Spectral Centrality Measures in Complex Networks

Review of the paper released by Nicola Perra and Santo Fortunato

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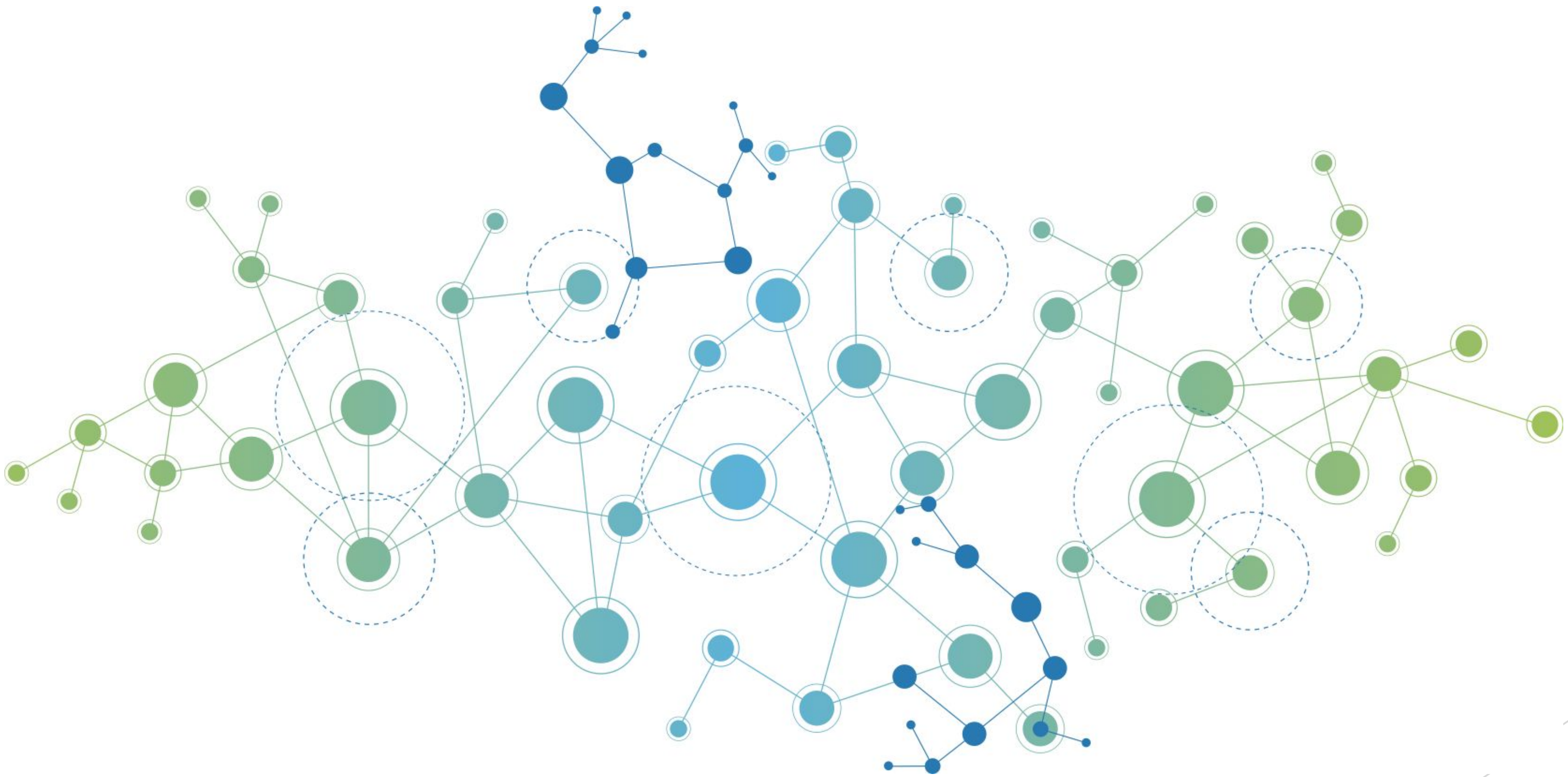
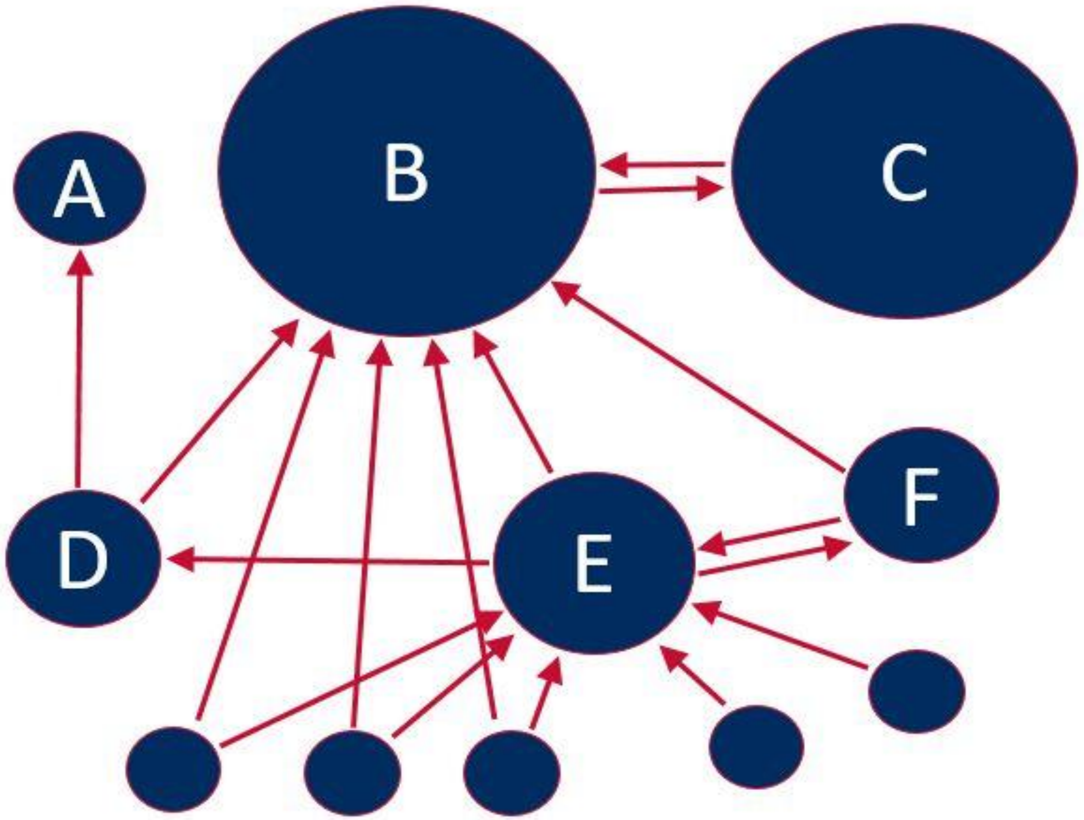
Introduction



