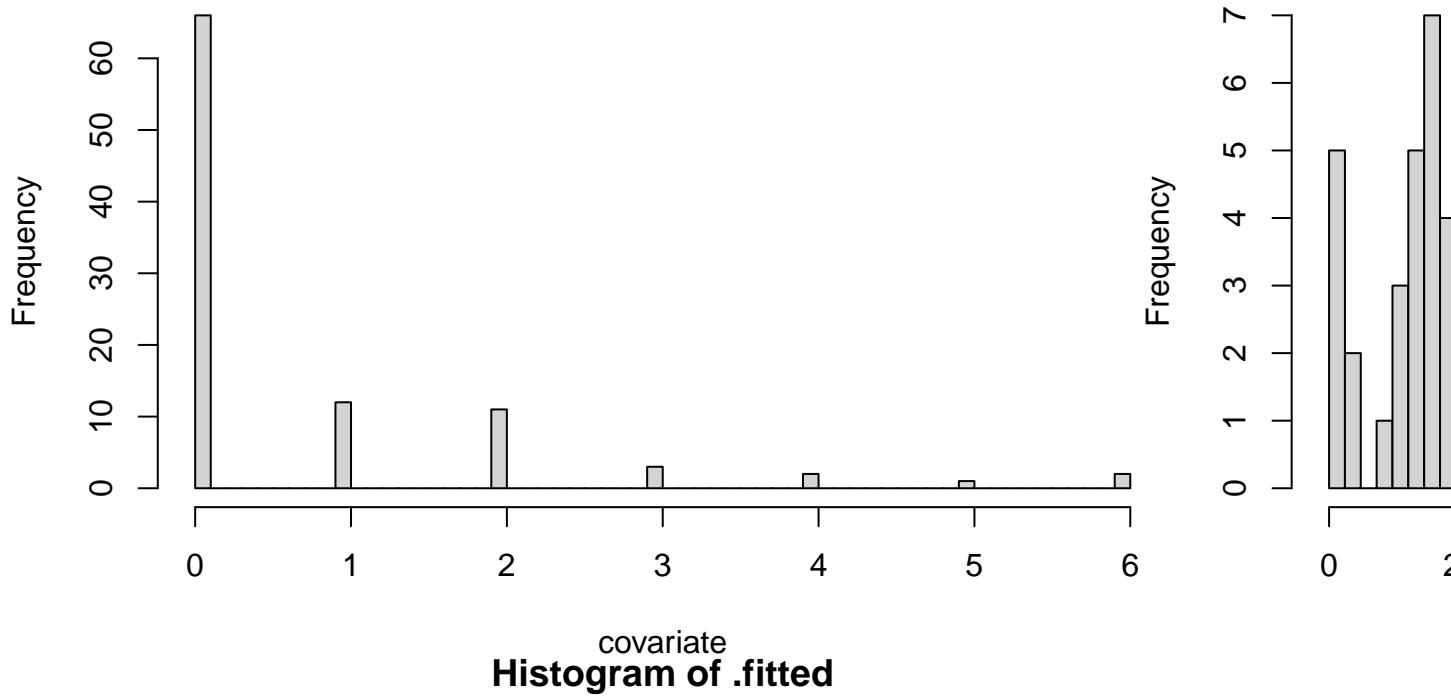
**Histogram of MARRIED**



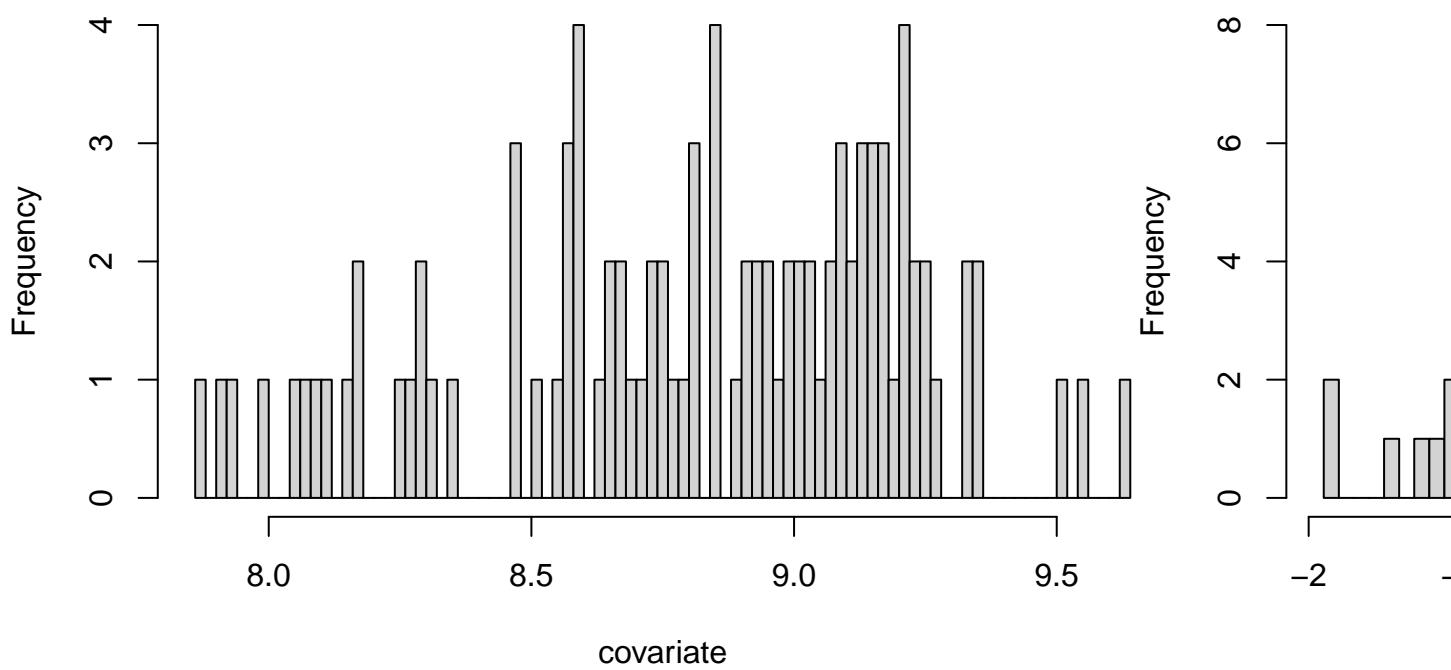
**covariate  
Histogram of ANNUAL\_MILEAGE**



