Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory
Harry L. Van Trees

ISBNs: 0-471-09390-4 (Hardback); 0-471-22110-4 (Electronic)

Chapter 1

Introduction

In Parts I, II, and III of *Detection, Estimation, and Modulation Theory* (DEMT) [VT68], [VT01a], [VT71a], [VT71b], [VT01b], we provide a reasonably complete discussion of several areas:

(i) Detection theory

In this case, we were concerned with detecting signals in the presence of Gaussian noise. The class of signals included known signals, signals with unknown parameters, and signals that are sample functions from Gaussian random processes. This problem was covered in Chapter I-4 and Chapters III-1 through III-5.

(ii) Estimation theory

In this case, we were concerned with estimating the parameters of signals in the presence of Gaussian noise. This problem was covered in Chapter I-4 and Chapters III-6 and III-7.

(iii) Modulation theory

In this case, we were concerned with estimating a continuous waveform (or the sampled version of it). If the signal has the waveform in it in a linear manner, then we have a linear estimation problem and obtain the Wiener filter or the Kalman-Bucy filter as the optimum estimator. This problem was covered in Chapter I-6. The case of nonlinear modulation is covered in Chapter I-5 and Volume II.

All of the results in the first three volumes consider signals and noises that could be characterized in the time domain (or equivalently, the frequency domain). In this book, we consider the case in which the signals and noises also have a spatial dependence. Therefore, we must characterize the signals and noises as space-time processes and solve the detection and estimation problems in the multidimensional space-time domain. This leads us to space-time processors. The spatial part of the processor is an **aperture** (or antenna) for the continuous space domain and an **array** for the discrete space domain. The focus of this book is on optimum array processing (and optimum aperture processing). The formal extension of the temporal results to the multidimensional problem is reasonably straightforward, but the implications of the results lead to a number of challenging questions.

In Section 1.1, we give a simple description of the array processing problem. In Section 1.2, we give a brief description of some representative applications in which arrays play a key role. In Section 1.3, we outline the structure of the array processing literature. In Section 1.4, we outline the organization of the book.

1.1 Array Processing

In this section, we introduce the array processing problem and discuss some of the issues that we will encounter in the text.

A representative array consisting of six sensors is shown in Figure 1.1. We can use this array to illustrate some of the issues. The four issues of interest are:

(A) Array configuration

(B) Spatial and temporal characteristics of the signal

(C) Spatial and temporal characteristics of the interference

(D) Objective of the array processing

The first issue is the array configuration. The array configuration consists of two parts. The first part is the antenna pattern of the individual elements. For example, in a transmitting RF array this will be a function of the physical configuration of the sensor and the current distribution on the elements. In many cases, we first assume that the elements have an **isotropic** pattern (i.e., uniform in all directions) and then incorporate the actual pattern later in the analysis.

The second part of the array configuration is the array geometry (i.e., the physical location of the elements). The array geometry is one part of the problem where we will focus our attention.

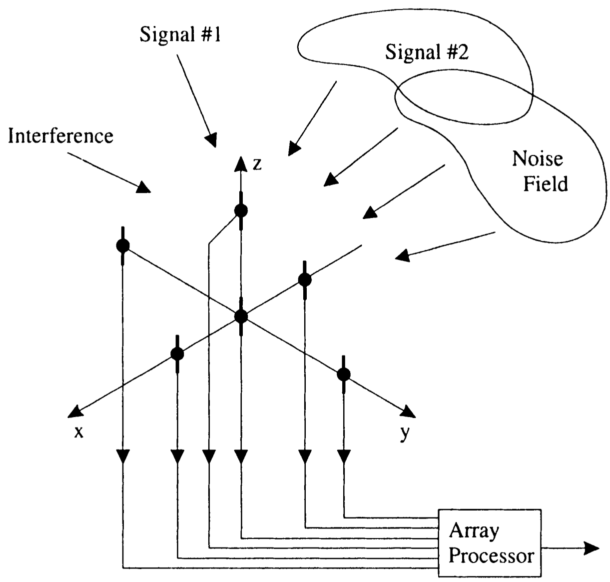

Figure 1.1 Array processing problem.

