Tracking With Debiased Consistent Converted Measurements Versus EKF

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In tracking applications target motion is usually best modeled in a simple fashion using Cartesian coordinates. Unfortunately, in most systems the target position measurements are provided in terms of range and azimuth (bearing) with respect to the sensor location. This situation requires either converting the measurements to a Cartesian frame of reference and working directly on *converted measurements* or using an extended Kalman filter (EKF) in *mixed coordinates.* An accurate means of tracking with *debiased consistent* converted measurements is presented which accurately accounts for the sensor inaccuracies over all practical geometries and accuracies. This method is compared with the mixed coordinates EKF approach as well as a converted measurement approach discussed in [1, 3] which is an acceptable approximation only for moderate cross-range errors. This new approach is shown to be more accurate in terms of position and velocity errors and provides consistent estimates (i.e., compatible with the filter calculated covariances) for all practical situations. The combination of parameters (range, range accuracy, and azimuth accuracy) for which debiasing is needed is presented in explicit form.

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I. INTRODUCTION

In active sonar and radar systems the measurements of the position of a target is reported in polar coordinates (its range and azimuth or bearing (as well as elevation angle in 3D radar) with respect to the sensor location). The inaccuracies of these measurements have a significant impact on the performance of a tracking system. In target tracking the target motion can be best modeled in Cartesian coordinates. Tracking in Cartesian coordinates using polar measurements can be handled in two ways. One method is to use a linear Kalman filter with measurements converted to a Cartesian frame of reference. In this case the Cartesian components of the errors in the converted measurements are correlated [3]. The other approach is to use an extended Kalman filter (EKF), which incorporates the original measurements in a nonlinear fashion into the target state estimation, resulting in a mixed coordinate filter. Using either technique the inaccuracy of the measurements when converted to a Cartesian frame of reference must be accounted for properly. In the first approach the *converted measurement* covariance is recalculated at each recursion of the filter. In the second (EKF) approach the intial state covariance depends on the accuracy of the initial *converted measurements* and the gains depend on the accuracy of the subsequent linearization; the overall performance depends critically on these accuracies.

There exist alternative approaches to the EKF which utilize mixed coordinate filters. One technique uses statistical linearization [4] as opposed to Taylor series expansion. Another technique, the modified gain extended Kalman filter [5], has been shown to guarantee stability and exponential convergence for a special class of nonlinearities. A "quasi-extended" Kalman filter [6] tracking in polar coordinates showed improvements when tracking maneuvering targets at close range. These suboptimal nonlinear methods, which rely on approximations, are not discussed here since the linear filter using consistent measurement conversion without approximation is demonstrated.

The accuracy of the converted measurements depends on the geometry (range and bearing) as well as the accuracy of the original measurements. The mean and covariance of the converted measurements are derived in Section II. It is shown that for certain levels of the cross-range measurement error (the bearing error multiplied by the range) the mean of the errors is significant and debiasing compensation is needed. Both the debiasing terms as well as the covariance terms of the converted measurements require knowledge of the true location of the target. Since this is obviously not practical, the evaluation of these terms is usually done at the measured values, which introduces "secondary errors." A procedure to account for these secondary errors in addition to the debiasing is the main contribution of this work.

It is shown in Section III that this procedure guarantees the consistency of the conversion over all practical values of interest. In other words, the modified expressions for the first two moments of the converted measurements (polar to Cartesian) match their actual statistics. These expressions are compared with the first two moment approximations presented in [3, 1]. Consequently, the new converted measurement Kalman filter (CMKF) is shown in Section IV to be consistent (has estimation errors compatible with the calculated covariance). This is unlike the EKF which is consistent only for small errors. Furthermore, since the CMKF has the correct covariance, it processes the measurements with a gain that is (nearly) optimal and, as shown, yields smaller errors than the EKF, even in the case of moderately accurate sensors (0.5° rms azimuth error). The EKF is shown to perform poorly at long range for rms azimuth error of 1.5° or more, while the CMKF is consistent even for 10° rms azimuth error.

The applicability of the debiasing procedure presented here is for active sonar systems or long range radar systems. The combination of parameters where debiasing is needed is given in explicit form in Section III.

