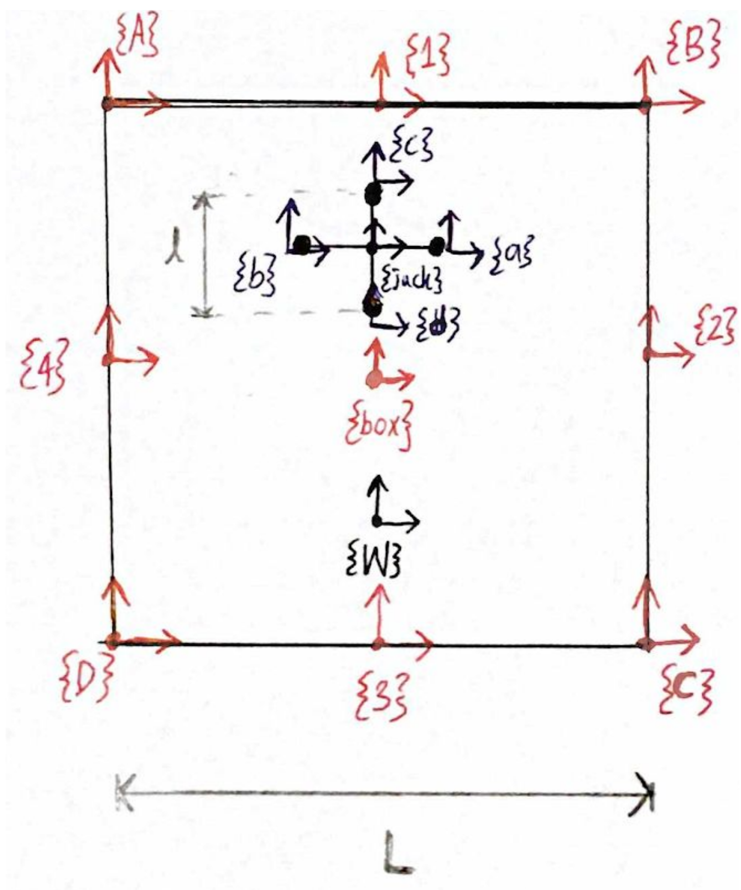
Dynamic Simulation of the Jack in the Box

ME 314
Dongho (DK) Kang

Introduction

I choose the default project which simulates a "jack" made of 4 point mass bouncing around in the rectangular box (or cup) experiencing collision and external forces.



Vertexes of the bot were represented as {A}, {B}, {C}, and {D} while the edges of it were done as {1}, {2}, {3}, and {4}. Four point masses of jack were shown as {a}, {b}, {c}, and {d} while the World frame is shown as {W}.

Nomenclature

L : *length of the box*
l : *length (diameter) of the jack*
m_b : *mass of the box*
m : *mass of the jack*
I_1 : *inertia tensor of the box*
I_2 : *inertia tensor of the jack*
V_1 = *velocity of the box*
V_2 = *velocity of the jack*

Calculation

Configuration:

Both the box and jack are free to move around in a xy-plane and rotate

$$q = \begin{bmatrix} \mathrm{x_{box}}(t) \\ \mathrm{y_{box}}(t) \\ \theta_{box}(t) \\ \mathrm{x_{jack}}(t) \\ \mathrm{y_{jack}}(t) \\ \theta_{jack}(t) \end{bmatrix} \quad \dot{q} = \begin{bmatrix} \frac{d}{dt}\mathrm{x_{box}}(t) \\ \frac{d}{dt}\mathrm{y_{box}}(t) \\ \frac{d}{dt}\theta_{box}(t) \\ \frac{d}{dt}\mathrm{x_{jack}}(t) \\ \frac{d}{dt}\mathrm{y_{jack}}(t) \\ \frac{d}{dt}\theta_{jack}(t) \end{bmatrix} \quad \ddot{q} = \begin{bmatrix} \frac{d^2}{dt^2}\mathrm{x_{box}}(t) \\ \frac{d^2}{dt^2}\mathrm{y_{box}}(t) \\ \frac{d^2}{dt^2}\theta_{box}(t) \\ \frac{d^2}{dt^2}\mathrm{x_{jack}}(t) \\ \frac{d^2}{dt^2}\mathrm{y_{jack}}(t) \\ \frac{d^2}{dt^2}\theta_{jack}(t) \end{bmatrix}$$

Lagrangian:

$$\widehat{V}_1 = \left(g_{boxW} * \tfrac{d}{dt} g_{Wbox}\right)$$
$$\widehat{V}_2 = \left(g_{jackW} * \tfrac{d}{dt} g_{Wjack}\right)$$

- Using $R^T\omega = R^T\widehat{\omega}R$ and $\widehat{\omega}p = -\widehat{p}\omega$ we can calculate V_1 and V_2

- I decided that gravity won't affect the box so that it will not fall along the jack, and this setup can be valid if we assume that someone is holding the box on his hand.

