A Combined Array Design Method Based on Normal Mode Decomposition

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***Abstract*—For the problem of array matched field processing localization performance of underwater targets in shallow sea environment, this paper proposes a method for analyzing the localization performance of arrays based on the decomposition of normal mode. In the process of establishing the method, by introducing the normal mode decomposition theory of the sound field, a receiving matrix is established by this theory, and the role of the formation in the target localization is characterized according to the matrix singularity. Based on the simulation calculations under the different formations, the effectiveness of the method is verified. The simulation results show that the more the number of the normal mode orders decomposed by the receiving array, the more abundant the sound field information is, and the better the matched field localization processing effect is. At the same time, according to the array effect analysis of this method, if the apertures are not complementary, the array's spatial resolution can hardly be improved, and at most improve the processing gain by increasing the array element. Therefore, for the design of combined arrays, in order to improve the localization performance, it is necessary to focus on increasing the effective aperture of the arrays.**

***Index Terms*—combined array; normal mode decomposition; matched field processing; effective aperture**

I. INTRODUCTION

In shallow-water multi-path propagation environment, due to the obvious congenital advantages of the matched field processing (MFP) technology, it has become a hot spot for acoustic researchers in recent years [1][2][3]. MFP is a sound field processing method based on generalized beamforming, which makes full use of the multi-path propagation characteristics of acoustic signals. It forms a copy field vector by calculating the amplitude and phase of the sound field of the receiving array through the underwater acoustic field model and performs correlation processing with the data received by the receiving array[4], thereby achieving the passive localization of the underwater target and the estimation of the marine environment parameters.

Due to the complexity of the marine environment, low noise targets must be identified in the context of strong interference and require high resolution detection performance. The inter-array coherent processor can treat arrays at different locations as a multi-dimensional array with a large aperture, so that higher spatial resolution and signal processing gain can be obtained. We call a variety of arrays to form a hydrophone array as a combined array, such as multiple vertical arrays, multiple horizontal arrays, vertical arrays + horizontal arrays, vertical arrays + circular arrays, and so on. Scholars have done in-depth research in combined arrays. The most famous is the Santa Barbara Channel Test in 2000, which used five uniform vertical arrays (VLA) to form a regular pentagon with 136m of bottom side. Based on this test, Tracey [5] and Zurk [6] used adaptive MFP technology to effectively distinguish between surface and underwater sound sources in the presence of water interference. Mingsian R. Bai [7] studied the 2.5-D combination of a circular array and a vertical array, and found that the combined array has good resolution of the direction of arrival (DOA). Liu Fengxia [8] studied the three-dimensional(3-D) localization method of a double spiral line array (DSLA). The array simultaneously estimate the acoustic source bearing, range and depth by using its horizontal and vertical aperture and increase the resolution and use broadband mid-frequency signals to improve the reliability of short DSLA localization.

Distributed multi-array detection systems use multiple elements to increase spatial sampling, which can increase the reliability of target localization. In addition, the use of combined arrays for target localization can use the synergy between different arrays to achieve 3-D localization of the target, thus providing better array processing space gain, depth, distance or azimuth resolution.

In order to design a combined array, we analyze the processing performance of the combined formation and form a relatively simple analysis method. For this reason, this paper analyzes the receiving matrix under the form of the decomposition of the normal mode, discusses the localization performance of the combined array, and proposes a localization performance judgment method based on the normal mode decomposition matrix.

The structure of this paper is as follows: Sec I introduce the research background; Sec II briefly introduces the theoretical model and discusses the MFP theory of 3-D space. This section also gives the L-shaped array ambiguity function and the derivation of the normal mode decomposition matrix. In sec III, the feasibility of this method is verified by simulating a single vertical array, an L-shaped array, and a double vertical array. Sec IV is the summary of the paper.

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II. THEORY

A. Normal Mode Decomposition of Space Line Arrays

For simple VLA and horizontal line arrays (HLA), there have been some MFP performance analysis methods and qualitative conclusions. Tantum and Nolte [9] discussed the how the array parameters (number of elements and aperture) affect the localization performance of MFP in HLA, and gave the design criterion of HLA based on the normal mode propagation model. Rachel M Hamson [10] studied the effect of array length on localization performance of VLA and showed that the VLA length should be longer than half sea depth.