We can divide array geometries into three categories:[1]

A1 Linear

A2 Planar

A3 Volumetric (3-D)

Within each category we can develop a taxonomy. In linear arrays, the following cases are of interest:

A1.1 Uniform spacing

A1.2 Non-uniform spacing

A1.3 Random spacing

We will find that the total length of the array and the array spacing determine how the array geometry affects the problem.

In a planar array, the boundary of the array and the element geometry are important. For example, we can have a planar array whose boundary is circular and the element geometry could be either square or triangular as shown in Figure 1.2.

[1]The indexing notation is for convenience. We will not use it in our later discussions.

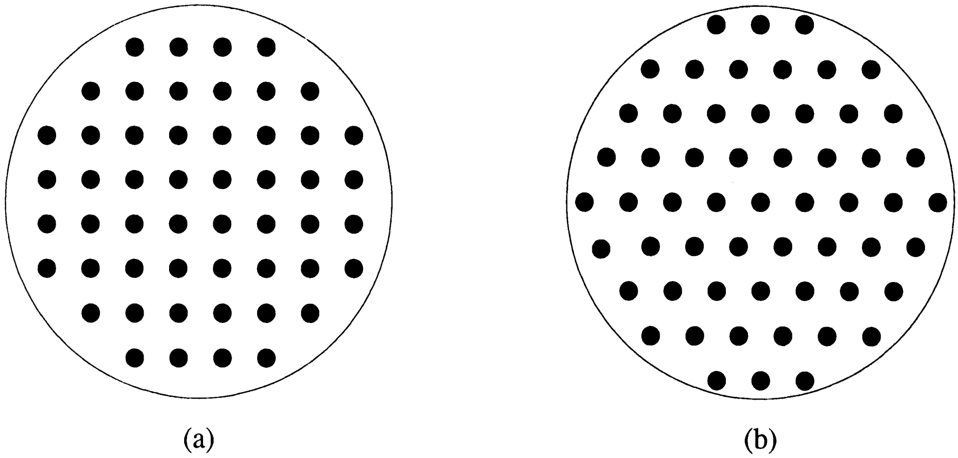

Figure 1.2 Array geometries.

The second issue is spatial and temporal structure of the signal. In the temporal domain we will encounter the same signal structures as in DEMT I and III:

BT1 Known signals

BT2 Signals with unknown parameters

BT3 Signals with known structure (e.g., QPSK)

BT4 Random signals

It is the spatial domain that is of most interest. The cases of interest here include:

BS1 Plane-wave signals from known directions

BS2 Plane-wave signals from unknown directions

BS3 Spatially spread signals

The third issue is the spatial and temporal structure of the noise (or interference). We will always include a "sensor noise" component that consists of a white Gaussian noise component that is statistically independent from sensor to sensor. The physical motivation for this inclusion is that noise of this type always exists. System design may reduce the sensor noise to a level that makes other noises dominant, but it will never be zero. From our results in DEMT I and III, we would anticipate that this white noise component will ensure that we do not obtain a singular solution.

The noise coming from external sources must be characterized both temporally and spatially. The temporal cases of most interest are:

CT1 Signals with unknown parameters (this includes the case where we know the modulation of an interfering signal, but not the information sequence)

CT2 Random signals

The spatial cases of most interest are:

CS1 One or more plane waves from known directions

CS2 One or more plane waves from unknown directions

CS3 Spatially spread interference

The fourth issue of interest is the objective of the array processing problem. Some representative objectives are:

D1 Detect the presence of a signal in the presence of noise and interference

D2 Demodulate the signal and estimate the information waveform (i.e., listen to the signal) in the presence of noise and interference

D3 A binary communication signal arrives over multiple paths, detect the information sequence

D4 Estimate the direction-of-arrival of multiple plane-wave signals in the presence of noise

D5 Construct temporal and spatial spectral estimate of the incoming signal and noise field

D6 Direct the transmitted signal to a specific spatial location

We will encounter other objectives in our discussion of applications in Section 1.2 and at various places in the text and problems. The above list serves to illustrate the wide variety of problems we will encounter.

We now consider various application areas in which arrays play an important role.

1.2 Applications

In this section we provide a brief discussion of seven areas in which arrays play an important role. These areas are:

(i) Radar

(ii) Radio astronomy

(iii) Sonar

(iv) Communications

(v) Direction-finding

(vi) Seismology

(vii) Medical diagnosis and treatment

Our discussion in each area is short. However, we provide a list of references that provide more detailed discussions. We also revisit these application areas in subsequent chapters and in the problems.

