PARTIAL SUBBLOCK WEIGHTING PTS SCHEME FOR PAPR REDUCTION OF OFDM SIGNALS

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Abstract

As one of the most promising schemes for peak-to-average power ratio (PAPR) of OFDM signals, partial transmit sequence (PTS) shows good PAPR reduction performance. However, large computational complexity is required for obtaining this good PAPR reduction performance in conventional PTS (CPTS). In this paper, a partial subblock weighting PTS (PSW-PTS) for PAPR reduction of OFDM signals is proposed. In the proposed PSW-PTS scheme, all the subblock sequences are divided into two parts: the original part and the weighted part, where only the subblock sequences in the weighted part are weighted by phase weighting factors. Moreover, two different sets of phase weighting factors are adopted to reduce computational complexity further. Simulation results show that compared with CPTS, the proposed PSW-PTS can obtain clear computational complexity reduction with similar PAPR reduction performance.

1 Introduction

Recently, orthogonal frequency division multiplexing (OFDM) has been one of the core techniques for wireless applications due to its high spectrum efficiency and good robustness against selective fading [1]. However, the transmitted OFDM signals have high peak-to-average power ratio (PAPR), which induces serious signal distortion and bit error rate. Therefore, it is very imperative to solve the PAPR problem of OFDM signals.

To improve the PAPR performance of OFDM signals, some schemes for PAPR reduction have been presented [2], such as block interleaving [3], window shaping [4], tone reservation (TR) [5], active constellation extension [6], clipping and filtering [7], companding [8,9], coding [10], phase optimization [11], selected mapping (SLM) [12] and partial transmit sequence (PTS) [13,14]. Among all the existing PAPR reduction schemes, PTS is one of probabilistic PAPR reduction schemes and the major aim is to reduce the probability of high PAPR existence. However, in order to achieve satisfactory PAPR reduction performance, large computational complexity is required in conventional PTS (CPTS) [2].

In this paper, a partial subblock weighting PTS (PSW-PTS) for PAPR reduction of OFDM signals is proposed. In the proposed PSW-PTS scheme, all the subblock sequences are divided into two parts, i.e., the original part and the weighted part. The subblock sequences in the original part are kept unchanged and only the subblock sequences in the weighted part are weighted by phase weighting factors. Moreover, two different sets of phase weighting factors are adopted in the weighted part for reducing computational complexity further. Simulation results show that compared with CPTS, the proposed PSW-PTS scheme can reduce computational complexity dramatically with similar PAPR reduction performance.

This paper is organised as follows. Section 2 gives the OFDM system model and conventional PTS. In Section 3, PSW-PTS scheme is proposed and its computational complexity is analysed. Section 4 gives simulation results to show the performance of PSW-PTS scheme. Finally, Section 5 gives a brief conclusion.

2 OFDM system model and conventional PTS

2.1 OFDM systems model

Assume $\boldsymbol{X} = [X_0, X_1, \cdots, X_{N-1}]$ represents an input data sequence modulated by M-ary phase shift keying (M-PSK) or M-ary quadrature amplitude modulation (M-QAM), where N denotes the number of subcarriers in an OFDM system. Thereupon, an OFDM signal $\boldsymbol{x} = [x_0, x_1, \cdots, x_{N-1}]$ in the discrete time domain can be expressed by

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, 0 \le n \le N-1 \tag{1}$$

where n is the discrete time index.

The PAPR of OFDM signal in the discrete time domain based on signal power can be defined by the ratio between the peak power and the average power of an OFDM signal, which can be expressed by

$$\mathrm{PAPR}(x_n) = 10\log_{10} \frac{\max\limits_{0\le n\le N-1} \{|x_n|^2\}}{E\{|x_n|^2\}} \mathrm{dB} \tag{2}$$

where $\max\{\cdot\}$ and $E\{\cdot\}$ represent the maximum operation and the expectation operation respectively.

