Applications of Natural Image Statistics: Noise Variance Estimation

Lecture material for IT530, DAIICT

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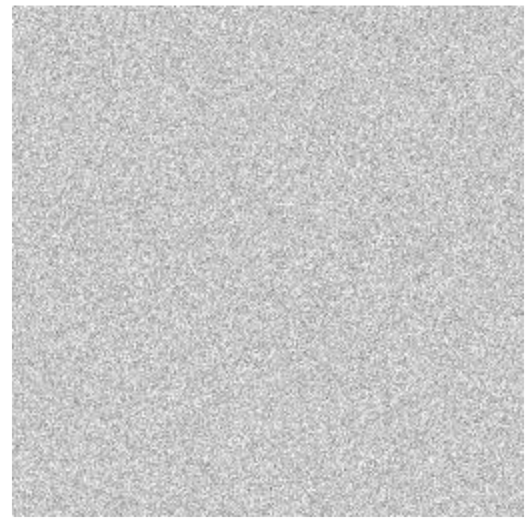
Based on a paper by Zoran and Weiss, “Scale Invariance and Noise in Natural Images”, ICCV 2011

Noise variance: a measure of how noisy a given image is

Consider a simple method

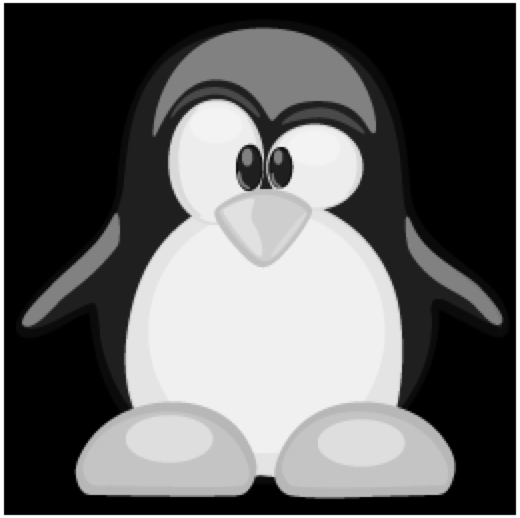
Constant intensity image with i.i.d. (additive) noise. How will you estimate the noise variance?

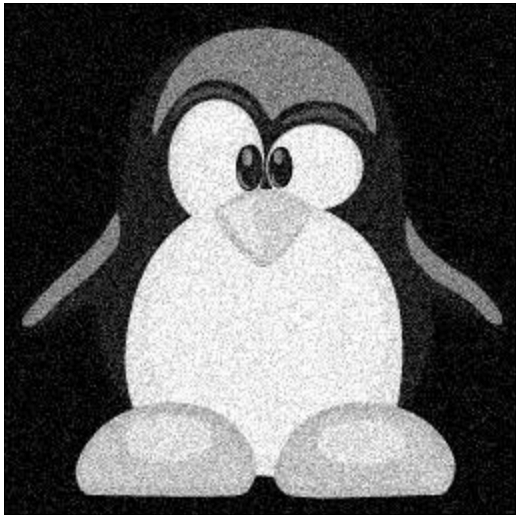


Noise variance = Variance of the image

Consider a simple method

Piece-wise flat intensity model (cartoon image): constant intensity regions separated by edges





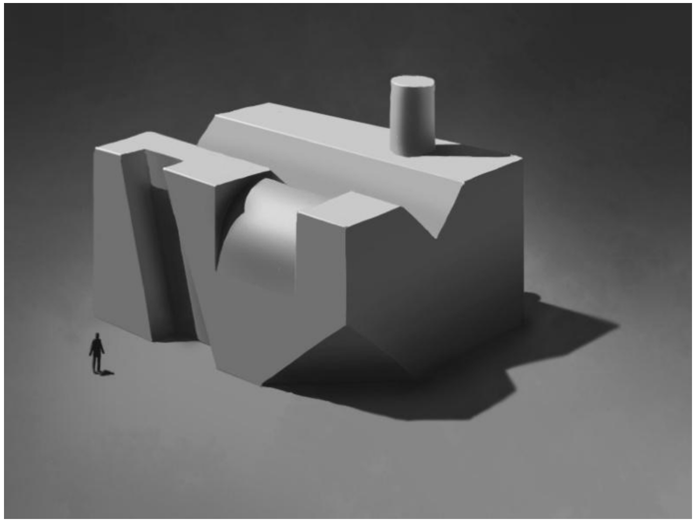
Compute variance of every (say) 16 x 16 patches in the image

Patches that contain edges will have higher variance than patches without edges

Variance of patches without edges = noise variance

Discard top K patches with highest variance – use the rest for estimating noise variance