PageRank



$$p(i) = \frac{q}{n} + (1 - q) \sum_{j:j \to i} \frac{p(j)}{k_{out}(j)}, \quad i = 1, 2, ..., n$$

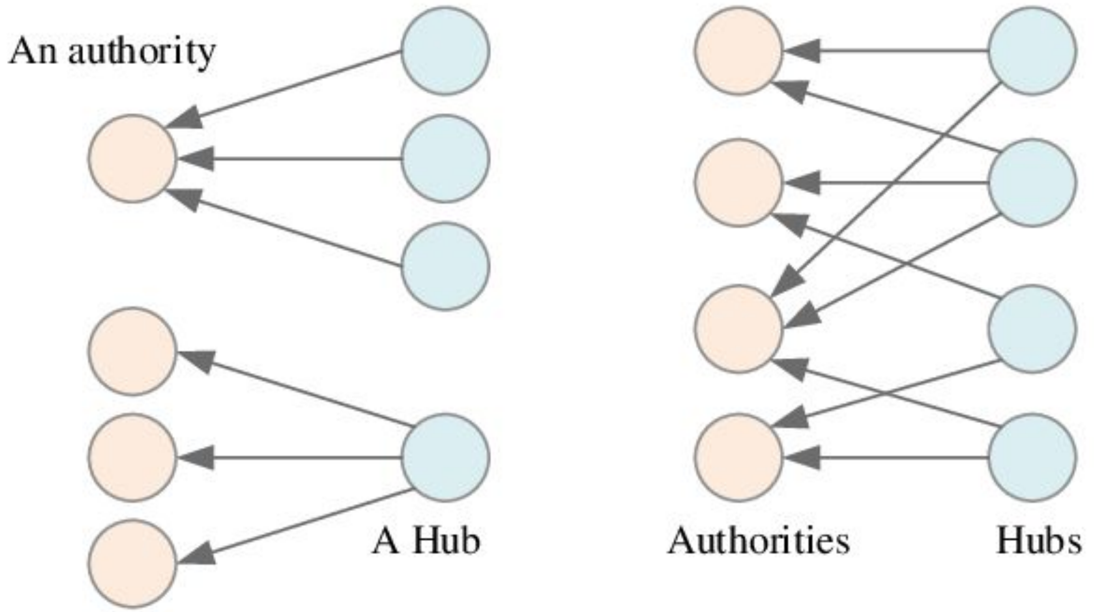
Eigenvector Centrality

$$\lambda x_i = \sum_{j:j\to i} x_j = \sum_j A_{ji} x_j = (\mathbf{A}^t \mathbf{x})_i$$

- Limitations

$$x_i = \alpha(\mathbf{A}^t \mathbf{x})_i + \epsilon$$

HITS Scores



$$\lambda y_i = \sum_{j:j\to i} x_j = \sum_j A_{ji} x_j = (\mathbf{A}^t \mathbf{x})_i$$