For the MFP, considering the requirements of the target localization, the requirement of the array design is different from the traditional plane wave beam forming processing requirements. Take the traditional Bartlett MFP algorithm as an example, considering 3-D space. We can write a normalized ambiguity function:

$$\boldsymbol{D}(r,z;\omega)=\frac{\boldsymbol{P}_r^{\mathrm{H}}(r,z;\omega)\,\mathrm{K}\,\boldsymbol{P}_r(r,z;\omega)}{|\boldsymbol{P}_r(r,z;\omega)|^2|\boldsymbol{P}(\omega)|^2}=\frac{|\boldsymbol{P}_r^{\mathrm{H}}(r,z;\omega)\boldsymbol{P}(\omega)|^2}{|\boldsymbol{P}_r^{\mathrm{H}}(r,z;\omega)|^2|\boldsymbol{P}(\omega)|^2} \tag{1}$$

Here matrix K means the data covariance matrix, here, $K=(\boldsymbol{P}(\omega)\boldsymbol{P}^H(\omega))$, $\boldsymbol{P}_r(r,z;\omega)$ means the normalized complex replica vectors on $\boldsymbol{N}$ hydrophones of the array $\boldsymbol{P}_r(r,z;\omega)=\begin{pmatrix}\boldsymbol{P}_{r1} & \boldsymbol{P}_{r2} & \cdots & \boldsymbol{P}_{rN}\end{pmatrix}^T$. $\boldsymbol{P}(\omega)$ means the normalized complex acoustic pressures measured on $\boldsymbol{N}$ hydrophones of an array $\boldsymbol{P}(\omega)=\begin{pmatrix}\boldsymbol{P}_1 & \boldsymbol{P}_2 & \cdots & \boldsymbol{P}_N\end{pmatrix}^T$ The superscript H represents the transposed conjugate and T represents the transpose. The source located at $(r_s,z_s)=(x_s,y_s,z_s)$, (r_n,z_n) represent the position of coordinates of receiving element.

Calculating the copy field using the normal mode algorithm, the copy field can be represented as a matrix

$$\boldsymbol{P}_r(\boldsymbol{r},z;\omega)=\boldsymbol{\Phi}_r\,\boldsymbol{A}(r,z) \tag{2}$$

Here

$$\boldsymbol{\Phi}=\begin{bmatrix}\boldsymbol{\Phi}_1(z_1)e^{i\xi_1\Delta r_1} & \cdots & \boldsymbol{\Phi}_M(z_1)e^{i\xi_1\Delta r_1}\\ \vdots & \ddots & \vdots\\ \boldsymbol{\Phi}_1(z_N)e^{i\xi_1\Delta r_N} & \cdots & \boldsymbol{\Phi}_M(z_N)e^{i\xi_M\Delta r_N}\end{bmatrix}$$

$$\boldsymbol{A}_r=\left(a_1(r_1)\Phi_1(z_s),\quad a_2(r_1)\Phi_2(z_s),\quad \cdots,\quad a_M(r_1)\Phi_M(z_s)\right)^T$$

Thus, the output of Bartlett MFP can be represented as

$$\boldsymbol{D}(r,z;\omega)\propto \boldsymbol{A}_r^H(\boldsymbol{\Phi}^H\boldsymbol{P}\boldsymbol{P}^H\boldsymbol{\Phi})\boldsymbol{A}_r=\boldsymbol{A}_r^H(\boldsymbol{B}\boldsymbol{B}^H)\boldsymbol{A}_r \tag{3}$$

Here, $\boldsymbol{B}=\boldsymbol{\Phi}^H\boldsymbol{P}$, which means the normal mode coefficient matrix obtained by normal mode decomposition from the received pressure.

The similarity from $\boldsymbol{B}$ and the normal mode coefficient $\boldsymbol{A}_r$, which is obtained from normal mode decomposition of the model, can directly determine the field localization performance and the spatial resolution. Without considering the mismatch of environmental parameters, the receiving array structure is the most of influence of the normal mode decomposition effect and the subsequent MFP performance. This is the starting point for our design.