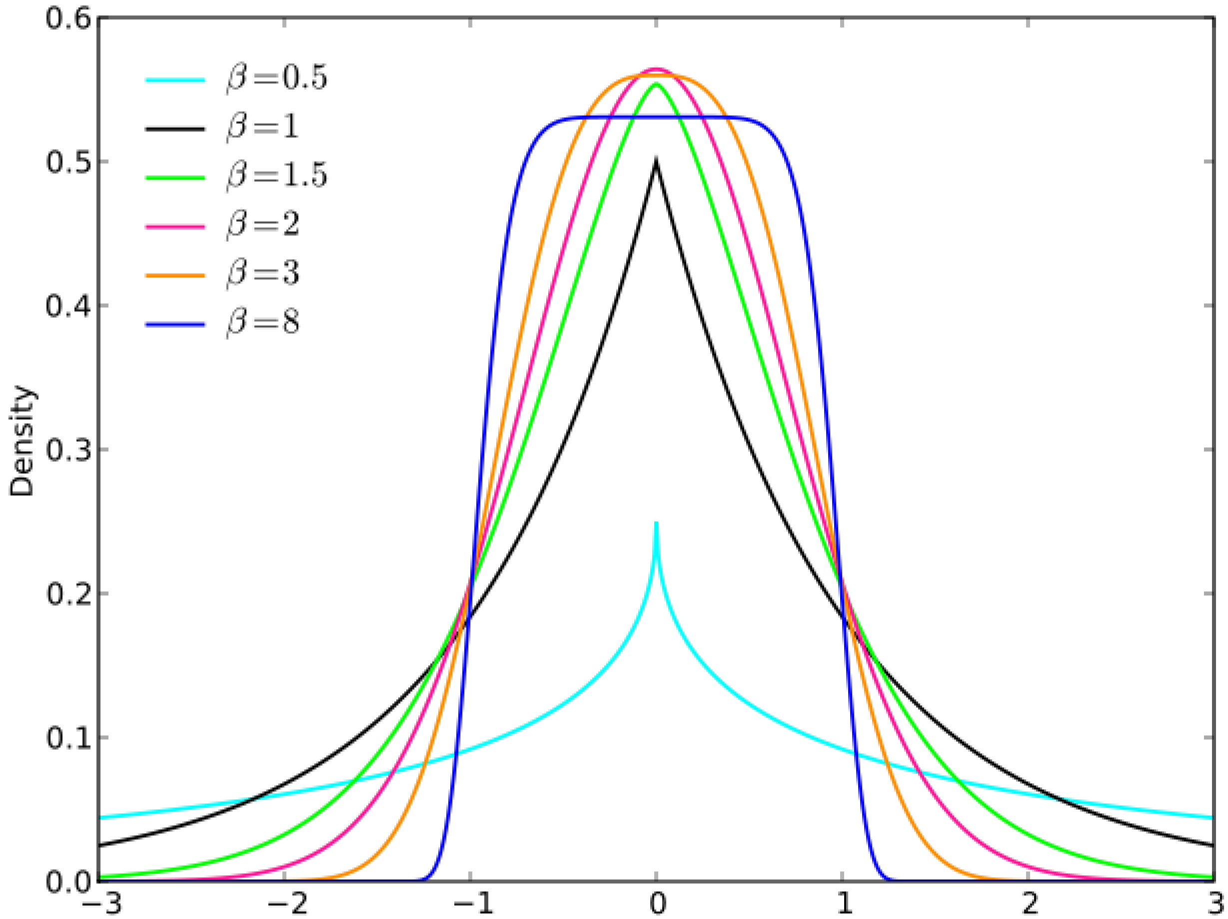
What's wrong with the method we just described?



Wavelet coefficients of natural images

- We know that they follow Generalized Gaussian Distribution with beta value between 0.5 and 1.
- Also true of DCT coefficients of natural images.

$$p(x) = \frac{\beta}{2\alpha\Gamma(1/\beta)} e^{-\left(\frac{|x-\mu|}{\alpha}\right)^{\beta}}$$



0.6
0.5
0.4
0.3
0.2
0.1
0.0
Density
−3
−2
−1
0
1
2
3
$\beta = 0.5$
$\beta = 1$
$\beta = 1.5$
$\beta = 2$
$\beta = 3$
$\beta = 8$

Kurtosis

- This is a measure of how peaky a distribution is, and how heavy its tails are.
- For random variable *x* with mean mu, kurtosis is defined as

$$\kappa_x = \frac{E[(x-\mu)^4]}{(E[(x-\mu)^2])^2}$$

- For $GG(x;\mu,\alpha,\beta)$, the kurtosis is given by

$$\kappa_x(\beta) = \frac{\Gamma(\frac{1}{\beta})\Gamma(\frac{5}{\beta})}{\Gamma(\frac{3}{\beta})^2}$$

- The higher the kurtosis, the heavier the tails. The Gaussian distribution has a kurtosis value of 3.