II. ANALYSIS OF CONVERTED MEASUREMENT ERRORS

The measured range and bearing are defined with respect to the true range r and bearing θ as

$$r_m = r + \tilde{r}; \qquad \theta_m = \theta + \tilde{\theta} \tag{1}$$

where the errors in range $\tilde{r}$ and bearing $\tilde{\theta}$ are assumed to be independent with zero mean and standard deviations σ_r and σ_θ, respectively. These polar measurements are transformed to Cartesian coordinate measurements using the classical conversion

$$x_m = r_m \cos\theta_m; \qquad y_m = r_m \sin\theta_m. \tag{2}$$

A. True Error Statistics

The errors in each coordinate can be found by expanding

$$\begin{aligned} x_m &= x + \tilde{x} = (r + \tilde{r})\cos(\theta + \tilde{\theta}); \\ y_m &= y + \tilde{y} = (r + \tilde{r})\sin(\theta + \tilde{\theta}) \end{aligned} \tag{3}$$

using trigonometric identities, where (x,y) is the true Cartesian position, to obtain

$$\begin{aligned} \tilde{x} &= r\cos\theta(\cos\tilde{\theta} - 1) - \tilde{r}\sin\theta\sin\tilde{\theta} \\ &\quad - r\sin\theta\sin\tilde{\theta} + \tilde{r}\cos\theta\cos\tilde{\theta} \end{aligned} \tag{4a}$$

$$\begin{aligned} \tilde{y} &= r\sin\theta(\cos\tilde{\theta} - 1) + \tilde{r}\cos\theta\sin\tilde{\theta} \\ &\quad + r\cos\theta\sin\tilde{\theta} + \tilde{r}\sin\theta\cos\tilde{\theta} \end{aligned} \tag{4b}$$

which are not independent, and each coordinate error depends on the true range and bearing as well as the errors in range and bearing.

The mean and covariance of the errors (4) can be made explicit assuming zero-mean Gaussian errors in the polar measurements (1). In this case

$$\begin{aligned} E[\cos\tilde{\theta}] &= e^{-\sigma_\theta^2/2} \\ E[\sin\tilde{\theta}] &= 0 \\ E[\cos^2\tilde{\theta}] &= 1/2(1 + e^{-2\sigma_\theta^2}) \\ E[\sin^2\tilde{\theta}] &= 1/2(1 - e^{-2\sigma_\theta^2}) \\ E[\sin\tilde{\theta}\cos\tilde{\theta}] &= 0. \end{aligned} \tag{5}$$

The mean error of (4) becomes

$$\mu_t(r,\theta) = \begin{bmatrix} E[\tilde{x} \mid r,\theta] \\ E[\tilde{y} \mid r,\theta] \end{bmatrix} = \begin{bmatrix} r\cos\theta(e^{-\sigma_\theta^2/2} - 1) \\ r\sin\theta(e^{-\sigma_\theta^2/2} - 1) \end{bmatrix}. \tag{6}$$

After some algebraic manipulation, the elements of the converted measurement covariance (with no approximations) are given by

$$\begin{aligned} R_t^{11} &= \mathrm{var}(\tilde{x} \mid r,\theta) \\ &= r^2 e^{-\sigma_\theta^2}[\cos^2\theta(\cosh(\sigma_\theta^2) - 1) + \sin^2\theta\sinh(\sigma_\theta^2)] \\ &\quad + \sigma_r^2 e^{-\sigma_\theta^2}[\cos^2\theta\cosh(\sigma_\theta^2) + \sin^2\theta\sinh(\sigma_\theta^2)] \end{aligned} \tag{7a}$$

$$\begin{aligned} R_t^{22} &= \mathrm{var}(\tilde{y} \mid r,\theta) \\ &= r^2 e^{-\sigma_\theta^2}[\sin^2\theta(\cosh(\sigma_\theta^2) - 1) + \cos^2\theta\sinh(\sigma_\theta^2)] \\ &\quad + \sigma_r^2 e^{-\sigma_\theta^2}[\sin^2\theta\cosh(\sigma_\theta^2) + \cos^2\theta\sinh(\sigma_\theta^2)] \end{aligned} \tag{7b}$$

$$\begin{aligned} R_t^{12} &= \mathrm{cov}(\tilde{x},\tilde{y} \mid r,\theta) \\ &= \sin\theta\cos\theta e^{-2\sigma_\theta^2}[\sigma_r^2 + r^2(1 - e^{\sigma_\theta^2})]. \end{aligned} \tag{7c}$$

Equations (6) and (7) are explicit expressions for the bias and covariance of the converted measurements. The converted measurements have a significant bias for large cross-range errors (long ranges and large bearing errors). The **true bias and covariance** which *depend on knowledge of the true range and bearing* are denoted as μ_t with elements (6) and R_t with elements (7), respectively.