$KE = 0.5 * V_1^T * I_1 * V_1 + 4 * (0.5 * V_2^T * I_2 * V_2)$ and $PE = 4 * (mgy_{jack})$

- Therefore, we have

$$L = KE - PE$$

External force:

- External force was applied assuming someone is shaking the box
- To ensure that jack hit all four edges of the box, the external force was applied in x-direction since gravitational force will take y-direction
- To make sure jack hit the all four edges of the box,
- Using cosine allows the force to change its direction to replicate shaking, and it can be expressed as:

$$F = F_0 cos(\omega t)$$

- Force of 10*t was applied to y-direction to mitigate the downward displacement of the box due to collision with the jack

- It can be shown in the matrix form accordingly with the configuration of this system

$$F_{ext} = [F,\ 10t,\ 0,\ 0,\ 0,\ 0,\ 0]^T$$

Constraint:

- The jack has the constraint that it has to stay within the boundaries of the box.
- Phi defines when the point masses of the jack make contacts with the boundaries of the box which are {1}, {2}, {3}, and {4}. There are multiple phi called phi_list in this system, and they are called in a for-loop so that it eliminates the necessity of making multiple impact update functions.
- Phi can be expressed with g matrix while $g_{ij}[3]$ represents x-component and $g_{ij}[7]$ does y-component

$$\begin{aligned}\phi_{list} = \ & [g1a[7], g1b[7], g1c[7], g1d[7],\\ & g2a[3], g2b[3], g2c[3], g2d[3],\\ & g3a[7], g3b[7], g3c[7], g3d[7],\\ & g4a[3], g4b[3], g4c[3], g4d[3]]\end{aligned}$$

$for\ \phi_{list}\ in\ \phi_n, \quad \phi = \phi_n$

Euler-Lagrangian:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = F_{ext} + \lambda \nabla \phi_n$$

and

$$\ddot{\phi}_n = 0$$

Impact Update Laws:

- Dummy variables and new configuration setup

$$q_{dummy} = [x_{box}\, y_{box}\, \theta_{box}\, x_{jack}\, y_{jack}\, \theta_{jack}]^T$$

$$q^- = [x_{box}^-\ y_{box}^-\ \theta_{box}^-\ x_{jack}^-\ y_{jack}^-\ \theta_{jack}^-\ ]^T$$
$$q^+ = [x_{box}^+\ y_{box}^+\ \theta_{box}^+\ x_{jack}^+\ y_{jack}^+\ \theta_{jack}^+\ ]^T$$

$$\dot{q}^- = \dot{q}.subs(q^-)$$
$$\dot{q}^+ = \dot{q}.subs(q^+)$$

- Hamiltonian

$$p = \frac{dL}{d\dot{q}}$$
$$H = p\dot{q} - L$$

$$H_{dummy} = H.subs(q_{dummy})$$
$$H^+ = H.subs(q^+)$$
$$\frac{d\phi}{dq_{dummy}} = \frac{d\phi}{dq} \cdot subs(q_{dummy})$$

- Impact equation

$$impact = \begin{bmatrix} \frac{dL}{d\dot{q}^+} - \frac{dL}{d\dot{q}_{dummy}} \\ H^+ - H_{dummy} \end{bmatrix} = \begin{bmatrix} \lambda\, d\phi / dq \\ 0 \end{bmatrix}$$