When the impact of mismatch is not considered, the result of the normal mode decomposition is

$$\begin{aligned}B_m&=\sum_{n=1}^{N}\Phi_{nm}^*P_n=\sum_{n=1}^{N}\Phi_{nm}^*\sum_{\ell=1}^{M}\Phi_{n\ell}A_\ell\\&=\sum_{n=1}^{N}\Phi_{nm}^*\Phi_{nm}A_m+\sum_{n=1}^{N}\sum_{\ell=1,\neq m}^{M}\Phi_{nm}^*\Phi_{n\ell}A_\ell\equiv B_m^{(1)}+B_m^{(2)}\end{aligned} \tag{4}$$

Here, the first part $B_m^{(1)}$ is non-cross item, $B_m^{(2)}$ is cross item. The cross item reflects the separability effect of the receiving array on the decomposition of the normal mode: the smaller it is, the better the normal mode decomposition effects and the better the corresponding matched field localization effects.

For the normal mode decomposition form involved before,

$$\boldsymbol{B}=\Phi^H\boldsymbol{P}=(\Phi^H\Phi)\boldsymbol{A}=\Theta\boldsymbol{A} \tag{5}$$

From this we can see that evaluating the quality of the normal mode decomposition, or the separability of the normal mode, can also be measured by the singularity of the decomposition matrix Θ . The closer the normal mode decomposing matrix is to the full-rank unit matrix, the more the normal mode of different orders can be distinguished. The final MFP will be stronger in the spatial resolution.

If the decomposition matrix is full rank, it is theoretically to distinguish all M-order normal modes. Of course, due to the large attenuation of high-order normal modes, the entire M-order normal modes excited by the sound source are generally not included in the long-distance sound field, so the rank of the decomposed matrix can be larger than the effective normal mode order. The effective orders will lead to better effect in final MFP and get better spatial resolution

From the traditional linear MFP, although the quality of the localization performance in the form is measured by the degree of matching degree between the measured and the copy field, it can be seen as a comparison between the normal mode coefficients. The more positive and accurate normal mode the receiving array can decompose, the better the effect of the MFP, and the signal-to-noise ratio(SNR) and the environmental mismatch will all affect the decomposition result.

B. Combination of horizontal array and vertical array

The combined arrays can sample a wide range of sound fields in space. By treating arrays at different locations as a processing method for multi-matrix arrays with large apertures, higher spatial gains can get higher spatial gain and increase target depth, distance or azimuth resolution.

For a combined array with a VLA and a HLA, note

$$\boldsymbol{\Phi}=\begin{pmatrix}\boldsymbol{\Phi}_1\\\boldsymbol{\Phi}_2\end{pmatrix}\quad \boldsymbol{P}=\begin{pmatrix}\boldsymbol{P}_1\\\boldsymbol{P}_2\end{pmatrix} \tag{6}$$

In this way, the output of the CMFP is

$$D(r,z,\phi)=\frac{\boldsymbol{P}_r^H(r,z,\phi)\boldsymbol{P}}{\Sigma}=\frac{1}{\Sigma}[\boldsymbol{P}_{1,r}^H(r,z,\phi)\boldsymbol{P}_1+\boldsymbol{P}_{2,r}^H(r,z,\phi)\boldsymbol{P}_2] \tag{7}$$

Here,

$$\sum=\sqrt{(\boldsymbol{P}_r^H\boldsymbol{P}_r)(\boldsymbol{P}^H\boldsymbol{P})}=\sqrt{(\boldsymbol{P}_{1,r}^H\boldsymbol{P}_{1,r}+\boldsymbol{P}_{2,r}^H\boldsymbol{P}_{2,r})(\boldsymbol{P}_1^H\boldsymbol{P}_1+\boldsymbol{P}_2^H\boldsymbol{P}_2)} \tag{8}$$