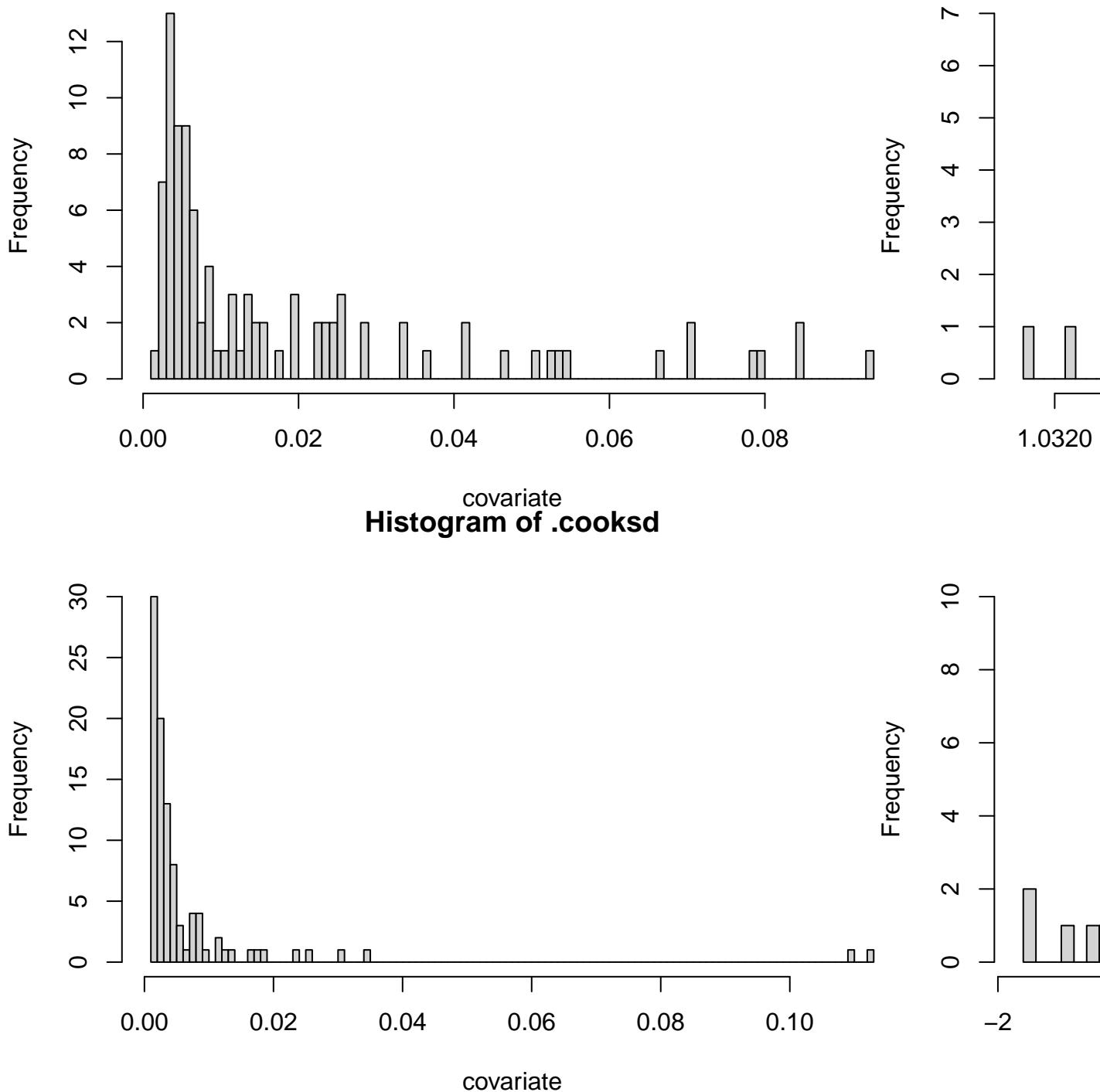
### Histogram of PAST\_ACCIDENTS



### covariate Histogram of .fitted



### Histogram of .hat



- There seems to be one case with an extremely high claim size (approx 6x the next biggest), which is likely to be a major influential point

```
# plotting influential points
ggplot(model_diagnostics, aes(x = .hat, y = .std.resid)) +
```

```

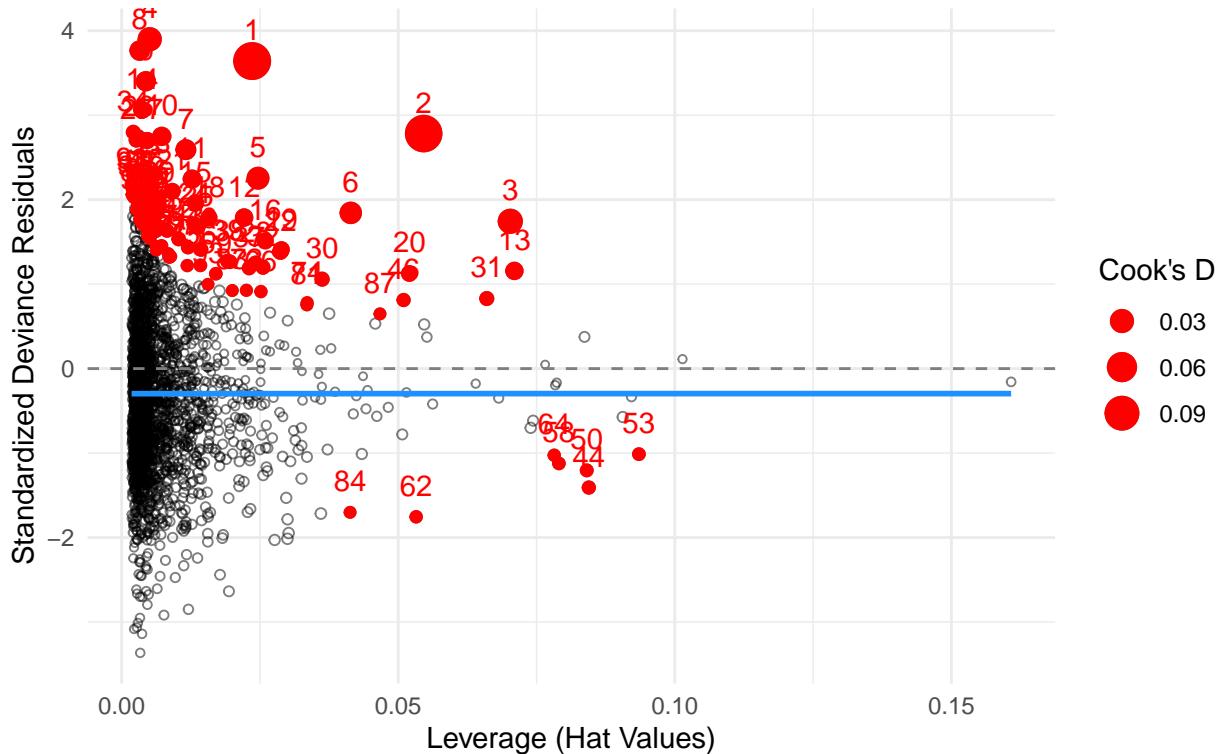
# Points are sized by their Cook's distance
geom_point(aes(size = .cooksdi), alpha = 0.5, shape = 1) +
  # Add a smoother to see the general trend
  geom_smooth(se = FALSE, col = "dodgerblue") +
  # Highlight the most influential points found earlier
  geom_point(data = influential_points, aes(size = .cooksdi), color = "red") +
  # Add labels to the influential points (e.g., by row number)
  geom_text(data = influential_points, aes(label = rownames(influential_points)),
            vjust = -1, color = "red") +
  # Add a horizontal line at 0
  geom_hline(yintercept = 0, linetype = "dashed", color = "gray50") +
  # Add labels and a title
  labs(
    title = "Influence Plot: Residuals vs. Leverage",
    subtitle = "Points sized by Cook's Distance. Red points are highly influential.",
    x = "Leverage (Hat Values)",
    y = "Standardized Deviance Residuals",
    size = "Cook's D"
  ) +
  theme_minimal()

## `geom_smooth()` using method = 'gam' and formula = 'y ~ s(x, bs = "cs")'

```

## Influence Plot: Residuals vs. Leverage

Points sized by Cook's Distance. Red points are highly influential.



opted for cooks distance as it combines the outlier and high leverage status of points into one metric - 96 influential points

```
influential_rows <- as.numeric(rownames(influential_points))
claimsize_df_no_influencers <- claimsize_df[-influential_rows, ]
model_without_influencers <- glm(formula(claimsize.2.gamma),
                                    family = Gamma(link = "log"),
                                    data = claimsize_df_no_influencers)

comparison <- cbind(Original = coef(claimsize.2.gamma),
                     No_Influencers = coef(model_without_influencers))

print(comparison)
```

investigating how influential points affect our model

	Original	No_Influencers
## (Intercept)	8.764295e+00	8.733942e+00
## GENDERmale	2.659746e-01	2.489130e-01
## DRIVING_EXPERIENCE10-19y	-5.953116e-01	-5.853716e-01
## DRIVING_EXPERIENCE20-29y	-5.479853e-01	-5.591693e-01
## DRIVING_EXPERIENCE30y+	-1.501714e-01	-2.972910e-01
## CREDIT_SCORE	4.418376e-01	4.076653e-01

```

## VEHICLE_YEARbefore 2015      2.311020e-01  2.237383e-01
## MARRIED                 2.449357e-01  2.563357e-01
## CHILDREN                  -7.110310e-01 -6.863137e-01
## ANNUAL_MILEAGE            -2.068753e-05 -1.630819e-05
## VEHICLE_TYPEsports car    7.186317e-01  7.650618e-01
## SPEEDING_VIOLATIONS       -4.698791e-02 -4.791772e-02
## PAST_ACCIDENTS             1.749107e-01  1.675351e-01
## GENDERmale:CHILDREN        2.082036e-01  1.905174e-01
## CHILDREN:ANNUAL_MILEAGE   4.510647e-05  4.385865e-05
## CREDIT_SCORE:PAST_ACCIDENTS -4.259676e-01 -3.929006e-01
## VEHICLE_YEARbefore 2015:MARRIED -3.099096e-01 -3.109545e-01
## GENDERmale:SPEEDING_VIOLATIONS 8.844577e-02  9.291959e-02
## CREDIT_SCORE:VEHICLE_TYPEsports car -1.066503e+00 -1.187627e+00

```

```
c(Original_Dispersion = summary(claimsize.2.gamma)$dispersion, No_Influencers_Dispersion = summary(mode
```

```

##          Original_Dispersion No_Influencers_Dispersion
##                1.202638           1.209761

```

- not much difference in dispersion
- influential points aren't doing too much to significantly affect the model fit

## Testing the Gamma and Lognormal model

```

# Helper: coerce factor levels in newdata to training levels stored in the fit
prep_newdata_for <- function(newdata, fit) {
  nd <- newdata
  if (!is.null(fit$xlevels)) {
    for (nm in names(fit$xlevels)) {
      if (nm %in% names(nd)) {
        nd[[nm]] <- factor(nd[[nm]], levels = fit$xlevels[[nm]])
      }
    }
  }
  droplevels(nd)
}

# 1) Test set for severity evaluation (only positive claims)
test_claimsiz_df <- test_data_imputed |>
  dplyr::filter(CLAIMS > 0)

# 2) Align test data to each model
test_for_gamma     <- prep_newdata_for(test_claimsiz_df, claimsize.2.gamma)
test_for_lognorm   <- prep_newdata_for(test_claimsiz_df, claimsize.2.lognormal)

# 3) Predictions
# Gamma(log): already on the dollar scale
pred_gamma <- predict(claimsize.2.gamma, newdata = test_for_gamma, type = "response")

```

```

# Lognormal (lm on log(CLAIMS)): use smearing correction
pred_log <- predict(claimsize.2.lognormal, newdata = test_for_lognorm) # predicts log(CLAIMS)

# Duan's smearing factor from TRAINING residuals of the log model
sf <- mean(exp(residuals(claimsize.2.lognormal)), na.rm = TRUE)
pred_lognormal <- exp(pred_log) * sf

# 4) Metrics
actual_claims <- test_for_lognorm$CLAIMS

rmse <- function(a, p) sqrt(mean((a - p)^2, na.rm = TRUE))
mae <- function(a, p) mean(abs(a - p), na.rm = TRUE)

rmse_gamma <- rmse(actual_claims, pred_gamma)
mae_gamma <- mae(actual_claims, pred_gamma)

rmse_lognormal <- rmse(actual_claims, pred_lognormal)
mae_lognormal <- mae(actual_claims, pred_lognormal)

comparison_df <- data.frame(
  Model = c("Gamma GLM", "Lognormal (lm + smearing)"),
  RMSE = c(rmse_gamma, rmse_lognormal),
  MAE = c(mae_gamma, mae_lognormal)
)
print(comparison_df)

##                               Model      RMSE      MAE
## 1             Gamma GLM 8950.423 5578.078
## 2 Lognormal (lm + smearing) 8970.584 5617.047

```

When we fit a lognormal severity model (lm on log(CLAIMS)), the model predicts the mean of the log outcome. Simply exponentiating those predictions gives the conditional median on the dollar scale and underestimates the mean (Jensen's inequality). To obtain unbiased mean severities, we apply a back-transformation correction: Duan's smearing factor (multiply by the average of exp(residuals)) or, under homoskedastic normal log-errors, multiply by  $\exp(\sigma^2/2)$ . Gamma GLM predictions (log link) already return the mean and need no correction.