$$\mu x_i = \sum_{j:i\to j} y_j = \sum_j A_{ij} y_j = (\mathbf{A}\mathbf{y})_i$$

$$\lambda\mu x_i = (\mathbf{A}\mathbf{A}^t \mathbf{x})_i$$

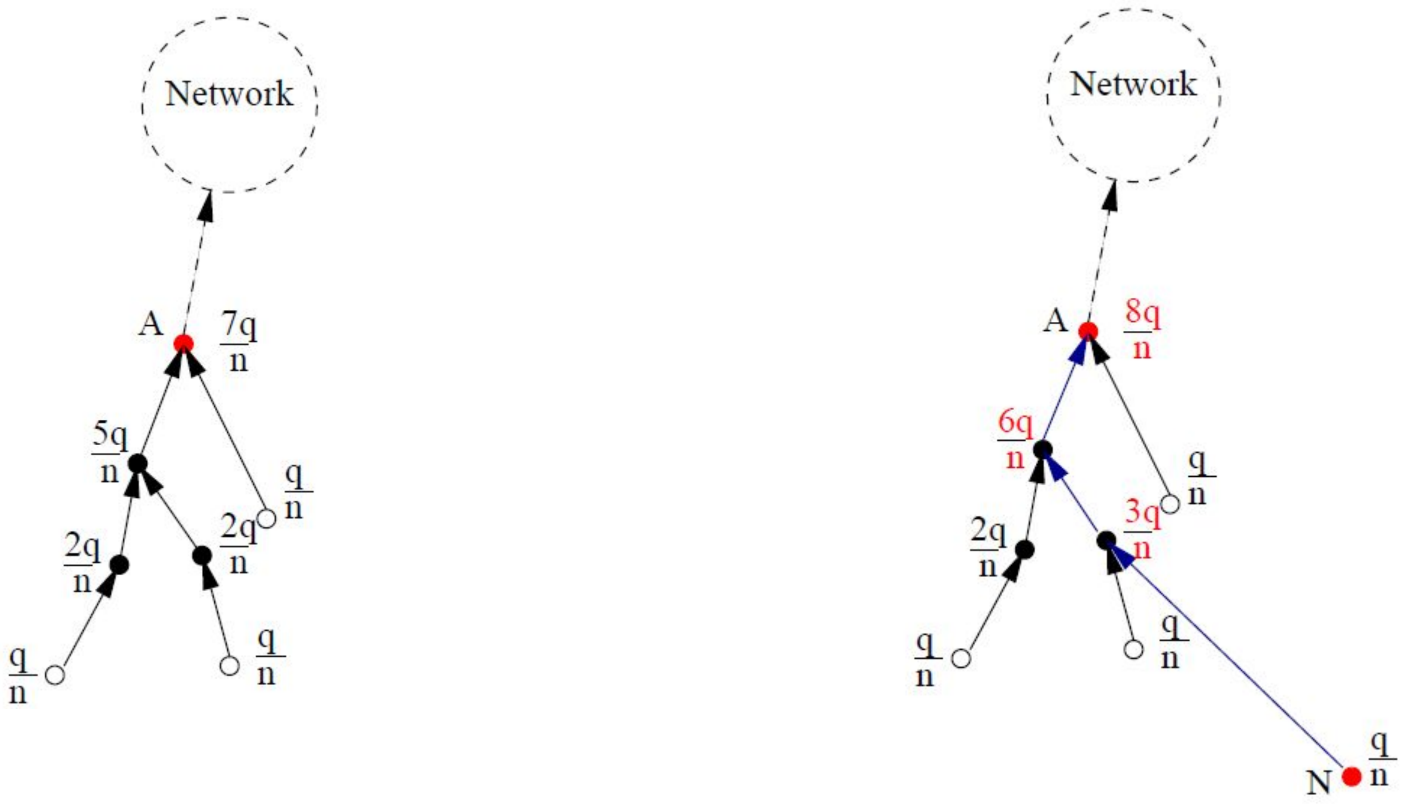
$$\lambda\mu y_i = (\mathbf{A}^t \mathbf{A}\mathbf{y})_i$$

Results for PageRank

- Two main limits, *q* ->0 and *q* -> 1
- Focus od DMS graph
- Expand to undirected graphs

$$\Pi(k_j, a) = \frac{a + k_j}{\sum_{l=1}^{i-1}(a + k_l)}$$

Attaching new node



$$p(i) \sim \frac{q}{n} + \sum_{j:j \to i} p(j) \qquad i = 1, 2, \ldots, n$$

q -> 0

- All PR values are multiples of the elementary unit q/n. When we say PR is increased by 1, that means it is increased by q/n
- PR increases if one moves from a node to another node following a link
- PR at each node is equal to the number of predecessors of that node

q -> 1

- Trivial since all nodes end up having PR value 1/n
- Defining the reduced PageRank