Observation

Ref: PhD thesis, Huang, Brown University

Histograms of DCT coefficients extracted from 8 x 8 patches of an image.

Barring the first coefficient, all the rest have distributions with very similar shapes (though the variances differ)

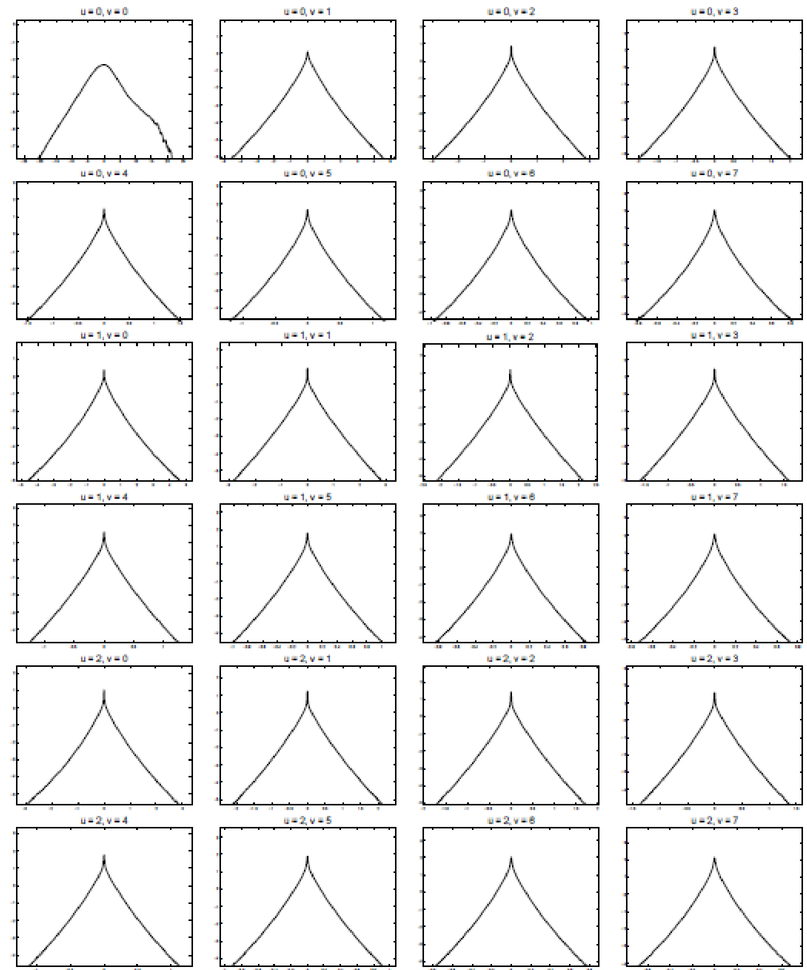
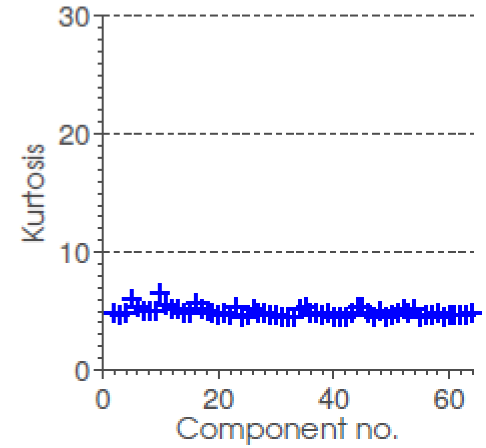


Figure 2.45: log(pdf) of DCT coefficients

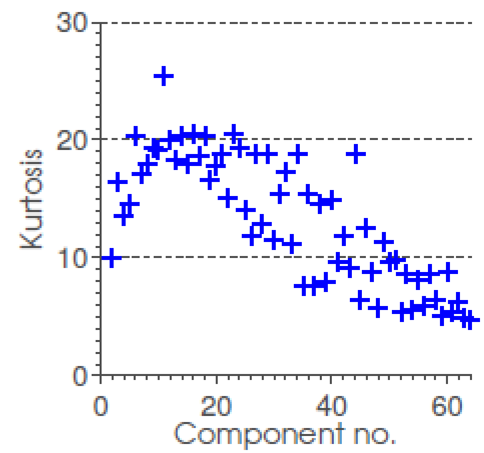
Ref: Zoran and Weiss, “Scale invariance and noise estimation in natural images”

Motivating Experiment (1)