B. Error Statistics Obtained From Linearization

Approximations for these expressions discussed in [3, 1] are accurate for limited target ranges and sensor accuracies. In Section III the usefulness of these approximations is quantified. These approximations are obtained by taking the first-order terms of a Taylor series expansion for the transformation (2) to approximate the Cartesian coordinate errors as

$$\begin{aligned}\tilde{x}_L &= \tilde{r}\cos\theta - \tilde{\theta} r \sin\theta \\ \tilde{y}_L &= \tilde{r}\sin\theta + \tilde{\theta} r \cos\theta\end{aligned} \tag{8}$$

instead of (4). Note that the "classical" approximation (8) consists of only the last two terms of (4) with the implicit assumption that $\tilde{\theta}$ is small. The ignored terms, however, can become significant if the cross-range error $r\tilde{\theta}$ is large.

The mean of the errors (8) is zero (because they are incomplete as indicated above) and the elements of their covariance matrix are

$$R_L^{11} = \mathrm{var}(\tilde{x}_L) = r^2\sigma_\theta^2 \sin^2\theta + \sigma_r^2\cos^2\theta \tag{9a}$$

$$R_L^{22} = \mathrm{var}(\tilde{y}_L) = r^2\sigma_\theta^2 \cos^2\theta + \sigma_r^2\sin^2\theta \tag{9b}$$

$$R_L^{12} = \mathrm{cov}(\tilde{x}_L, \tilde{y}_L) = (\sigma_r^2 - r^2\sigma_\theta^2)\sin\theta\cos\theta. \tag{9c}$$

The use of zero-mean expressions (8) does not account for the bias at long ranges with significant bearing error which is accounted for in (4), and the covariance approximation (9) is also shown in Section III to be poor under the same conditions. The bias and covariance based on the *linear approximation* (8) are denoted as μ_L and R_L, respectively. Note that $\mu_L = 0$, even though the standard transformation (2) is biased; however the approximation (8) "shows" it as unbiased.

C. Use of True Error Statistics in Practice

Expressions (6) and (7) cannot be used due to the fact that they are conditioned on thc true values of range and bearing which are not available in practice. To make these results useful, *the expected value of these true moments* are evaluated conditioned on the measured position. Therefore it is of interest to examine the expected bias and covariance

$$E[\mu_t(r,\theta) \mid r_m, \theta_m] = \mu_a \tag{10}$$

$$E[R_t(r,\theta) \mid r_m, \theta_m] = R_a \tag{11}$$

i.e., the expected value of the true bias with elements (8) and the true covariance with elements (9), conditioned on the measured position. These are called the **average true bias** and the **average true covariance.** Expanding (10) and (11) using (1) then applying trigonometric identities it can be shown that the mean (10) has elements

$$\mu_a = \begin{bmatrix} r_m\cos\theta_m(e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}) \\ r_m\sin\theta_m(e^{-\sigma_\theta^2} - e^{-\sigma_\theta^2/2}) \end{bmatrix} \tag{12}$$

and the covariance (11) has elements

$$\begin{aligned} R_a^{11} = {} & r_m^2 e^{-2\sigma_\theta^2}[\cos^2\theta_m(\cosh 2\sigma_\theta^2 - \cosh\sigma_\theta^2) \\ & + \sin^2\theta_m(\sinh 2\sigma_\theta^2 - \sinh\sigma_\theta^2)] \\ & + \sigma_r^2 e^{-2\sigma_\theta^2}[\cos^2\theta_m(2\cosh 2\sigma_\theta^2 - \cosh\sigma_\theta^2) \\ & + \sin^2\theta_m(2\sinh 2\sigma_\theta^2 - \sinh\sigma_\theta^2)] \end{aligned} \tag{13a}$$

$$\begin{aligned} R_a^{22} = {} & r_m^2 e^{-2\sigma_\theta^2}[\sin^2\theta_m(\cosh 2\sigma_\theta^2 - \cosh\sigma_\theta^2) \\ & + \cos^2\theta_m(\sinh 2\sigma_\theta^2 - \sinh\sigma_\theta^2)] \\ & + \sigma_r^2 e^{-2\sigma_\theta^2}[\sin^2\theta_m(2\cosh 2\sigma_\theta^2 - \cosh\sigma_\theta^2) \\ & + \cos^2\theta_m(2\sinh 2\sigma_\theta^2 - \sinh\sigma_\theta^2)] \end{aligned} \tag{13b}$$

$$R_a^{12} = \sin\theta_m\cos\theta_m e^{-4\sigma_\theta^2}[\sigma_r^2 + (r_m^2 + \sigma_r^2)(1 - e^{\sigma_\theta^2})]. \tag{13c}$$

Note that the average covariance (13) is *larger* than the covariance (7) conditioned on the exact position; it accounts for the additional errors incurred by evaluating it at the measured position. This is critical in showing the consistency later. This bias, and the above mentioned increase in the covariance, are significant only for long ranges and/or large bearing errors (i.e., large cross-range errors), as one would expect. Nevertheless, such situations are of practical interest.