$$p_r(i) = p(i) - \frac{q}{n} \quad i = 1, 2, \ldots, n$$

$$p_r(i) = \frac{q(1-q)}{mn} k_{in}(i), \qquad i = 1, 2, \ldots, n$$

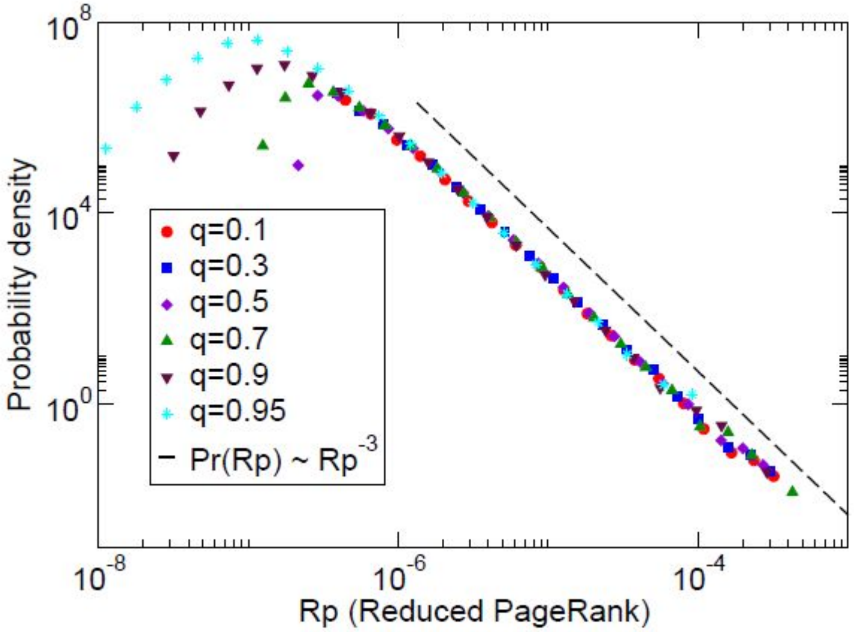
Extension to undirected graphs

- Primary idea: extension to random walk, undirected links can be crossed in both directions

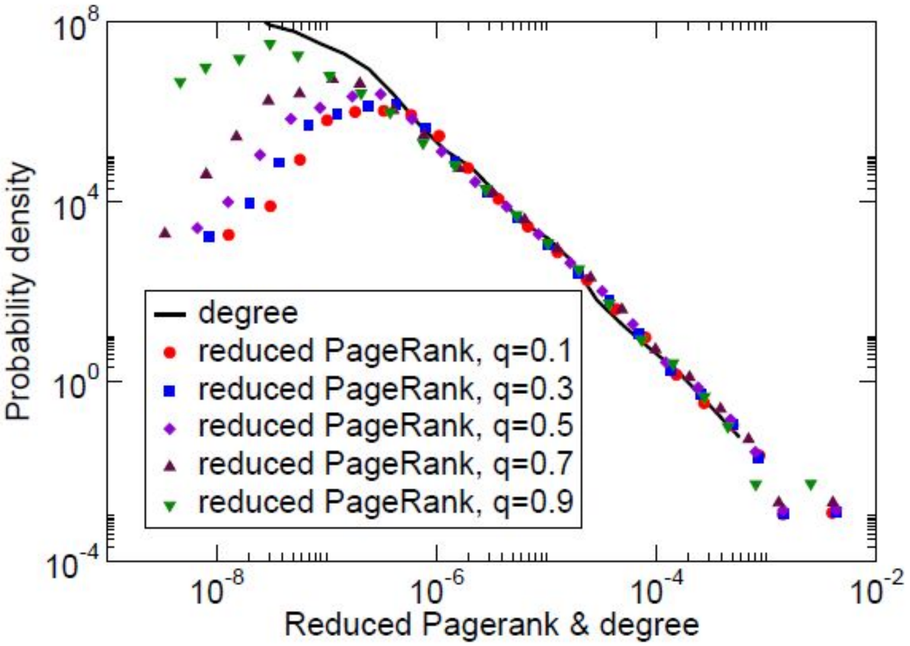
$$p(i) = \frac{q}{n} + (1-q) \sum_{j:j \leftrightarrow i} \frac{p(j)}{k_j}. \qquad i = 1, 2, \ldots, n$$

- Contribution of random jumping is present and this mixed process is still hard to solve