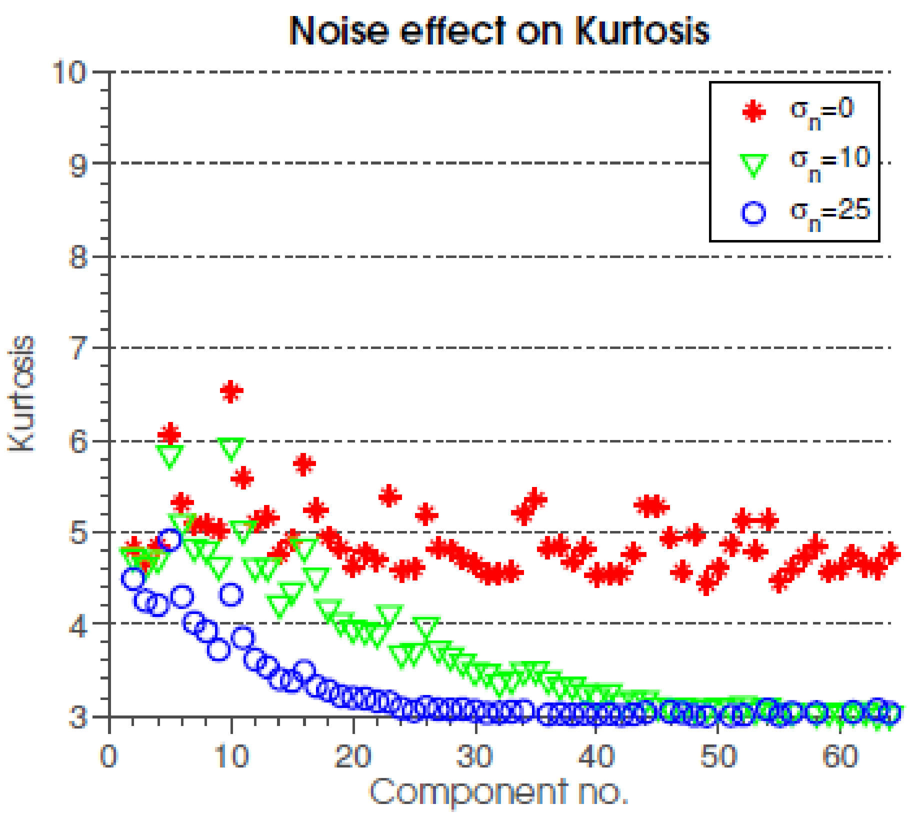
“Clean” natural image snapped under good lighting and with a good camera – kurtosis of all DCT coefficients is nearly the same



Existing natural image (slightly noisy): kurtosis of higher frequency components is much lower



Motivating Experiment (2)



More noise = sharper drop in kurtosis of higher frequency DCT coefficients

Conjecture!

- Kurtosis in clean natural images is constant across scale (frequency).
- Noise added to a clean image causes deviations from this behavior.
- This can be used to estimate variance of the noise present in a given image (e.g. Lena).

Method

Assume i.i.d. additive zero-mean Gaussian noise

$$x \sim GG(\mu, \sigma_x^2, \alpha)$$

$$y = x + \eta$$

Unknown ☹

$$\kappa = \frac{\mu_4}{\sigma^4}$$

$$\sigma_y^2 = \sigma_x^2 \left(1 + \frac{\sigma_n^2}{\sigma_x^2}\right)$$

$$\mu_4(y) = 3\sigma_x^4 \left(1 + \frac{\sigma_n^2}{\sigma_x^2}\right)^2 + \sigma_x^4 \left(\kappa_x(\alpha) - 3\right)$$

Can be measured from noisy image

$$\kappa_y = \frac{\kappa_x(\alpha) - 3}{\left(1 + \frac{\sigma_n^2}{\sigma_x^2}\right)^2} + 3$$

To be estimated

Method (continued)

- Divide the noisy image into N x N blocks and compute the DCT coefficients of each block.
- Find the statistics of each of the i=2 to N*N different DCT coefficients (across blocks)

$$\sigma_{y_i}^2 \qquad \kappa_{y_i}$$

- Solve the following:

Note that this is independent of the index i

$$\hat{\kappa}_x, \hat{\sigma}_n^2 = \underset{\kappa_x, \sigma_n^2}{\arg\min} \sum_{i=2}^{N^2} \left| \frac{\kappa_x - 3}{\left(1 + \frac{\sigma_n^2}{\hat{\sigma}_{y_i}^2 - \sigma_n^2}\right)^2} + 3 - \hat{\kappa}_{y_i} \right|$$