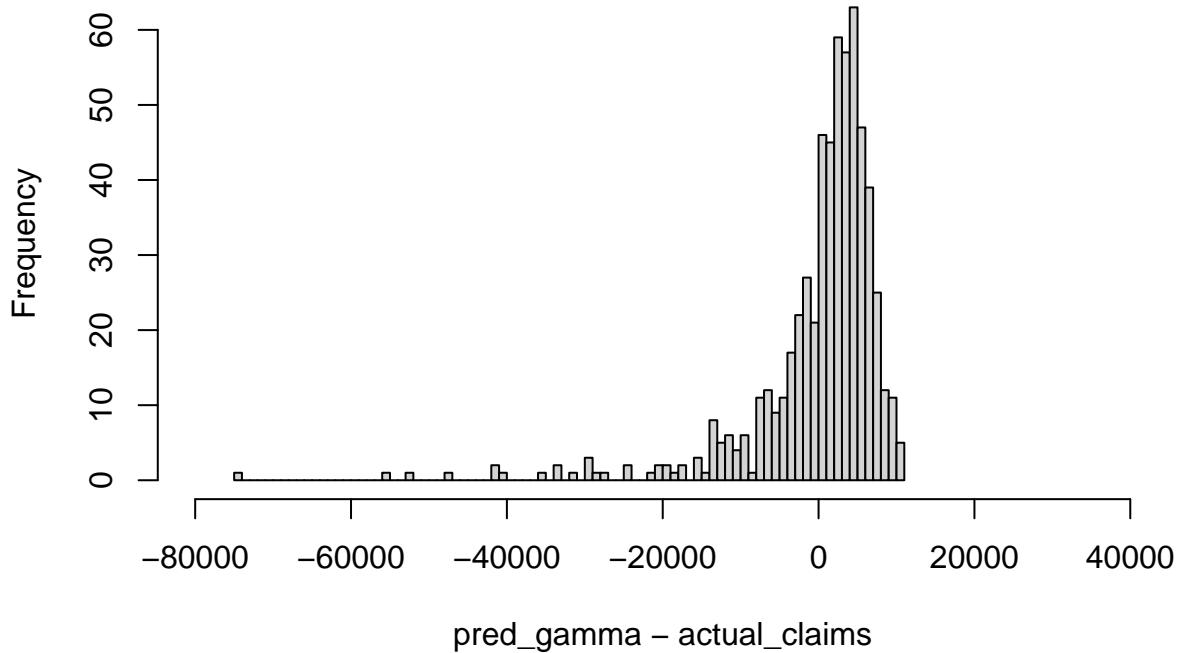
```

# plot errors

hist(pred_gamma - actual_claims, breaks = 100, xlim = c(-80000, 40000),
     main = 'gamma model errors')

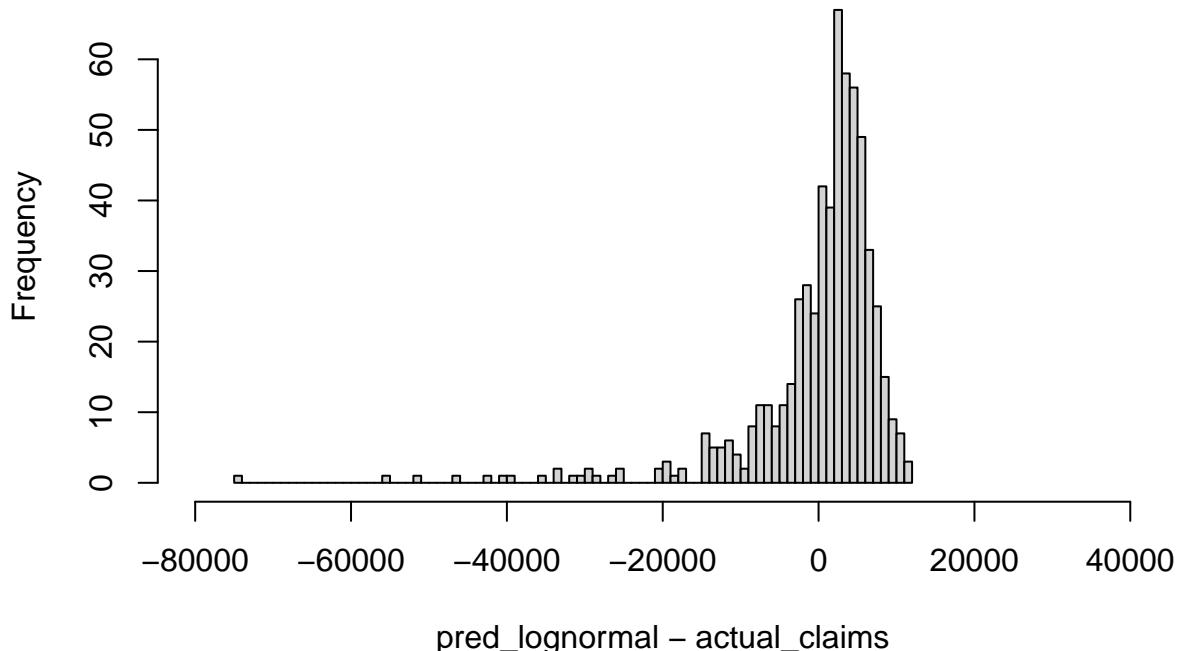
```

## gamma model errors



```
hist(pred_lognormal - actual_claims, breaks = 100, xlim = c(-80000, 40000),  
main = 'lognormal model errors')
```

## lognormal model errors



There are some extreme tail cases that our models mis, leading to a skewed error distribution

```

# computing average errors
print(paste("gamma model", mean(pred_gamma - actual_claims)))

## [1] "gamma model -2.47898757513912"

print(paste("lognormal model", mean(pred_lognormal - actual_claims)))

## [1] "lognormal model -3.15019208632837"

# observing tail behaviours of the errors
gamma_model_errors <- pred_gamma - actual_claims
gamma_model_errors_tail <- abs(gamma_model_errors[gamma_model_errors < (-10)])
lognormal_model_errors <- pred_lognormal - actual_claims
lognormal_model_errors_tail <- abs(lognormal_model_errors[lognormal_model_errors < (-10)])

# --- 1. Define Your Data and Names ---
# Replace these placeholder vectors with your actual data
vector1 <- gamma_model_errors_tail # e.g., errors_gamma
vector2 <- lognormal_model_errors_tail # e.g., errors_lognormal

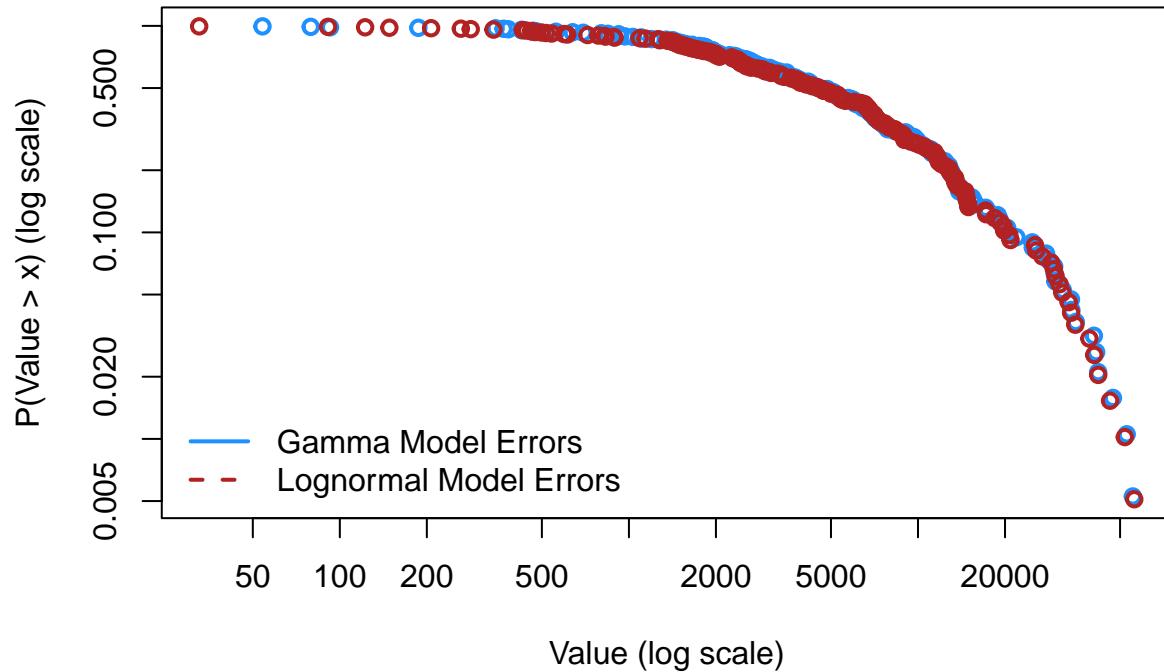
# Define names for the legend
name1 <- "Gamma Model Errors"
name2 <- "Lognormal Model Errors"

# --- Plot 1: Log-Log Survival Plot ---

ecdf_x <- ecdf(vector1)
data_x <- data.frame(val = sort(unique(vector1))) %>% mutate(prob = 1 - ecdf_x(val)) %>% filter(prob > 0)
ecdf_y <- ecdf(vector2)
data_y <- data.frame(val = sort(unique(vector2))) %>% mutate(prob = 1 - ecdf_y(val)) %>% filter(prob > 0)
xlim_range <- range(c(data_x$val, data_y$val))
ylim_range <- range(c(data_x$prob, data_y$prob))
plot(data_x$val, data_x$prob, type = "p", log = "xy", col = "dodgerblue", lwd = 2,
      xlim = xlim_range, ylim = ylim_range, main = "Log-Log Survival Plot",
      xlab = "Value (log scale)", ylab = "P(Value > x) (log scale)")
points(data_y$val, data_y$prob, col = "firebrick", lwd = 2, lty = 2)
legend("bottomleft", legend = c(name1, name2), col = c("dodgerblue", "firebrick"), lty = c(1, 2), lwd = 2)

```

## Log-Log Survival Plot



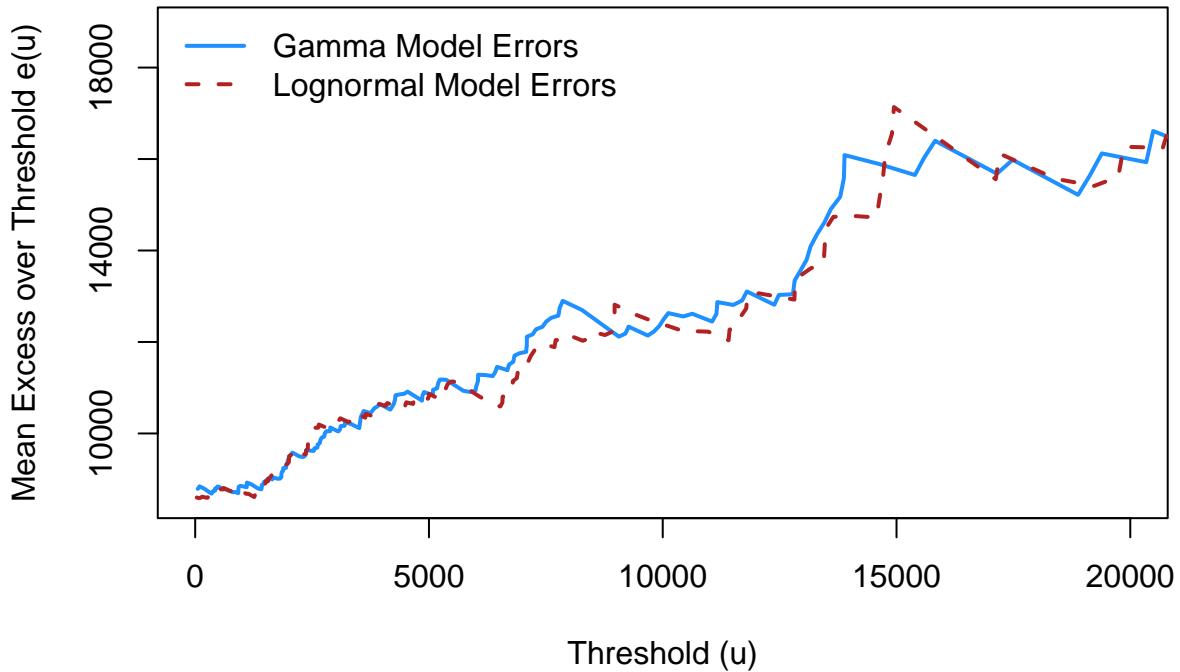
```
# --- Plot 2: Mean Excess Plot ---

mean_excess_func <- function(data) {
  thresholds <- unique(sort(data))
  excess <- sapply(thresholds, function(u) { mean(data[data > u] - u) })
  return(data.frame(threshold = thresholds, mean_excess = excess))
}

me_x <- mean_excess_func(vector1)
me_y <- mean_excess_func(vector2)

xlim_range_me <- range(c(me_x$threshold, me_y$threshold))
ylim_range_me <- range(c(me_x$mean_excess, me_y$mean_excess), na.rm = TRUE)
plot(me_x$threshold, me_x$mean_excess, type = "l", col = "dodgerblue", lwd = 2,
     xlim = c(0, 20000), ylim = ylim_range_me, main = "Mean Excess Plot",
     xlab = "Threshold (u)", ylab = "Mean Excess over Threshold e(u)")
lines(me_y$threshold, me_y$mean_excess, col = "firebrick", lwd = 2, lty = 2)
legend("topleft", legend = c(name1, name2), col = c("dodgerblue", "firebrick"), lty = c(1, 2), lwd = 2,
```