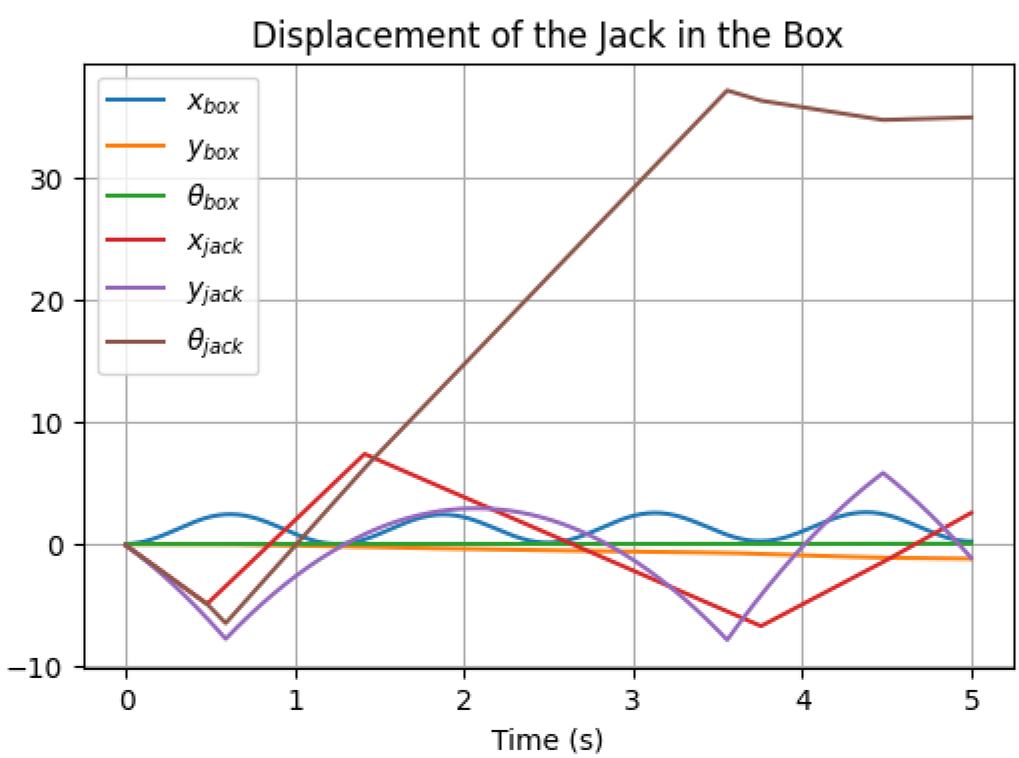
- Function ‘impact_condition’ takes phi_list and initial conditions which includes initial positions and velocities, lambdifies phi values, and tolerance to decide the impact condition.
- Function ‘impact_update takes initial conditions and impact equations to numerically update impacts

Discussion

Physics Laws in this Simulation:

- Gravity applied to the jack
- No gravity applied to the box
- No friction
- Elastic collision

I believe that the result of the simulation is accurate. The behaviors of the box and jack followed the laws of physics that I set in the simulation most of the time. Despite the overall success of the simulation, it sometimes showed abnormality under specific conditions. When the relative speed between the jack and the box is large, the jack often gets stuck at the boundary lines (edges) of the box or even breaks its constraint and escapes from the box. It happened when the jack gained large acceleration from frequent collisions or a large amount of the external force were applied.



The graph above represents the 2D positions and rotations of the jack and the box during the simulation for 5 seconds. The initial condition was set to [0, 0, 0, 0, 0, 0, 0, -10, 0, -10, 0, -10,-5] so that jack will have some initial velocities and rotations to prevent the jack from bouncing without rotating and hitting the all four edges of the box. From the drastic changes of the slopes of x_jack and y_jack, we can distinctly know that the jack experiences impacts. When the jack hits the top edge, the y_jack slope shows a sharp change while it shows a smooth curve when it doesn't collide with the top, and we can see that gravity is properly acting on the jack.Moreover, we can see from the theta_jack slope that the jack experiences rotation, and its direction can change upon impact. X-displacement of the box appears accordingly with the external force applied which was (12,000 cos(5t)) N in x-direction. Since the mass of the box is 400 kg while the mass of the jack is 1 kg, the jack's collision barely affects displacement and rotation of the box, and y_box does not show significant changes in its displacement since there is no gravitational force applied to the box. However, the y-displacement of the box experiences some gradual and small changes with a negative rate of change, and this is due to gravitational force acting on the jack makes the jack to collide with the bottom edge more than the top edge. These behaviors are expected.

The animation of the simulation demonstrated reasonable results. The jack starts to fall down to the bottom left, as negative initial velocities were set to x and y directions. The jack successfully demonstrates impact with all four edges of the box showing that constraint is working properly. While the box oscillates in x-direction due to the external force, it gradually moves downward as the jack collides with the bottom edge. The animation accurately demonstrates the result of the simulation.