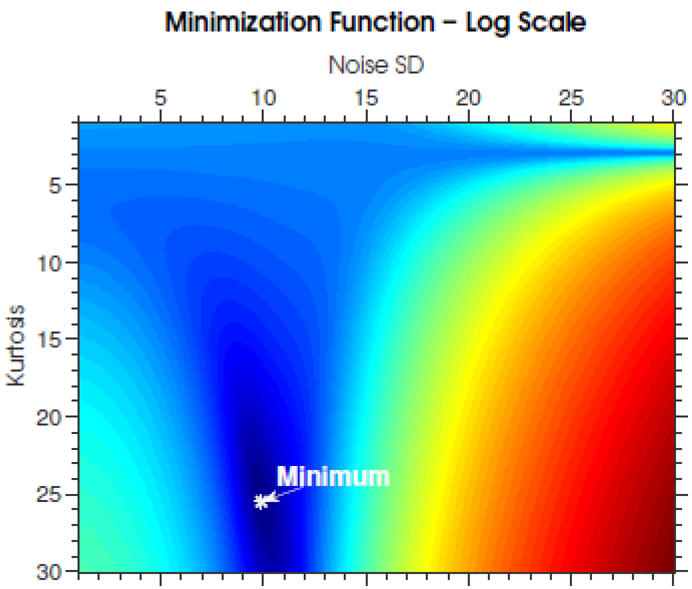
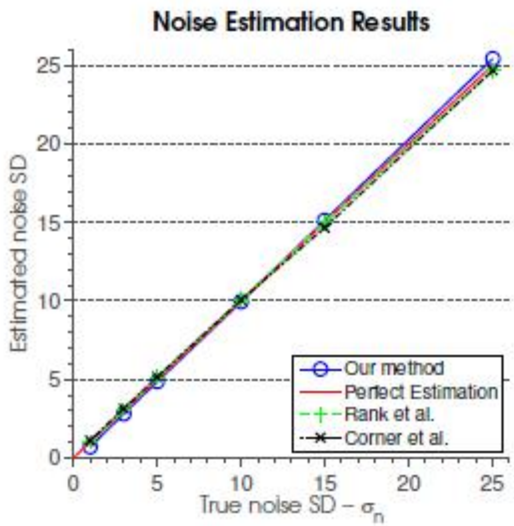
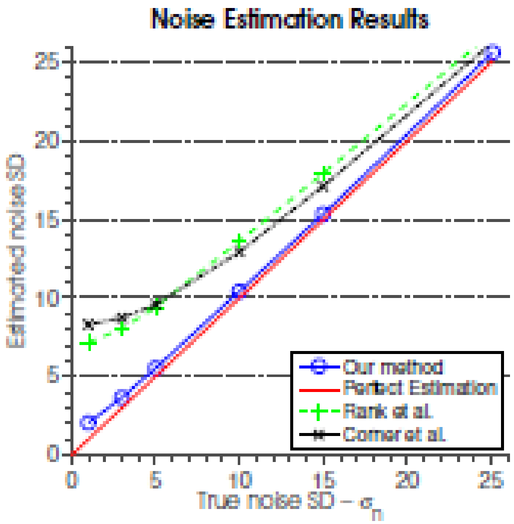


Figure 3: Function space for Eq. 3. The minimum is the actual point found numerically. Noise added to the image had a standard deviation of 10.

Results



Original Image



Original Image

Results

	BIRD			FIELD			Van Hateren			Berkeley		
σ_n	$\hat{\sigma}_n$	$\hat{\sigma}_n^R$	$\hat{\sigma}_n^C$	$\hat{\sigma}_n$	$\hat{\sigma}_n^R$	$\hat{\sigma}_n^C$	$\hat{\sigma}_n$	$\hat{\sigma}_n^R$	$\hat{\sigma}_n^C$	$\hat{\sigma}_n$	$\hat{\sigma}_n^R$	$\hat{\sigma}_n^C$
1	0.68	1.01	1.1	2.06	7.16	8.3	0.89 ± 0.71	2.02 ± 1.19	2.25 ± 1.1	1.5 ± 1.8	2.9 ± 2.8	3.7 ± 2.8
3	2.8	3.05	3.13	3.67	8.04	8.76	2.92 ± 0.62	3.64 ± 0.89	3.75 ± 0.72	2.9 ± 1.8	4.5 ± 2.3	4.9 ± 2.3
5	4.83	5.11	5.14	5.52	9.43	9.56	4.96 ± 0.59	5.47 ± 0.72	5.55 ± 0.52	4.9 ± 1.9	6.4 ± 2	6.6 ± 1.9
10	9.98	10.11	10.08	10.35	13.56	12.89	10.13 ± 0.89	10.2 ± 0.49	10.2 ± 0.48	9.7 ± 1.8	11.1 ± 1.6	11.1 ± 1.3
15	15.15	14.97	14.98	15.34	17.9	17.05	15.32 ± 1.13	15 ± 0.38	14.87 ± 0.62	14.7 ± 1.8	15.9 ± 1.3	15.7 ± 1.1
25	25.47	24.75	23.73	25.57	26.8	26.27	26.33 ± 1.81	24.65 ± 0.28	23.9 ± 1.47	24.8 ± 2.0	25 ± 1.0	25 ± 1.2
$\langle \varepsilon \rangle$	7%	**1%**	4%	**24%**	155%	176%	**3%**	22%	28%	**9.8%**	49%	65%

Table 1: Results summary for images and methods presented, for each image the first column is our method estimation $\hat{\sigma}_n$, the second Rank et al. $\hat{\sigma}_n^R$ and finally Corner et al. $\hat{\sigma}_n^C$. Last row is the average relative estimation error. On 50 images from the Van Hateren natural image database we obtain an average 3% error rate, while current state-of-the art method obtain 22% and 28%. Results for the Berkeley database (100 images) are similar, our method out-performs current state-of-the-art methods.