## Mean Excess Plot



```

# AIC for the Gamma(GLM) model (already on Y-scale)
aic_gamma <- AIC(claimsizes.2.gamma)

# AIC for the lognormal model from lm(log(CLAIMS) ~ ...)
# Convert the lm to a lognormal likelihood on Y
mf_ln <- model.frame(claimsizes.2.lognormal)
y_log <- model.response(mf_ln)                      # = log(CLAIMS)
y      <- exp(y_log)                                # original scale
mu_log <- fitted(claimsizes.2.lognormal)           # mean on log scale
sdlog <- sigma(claimsizes.2.lognormal)              # residual SD on log scale

ll_lognorm <- sum(dlnorm(y, meanlog = mu_log, sdlog = sdlog, log = TRUE))
k_lognorm <- length(coef(claimsizes.2.lognormal)) + 1 # +1 for sd
aic_lognorm <- -2 * ll_lognorm + 2 * k_lognorm

# (Equivalent shortcut: aic_lognorm <- AIC(claimsizes.2.lognormal) + 2 * sum(log(y)))

# Compare
aics <- c(Gamma = aic_gamma, Lognormal = aic_lognorm)
delta <- aics - min(aics)
akaike_wt <- exp(-0.5 * delta) / sum(exp(-0.5 * delta))

data.frame(Model = names(aics), AIC = aics, DeltaAIC = delta, AkaikeWeight = akaike_wt)

##          Model      AIC DeltaAIC AkaikeWeight
## Gamma      Gamma 50045.40    0.0000 1.000000e+00
## Lognormal Lognormal 50253.18 207.7781 7.612986e-46

```

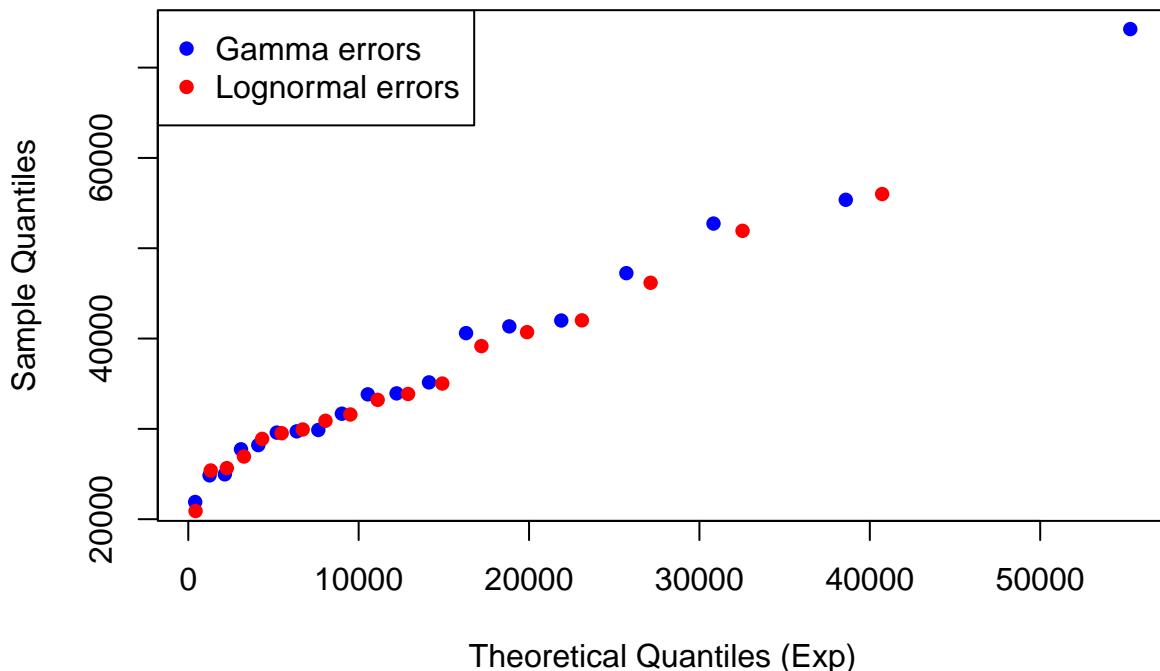
```

# QQ Plot against Exponential (for tail diagnosis)
qqtail <- function(vec, col, add=FALSE, ...) {
  n <- length(vec)
  sorted <- sort(vec)
  k <- floor(0.1 * n)
  tail <- sorted[(n-k+1):n]
  qexp <- qexp(ppoints(k), rate=1/mean(tail - min(tail)))
  if (!add) {
    plot(qexp, tail, col=col, pch=16, xlab="Theoretical Quantiles (Exp)",
          ylab="Sample Quantiles", main="QQ Plot (Tail)", ...)
  } else {
    points(qexp, tail, col=col, pch=16)
  }
}

qqtail(gamma_model_errors_tail, "blue")
qqtail(lognormal_model_errors_tail, "red", add=TRUE)
legend("topleft", legend=c("Gamma errors", "Lognormal errors"), col=c("blue", "red"), pch=16)

```

## QQ Plot (Tail)



```

# Hill Plot (Pareto tail index estimator)
hillplot <- function(vec, col, add=FALSE, ...) {
  sorted <- sort(vec, decreasing=TRUE)
  n <- length(vec)
  k <- 2:(n-1)
  hill <- sapply(k, function(i) mean(log(sorted[1:i])) - log(sorted[i]))
  if (!add) {
    plot(k, hill, type="l", col=col, lwd=2,
          xlab="Order Statistics k", ylab="Hill Estimator", main="Hill Plot", ...)
  }
}

```

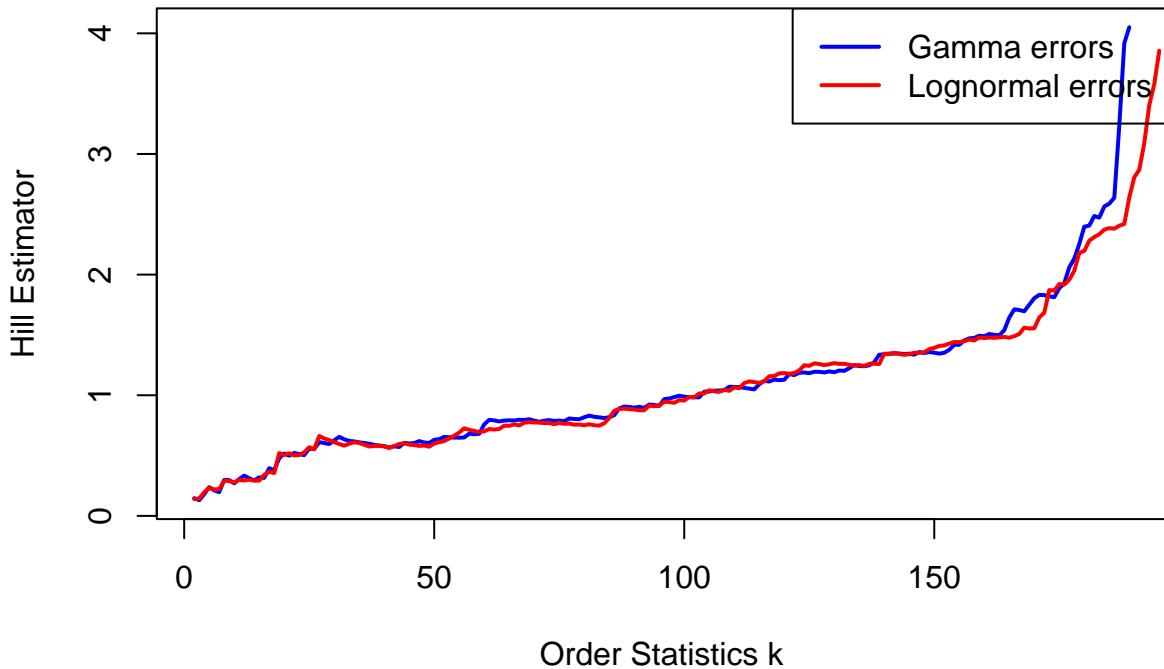
```

} else {
  lines(k, hill, col=col, lwd=2)
}
}

hillplot(gamma_model_errors_tail, "blue")
hillplot(lognormal_model_errors_tail, "red", add=TRUE)
legend("topright", legend=c("Gamma errors", "Lognormal errors"), col=c("blue", "red"), lwd=2)