We can also analysis the result of matched field of VLA and HLA as followed:

$$D^{(1)}(r,z,\phi)=\frac{1}{\Sigma_1}\boldsymbol{P}_{1,r}^H(r,z,\phi)\boldsymbol{P}_1=\frac{1}{\Sigma_1}\boldsymbol{A}_r^H\boldsymbol{\Phi}_{1,r}^H\boldsymbol{P}_1 \tag{9}$$

$$D^{(2)}(r,z,\phi)=\frac{1}{\Sigma_2}\boldsymbol{P}_{2,r}^H(r,z,\phi)\boldsymbol{P}_2=\frac{1}{\Sigma_2}\boldsymbol{A}_r^H\boldsymbol{\Phi}_{2,r}^H\boldsymbol{P}_2 \tag{10}$$

And

$$\boldsymbol{B}^{(1)}=\boldsymbol{\Phi}_{1,r}^H\boldsymbol{P}_1;\ \boldsymbol{B}^{(2)}=\boldsymbol{\Phi}_{2,r}^H\boldsymbol{P}_2;$$

$$\Sigma_1=\sqrt{(\boldsymbol{P}_{1,r}^H\boldsymbol{P}_{1,r})(\boldsymbol{P}_1^H\boldsymbol{P}_1)};\ \Sigma_2=\sqrt{(\boldsymbol{P}_{2,r}^H\boldsymbol{P}_{2,r})(\boldsymbol{P}_2^H\boldsymbol{P}_2)}$$

So, the localization function of the combined array is as followed :

$$D(r,z,\varphi)\equiv\frac{\Sigma_1}{\Sigma}D^{(1)}(r,z,\varphi)+\frac{\Sigma_2}{\Sigma}D^{(2)}(r,z,\varphi)=\boldsymbol{A}_r^H\left[\frac{\Sigma_1}{\Sigma}\boldsymbol{B}^{(1)}+\frac{\Sigma_2}{\Sigma}\boldsymbol{B}^{(2)}\right] \tag{11}$$

We can draw conclusions from the above formula. For coherent processors, the localization function of the array is a form of weighted summation. For non-coherent processors, the localization function is added directly by the processing of the two sub-arrays.

If we look directly at the decomposition of the normal mode, the corresponding normal mode decomposition matrix for the composite array is

$$\boldsymbol{\Theta}=\boldsymbol{\Phi}^H\boldsymbol{\Phi}=\boldsymbol{\Phi}_1^H\boldsymbol{\Phi}_1+\boldsymbol{\Phi}_2^H\boldsymbol{\Phi}_2=\boldsymbol{\Theta}_1+\boldsymbol{\Theta}_2 \tag{12}$$

$$\boldsymbol{B}=\boldsymbol{\Phi}^H\boldsymbol{P}=\boldsymbol{\Phi}^H(\boldsymbol{\Phi}\boldsymbol{A})=\boldsymbol{\Theta}\boldsymbol{A}=(\boldsymbol{\Theta}_1+\boldsymbol{\Theta}_2)\boldsymbol{A} \tag{13}$$

From Formula 12 and 13, it can be concluded that the decomposition result of the combined array is the result of a normal mode decomposition of the each sub-array. In principle, as long as the normal mode decomposition matrix of the combined array is full rank, no matter whether the decomposition matrix corresponding to each sub-arrays is full or not, it can have better normal mode decomposition effect and further localization effect.

For a combined array, it can also be considered as a comparison of the decomposition coefficients of a normal mode from the form of an inter-array coherent matching processor. The quality of the localization performance of the combined array can also be determined by the performance of the normal mode decomposition matrix to establish a qualitative or even quantitative relationship with each other.

III. Simulation Experiment

A. Verification of Normal Mode Decomposition Method

In this section, taking a single VLA as an example, the localization performance of VLA in different lengths is analyzed to verify the feasibility of the method and to illustrate the required array design.

Numerical simulation uses horizontal shallow sea channels as shown in Fig.1. The sea depth is 100m, the sound source position is 7.5km and the depth is 50m.