Moreover, in order to evaluate and compare the performance of PAPR reduction schemes, complementary cumulative distribution function (CCDF) [15] is usually employed, which can be expressed by

$$\begin{aligned}\mathrm{CCDF}(N,\mathrm{PAPR}_0) &= \Pr\{\mathrm{PAPR} > \mathrm{PAPR}_0\} \\ &= 1-(1-e^{-\mathrm{PAPR}_0})^N\end{aligned} \tag{3}$$

where PAPR_0 denotes a given value of PAPR.

To approach the real PAPR performance, the oversampling is usually adopted in the discrete time domain, which can be achieved by LN-point inverse fast Fourier transform (IFFT) of symbol sequence with $(L-1)N$ zero-padding. When the oversampling factor L is four [16], the PAPR performance of discrete-time OFDM signals is almost the same as that of continuous-time OFDM signals.

2.2 Conventional PTS

In CPTS, the input symbol sequence in the frequency domain is firstly partitioned into several subblock sequences by employing the corresponding subblock partition methods, where only a part of subcarrier signals exist and the other ones are padded by zeros. Let $\boldsymbol{X}$ and V be the input symbol sequence and the number of subblock sequences respectively. After the subblock partition is completed, the input sequence can be expressed by

$$\boldsymbol{X} = \sum_{i=1}^{V} \boldsymbol{X}_i \tag{4}$$

where $\boldsymbol{X}_i$ denotes the ith subblock sequence.

For each subblock, a phase weighting factor is adopted for weighting it. Then, by employing IFFT operation, the candidate sequence $\boldsymbol{x}'$ can be obtained, given by

$$\boldsymbol{x}' = \mathrm{IFFT}\left\{\sum_{i=1}^{V} b_i \boldsymbol{X}_i\right\} = \sum_{i=1}^{V} b_i \cdot \mathrm{IFFT}\{\boldsymbol{X}_i\} = \sum_{i=1}^{V} b_i \boldsymbol{x}_i \tag{5}$$

where b_i denotes the phase weighting factor for ith subblock sequence and $\boldsymbol{x}_i$ represents the ith subblock sequence in the time domain.

Finally, among all the candidate sequences, the one with the lowest PAPR is chosen for transmitted. The block diagram of CPTS scheme is shown in Figure 1.

At the receiver, in order to recover the original data sequence successfully, the side information must be required. Assume there are W allowed phase weighting factors in CPTS. Thereupon, we can obtain W^{V-1} candidate sequences, where

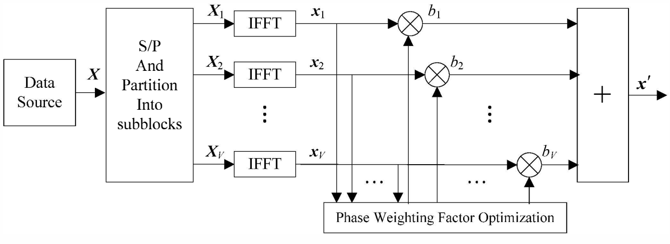

Figure 1: Block diagram of CPTS scheme

V denotes the number of subblock sequences. Thus, $\lceil \log_2 W^{V-1} \rceil$ bits are needed to represent this side information, where $\lceil\cdot\rceil$ denotes the element to the nearest integers toward infinity.

3 Partial subblock weighting PTS

In this section, a partial subblock weighting PTS (PSW-PTS) scheme for improving the PAPR problem of OFDM signals is proposed, which aims to obtain good PAPR reduction performance and clear computational complexity reduction compared with CPTS. In proposed PSW-PTS scheme, all the subblocks are divided into two parts, i.e., the original part and the weighted part. Moreover, two different sets of phase weighting factors are adopted for reducing computational complexity.