```

## Hill Plot



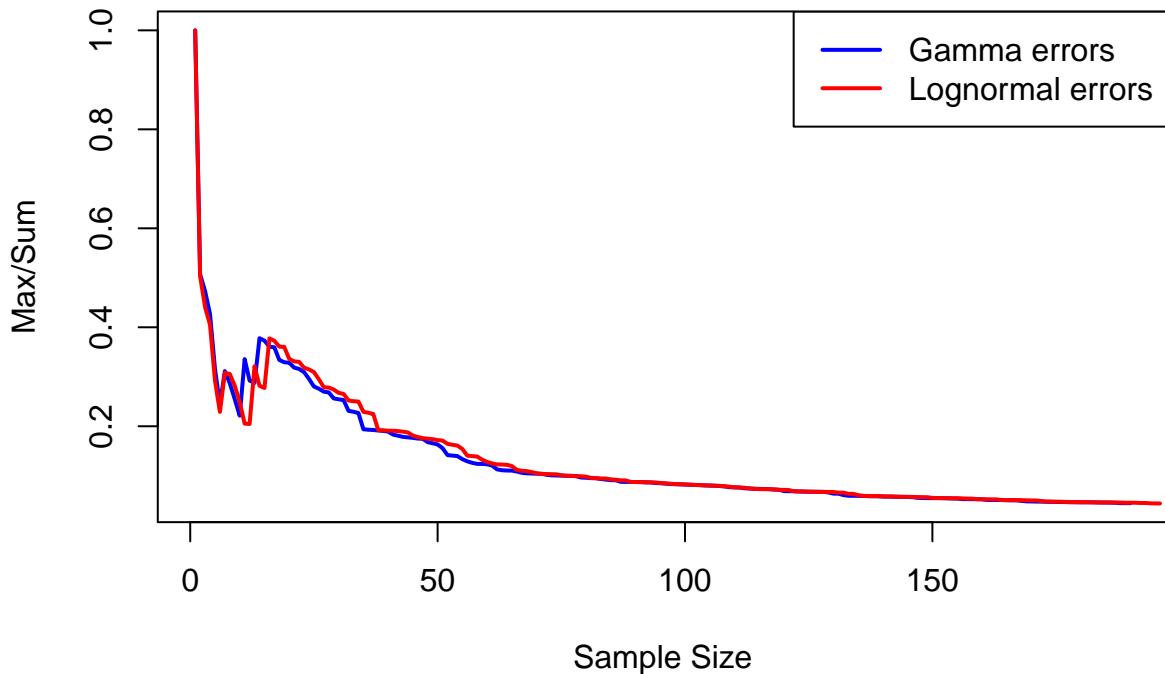
```

# Maximum-to-Sum Plot (finite moments check)
max2sumplot <- function(vec, col, add=FALSE, ...) {
  n <- length(vec)
  k <- 1:n
  maxvals <- sapply(k, function(i) max(vec[1:i]))
  sumvals <- sapply(k, function(i) sum(vec[1:i]))
  ratio <- maxvals/sumvals
  if (!add) {
    plot(k, ratio, type="l", col=col, lwd=2,
         xlab="Sample Size", ylab="Max/Sum", main="Maximum-to-Sum Plot", ...)
  } else {
    lines(k, ratio, col=col, lwd=2)
  }
}

max2sumplot(gamma_model_errors_tail, "blue")
max2sumplot(lognormal_model_errors_tail, "red", add=TRUE)
legend("topright", legend=c("Gamma errors", "Lognormal errors"), col=c("blue", "red"), lwd=2)

```

## Maximum-to-Sum Plot



- Not much difference in the tail, however we choose Gamma model largely due to the difference in AIC.
- Theory says lognormal has a heavier tail than gamma; diagnostics are consistent but the difference is small in this dataset.
- Both models exhibit very similar tail behaviour on residuals and near-identical predictive accuracy.
- the gamma model is a reasonable, slightly better-scoring choice.

## Investigation of claim outcome

```
claimoutcome_df <- train_data_imputed %>% select(-CLAIMS)
mean(claimoutcome_df$OUTCOME)
```

```
## [1] 0.31675
```

### logistic regression

```
# full basic model without interaction terms
outcome.0.logistic <- glm(OUTCOME ~ .,
                           family = binomial(link = "logit"),
                           data = claimoutcome_df)

# full model with interaction terms
outcome.1.logistic <- glm(OUTCOME ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION +
```

```

CREDIT_SCORE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +
MARRIED + CHILDREN + ANNUAL_MILEAGE +
VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2,
family= binomial(link = "logit"),
data = claimoutcome_df)

```

## Variable selection

Stepwise selection based on AIC

```

## Call:
## glm(formula = OUTCOME ~ DRIVING_EXPERIENCE + VEHICLE_OWNERSHIP +
##      VEHICLE_YEAR + GENDER + MARRIED + PAST_ACCIDENTS + ANNUAL_MILEAGE +
##      SPEEDING_VIOLATIONS + DRIVING_EXPERIENCE:VEHICLE_YEAR + VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS +
##      GENDER:MARRIED, family = binomial(link = "logit"), data = claimoutcome_df)
##
## Coefficients:
##                               Estimate Std. Error z value
## (Intercept)                -8.220e-01  2.229e-01 -3.687
## DRIVING_EXPERIENCE10-19y    -1.533e+00  1.808e-01 -8.477
## DRIVING_EXPERIENCE20-29y    -2.237e+00  2.768e-01 -8.082
## DRIVING_EXPERIENCE30y+     -2.634e+00  4.387e-01 -6.004
## VEHICLE_OWNERSHIP          -1.897e+00  8.301e-02 -22.847
## VEHICLE_YEARbefore 2015     2.093e+00  1.152e-01 18.169
## GENDERmale                 8.538e-01  8.904e-02  9.588
## MARRIED                     -6.910e-01  1.087e-01 -6.355
## PAST_ACCIDENTS              -2.149e-01  3.877e-02 -5.542
## ANNUAL_MILEAGE               6.917e-05  1.440e-05  4.803
## SPEEDING_VIOLATIONS         4.719e-03  3.758e-02  0.126
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 -4.730e-01  1.903e-01 -2.486
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 -1.423e+00  2.936e-01 -4.847
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015   -2.037e+00  5.280e-01 -3.859
## VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS           1.282e-01  3.976e-02  3.224
## GENDERmale:MARRIED                  4.189e-01  1.381e-01  3.032
##
## Pr(>|z|)
## (Intercept)            0.000227 ***
## DRIVING_EXPERIENCE10-19y < 2e-16 ***
## DRIVING_EXPERIENCE20-29y 6.35e-16 ***
## DRIVING_EXPERIENCE30y+  1.92e-09 ***
## VEHICLE_OWNERSHIP        < 2e-16 ***
## VEHICLE_YEARbefore 2015 < 2e-16 ***
## GENDERmale               < 2e-16 ***
## MARRIED                   2.08e-10 ***
## PAST_ACCIDENTS            2.98e-08 ***
## ANNUAL_MILEAGE             1.56e-06 ***
## SPEEDING_VIOLATIONS       0.900080
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 0.012923 *
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 1.25e-06 ***
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015  0.000114 ***
## VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS          0.001263 **
## GENDERmale:MARRIED                  0.002426 **
##
---
```

```

## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 9990.3  on 7999  degrees of freedom
## Residual deviance: 5717.5  on 7984  degrees of freedom
## AIC: 5749.5
##
## Number of Fisher Scoring iterations: 6

```

### hypothesis tests

```

# Align both models to the exact same rows used in the full model
mf <- model.frame(outcome.1.logistic)
reduced <- update(outcome.2.logistic, data = mf)

# Likelihood-ratio test (valid when reduced is nested in full)
anova(reduced, outcome.1.logistic, test = "Chisq")

```

```

## Analysis of Deviance Table
##
## Model 1: OUTCOME ~ DRIVING_EXPERIENCE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +
##           GENDER + MARRIED + PAST_ACCIDENTS + ANNUAL_MILEAGE + SPEEDING_VIOLATIONS +
##           DRIVING_EXPERIENCE:VEHICLE_YEAR + VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS +
##           GENDER:MARRIED
## Model 2: OUTCOME ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION + CREDIT_SCORE +
##           VEHICLE_OWNERSHIP + VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE +
##           VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7984    5717.5
## 2      7845    5542.3 139    175.15  0.0205 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# AIC and BIC (smaller is better)
AIC(reduced, outcome.1.logistic)

```

```

##                  df      AIC
## reduced          16 5749.453
## outcome.1.logistic 155 5852.303

```

```

n <- nobs(outcome.1.logistic)
AIC(reduced, outcome.1.logistic, k = log(n)) # BIC

```

```

##                  df      AIC
## reduced          16 5861.248
## outcome.1.logistic 155 6935.318

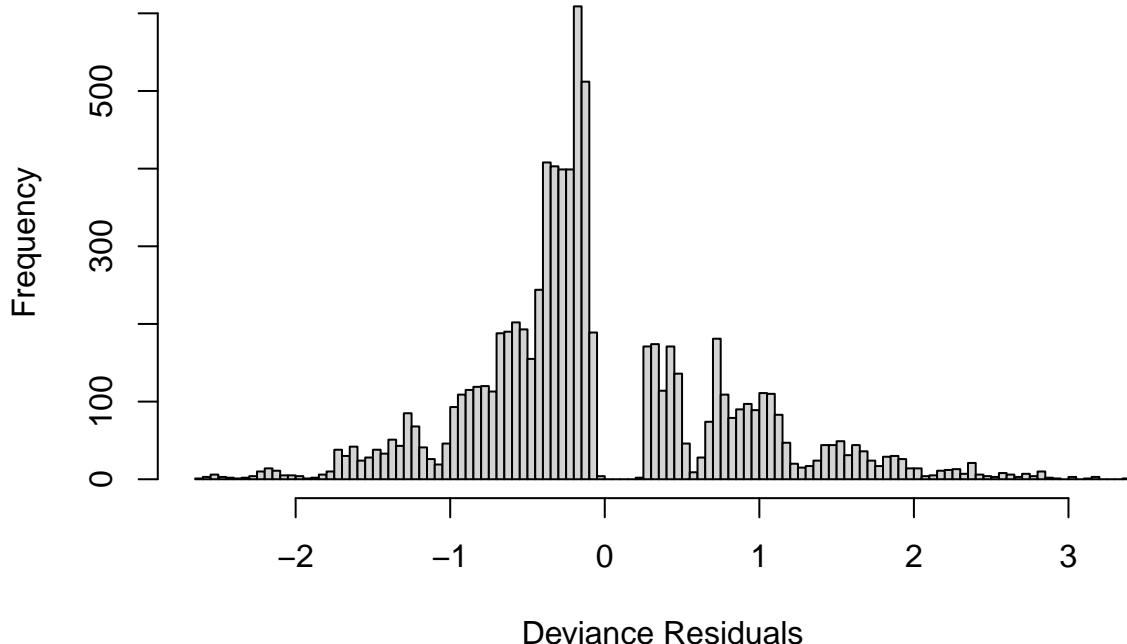
```

- The LR test suggests some in-sample gain from the full model, but parsimony (AIC/BIC) and predictive validation likely favor the reduced model unless you see a clear lift on a hold-out set.