Thus the new polar-to-Cartesian **unbiased consistent conversion** with correction for the average bias is, instead of (2), given by

$$z^c = \begin{bmatrix} x_m^c \\ y_m^c \end{bmatrix} = \begin{bmatrix} r_m\cos\theta_m \\ r_m\sin\theta_m \end{bmatrix} - \mu_a \tag{14}$$

where the elements of μ_a are given in (12). The average covariance of the conversion is R_a with elements (13).

III. CONSISTENCY ANALYSIS OF NEW AND CLASSICAL LINEARIZED CONVERSIONS

The covariance R_a is shown in this section to be the *only* covariance consistent with the converted measurements when using measured values of range and bearing for all target locations and sensor accuracies. The (ideal) true covariance R_t is always consistent provided the true position is used in its evaluation, but it becomes optimistic when it relies on the measured position. The limitations of the covariance R_L are discussed. The analysis in this section examines only the single scan converted measurement errors. The use of the converted measurements in tracking is discussed in Section IV.

The validity of a covariance can be determined by performing the statistical consistency check [2] outlined below.

For any zero mean random vector $\tilde{z}$ of dimension n, the quadratic form

$$\psi = \tilde{z}' A \tilde{z} \tag{15}$$

will have an expected value

$$E[\tilde{z}' A \tilde{z}] = \mathrm{tr}\{A E[\tilde{z}\tilde{z}']\} = \mathrm{tr}\{A P_{\tilde{z}\tilde{z}}\}. \tag{16}$$

If A is the inverse covariance of $\tilde{z}$, $P_{\tilde{z}\tilde{z}}^{-1}$, then

$$E[\tilde{z}' P_{\tilde{z}\tilde{z}}^{-1} \tilde{z}] = \mathrm{tr}[I_n] = n. \tag{17}$$

Therefore the expected normalized error squared should equal n, the dimension of the random vector $\tilde{z}$. This fact can be utilized to check the accuracy of a covariance approximation. The sample mean of (16)

$$\overline{\psi} = \frac{1}{N} \sum_{i=1}^{N} \tilde{z}_i' P_{\tilde{z}\tilde{z}}^{-1} \tilde{z}_i \tag{18}$$

is used to perform the consistency test, where $\tilde{z}_i$ is the 2-dimensional vector of converted measurement errors in sample i; $\mu = 0$ and P are the hypothesized mean and covariance of the errors to be tested, and N is the total number of samples used in the test. In this case (18) yields the average normalized error squared (NES).

The above test was performed using $N = 1000$ converted position measurements for each position and accuracy considered. The mean value of the sample test statistic is 2 when the assumed covariance is matched to the actual error covariance. It should be noted that if the errors are jointly Gaussian then the distribution of (18) multiplied by N is chi-square with $2N = 2000$ deg of freedom and an acceptance region is obtained for the test. The average NES for the unbiased conversion (14), compensated for the bias μ_a with covariance R_a evaluated at the measured position, is plotted as a function of the bearing error in Fig. 1. The true target position is at 10^5 m with azimuth 45° and the standard deviation of range error is 50 m. The plot also indicates the chi-square 0.99 probability bounds. The same results for the classical conversion (2) and covariance R_L are shown in Fig. 2. The covariance R_a is consistent with the errors even though the variability in the plot marginally violates the chi-square bounds since the errors are not Gaussian. The linear approximation for the classical conversion results in an optimistic covariance R_L, which is clearly inconsistent when the bearing error exceeds 1.0° at this range.

Nearly identical results are obtained at different bearing angles, since each covariance discussed above properly accounts for the errors in each coordinate and the cross errors as a function of bearing angle. The remaining three parameters, r, σ_r, and σ_θ must be considered together to determine when R_L will provide a consistent covariance for converted measurement errors. Using the linear approximation approach, longer ranges require more accurate bearing measurements and for larger standard deviation of range error a larger bearing error is tolerated. By examining the significance of the bias the validity limit for the classical conversion in terms of these parameters can be defined.

