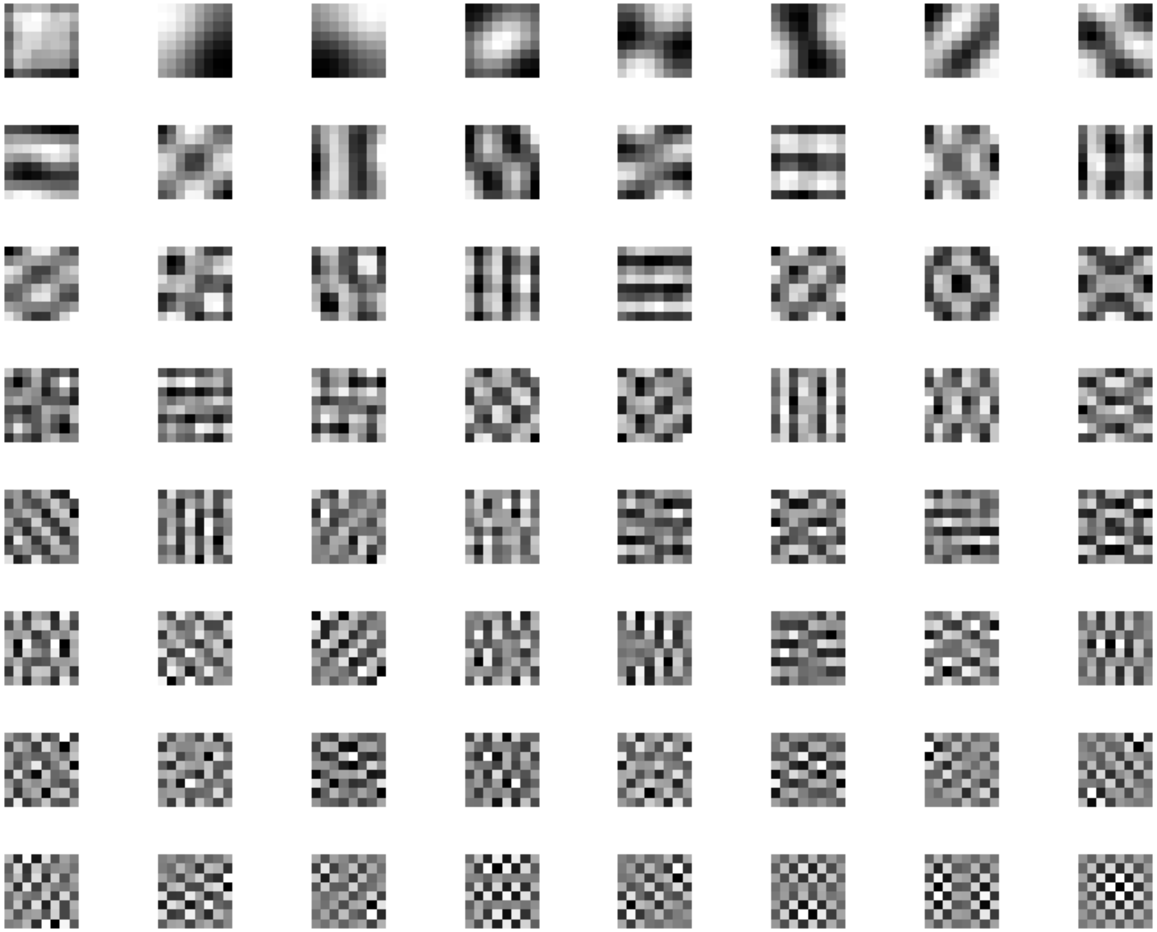
Principal Components of Natural Images

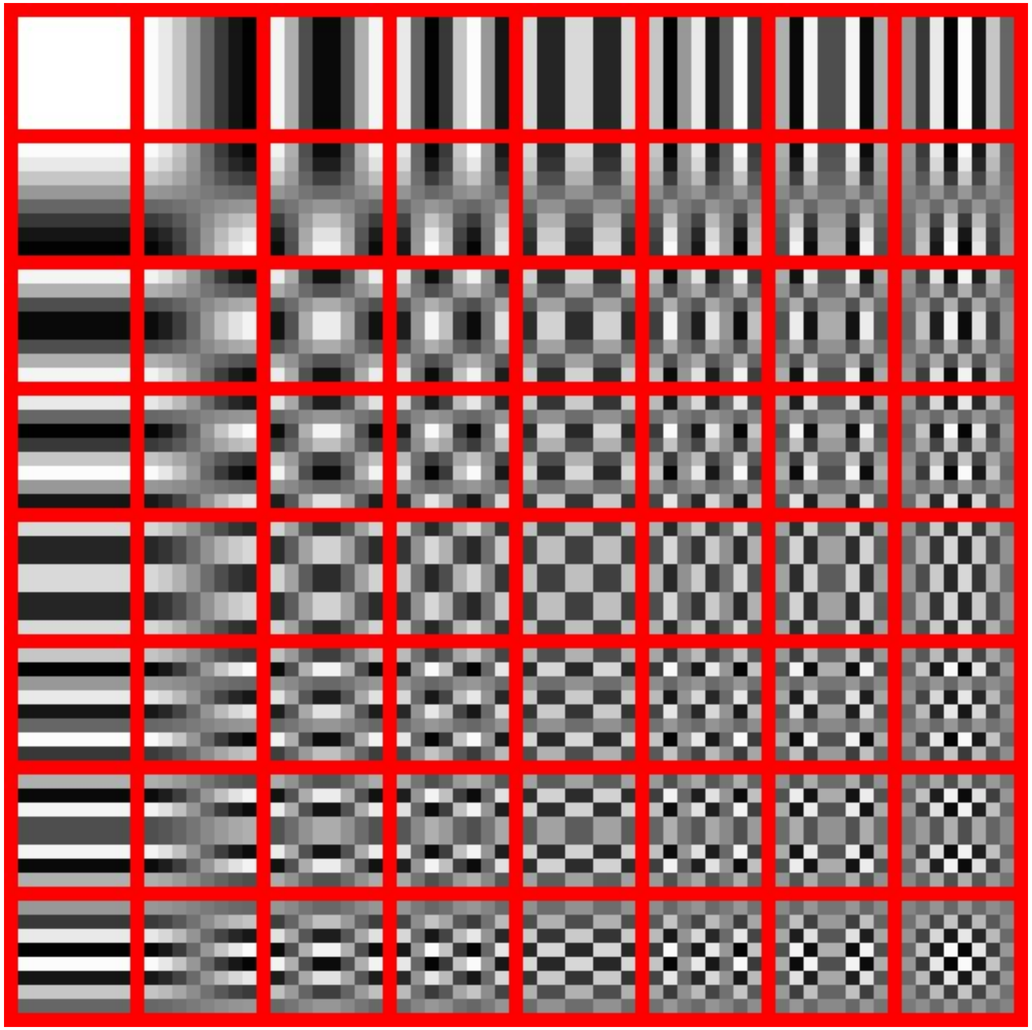
IT 530, Lecture Notes

Experiment

- Take several (more than 10^6) 8 x 8 patches from a database of natural images.
- Compute their covariance matrix (of size 64 x 64).
- Perform an eigen-decomposition of this matrix.
- Plot each of the 64 eigenvectors in the form of an 8 x 8 image (each image rescaled to the [0,1] intensity range).



These look very similar to Fourier (DCT) Bases.



DCT bases: source http://en.wikipedia.org/wiki/JPEG

Why?

- Natural images are translation-invariant: the correlations between two pixels depends only on the distance between them.

$$\mathrm{cov}(I(x,y), I(x',y')) = f((x-x')^2 + (y-y')^2)$$

Why?

- We will try to prove that the eigenvectors of a covariance matrix will be sinusoids, assuming the covariance matrix has the translation invariance property.
- To this end:

$$\sum_x \mathrm{cov}(x,x')\sin(x+\alpha)=\sum_x c(x-x')\sin(x+\alpha)=\sum_z c(z)\sin(z+x'+\alpha)$$

$z=x-x'$;

Assume 1D space

$$\sum_z c(z)\sin(z+x'+\alpha) = \sum_z c(z)(\sin(z)\cos(x'+\alpha)+\cos(z)\sin(x'+\alpha))$$
$$= [\sum_z c(z)\sin(z)]\cos(x'+\alpha) + [\sum_z c(z)\cos(z)]\sin(x'+\alpha)$$

$$\sum_x \mathrm{cov}(x,x')\sin(x+\alpha) = [\sum_z c(z)\cos(z)]\sin(x'+\alpha)$$

Eigenvector

Eigenvalue

Note that ***c(z)*** is even symmetric, whereas sin(***z***) is odd symmetric. So the first sum cancels out (***z*** takes positive as well as negative values)

$$\xi = x - x'$$
$$\eta = y - y'$$

Assume 2D space

$$\sum_{x,y} c((x-x')^2 + (y-y')^2)\sin(ax + by + c)$$
$$= \sum_{\xi,\eta} c(\xi,\eta)\sin(a\xi + b\eta + ax' + by' + c)$$
$$= \sum_{\xi,\eta} c(\xi,\eta)[\sin(a\xi + b\eta)\cos(ax' + by' + c) + \cos(a\xi + b\eta)\sin(ax' + by' + c)]$$
$$= 0 + [\sum_{\xi,\eta} c(\xi,\eta)\cos(a\xi + b\eta)]\sin(ax' + by' + c) \quad (5.35)$$