Related method

- Due to De Stefano et al, “Training methods for image noise level estimation on wavelet components”
- Assume wavelet coefficients follow a Laplacian distribution.

$$p(x) = \frac{1}{2\alpha} e^{-\left(\frac{|x-\mu|}{\alpha}\right)}$$

$$\kappa_x = 6 \because \beta = 1$$

Related method

$$y = x + \varepsilon$$

$$\sigma_y^2 = \sigma_x^2 + \sigma^2$$

$$\sigma_y^4 = 6\sigma_x^4 + 3\sigma^2 + 6\sigma_x^2\sigma^2$$

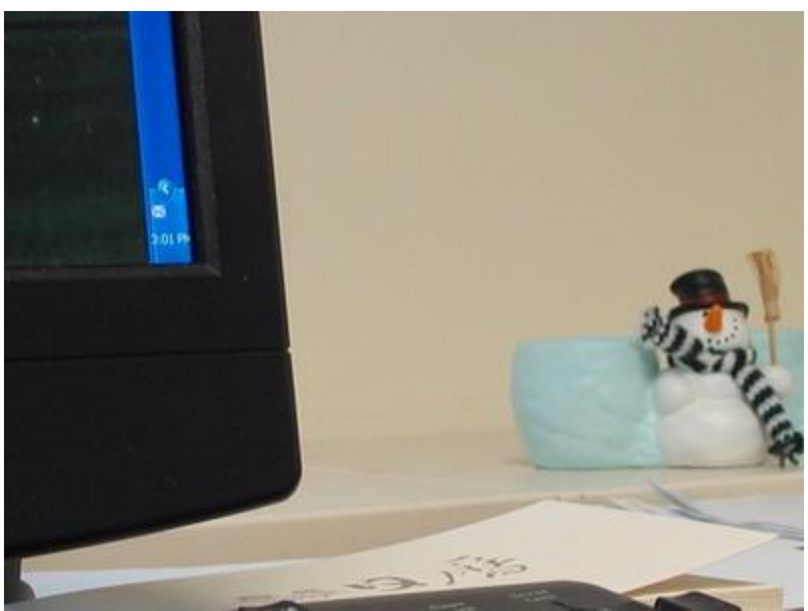
$$\therefore \sigma^2 = \sigma_y^2(1 - \sqrt{\frac{1}{3}\frac{\sigma_y^4}{\sigma_y^2} - 1})$$

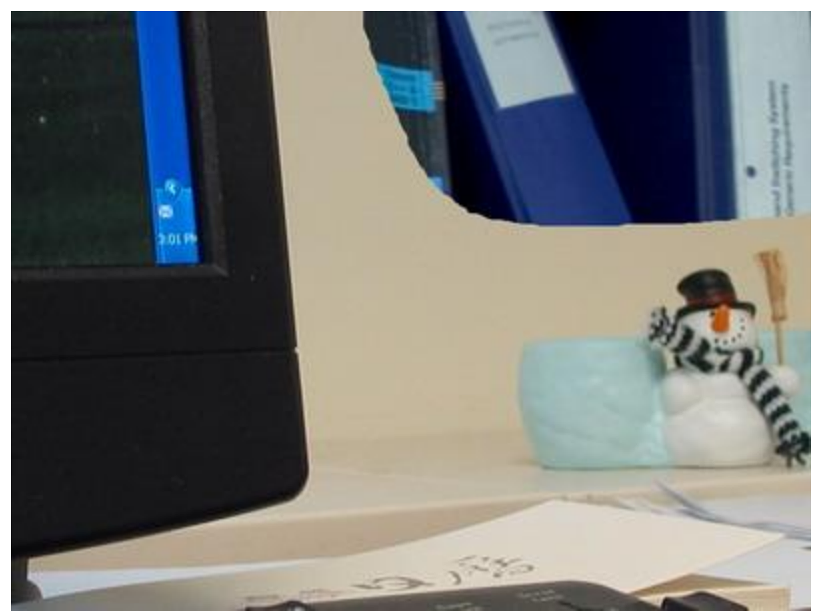
How does this method compare to that proposed by Zoran and Weiss?

