MTAT.05.008 Fou Math

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Homework # 02

Handed out:	Thu, Sep 12, 2022
Due:	Mon, Sep 26, 2022 (by 16:15)
In TeX as PDF by email to	`dotheis@ut.ee`
Subject:	`FOUMATH-HW02-`*lastname*

Problem 1

Simplify

(a) $(3-8i)(9+2i)$

(b) $(3-4i)^2$

(c) $\frac{1}{3-7i}$

(d) $\frac{9+2i}{i-4}$

(e) $(1-i)^3$

(f) $\frac{1}{3i}$

(g) $\frac{3i-2}{7-3i}$

(h) i^k, for $k=-1,0,1,2,3,4,5,15,25,34$

(i) i^k, for $k=-1,-2,-3,-4,-5$

(j) $\left(\frac{1}{3-i}\right)^2$

Let $z=3-4i$, $w=3i-7$. Find:

(k) $z+w$

(l) zw

(m) z/w

(n) w^*, z^*

(o) $|w|$, $|z|$

Problem 2

Solve the following system of equations in $z, w \in \mathbb{C}$:

$$\frac{z - w}{z^2 - w^2} = 1 + i \tag{2a}$$

$$\frac{z - iw}{z^2 + w^2} = 1 - i. \tag{2b}$$

(I.e., give complex number $z, w \in \mathbb{C}$ satisfying both equations.)

Problem 3

A mapping $f\colon \mathbb{C}^m \to \mathbb{C}^n$ is called $\mathbb{R}$-*linear,* if for all $\alpha, \beta \in \mathbb{R}$ and all $z, w \in \mathbb{C}^m$ we have

$$f(\alpha \cdot z + \beta \cdot w) = \alpha \cdot f(z) + \beta \cdot f(w) \tag{3a}$$

(where addition of vectors, and multiplication of vectors by scalars is defined as usual).

A mapping $f\colon \mathbb{C}^m \to \mathbb{C}^n$ is called $\mathbb{C}$-*linear,* if for all $\alpha, \beta \in \mathbb{C}$ and all $z, w \in \mathbb{C}^m$ we have

$$f(\alpha \cdot z + \beta \cdot w) = \alpha \cdot f(z) + \beta \cdot f(w) \tag{3b}$$

(where addition of vectors, and multiplication of vectors by scalars is defined as usual).

(a) Show that every $\mathbb{C}$-linear mapping is $\mathbb{R}$-linear.

(b) Show that for all $n \geq 1$, the mapping

$$c\colon \mathbb{C}^n \to \mathbb{C}^n \colon z \mapsto \left(z_j^*\right)_{j=1,\dots,n} \tag{3c}$$

("entry-wise complex conjugate") is $\mathbb{R}$-linear.

(c) Show that, for all $n \geq 1$, the mapping c is not $\mathbb{C}$-linear.

Problem 4

Prove the following.

(a) For all $z \in \mathbb{C}$: $\Re(z) = \dfrac{z + z^*}{2}$.

(b) For all $z \in \mathbb{C}$: $\Im(z) = \dfrac{z - z^*}{2i}$.

(c) For all $z, w \in \mathbb{C}$: $w^* \cdot z = 0$ implies $w = 0$ or $z = 0$.

P.S.: These equations/facts are important, they should be committed to memory.