3.1 The ideas of proposed PSW-PTS

In proposed PSW-PTS scheme, just like CPTS, the input data sequence is firstly partitioned into several subblock sequences. After the subblock partition is completed, all the subblock sequences are divided into two parts, i.e., the original part and the weighted part. It is known that without any performance loss, the phase weighting factor for the first subblock sequence is one. Hence, for all the subblock sequences, the first half can be viewed as the original part and the weighted part consists of the rest subblocks. For example, when the number of subblock sequences is four (i.e., $\boldsymbol{x}_1$, $\boldsymbol{x}_2$, $\boldsymbol{x}_3$, $\boldsymbol{x}_4$), the two parts can be given as follows.

The original part: $\boldsymbol{x}_1$, $\boldsymbol{x}_2$
The weighted part: $\boldsymbol{x}_3$, $\boldsymbol{x}_4$

Then, for each part, the subcandidate sequences could be generated. In the original part, since the subblock sequences remain unchanged, we can obtain the only one subcandidate sequence, which can be directly obtained by the combination of subblock sequences in this part. For the weighted part, since the subblock sequences need to be weighted by phase weighting factors, the corresponding phase weighting factors should be generated. Here, two different sets of phase weighting factors are adopted, i.e., $\{1, -1\}$ and $\{j, -j\}$. Let $\boldsymbol{h}_i$, $i = 1, 2, \cdots, M$ and $\boldsymbol{k}_l$, $l = 1, 2, \cdots, P$ be the subcandidate sequences generated by using two different sets of phase weighting factors, where M and P represent the number of subcandidate sequences. Based on the linearity of IFFT, the subcandidate sequences $\boldsymbol{h}_i$ and $\boldsymbol{k}_l$ can be directly used for obtaining an additional subcandidate sequence without any phase weighting process, given by

$$\boldsymbol{y}_{i,l} = \frac{\sqrt{2}}{2}(\boldsymbol{h}_i + \boldsymbol{k}_l) \quad (i = 1, 2, \cdots, M; l = 1, 2, \cdots P) \tag{6}$$

where $\boldsymbol{y}_{i,l}$ denotes a subcandidate sequence generated by the combination of $\boldsymbol{h}_i$ and $\boldsymbol{k}_l$.

For the subcandidate sequences $\boldsymbol{h}_i$ and $\boldsymbol{k}_l$, phase weighting factors for generating them come from the sets of phase weighting factors $\{1, -1\}$ and $\{j, -j)$ respectively. Hence, for the subcandidate sequences $\boldsymbol{y}_{i,l}$, if phase weighting factors are equivalently incorporated, the factors should be come from the set of phase weighting factors {1+*j*, 1−*j*, −1+*j*, −1−*j*}. However, because the factors for weighting the subblock sequences have the non-unit magnitude, $\sqrt{2}/2$ must be multiplied to revise the magnitude of phase weighting factor and obtain the unit magnitude, shown in Eq.(6).

After all the subcandidate sequences are obtained, the subcandidate sequence from the original part is combined with each subcandidate sequence from the weighted part to obtain OFDM candidate sequence. Finally, among all the OFDM candidate sequences, the one with the minimum PAPR is selected for transmitting.

For easily understanding the basic ideas of proposed PSW-PTS, we still take the number of subblocks $V = 4$ as an example. Based on the above discussions, when the number of subblock sequences is four, the subblocks $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ are in the original part and the weighted part consists of the rest ones. Hence, we can achieve the subcandidate sequences of these two parts as follows.

The original part: $\boldsymbol{y}' = \boldsymbol{x}_1 + \boldsymbol{x}_2$

The weighted part: $\boldsymbol{h}_1 = \boldsymbol{x}_3 + \boldsymbol{x}_4$, $\boldsymbol{h}_2 = \boldsymbol{x}_3 - \boldsymbol{x}_4$, $\boldsymbol{h}_3 = -\boldsymbol{x}_3 + \boldsymbol{x}_4$, $\boldsymbol{h}_4 = -\boldsymbol{x}_3 - \boldsymbol{x}_4$, $\boldsymbol{k}_1 = j\boldsymbol{x}_3 + j\boldsymbol{x}_4$, $\boldsymbol{k}_2 = j\boldsymbol{x}_3 - j\boldsymbol{x}_4$, $\boldsymbol{k}_3 = -j\boldsymbol{x}_3 + j\boldsymbol{x}_4$, $\boldsymbol{k}_4 = -j\boldsymbol{x}_3 - j\boldsymbol{x}_4$,

$$\boldsymbol{y}_{i,l} = \sqrt{2}\left(\boldsymbol{h}_i + \boldsymbol{k}_l\right)/2, i,l = 1,2,3,4$$