Image Splicing

- Digital era: abundance of images and videos on the web.
- Image manipulation is rampant – one example is splicing.

http://www.ee.columbia.edu/ln/dvmm/downloads/authsplcuncmp/






WHIZZ
Germicidal
media 2004
speed. power.

How to detect splicing?

- Conjecture: noise within a given authentic image will have (more or less) constant variance – at least no abrupt changes in variance.
- Conjecture: noise within the spliced region will (usually) have hugely different variance from the noise outside the spliced region.

How to detect splicing?

- Consider a region around each pixel in an image. Estimate noise variance in that region using the local kurtosis and mean.
- Repeat for all pixels.
- You get an image of “noise variance” values.
- Presence of distinct segments in such an image will indicate splicing.

Ref: Pan et al, “Exposing image splicing with inconsistent local noise variances”

Noise Variance Estimation

- Very similar to the method by Zoran and Weiss we discussed earlier.

$$\tilde{\kappa}_k = \kappa_k \left(\frac{\tilde{\sigma}_k^2 - \sigma^2}{\tilde{\sigma}_k^2} \right)^2 . \qquad \sqrt{\tilde{\kappa}_k} = \sqrt{\kappa_k} \left(\frac{\tilde{\sigma}_k^2 - \sigma^2}{\tilde{\sigma}_k^2} \right) .$$

$$\sqrt{\kappa} = \frac{\left\langle \sqrt{\tilde{\kappa}_k} \right\rangle_k \left\langle \frac{1}{(\tilde{\sigma}_k^2)^2} \right\rangle_k - \left\langle \frac{\sqrt{\tilde{\kappa}_k}}{\tilde{\sigma}_k^2} \right\rangle_k \left\langle \frac{1}{\tilde{\sigma}_k^2} \right\rangle_k}{\left\langle \frac{1}{(\tilde{\sigma}_k^2)^2} \right\rangle_k - \left\langle \frac{1}{\tilde{\sigma}_k^2} \right\rangle_k^2}$$

$$\sigma^2 = \frac{1}{\left\langle \frac{1}{\tilde{\sigma}_k^2} \right\rangle_k} - \frac{1}{\sqrt{\kappa}} \frac{\left\langle \sqrt{\tilde{\kappa}_k} \right\rangle_k}{\left\langle \frac{1}{\tilde{\sigma}_k^2} \right\rangle_k},$$

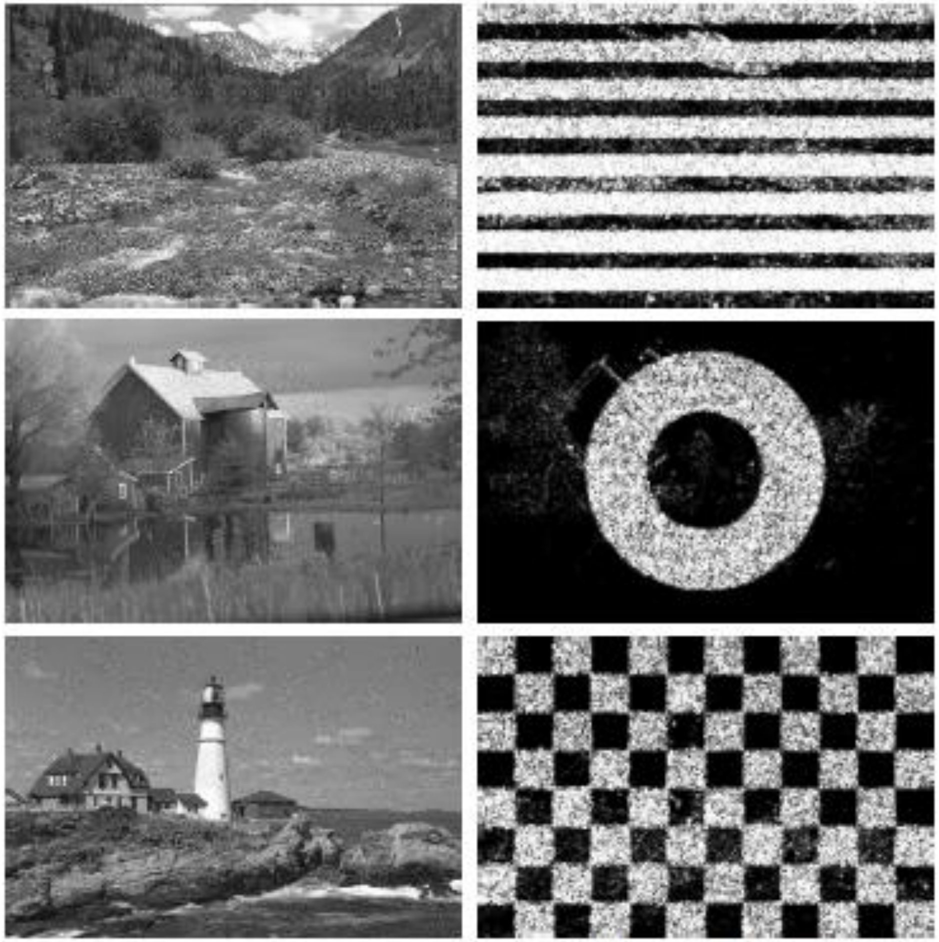


Figure 2. *Local noise variance estimation for three different images with different additive white Gaussian noise patterns.*