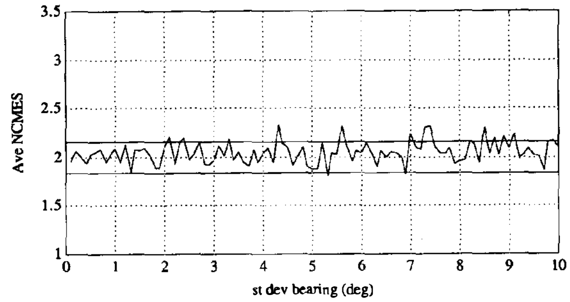

Fig. 1. Average NES for debiased conversion evaluated at measured position ($\sigma_r = 50$ m; true range 10^5 m; true bearing 45°).

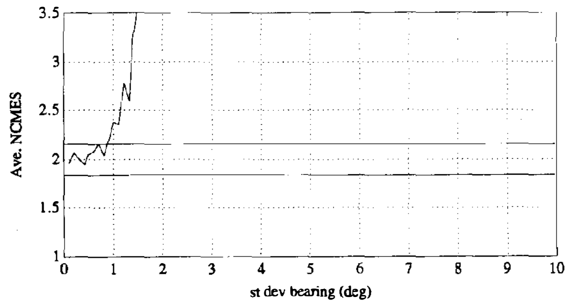

Fig. 2. Average NES for classical conversion evaluated at measured position ($\sigma_r = 50$ m; true range 10^5 m; true bearing 45°).

A. Validity Limit of Classical Linearized Conversion

The bias (12) can be approximated with a Taylor series expansion as

$$\mu = \begin{bmatrix} r\cos\theta\sigma_\theta^2/2 \\ r\sin\theta\sigma_\theta^2/2 \end{bmatrix}. \tag{19}$$

The maximum bias magnitude is, from (19),

$$\|\mu\|_{\max} = \frac{r\sigma^2}{2} \tag{20}$$

where σ_θ is defined in radians. The minimum standard deviation from covariance matrix (13) is

$$\sqrt{\lambda_{\min}(R_a)} = \min(\sigma_r, r\sigma_\theta) \tag{21}$$

where $\lambda_{\min}(R_a)$ denotes the minimum eigenvalue.

Define the **bias significance** as the ratio of the above two quantities

$$\beta = \frac{r\sigma_\theta^2}{2\min(\sigma_r, r\sigma_\theta)}. \tag{22}$$

Based on the results of Fig. 2, the maximum bias significance that can be tolerated is

$$\beta_{\max} = \frac{10^5(0.8\pi/180)^2}{2(50)} \approx 0.2. \tag{23}$$

In general, the **validity limit of the classical linearized conversion** is

$$\frac{r\sigma_\theta^2}{\sigma_r} < 0.4 \tag{24a}$$

$$\sigma_\theta < 0.4 \text{ rad} \approx 23^\circ. \tag{24b}$$

In practically all systems (24b) is satisified. In most radar systems, inequality (24a) is satisfied, but this is not always the case in sonar systems.

In conclusion, a useful guideline for the limit of validity of the linearized conversion is provided by (24a) and (24b), whereas, the main result of Section II provides a consistent conversion from polar to Cartesian measurements for *all* practical parameters.

IV. TRACKING WITH CONVERTED MEASUREMENTS

The results from the previous section provide a consistent covariance R_a to account for the measurement errors when converting polar measurements into a Cartesian frame of reference given that the true position is not available. This is important in long range target tracking where the bearing inaccuracy dominates the estimation errors and causes problems in target localization accuracy and filter consistency. This measurement covariance is utilized in a CMKF with the debiased conversion (14), which yields the CMKF-D. The improved performance using the CMKF-D is shown. The results are compared with EKF tracking in mixed coordinates as well as a CMKF using the classical linearized conversion (CMKF-L).

A. Converted Measurement Filter

With the CMKF, the polar position measurement at time k, $z^p(k)$, is converted to a Cartesian coordinate measurement $z^c(k)$ using the nonlinear transformation (14)

$$z^c(k) = \eta[z^p(k)] - \mu(k) = \begin{bmatrix} r_m(k)\cos\theta_m(k) \\ r_m(k)\sin\theta_m(k) \end{bmatrix} - \mu_k \tag{25}$$

where $\mu(k)$ is the bias compensation which will be $\mu_L = 0$ or μ_a according to the case considered, CMKF-L and CMKF-D, respectively.