Reduced PageRank on undirected graphs



Results on DMS graph



Results on .gov domain

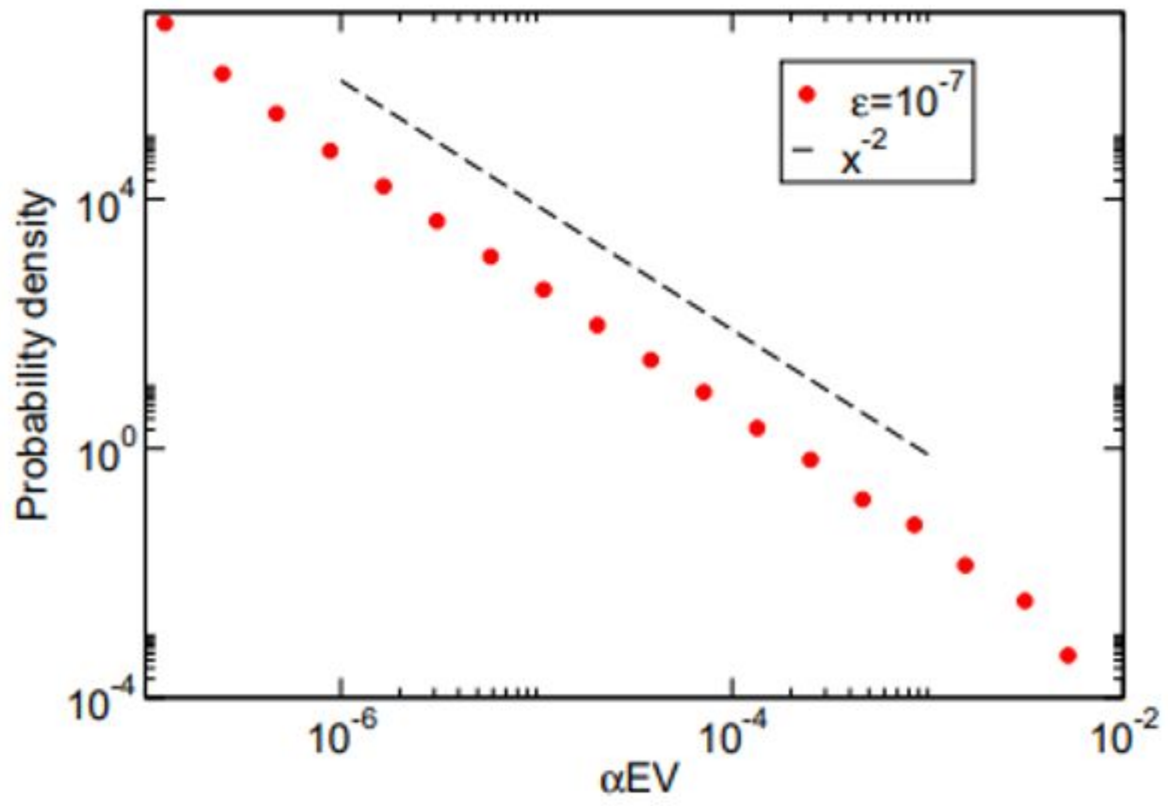
Eigenvector centrality. Directed Graphs

Similarity between Eigenvector centrality and Page Rank

$$x_i = \alpha(\mathbf{A}^t\mathbf{x})_i + \epsilon.$$

$$p(i) \sim \frac{q}{n} + \sum_{j:j\to i} p(j) \qquad i = 1, 2, \ldots, n$$

Eigenvector centrality. Directed Graphs



Distribution on DMS graph

Eigenvector centrality. Undirected graphs

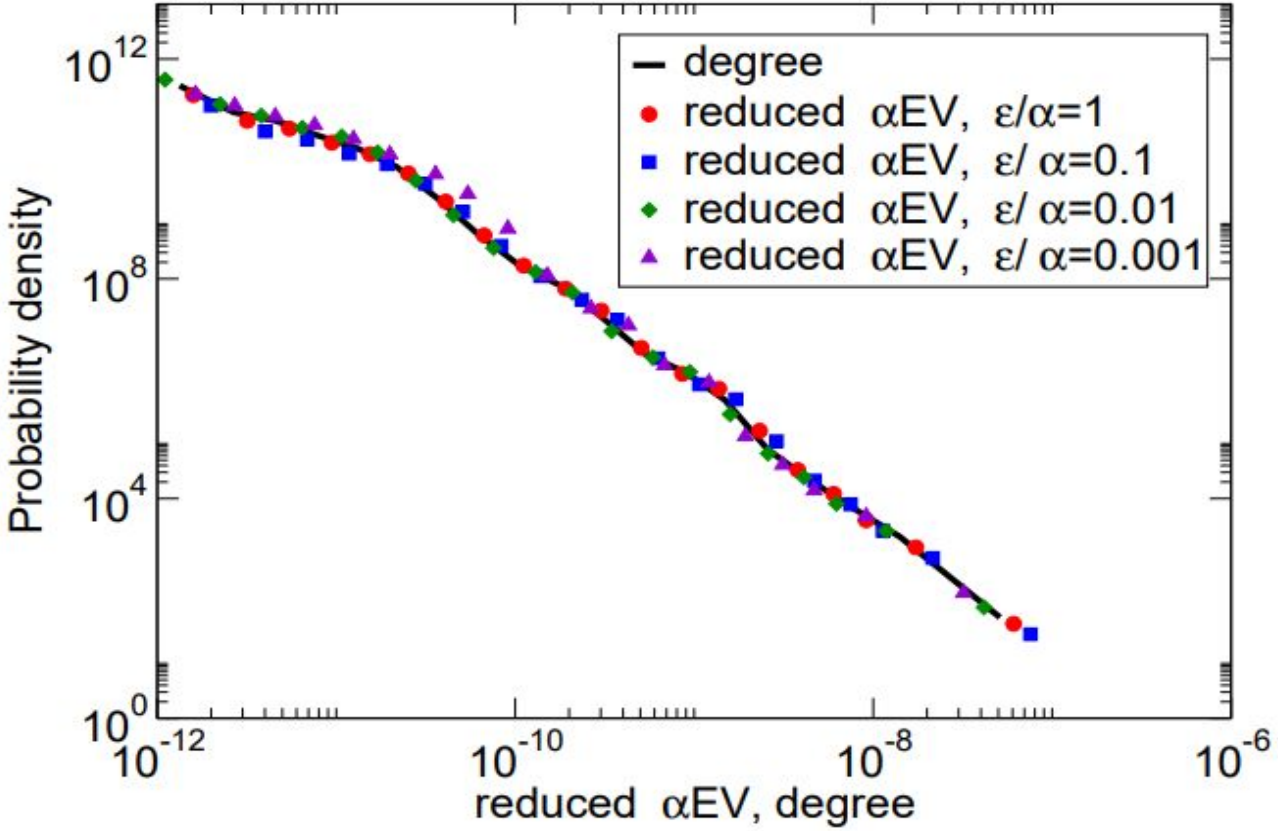
Formula for reduced Eigenvector centrality on Undirected Graphs

