MTAT.05.008 Fou Math

Assoc. Prof. Dirk Oliver Theis

# Homework # 03

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| Handed out: | 3 Oct, 2022 |
| Due: | 17 Oct, 2022 |
| In TeX as PDF by email to | `dotheis@ut.ee` |
| Subject: | `FOUMATH-HW03`–*lastname* |

## ***Multiplication with Special Matrices***

*You should thoroughly understand and memorize all the facts you're asked to figure out on this problem sheet.*

## Problem 1

For given $\alpha \in \mathbb{C}$, $n \in \mathbb{N}$, $k_0, \ell_0 \in \{1, \dots, n\}$, $k_0 \neq \ell_0$, consider the following matrix, $E^{(k_0,\ell_0)}$:

$$E^{(k_0,\ell_0)}_{k,\ell} = \begin{cases} 1, & \text{if } k = \ell; \\ \alpha & \text{if } k = k_0 \textit{ and } \ell = \ell_0; \\ 0, & \text{otherwise.} \end{cases}$$

Describe the effects of the following matrix multiplications, in one sentence each.

(a) Multiplying $E^*$ on the left to a generic complex $n$-by-$m$ matrix $X$, i.e., $Y = E^{(k_0,\ell_0)} \cdot X$;

(b) Multiplying $E^*$ on the right to a generic complex $m$-by-$n$ matrix $X$, i.e., $Y = X \cdot E^{(k_0,\ell_0)}$.

## Problem 2

A positive integer $n \in \mathbb{N}$ and a permutation $\pi$ on $\{1, \dots, n\}$ (i.e., $\pi\colon \{1, \dots, n\} \to \{1, \dots, n\}$ is a bijection) are given. Consider the following matrix, $E^{(\pi)}$:

$$E^{(\pi)}_{k,\ell} = \begin{cases} 1, & \text{if } k = \pi(\ell); \\ 0, & \text{otherwise.} \end{cases}$$

Describe the effects of the following matrix multiplications, in one sentence each.

(a) Multiplying $E^*$ on the left to a generic complex $n$-by-$m$ matrix $X$, i.e., $Y = E^{(\pi)} \cdot X$;

(b) Multiplying $E^*$ on the right to a generic complex $m$-by-$n$ matrix $X$, i.e., $Y = X \cdot E^{(\pi)}$.

# Problem 3

With the notations of Problem 2, for two permutations $\pi, \sigma$ describe the matrix $E^{(\pi)} \cdot E^{(\sigma)}$.

# Problem 4

Consider the $(n \times n)$-matrix $A$ which has, as its diagonal entries $A_{1,1} = \lambda_1$, $\dots$, $A_{n,n} = \lambda_n$, for complex numbers $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{C}$; all non-diagonal entries of $A$ are zero.

Describe the matrix $B := A^r$, for each $r = 0, 1, 2, \dots$.