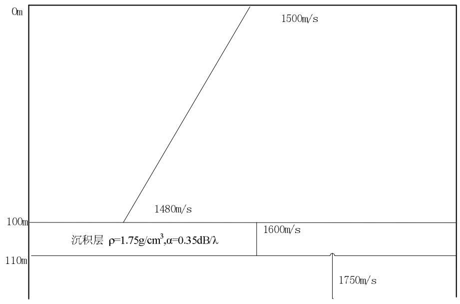
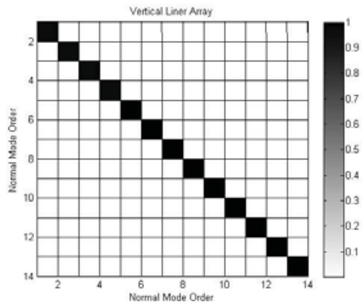
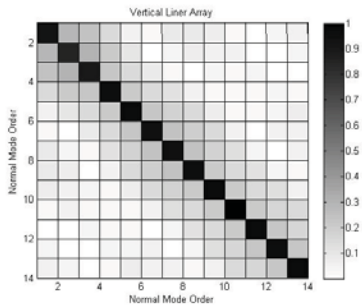


Fig.1. Ocean Environment Model.

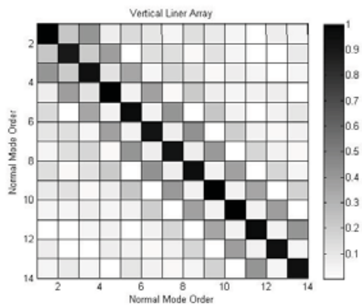
Fig.2 shows the value of normal mode decomposition matrices with different length of VLA at 400Hz, there are 11th order waveguide normal modes, and the array length varies from approximately 98m in the full array to slightly less than 44m as a half array.



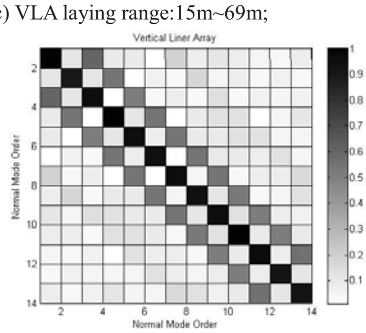
(a) VLA laying range:1m~99m;



(b) VLA laying range:15m~89m;



(c) VLA laying range:15m~69m;



(d) VLA laying range:15m~59m;

Fig.2 The value of normal mode decomposition matrices with different length of VLA

It can be seen from Fig.2 that the reduction of the array length makes the diagonalization feature of the normal mode decomposition matrix gradually disappear (corresponding to the decomposing effect of the normal mode) but the contribution of the low-order normal mode is still significantly greater than that of the other order normal modes. This is because in the case of a negative gradient sound speed profile, low-order normal mode is generally limited to the depth of the water layer near the seabed, and only a relatively short receiving array can satisfy the requirement of “full sampling”.

TABLE I. Eigenvalue Distribution of Normal mode Decomposition Matrices of Different Length VLA

Mode	1-99m	15-89m	15-69m	15-59m
Length(m)	98	74	54	44
1	1.00	1.00	1.00	1.00
2	0.91	0.71	0.71	0.67
3	0.90	0.60	0.61	0.57
4	0.90	0.49	0.55	0.52
5	0.90	0.40	0.45	0.43
6	0.90	0.36	0.42	0.40
7	0.90	0.35	0.32	0.32
8	0.90	0.34	0.29	0.30
9	0.90	0.33	0.24	0.22
10	0.90	0.31	0.22	0.21
11	0.90	0.31	0.21	0.00
12	0.90	0.30	0.00	0.00
13	0.90	0.26	0.00	0.00
14	0.90	0.24	0.00	0.00

Table 1 shows the eigenvalue distribution of normal mode decomposition matrices of different length of VLA. From Table 1, it can be seen that due to the reduction of the array length, some of the feature values have changed from nearly 1 to nearly 0. The appearance of small eigenvalues indicates that the normal mode decomposition performance of the receive array has been significantly reduced, which will also lead to a decrease in the MFP localization performance.

From Table 1, it can be seen that due to the reduction of the array length, some of the feature values have changed from nearly 1 to nearly 0. The appearance of small eigenvalues indicates that the normal mode decomposition performance of the receive array has been significantly reduced, which will also lead to a decrease in the MFP localization performance.