The filter equations are described entirely in Cartesian coordinates. The predicted state is

$$\hat{x}(k \mid k-1) = F\hat{x}(k-1 \mid k-1) \tag{26}$$

where F is assumed to be the white noise acceleration state transition matrix

$$F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

and T is the time between measurements. The predicted measurement is

$$\hat{z}^c(k \mid k-1) = H\hat{x}(k \mid k-1) \tag{28}$$

where the measurement matrix is

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{29}$$

The updated state estimate is

$$\hat{x}(k \mid k) = \hat{x}(k \mid k-1) + W(k)[z^c(k) - \hat{z}^c(k \mid k-1)] \tag{30}$$

where $W(k)$ is the Kalman gain. The predicted state covariance is

$$P(k \mid k-1) = FP(k-1 \mid k-1)F' + Q \tag{31}$$

where $P(k-1 \mid k-1)$ is the state error covariance at time $k-1$ and Q is the assumed process noise covariance. The predicted measurement covariance is

$$S(k) = HP(k \mid k-1)H' + R^c(k) \tag{32}$$

where $R^c(k)$ is the *converted measurement error covariance* which will be either R_L or R_a according to the case evaluated, CMKF-L and CMKF-D, respectively. The Kalman gain is

$$W(k) = P(k \mid k-1)H'S(k)^{-1} \tag{33}$$

and the updated state covariance is

$$P(k \mid k) = P(k \mid k-1) - W(k)S(k)W(k)'. \tag{34}$$

The converted measurement covariance $R^c(k)$ in (32) was shown in Section II to be a function of the target range and bearing as well as the error in their respective measurements. Therefore the range and bearing information used to evaluate the covariance should be the best estimate available at time k. In general this estimate is determined by the predicted state (26) by applying the inverse mapping of η from (25). The only time this does not hold is at initialization, where the first few time steps this provides poor predictions of the target position relative to the measurement accuracy: then the measurements themselves should be chosen to evaluate the converted covariance. This provides more accurate results. A simple rule determines when to use the current measurements versus the predicted state. The measurements are used if

$$\det[HP(k \mid k-1)H'] \geq \det[R^c(k)] \tag{35}$$

where "det" is the determinant. The test is: if the "size" of the predicted measurement covariance based on the state estimation error alone exceeds the "size" of the measurement error covariance then the "more accurate" measurements should be used to evaluate the converted measurement error covariance.

B. Extended Kalman Filter

In the "mixed coordinate" EKF the state is maintained in Cartesian coordinates and the measurements are utilized in their polar form. Therefore, there is a nonlinear measurement prediction equation and linearization of the measurement equation is required for the state and covariance update. The EKF is described as follows.

The measurements at time k are

$$z^p(k) = \begin{bmatrix} r_m(k) \\ \theta_m(k) \end{bmatrix}. \tag{36}$$

The state prediction proceeds as in (26) but the measurement prediction is the nonlinear mapping

$$\hat{z}^p(k \mid k-1) = h[\hat{x}(k \mid k-1)] \tag{37}$$

where $h = \eta^{-1}$ from (25) and the updated state is

$$\begin{aligned} \hat{x}(k \mid k) = {} & \hat{x}(k \mid k-1) \\ & + W(k)[z^p(k) - \hat{z}^p(k \mid k-1)] \end{aligned} \tag{38}$$

where the "mixed coordinate" gain $W(k)$ is different from the gain in (30) since it is obtained from a different covariance update and is applied to the polar measurements. The predicted state covariance is the same as (31) and the predicted measurement covariance is

$$S(k) = H_x(k)P(k \mid k-1)H_x(k)' + R^p \tag{39}$$

where $H_x(k)$ is the Jacobian of the transformation h evaluated at the predicted state estimate $\hat{x}(k \mid k-1)$ and R^p, is the polar measurement covariance

$$R^p = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}. \tag{40}$$

The gain is

$$W(k) = P(k \mid k-1)H_x(k)'S(k)^{-1} \tag{41}$$

and the updated state covariance is the same as (34) using (39) and (41).

The use of the Jacobian, a first-order approximation, can cause inconsistency of the covariance update (39) when the linearization is poor which results in an improper gain (41).

Both the CMKF and the EKF require proper initialization. The difference between these two filters is that the CMKF updates its converted measurement covariance at time k while the EKF updates a Jacobian for the nonlinear transformation among the coordinates for the filter covariance calculation. Both these "measurement dependent" updates requires the use of the most recent predicted state estimate.