By combining the subcandidate sequences from these two parts, all the OFDM candidate sequences can be obtained, given by

$$\begin{cases} \boldsymbol{y}' + \boldsymbol{h}_i, i = 1,2,3,4 \\ \boldsymbol{y}' + \boldsymbol{k}_i, i = 1,2,3,4 \\ \boldsymbol{y}' + \boldsymbol{y}_{i,l}, i,l = 1,2,3,4 \end{cases} \tag{7}$$

Thus, when the number of subblock sequences is four, we can obtain twenty-four OFDM candidate sequences at most. It is worth mentioning that the number of the generated subcandidate sequences in the weighted part can be adjusted in terms of the practical requirements. Moreover, the number of OFDM candidate sequences is mainly dominated by the number of subcandidate sequences in the weighted part.

For proposed PSW-PTS scheme, the side information is also required to realize the successful recovery of the original data sequence, which can be transmitted by accompanying with the transmitted OFDM signal. We assume that C subcandidate sequences are generated in the weighted part, which means that C OFDM candidate sequences can be obtained in the proposed PSW-PTS. Thus, $\lceil \log_2 C \rceil$ bits are required to represent this side information.

3.2 Computational complexity analysis

In proposed PSW-PTS scheme, the computational complexity reduction is mainly dominated by the generation of additional subcandidate sequences in the weighted part. That is to say, the more additional subcandidate sequences are generated, the more reduction in computational complexity can be obtained. Since the number of IFFT operations needed in proposed PSW-PTS scheme is the same as that in CPTS, the number of complex multiplications and complex additions needed in the generations of OFDM candidate sequences is only taken into account.

For CPTS, let V and W be the number of subblock sequences and the number of allowed phase weighting factors respectively. The computational complexity of CPTS can be given in terms of complex multiplication and complex additions as follows.

Complex Mul.: $LN(V-1)W^{V-1}$

Complex Add.: $LN(V-1)W^{V-1}$

where N is the number of subcarriers in an OFDM system and L is the oversampling factor.

For proposed PSW-PTS scheme, assume the weighted part consists of U subblocks and C subcandidate sequences are generated in this part, where the number of additional subcandidate sequences is D. Based on the above assumption, C OFDM candidate sequences can be obtained and the computational complexity of proposed PSW-PTS can be shown in terms of complex multiplication and complex addition as follows.

Complex Mul.: $LN(CU-DU+D)$

Complex Add.: $LN(V-U-1+CU-DU+2D)$

To show the advantage of proposed PSW-PTS scheme in computational complexity reduction against CPTS, computational complexity reduction ratio (CCRR) [14] is usually adopted, defined by

$$\mathrm{CCRR}(\%) = \left(1 - \frac{\text{Complexity of Proposed PSW-PTS}}{\text{Complexity of CPTS}}\right) \times 100\% \tag{8}$$

4 Simulation results

In this section, extensive simulations for showing the performance of proposed PSW-PTS scheme are done. In simulations, an OFDM system with 256 subcarriers ($N = 256$) is considered and the oversampling factor L is four. In CPTS, the set of phase weighting factors is {1, −1} (i.e., $W = 2$).

Figure 2 shows the CCDFs of proposed PSW-PTS and CPTS with the number of subblocks $V = 4$, 6 and 8 in the OFDM system employing QPSK modulation. For a comparison purpose, when the number of subblocks V is fixed, the same number of OFDM candidate sequences is generated in both proposed PSW-PTS and CPTS.