From Fig. 2 and Table 1, we can find that in order to effectively use the VLA for localization, the array length is generally required to be close to the full array level and the length must be at least more than half the sea depth.

B. Combined Array Localization Performance Analysis

In the previous section, the localization performance of a single VLA has been discussed through a simple positive wave decomposition matrix, which verifies the feasibility of this method. In this section, we will continue to use this method to study the localization performance of combined arrays.

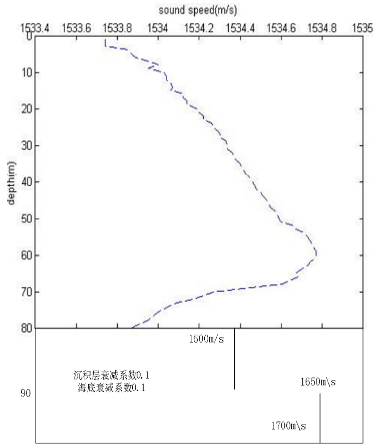


Fig.3 Ocean environment

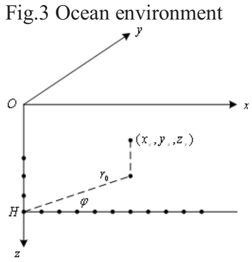
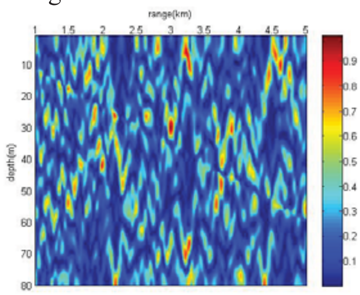
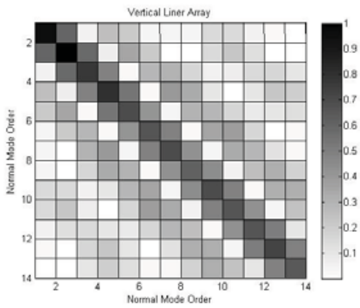


Fig.4 L-shaped array model

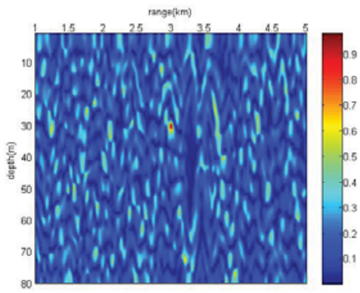
In the numerical simulation, firstly, the comparison of the localization performance between a single VLA (half array) and the L-shaped array is performed. The ocean environment and the the array model are showed as Fig.3 and Fig. 4.



(a) Localization ambiguity surface of VLA

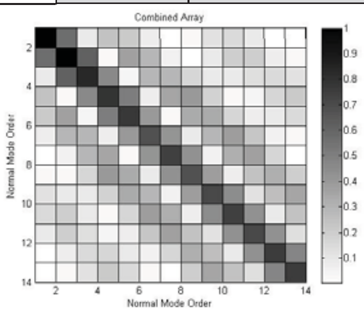


(b) The value distribution of normal mode decomposition matrices with VLA



(c) Localization ambiguity surface of L-shaped array

Mode	No coincident	Partial coincidence	Completely coincide
1	1.00	1.00	1.00
2	0.56	0.61	0.57
3	0.46	0.49	0.42
4	0.40	0.42	0.38
5	0.38	0.35	0.33
6	0.36	0.29	0.28
7	0.36	0.25	0.21
8	0.33	0.22	0.17
9	0.30	0.21	0.15
10	0.27	0.20	0.10
11	0.25	0.16	0.02
12	0.22	0.13	0.00
13	0.16	0.10	0.00
14	0.11	0.07	0.00



(d) The distribution of normal mode decomposition matrices with L-shaped array

Fig. 5 Localization Performance of Different Arrays

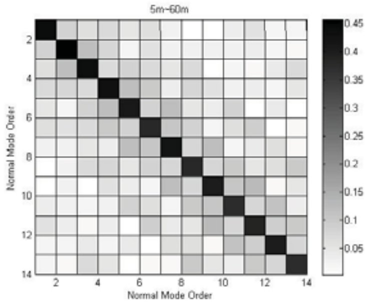
The experiment simulated at 400Hz, the source located at range 3km and 30m deep. The ratio of the mainlobe to the sidelobe of VLA and L-shaped array are 6.89 and 8.37. We can directly find the difference between both arrays. The value distribution of normal mode decomposition matrices are showed in Table 2.