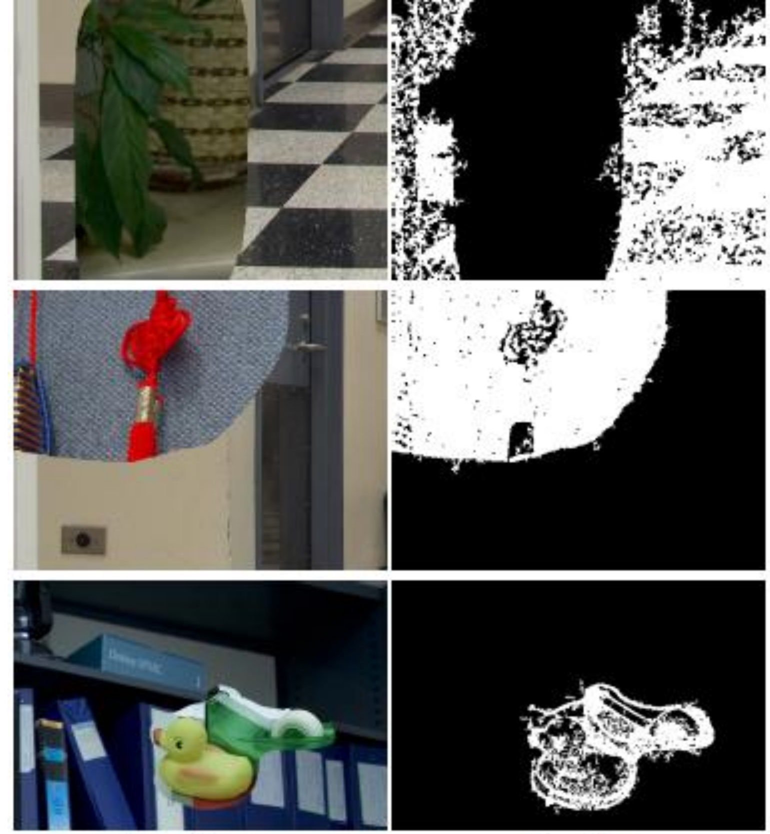


Figure 3. *Detection results of our method for three forged images with image splicing from the* Columbia uncompressed image splicing detection evaluation dataset *[13]. See text for details.*

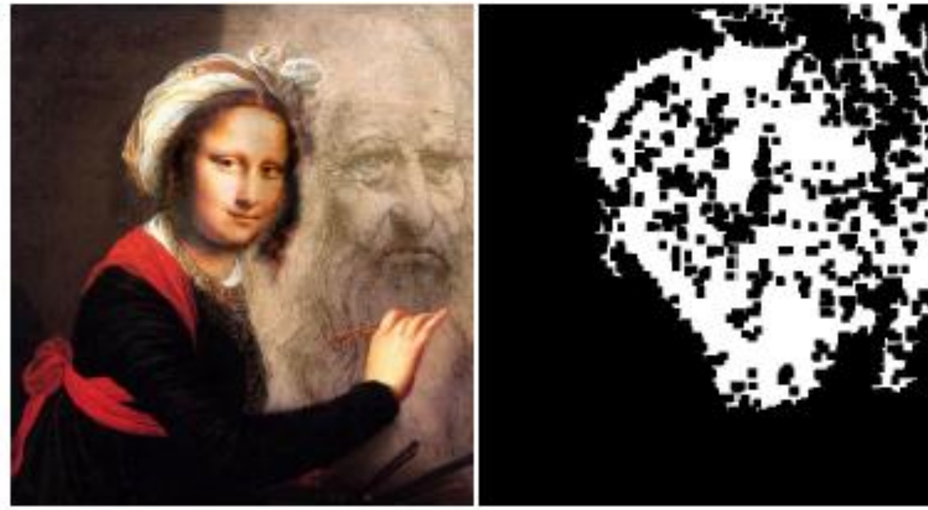


Figure 4. *Detection results of our method for a set of image splicing image forgeries from* `Worth1000.com`*. See text for details.*

Limitations

- Assume that (1) noise within an authentic image has (nearly) constant statistics, and (2) noise within the forged image is statistically different from that within the authentic image.
- JPEG compression or local changes in illumination can affect the stationarity of the noise within the image.