$$x_i^r = \alpha(\mathbf{A}\mathbf{x}^\mathbf{r})_i + k_i \alpha \epsilon$$

Approximated sum of neighbors alpha centralities

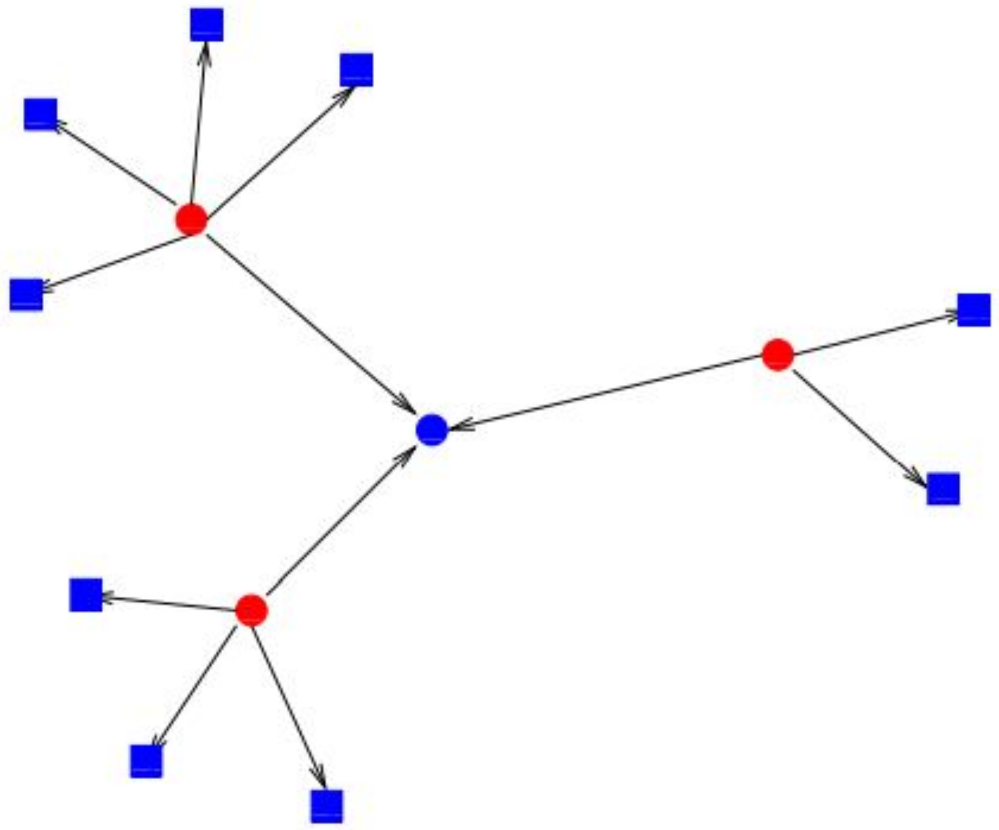
$$k_i \langle x^r \rangle$$

Eigenvector centrality. Undirected graphs



Distribution on .gov domain

HITS scores