C. Simulation Results

A long range target tracking application is simulated to examine the effects of converted measurements on target state estimation for different standard deviations of bearing error. The initial target location is at a range of 70 km and azimuth 45° with initial velocity of 15 m/s heading due North. The sensor is assumed fixed at the origin. The target trajectory is modeled by the second-order kinematic model [2, eq. (2-297)] and includes process noise (white noise acceleration) with a standard deviation 0.01 m/s^2 in each coordinate. Tracking is performed using 50 measurements obtained with sampling interval of $T = 60$ s. The above target trajectory was chosen since it opens in range significantly and also provides a moderate change in bearing angle. Tracks are initiated with two point differencing to obtain the initial velocity estimate [2]. The standard deviation of range errors is assumed to be 50 m and two standard deviations of bearing error were used $\sigma_\theta = 1.5°$, and 2.5°.

Two converted measurement filters, the CMKF-D using the debiased conversion and the classical conversion CMKF-L with linearized measurement covariance are examined. These algorithms are compared with the mixed coordinate EKF. All results presented are based on 1000 Monte Carlo runs.

The rms position error and the rms velocity error for the three filters are shown in Figs. 3 and 4, respectively, for $\sigma_\theta = 1.5°$. Figs. 5 and 6 present the normalized estimation error squared (NEES) [2] for position and velocity with their associated 99% probability regions. The CMKF-D provides the best accuracy and is consistent. The other two filters are close in accuracy (except for the velocity) but the EKF is inconsistent over the first 40 scans and the CMKF-L is inconsistent for the first 15 scans. Figs. 7–10 present the same results with $\sigma_\theta = 2.5°$.

As can be seen, the unbiased converted measurement filter using the bias correction and measurement covariance derived in Section II clearly outperforms the other two techniques in terms of accuracy and consistency. The increase in bearing error, hence cross-range error, causes significant degradation to the EKF and CMKF-L consistency which is reflected in large rms position errors.

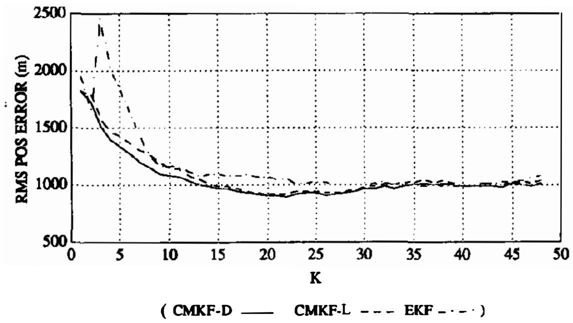

Fig. 3. Comparison of rms position errors, $\sigma_\theta = 1.5$ deg.

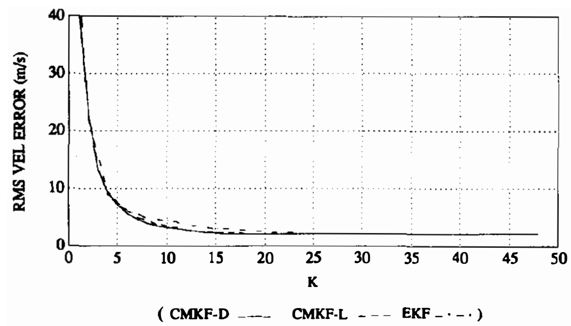

Fig. 4. Comparison of rms velocity errors, $\sigma_\theta = 1.5$ deg.

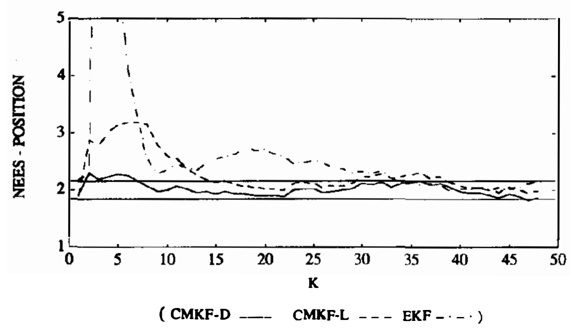

Fig. 5. Comparison of NEES—position, $\sigma_\theta = 1.5$ deg.