It can be seen from Figure 2 that, compared with CPTS, the proposed PSW-PTS has a slight degradation of PAPR reduction performance with the same number of OFDM candidate sequences. That is due to the fact that the additional subcandidate sequences in the weighted part are obtained by the combination of two subcandidate sequences, which increases the correlation of candidate sequences. However, this negligible degradation of PAPR reduction performance is

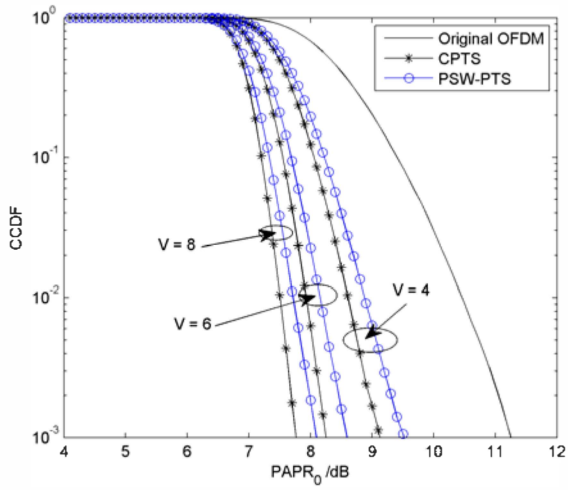

Figure 2: CCDFs of proposed PSW-PTS and CPTS in OFDM system with QPSK

the price for significant reduction in computational complexity.

Meanwhile, Figure 3 gives the CCDFs of proposed PSW-PTS and CPTS in OFDM system employing 16QAM modulation. The similar results can be achieved by analysing the curves of CCDF in Figure 3.

Based on the parameters of OFDM system in the above simulations, a comparison of computational complexity between proposed PSW-PTS and CPTS is considered in terms of CCRR, shown in Table 1.

As we can seen in Table 1, compared with CPTS, the proposed PSW-PTS can obtain clear computational complexity reduction with the same number of OFDM candidate sequences. For example, when the number of subblock sequences is eight, 78.8% reduction in complex multiplication and 66.4% reduction in complex addition can be obtained. With the number of subblock sequences and the

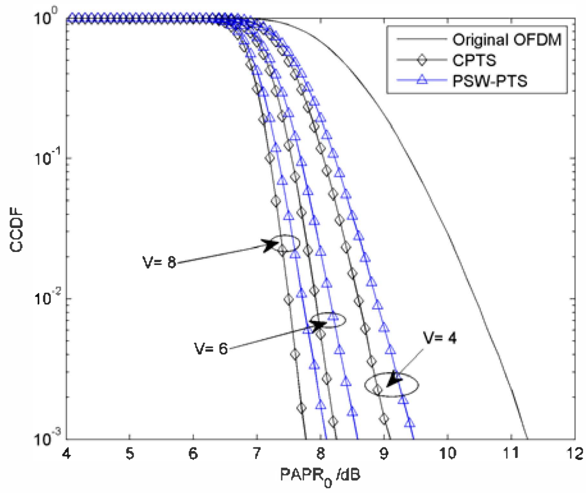

Figure 3: CCDFs of proposed PSW-PTS and CPTS in OFDM system with 16QAM

	$V = 4$	$V = 6$	$V = 8$
CCRR(Complex Mul.)	50%	67.5%	78.8%
CCRR(Complex Add.)	29.2%	52.5%	66.4%

Table 1: CCRRs of proposed PSW-PTS and CPTS

number of candidate sequences increasing, more and more reduction in computational complexity can be obtained.

5 Conclusion

In this paper, a PSW-PTS scheme for PAPR reduction of OFDM signals is proposed. In proposed PSW-PTS scheme, all the subblock sequences are divided into two parts: the original part and the weighted part. Only the subblock sequences in the weighted part are weighted by phase weighting factors. Moreover, two different sets of phase weighting factors are adopted in the weighted part for reducing computational complexity further. Simulation results show that compared with CPTS, the proposed PSW-PTS scheme can achieve clear computational complexity reduction with similar PAPR reduction performance.

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