HITS scores

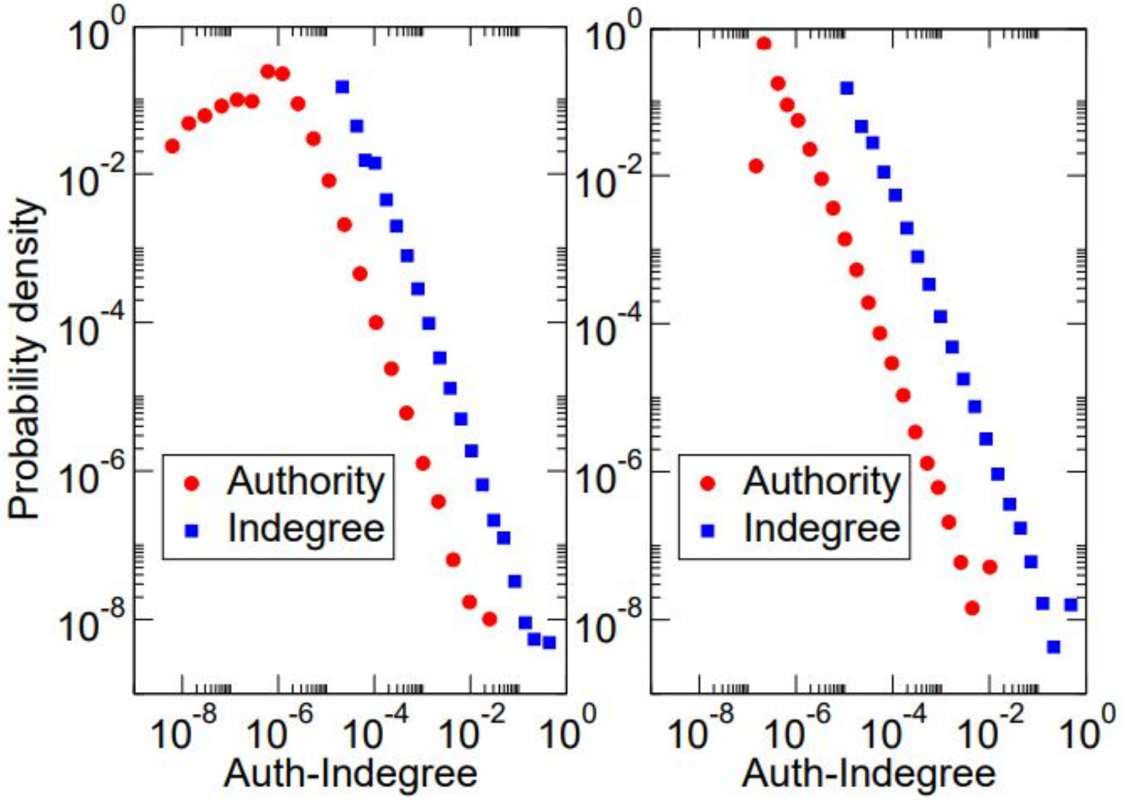
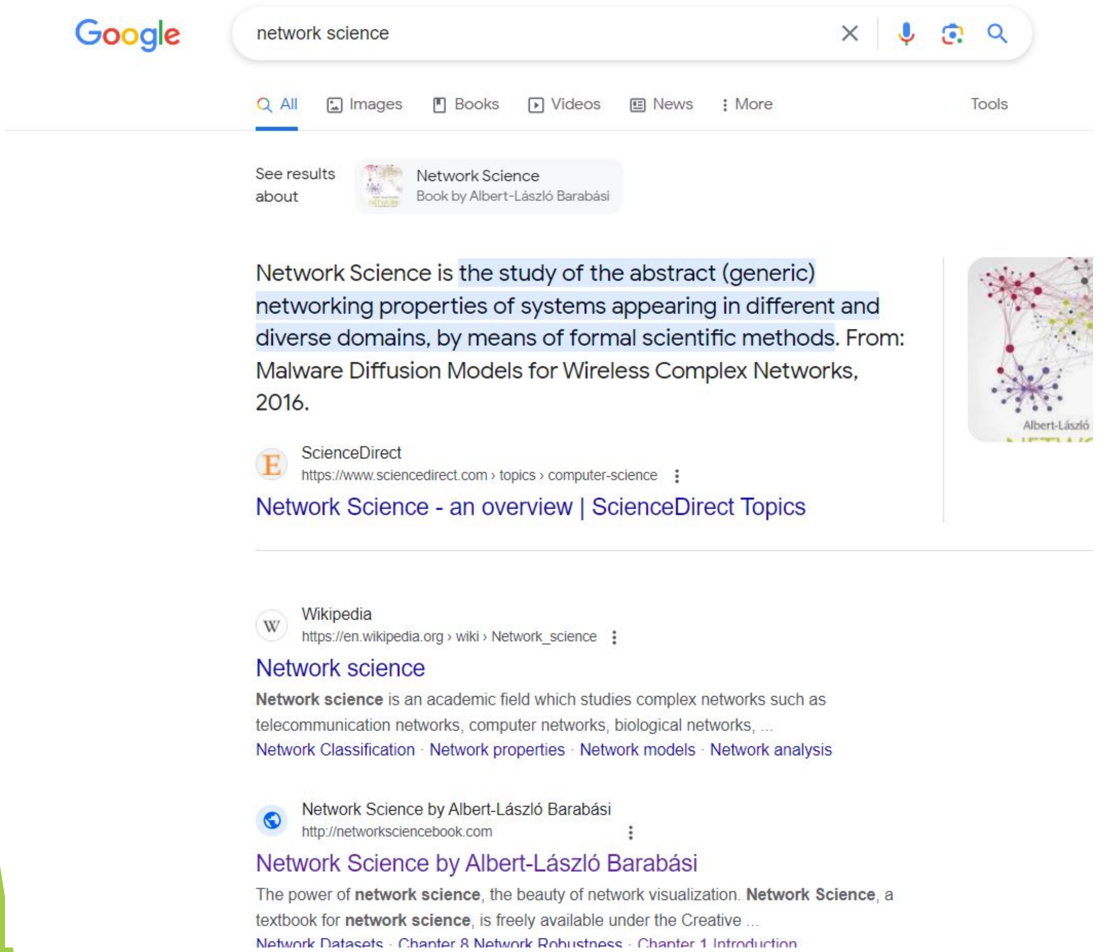


FIG. 10: Distribution of the authority scores versus indegree distribution. (Left) DMS graph with 10^5 nodes, $m = 10$ and $a = 1$. (Right) DMS graph with 10^5 nodes, $m = 50$ and $a = 1$.