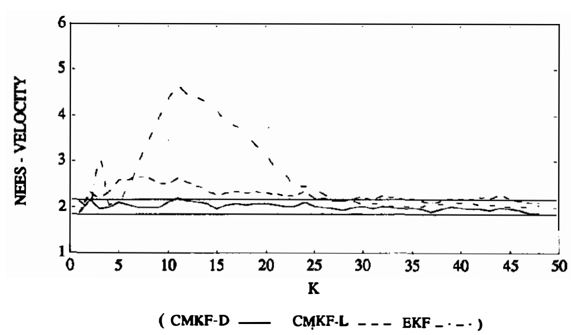

Fig. 6. Comparison of NEES—velocity $\sigma_\theta = 1.5$ deg.

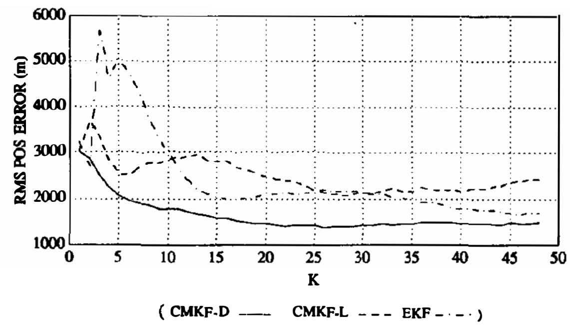

Fig. 7. Comparison of rms position errors, $\sigma_\theta = 2.5$ deg.

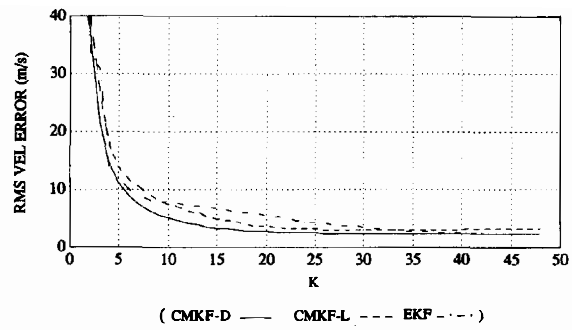

Fig. 8. Comparison of rms velocity errors, $\sigma_\theta = 2.5$ deg.

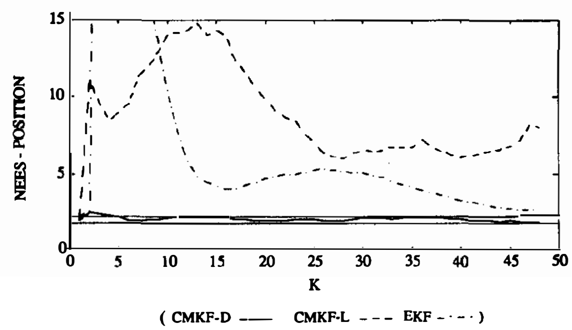

Fig. 9. Comparison of NEES—position, $\sigma_\theta = 2.5$ deg.

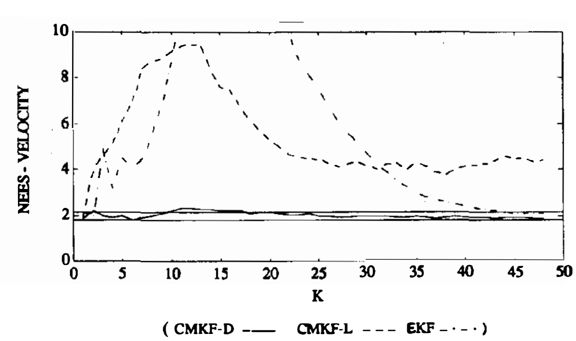

Fig. 10. Comparison of NEES—velocity, $\sigma_\theta = 2.5$ deg.

V. CONCLUSIONS

The mean and covariance of the errors of Cartesian measurements which are obtained by converting polar measurements have been derived. A procedure is presented which guarantees consistency of the conversion for any target geometry or polar measurement accuracy using the measured target position. This procedure is only slightly more complex than the commonly used classical conversion. It is demonstrated that this procedure, when used in target tracking, clearly provides better estimation accuracy than the classical conversion filter and the mixed coordinate EKF. Furthermore, this filter is the only filter which is consistent over all practical geometries and sensor inaccuracies.

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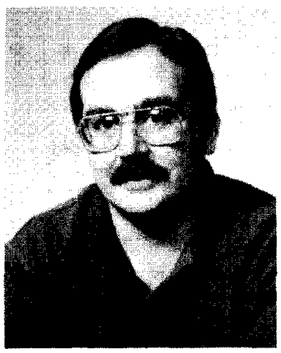

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