## Residual analysis and finding influential points

```
hist(residuals(outcome.2.logistic, type = "deviance"), main = "Histogram of Deviance Residuals", xlab =
```

**Histogram of Deviance Residuals**

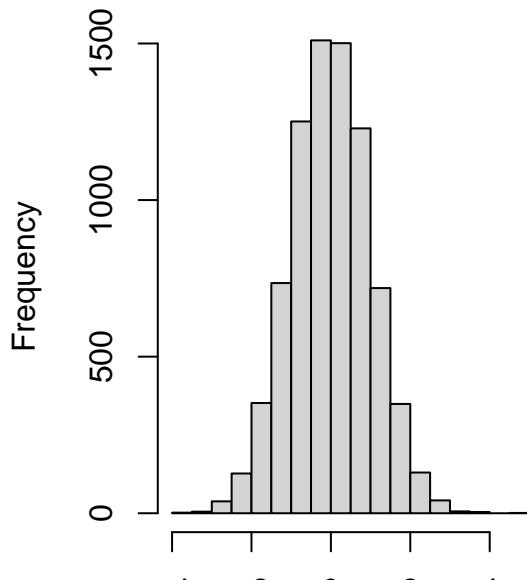


non normal shape as the response variable is discrete and has a high number of 0's - look at randomised quantile residuals instead

```
qres <- qresiduals(outcome.2.logistic)
par(mfrow = c(1, 2)) # Set up a 1x2 plotting area
hist(qres, main = "Histogram of Quantile Residuals", xlab = "Quantile Residuals")
qqnorm(qres, main = "Normal Q-Q Plot")
qqline(qres, col = "red", lty = 2)
```

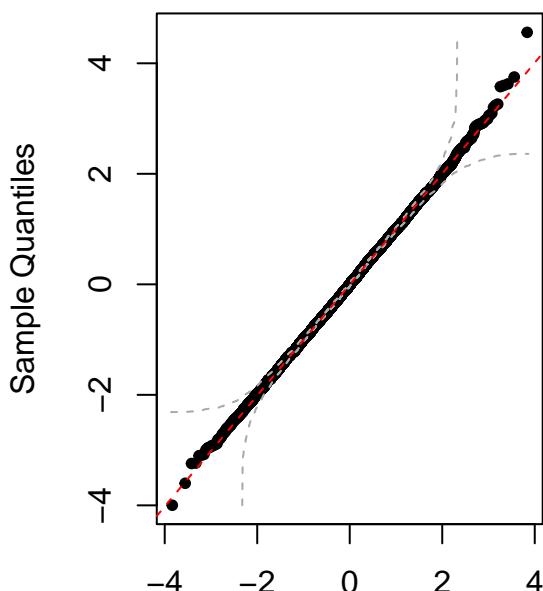
- we see a

**Histogram of Quantile Residuals:**



**Quantile Residuals**

**Normal Q-Q Plot**



**Standard Normal Quantiles** - We

see that the quantile residuals are normally distributed, and the qq plot shows a very close fit to the normal distribution, suggesting that our logistic regression model is appropriate for the data.

## Probit regression

- using binomial with probit link

```
# full basic model without interaction terms
outcome.0.probit <- glm(OUTCOME ~ .,
                        family = binomial(link = "probit"),
                        data = claimoutcome_df)

# full model with interaction terms
outcome.1.probit <- glm(OUTCOME ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION +
                                      CREDIT_SCORE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +
                                      MARRIED + CHILDREN + ANNUAL_MILEAGE +
                                      VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2,
                        family = binomial(link = "probit"),
                        data = claimoutcome_df)
```

## Variable selection

```
##
## Call:
## glm(formula = OUTCOME ~ DRIVING_EXPERIENCE + VEHICLE_OWNERSHIP +
##      VEHICLE_YEAR + GENDER + MARRIED + PAST_ACCIDENTS + ANNUAL_MILEAGE +
##      SPEEDING_VIOLATIONS + DRIVING_EXPERIENCE:VEHICLE_YEAR + DRIVING_EXPERIENCE:VEHICLE_OWNERSHIP +
##      VEHICLE_OWNERSHIP:PAST_ACCIDENTS, family = binomial(link = "probit"),
```

```

##      data = claimoutcome_df)
##
## Coefficients:
##                               Estimate Std. Error z value
## (Intercept)                -5.178e-01  1.279e-01 -4.048
## DRIVING_EXPERIENCE10-19y   -9.270e-01  1.131e-01 -8.197
## DRIVING_EXPERIENCE20-29y   -1.676e+00  1.702e-01 -9.850
## DRIVING_EXPERIENCE30y+    -2.302e+00  3.638e-01 -6.329
## VEHICLE_OWNERSHIP         -1.105e+00  6.052e-02 -18.265
## VEHICLE_YEARbefore 2015    1.234e+00  6.595e-02 18.708
## GENDERmale                 5.655e-01  4.024e-02 14.051
## MARRIED                    -2.609e-01  4.308e-02 -6.056
## PAST_ACCIDENTS              4.949e-02  3.117e-02 -1.588
## ANNUAL_MILEAGE               3.933e-05  8.090e-06  4.861
## SPEEDING_VIOLATIONS        4.211e-02  1.370e-02  3.073
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 -3.399e-01  1.043e-01 -3.258
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 -8.933e-01  1.392e-01 -6.417
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015  -1.128e+00  2.288e-01 -4.931
## DRIVING_EXPERIENCE10-19y:VEHICLE_OWNERSHIP       1.597e-01  9.360e-02  1.707
## DRIVING_EXPERIENCE20-29y:VEHICLE_OWNERSHIP       8.736e-01  1.504e-01  5.809
## DRIVING_EXPERIENCE30y+:VEHICLE_OWNERSHIP        1.456e+00  3.472e-01  4.193
## VEHICLE_OWNERSHIP:PAST_ACCIDENTS                -1.342e-01  4.131e-02 -3.248
## Pr(>|z|)
## (Intercept)                  5.16e-05 ***
## DRIVING_EXPERIENCE10-19y     2.46e-16 ***
## DRIVING_EXPERIENCE20-29y     < 2e-16 ***
## DRIVING_EXPERIENCE30y+       2.47e-10 ***
## VEHICLE_OWNERSHIP            < 2e-16 ***
## VEHICLE_YEARbefore 2015      < 2e-16 ***
## GENDERmale                   < 2e-16 ***
## MARRIED                      1.39e-09 ***
## PAST_ACCIDENTS                0.11230
## ANNUAL_MILEAGE                1.17e-06 ***
## SPEEDING_VIOLATIONS          0.00212 **
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 0.00112 **
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 1.39e-10 ***
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015  8.20e-07 ***
## DRIVING_EXPERIENCE10-19y:VEHICLE_OWNERSHIP        0.08791 .
## DRIVING_EXPERIENCE20-29y:VEHICLE_OWNERSHIP        6.29e-09 ***
## DRIVING_EXPERIENCE30y+:VEHICLE_OWNERSHIP         2.76e-05 ***
## VEHICLE_OWNERSHIP:PAST_ACCIDENTS                0.00116 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 9990.3  on 7999  degrees of freedom
## Residual deviance: 5711.9  on 7982  degrees of freedom
## AIC: 5747.9
##
## Number of Fisher Scoring iterations: 7

```

## hypothesis tests

```
# Ensure both models use identical rows
mf <- model.frame(outcome.1.probit)
reduced <- update(outcome.2.probit, data = mf)

# Likelihood-ratio test (Model 1 = reduced, Model 2 = full)
anova(reduced, outcome.1.probit, test = "Chisq")

## Analysis of Deviance Table
##
## Model 1: OUTCOME ~ DRIVING_EXPERIENCE + VEHICLE_OWNERSHIP + VEHICLE_YEAR +
##           GENDER + MARRIED + PAST_ACCIDENTS + ANNUAL_MILEAGE + SPEEDING_VIOLATIONS +
##           DRIVING_EXPERIENCE:VEHICLE_YEAR + DRIVING_EXPERIENCE:VEHICLE_OWNERSHIP +
##           VEHICLE_OWNERSHIP:PAST_ACCIDENTS
## Model 2: OUTCOME ~ (AGE + GENDER + DRIVING_EXPERIENCE + EDUCATION + CREDIT_SCORE +
##           VEHICLE_OWNERSHIP + VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE +
##           VEHICLE_TYPE + SPEEDING_VIOLATIONS + PAST_ACCIDENTS)^2
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1      7982    5711.9
## 2      7845    5544.3 137    167.56  0.03888 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# AIC and BIC comparison (smaller is better)
AIC(reduced, outcome.1.probit)

##
##          df      AIC
## reduced      18 5747.855
## outcome.1.probit 155 5854.298

n <- nobs(outcome.1.probit)
AIC(reduced, outcome.1.probit, k = log(n)) # BIC

##
##          df      AIC
## reduced      18 5873.625
## outcome.1.probit 155 6937.313
```

## Comparison between logit and probit link functions

```
# Helpers
prep_newdata_for <- function(newdata, fit) {
  nd <- newdata
  if (!is.null(fit$xlevels)) {
    for (nm in names(fit$xlevels)) if (nm %in% names(nd)) {
      nd[[nm]] <- factor(nd[[nm]], levels = fit$xlevels[[nm]])
    }
  }
  droplevels(nd)
```

```

}

log_loss <- function(y, p, eps = 1e-15) {
  p <- pmin(pmax(p, eps), 1 - eps)
  -mean(y * log(p) + (1 - y) * log(1 - p), na.rm = TRUE)
}

brier <- function(y, p) mean((p - y)^2, na.rm = TRUE)

calibration_df <- function(y, p, bins = 10, label = "model") {
  brks <- quantile(p, probs = seq(0, 1, length.out = bins + 1), na.rm = TRUE)
  g <- cut(p, breaks = unique(brks), include.lowest = TRUE)
  dplyr::summarise(dplyr::group_by(data.frame(y, p, g), g),
    mean_p = mean(p, na.rm = TRUE),
    obs = mean(y, na.rm = TRUE),
    n = dplyr::n()) |>
    mutate(model = label)
}

# Prepare test data for each model (handles factors/levels)
test_logit <- prep_newdata_for(test_data_imputed, outcome.2.logistic)
test_probit <- prep_newdata_for(test_data_imputed, outcome.2.probit)

# Predictions
y <- test_logit$OUTCOME
p_logit <- predict(outcome.2.logistic, newdata = test_logit, type = "response")
p_probit <- predict(outcome.2.probit, newdata = test_probit, type = "response")

# Keep rows where both models produce probabilities
keep <- is.finite(p_logit) & is.finite(p_probit) & !is.na(y)
y <- y[keep]; p_logit <- p_logit[keep]; p_probit <- p_probit[keep]

# Build ROC objects with pROC
roc_logit <- pROC::roc(y, p_logit, quiet = TRUE)
roc_probit <- pROC::roc(y, p_probit, quiet = TRUE)

# Get numeric AUCs (either of these styles works)
auc_logit <- as.numeric(pROC::auc(roc_logit))
auc_probit <- as.numeric(pROC::auc(roc_probit))
# or:
# auc_logit <- as.numeric(roc_logit$auc)
# auc_probit <- as.numeric(roc_probit$auc)

metrics <- data.frame(
  Model = c("Logit", "Probit"),
  AUC = c(auc_logit, auc_probit),
  LogLoss = c(log_loss(y, p_logit), log_loss(y, p_probit)),
  Brier = c(brier(y, p_logit), brier(y, p_probit))
)
print(metrics, row.names = FALSE)

##   Model      AUC  LogLoss      Brier
##   Logit  0.8811706  0.3921841  0.1246430
##   Probit 0.8817748  0.3903196  0.1242813