Rankings



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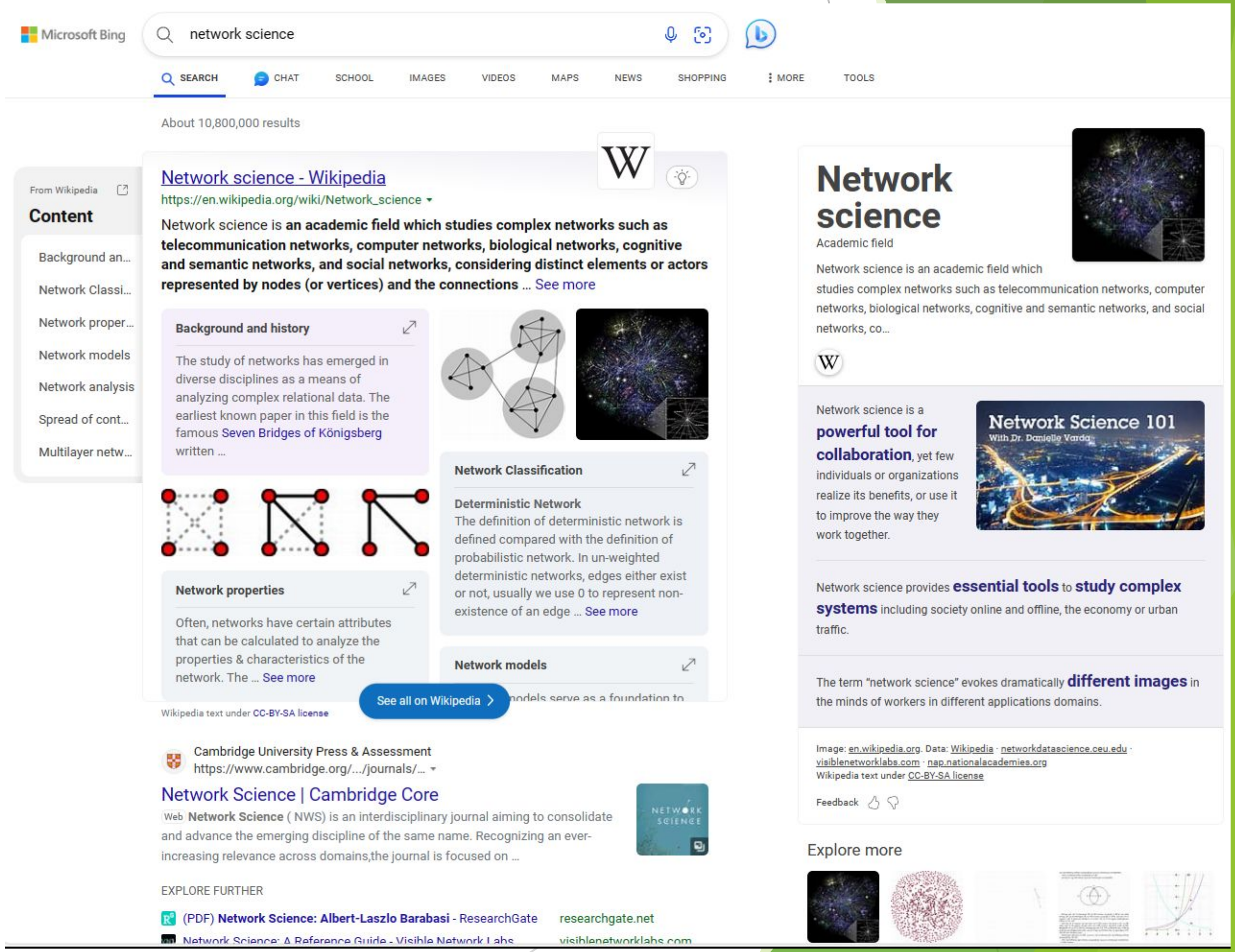

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Background and history
The study of networks has emerged in diverse disciplines as a means of analyzing complex relational data. The earliest known paper in this field is the famous Seven Bridges of Königsberg written ...
Network Classification
Deterministic Network
The definition of deterministic network is defined compared with the definition of probabilistic network. In un-weighted deterministic networks, edges either exist or not, usually we use 0 to represent non-existence of an edge ... See more
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The term "network science" evokes dramatically different images in the minds of workers in different applications domains.
Feedback
Explore more

Rankings

Measures	τ
PR-αEV	0.8192
PR-AUTH	0.5774
PR-HUBS	0.1213
PR-IN	0.6444
PR-OUT	-0.3012
αEV-AUTH	0.5788
αEV-IN	0.6487
αEV-HUBS	0.1220
αEV-OUT	0.5788
AUTH-IN	0.5458
AUTH-HUBS	0.1076
AUTH-OUT	-0.2611
HUBS-IN	0.1142
HUBS-OUT	-0.2126
IN-OUT	-0.2507

TABLE I: Kendall's τ for each pair of centrality measures computed for a DMS directed graph, with $n = 10^6$, $m = 3$ and $a = 3$.

Rankings

Measures	τ
PR-αEV	0.09
PR-AUTH	0.14
PR-HUBS	0.04
PR-IN	0.14
PR-OUT	0.02
αEV-AUTH	0.12
αEV-IN	0.07
αEV-HUBS	0.08
αEV-OUT	0.01
AUTH-IN	0.12
AUTH-HUBS	0.07
AUTH-OUT	0.01
HUBS-IN	0.02
HUBS-OUT	0.07
IN-OUT	0.07

TABLE II: Kendall's τ for each pairs of centrality measures for the network of political blogs studied by Adamic and Glance.

Conclusion

Thank you for your attention

Questions?