TABLE II The value distribution of normal mode decomposition matrices with different arrays

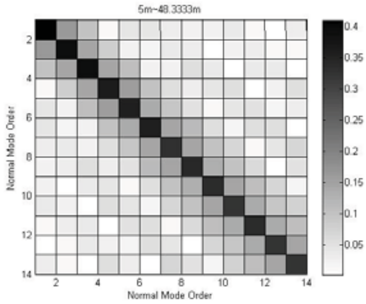
Although the two arrays have many small eigenvalues, the eigenvalues of the normal mode decomposition matrix of the VLA (half-array) are closer to the "0" value, indicating that the localization in a VLA (half array) makes it easier to "mismatch" in the same marine environment.

The combined array design should aim to increase the effective vertical or horizontal aperture. If the aperture repeats, the localization performance does not improve significantly. In addition, horizontal and vertical apertures cannot be directly complementary because the decomposition mechanism is different and it belongs to two orthogonal latitudes. For VLA, the normal mode decomposition matrix uses the orthogonality of its eigenfunction. In principle, when it is used in full sea depth, it has the best decomposition effect and the aperture efficiency is the highest. The horizontal array employs a similar beamforming method. In principle, it requires much larger aperture to ensure the beam space resolution.

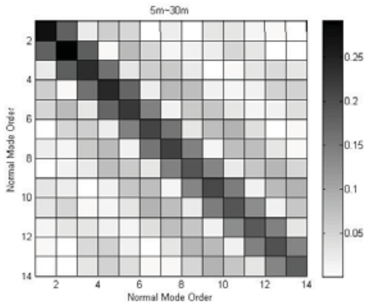
Taking a dual VLA as an example, when the combined array is complemented by the aperture, the effective aperture is increased, thereby improving the localization performance. Here we simulate 3 situations: the aperture of the VLA does not coincide, the aperture of the array overlaps partially, and the aperture of the array completely overlaps. The distance between two VLAs is 100m, the simulation frequency is 400Hz, the source location is 5km, the depth is 5m, and the array element spacing is 1.67m. The simulation results are shown in Fig.6. As the small eigenvalues gradually increase, the diagonalization trend becomes more dispersed.



(a) No coincident apertures



(b) A part of the apertures that coincide



(c)array aperture is completely coincident

Fig. 6 The distribution of normal mode decomposition matrices with different situation.

TABLE III Dual VLA normal mode decomposition matrix eigenvalue

Mode\Array	L-shape array	VLA
1	1.00	1.00
2	0.54	0.55
3	0.36	0.38
4	0.34	0.36
5	0.27	0.28
6	0.19	0.24
7	0.14	0.16
8	0.11	0.12
9	0.07	0.08
10	0.07	0.06

11	0.02	0.00
12	0.01	0.00
13	0.00	0.00
14	0.00	0.00
15	0.00	0.00

In the case of complementary apertures, the combined array can improve the singularity of the normal mode decomposition matrix, that is, it can improve the localization performance. When it is not complementary, it can hardly improve the depth resolution. At most, it increases the number of array elements and improves spatial processing gain.

IV. SUMMARY

In the shallow sea environment, the sound receiving array based on the MFP theory can effectively achieve the goal of localization, and the localization performance receives the influence of parameters such as array shape and effective array element. For this reason, this paper proposes a qualitative conclusion for the design of combined arrays based on the normal mode decomposition method. The design of formations should be focus on increasing the effective aperture, and the horizontal and vertical apertures cannot be directly complementary, because the decomposition mechanism is different and it belongs to two orthogonal dimensions. From the decomposing matrix of the normal mode, the localization performance of the MFP is also corresponding to the decomposition performance of the normal mode. Therefore, we can design the combined array by the singularity of the normal mode decomposing matrix.

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