```

```

# AUC difference test
auc_test <- pROC::roc.test(roc_logit, roc_probit, method = "delong")
cat(sprintf("\nDeLong test for AUC difference: z = %.3f, p = %.4f\n",
           auc_test$statistic, auc_test$p.value))

## 
## DeLong test for AUC difference: z = -0.580, p = 0.5618

# ROC curves (overlaid)
df_roc_logit <- data.frame(
  fpr = rev(1 - roc_logit$specificities),
  tpr = rev(roc_logit$sensitivities),
  model = "Logit"
)
df_roc_probit <- data.frame(
  fpr = rev(1 - roc_probit$specificities),
  tpr = rev(roc_probit$sensitivities),
  model = "Probit"
)
df_roc <- rbind(df_roc_logit, df_roc_probit)

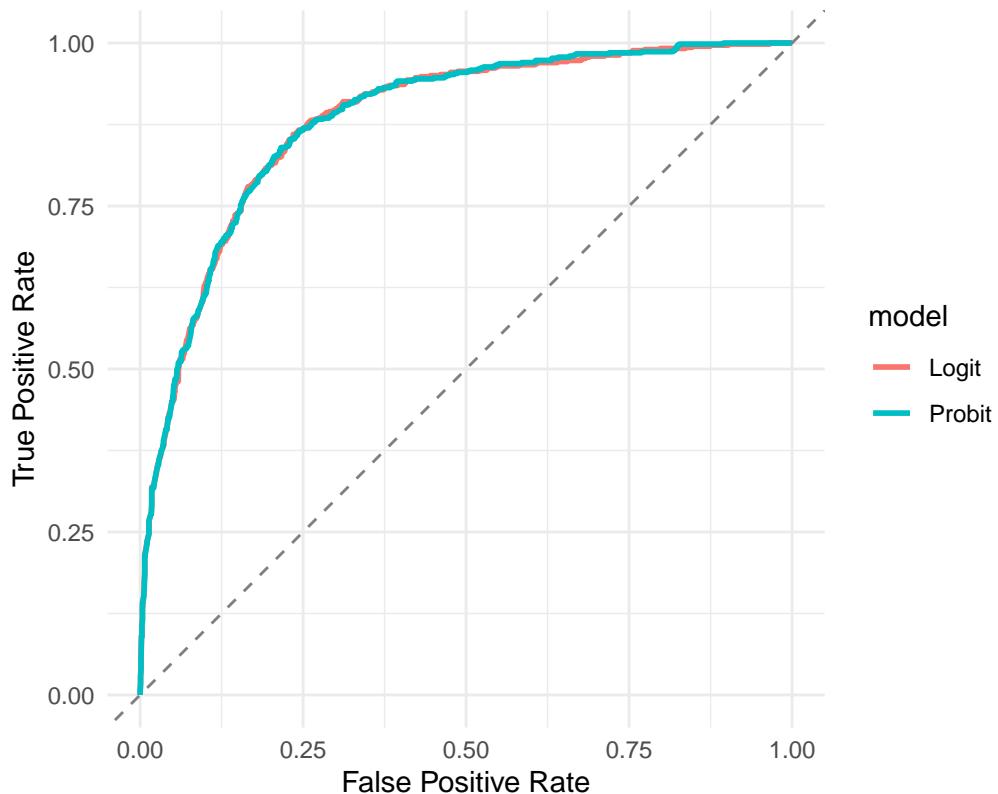
p_roc <- ggplot(df_roc, aes(x = fpr, y = tpr, color = model)) +
  geom_line(size = 1) +
  geom_abline(slope = 1, intercept = 0, linetype = 2, color = "gray50") +
  labs(title = sprintf("ROC on test (AUC: logit=%.3f, probit=%.3f)",
                       metrics$AUC[metrics$Model=="Logit"],
                       metrics$AUC[metrics$Model=="Probit"]),
       x = "False Positive Rate", y = "True Positive Rate") +
  theme_minimal() + coord_equal()

## Warning: Using 'size' aesthetic for lines was deprecated in ggplot2 3.4.0.
## i Please use 'linewidth' instead.
## This warning is displayed once every 8 hours.
## Call 'lifecycle::last_lifecycle_warnings()' to see where this warning was
## generated.

print(p_roc)

```

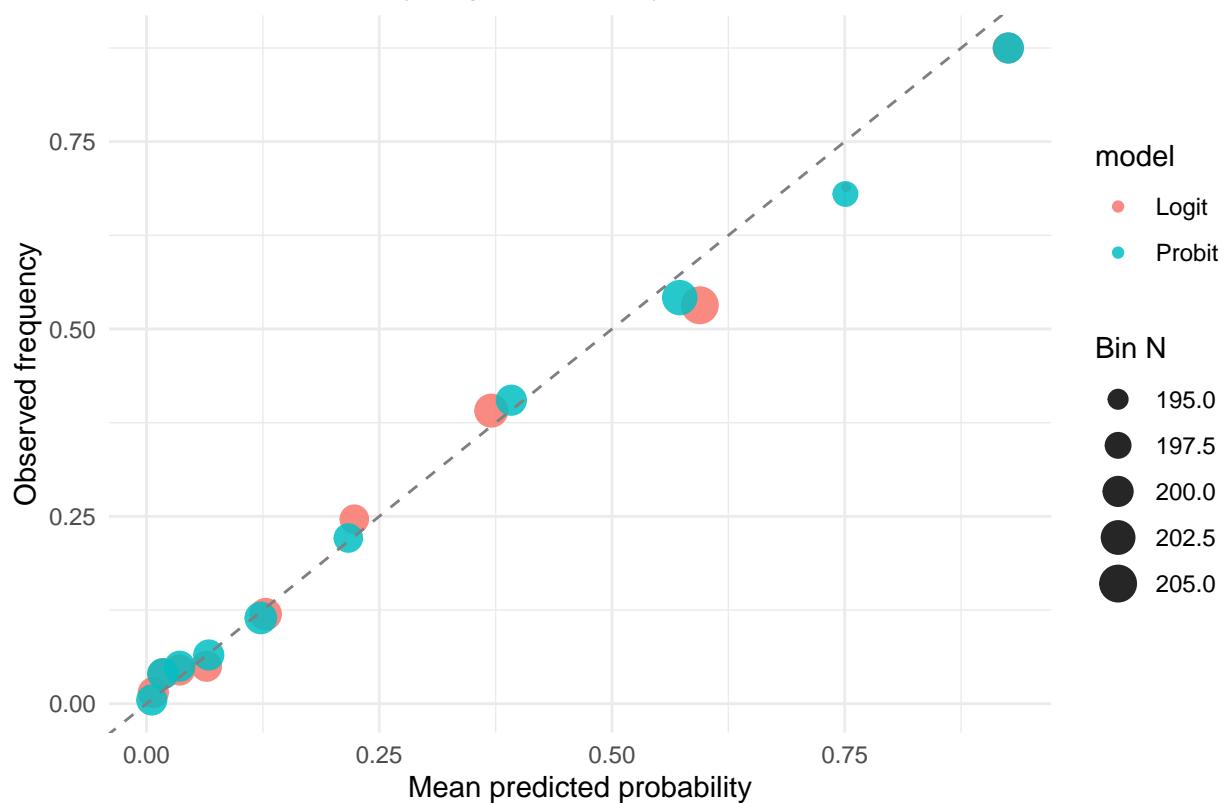
ROC on test (AUC: logit=0.881, probit=0.882)



```
# Calibration (reliability) plot
cal_logit <- calibration_df(y, p_logit, bins = 10, label = "Logit")
cal_probit <- calibration_df(y, p_probit, bins = 10, label = "Probit")
cal <- rbind(cal_logit, cal_probit)

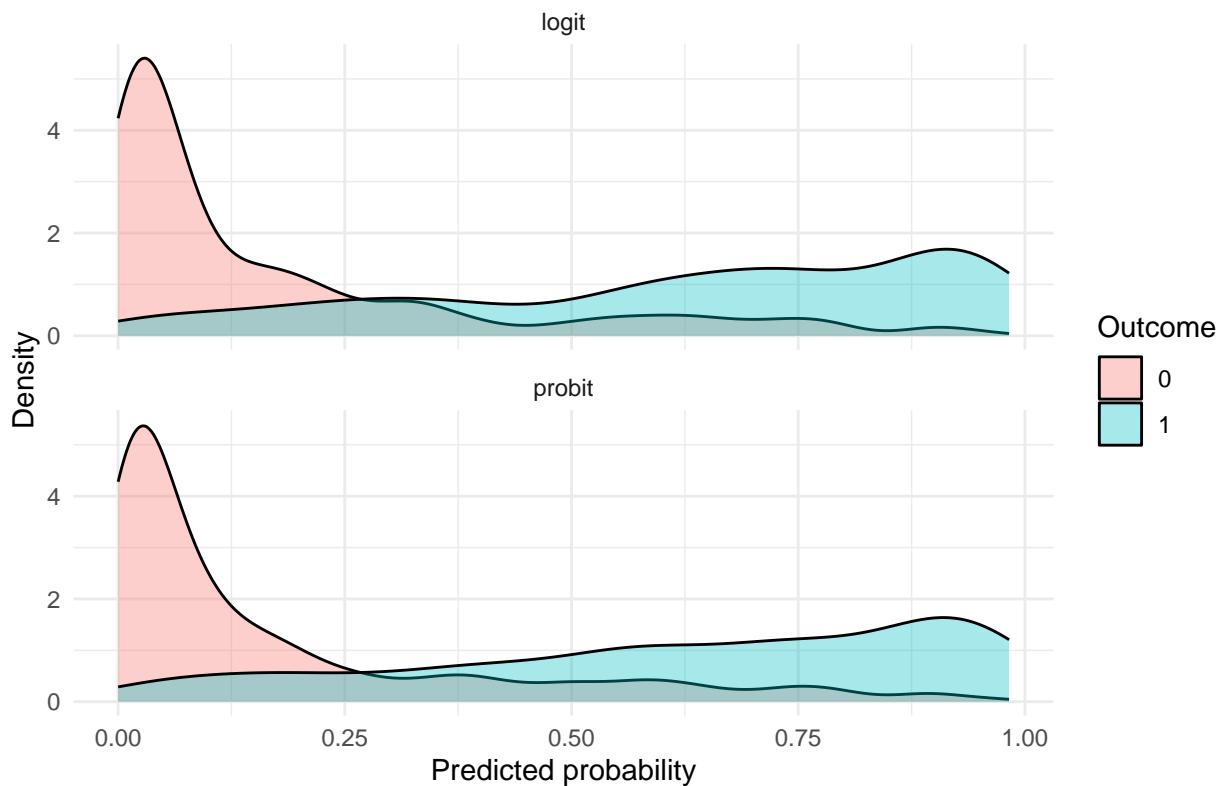
p_cal <- ggplot(cal, aes(x = mean_p, y = obs, color = model, size = n)) +
  geom_point(alpha = 0.85) +
  geom_abline(slope = 1, intercept = 0, linetype = 2, color = "gray50") +
  scale_size_continuous(name = "Bin N") +
  labs(title = "Calibration on test (10 quantile bins)",
       x = "Mean predicted probability", y = "Observed frequency") +
  theme_minimal()
print(p_cal)
```

## Calibration on test (10 quantile bins)



```
# Optional: density of predicted probabilities by class (separation plot)
df_prob <- data.frame(y = factor(y, levels = c(0,1)),
                       logit = p_logit, probit = p_probit) |>
  tidyr::pivot_longer(cols = c(logit, probit), names_to = "model", values_to = "p")
p_den <- ggplot(df_prob, aes(x = p, fill = y)) +
  geom_density(alpha = 0.35) +
  facet_wrap(~ model, ncol = 1) +
  labs(title = "Predicted probability densities by outcome (test)",
       x = "Predicted probability", y = "Density", fill = "Outcome") +
  theme_minimal()
print(p_den)
```

## Predicted probability densities by outcome (test)



- Predicted-probability densities by outcome Clear separation: class 0 probabilities are heavily concentrated near 0; class 1 probabilities are mostly 0.5–1.0 with a peak near 0.8–0.95. Overlap is mainly in 0.1–0.3, which is where most errors will occur. Logit vs probit look almost identical; probit shows a hair more mass near very high probabilities, but the difference is tiny.
- Calibration (10 quantile bins) Points lie very close to the 45° line for both models across the range → well-calibrated probabilities. Minor, likely noise-level deviations: around 0.7–0.8 the probit is slightly underconfident (observed > predicted), around ~0.55 the logit is slightly overconfident. Bin sizes are ~200, so sampling variation can explain this.
- ROC and AUC Curves overlap almost perfectly; test AUCs are 0.881 (logit) vs 0.882 (probit). The 0.001 gap is negligible and would not be statistically or practically significant (DeLong test would almost surely be non-significant). Bottom line
- On the test set, logit and probit are essentially indistinguishable
- Choose Logit due to its superior explanatory power

## Final model

- Binomial with logit link for claim occurrence
- Gamma GLM with log link for claim severity

```
# kNN-impute numeric predictors and refit models on data_imputed
```

```
# Predictors used by either model (exclude responses)
```

```

pred_vars <- setdiff(
  unique(c(all.vars(formula(outcome.2.logistic))[-1],
    all.vars(formula(claimsize.2.gamma))[-1])),
  c("OUTCOME", "CLAIMS")
)

# Numeric predictors to impute
num_vars <- intersect(pred_vars, names(Filter(is.numeric, data)))

# Train kNN imputer (k = 5) on training numerics and impute
rec <- recipe(~ ., data = data[, num_vars, drop = FALSE]) |>
  step_impute_knn(all_predictors(), neighbors = 5)
imp <- prep(rec, training = data[, num_vars, drop = FALSE], retain = TRUE)

data_imputed <- data
data_imputed[, num_vars] <- bake(imp, new_data = data[, num_vars, drop = FALSE])

# Refit models with identical specs on imputed data
claimsize.3.gamma <- update(claimsize.2.gamma, data = data_imputed[data_imputed$CLAIMS > 0, ])
outcome.3.logistic <- update(outcome.2.logistic, data = data_imputed)

```

## summaries of final models

```
summary(claimsize.3.gamma)
```

```

##
## Call:
## glm(formula = CLAIMS ~ GENDER + DRIVING_EXPERIENCE + CREDIT_SCORE +
##       VEHICLE_YEAR + MARRIED + CHILDREN + ANNUAL_MILEAGE + VEHICLE_TYPE +
##       SPEEDING_VIOLATIONS + PAST_ACCIDENTS + GENDER:CHILDREN +
##       CHILDREN:ANNUAL_MILEAGE + CREDIT_SCORE:PAST_ACCIDENTS + VEHICLE_YEAR:MARRIED +
##       GENDER:SPEEDING_VIOLATIONS + CREDIT_SCORE:VEHICLE_TYPE, family = Gamma(link = "log"),
##       data = data_imputed[data_imputed$CLAIMS > 0, ], control = glm.control(maxit = 100))
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)                8.670e+00  2.119e-01  40.917 < 2e-16 ***
## GENDERmale                 2.873e-01  5.923e-02   4.851 1.29e-06 ***
## DRIVING_EXPERIENCE10-19y   -6.185e-01  6.432e-02  -9.616 < 2e-16 ***
## DRIVING_EXPERIENCE20-29y   -6.235e-01  1.387e-01  -4.494 7.23e-06 ***
## DRIVING_EXPERIENCE30y+     -2.508e-01  2.765e-01  -0.907  0.36448
## CREDIT_SCORE                 4.180e-01  1.755e-01   2.382  0.01728 *
## VEHICLE_YEARbefore 2015      1.805e-01  8.512e-02   2.120  0.03409 *
## MARRIED                      1.746e-01  1.275e-01   1.369  0.17098
## CHILDREN                     -6.274e-01  2.178e-01  -2.881  0.00399 **
## ANNUAL_MILEAGE                -9.846e-06 1.211e-05  -0.813  0.41620
## VEHICLE_TYPEsports car        5.836e-01  3.300e-01   1.768  0.07709 .
## SPEEDING_VIOLATIONS          -3.992e-02  4.470e-02  -0.893  0.37195
## PAST_ACCIDENTS                2.318e-01  1.006e-01   2.304  0.02127 *
## GENDERmale:CHILDREN           2.179e-01  8.137e-02   2.677  0.00746 **

```

```

## CHILDREN:ANNUAL_MILEAGE      3.811e-05  1.619e-05  2.354  0.01862 *
## CREDIT_SCORE:PAST_ACCIDENTS -4.728e-01  2.033e-01 -2.325  0.02013 *
## VEHICLE_YEARbefore 2015:MARRIED -2.353e-01  1.335e-01 -1.762  0.07817 .
## GENDERmale:SPEEDING_VIOLATIONS  8.007e-02  4.358e-02  1.837  0.06629 .
## CREDIT_SCORE:VEHICLE_TYPEsports car -8.767e-01  6.876e-01 -1.275  0.20239
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for Gamma family taken to be 1.223407)
##
## Null deviance: 3675.6  on 3132  degrees of freedom
## Residual deviance: 3338.0  on 3114  degrees of freedom
## AIC: 61846
##
## Number of Fisher Scoring iterations: 6

```

```
summary(outcome.3.logistic)
```

```

##
## Call:
## glm(formula = OUTCOME ~ DRIVING_EXPERIENCE + VEHICLE_OWNERSHIP +
##       VEHICLE_YEAR + GENDER + MARRIED + PAST_ACCIDENTS + ANNUAL_MILEAGE +
##       SPEEDING_VIOLATIONS + DRIVING_EXPERIENCE:VEHICLE_YEAR + VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS +
##       GENDER:MARRIED, family = binomial(link = "logit"), data = data_imputed)
##
## Coefficients:
##                               Estimate Std. Error z value
## (Intercept)                -6.576e-01  1.958e-01 -3.359
## DRIVING_EXPERIENCE10-19y    -1.518e+00  1.611e-01 -9.421
## DRIVING_EXPERIENCE20-29y    -2.201e+00  2.427e-01 -9.069
## DRIVING_EXPERIENCE30y+     -2.693e+00  4.046e-01 -6.657
## VEHICLE_OWNERSHIP          -1.821e+00  7.295e-02 -24.961
## VEHICLE_YEARbefore 2015     1.990e+00  1.011e-01 19.679
## GENDERmale                 8.221e-01  7.876e-02 10.437
## MARRIED                     -6.517e-01  9.515e-02 -6.849
## PAST_ACCIDENTS              -2.217e-01  3.470e-02 -6.388
## ANNUAL_MILEAGE               5.524e-05  1.253e-05  4.408
## SPEEDING_VIOLATIONS         1.956e-03  3.370e-02  0.058
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 -3.851e-01  1.692e-01 -2.276
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 -1.396e+00  2.583e-01 -5.405
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015   -1.832e+00  4.769e-01 -3.841
## VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS           1.265e-01  3.564e-02  3.550
## GENDERmale:MARRIED             3.524e-01  1.217e-01  2.896
##
## Pr(>|z|)
## (Intercept)                  0.000781 ***
## DRIVING_EXPERIENCE10-19y      < 2e-16 ***
## DRIVING_EXPERIENCE20-29y      < 2e-16 ***
## DRIVING_EXPERIENCE30y+        2.79e-11 ***
## VEHICLE_OWNERSHIP            < 2e-16 ***
## VEHICLE_YEARbefore 2015       < 2e-16 ***
## GENDERmale                   < 2e-16 ***
## MARRIED                      7.42e-12 ***
## PAST_ACCIDENTS                1.68e-10 ***
## ANNUAL_MILEAGE                 1.04e-05 ***

```

```

## SPEEDING_VIOLATIONS          0.953723
## DRIVING_EXPERIENCE10-19y:VEHICLE_YEARbefore 2015 0.022842 *
## DRIVING_EXPERIENCE20-29y:VEHICLE_YEARbefore 2015 6.48e-08 ***
## DRIVING_EXPERIENCE30y+:VEHICLE_YEARbefore 2015    0.000122 ***
## VEHICLE_OWNERSHIP:SPEEDING_VIOLATIONS        0.000385 ***
## GENDERmale:MARRIED                  0.003785 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ',' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
## Null deviance: 12434.3  on 9999  degrees of freedom
## Residual deviance: 7286.1  on 9984  degrees of freedom
## AIC: 7318.1
##
## Number of Fisher Scoring iterations: 6

dir.create("models", showWarnings = FALSE)
dir.create("data",   showWarnings = FALSE)

saveRDS(outcome.3.logistic, "models/outcome_3_logistic.rds")
saveRDS(claimsize.3.gamma,   "models/claimsizes_3_gamma.rds")
saveRDS(data,               "data/training_data.rds")

```