1.2.1 Radar

The radar area is the area in which antenna arrays were first used.[2] Skolnik has a good discussion of the use of phased arrays in radar in Chapter 8 of his book, *Introduction to Radar Systems* [Sko80], and some of our specific examples are taken from it.

Most radar systems are active systems and the antenna array is used for both transmission and reception of signals.

Although the concept of phased array antennas was known during World War I [Sou62], the first usage was in World War II. Examples of United States systems included fire control radars for Navy ships [OK72] and high resolution navigation and bombing radars [Rid47]. Examples of British usage included height-finding radars [Smi49]. The work at the M.I.T. Radiation Lab is described in the Radlab Series.

Current military systems include the PAVE PAWS radar, which is used for ballistic missile detection [Bro85], the AEGIS phased array antenna [Bro91], and numerous airborne systems. Non-military systems include air traffic control radars [EJS82].

[2] The usage of phased arrays in communication systems evolved in the same time period. We discuss communication applications in Section 1.2.4.

Ground- and ship-based radars can generally use a model in which both the signal and interference can be modeled as plane waves impinging on the array. In some environments we encounter multipath, but the plane-wave model is still valid.

Airborne radars looking at the ground have the additional complexity of reflections from the ground that is referred to as clutter (clutter models were discussed in Chapter 13 of DEMT III [VT71b], [VT01b]). Models must now include spatially spread interference. A discussion of this type of system is given in Ward [War94].

Other references that discuss various aspects of radar systems include Allen [All63], Reintjes and Coate [RC52], Skolnik [Sko70], Barton [Bar65], Berkowitz [Ber65], Di Franco and Rubin [FR68], Cook and Bernfeld [CB67], Barton and Ward [BW69], Reed [Ree69], and Haykin ([Hay80], Chapter 4 of [Hay85] and [SH92]).

1.2.2 Radio Astronomy

Antenna arrays are widely used in the radio (or radar) astronomy area. Books that provide good introductions to the area include Kraus [Kra66], Evans and Hagfors [EH68], Christiansen and Högbom [CH69], and Kellerman and Verschuur [KV73]. Yen in Chapter 5 of [Hay85] provides a good introduction.

A radio astronomy system is a passive system that is used to detect celestial objects and estimate their characteristics. These systems usually employ arrays with very long baselines. These baselines range from tens of kilometers to thousands of kilometers. Representative systems include the very large array (VLA) of the National Radio Astronomy Observatory [TCWN80] and Cambridge telescope [Ryl73].

Typical array configurations include:

- Linear arrays with unequal spacing
- Parallel linear arrays
- Circular arrays
- Arrays with three linear arms spaced at 120° with a common center

Some of the issues that must be considered in radio astronomy include the rotation of the earth during the signal processing period, different propagation characteristics through the ionosphere and troposphere at different array elements, and synchronization over long distances. We encounter various radio astronomy examples at different points in the text.

1.2.3 Sonar

Arrays are widely used in sonar systems. A good discussion of array processing in sonar systems is given by Baggeroer in Chapter 6 of [Opp78]. Several of our examples are taken from this reference. The journal article by Knight et al. [KPK81] (this article contains 253 references) and Owsley's chapter in [Hay85] also provide good introductions. It is useful to consider active and passive sonars separately.

An active sonar transmits acoustic energy into the water and processes received echos. The theory of active sonars has much in common with radars. However, a fundamental important difference between sonar and radar is that the propagation of acoustic energy in the ocean is significantly more complicated than the propagation of electromagnetic energy in the atmosphere. These propagation characteristics have a major influence on the design of sonar systems.

Propagation factors include spreading loss, absorption, and ducting. These factors will vary depending on the ranges of interest, the depth of the water, and the nature of the boundaries. Discussion of sound propagation is given in Urick [Uri67], Albers [Alb60], Burdic [Bur91], Horton [Hor57], and Hassab [Has89], as well as Section 6.2 of Baggeroer's chapter of [Opp78].

The noise background includes ambient noise, self noise, and reverberation noise. Ambient noise is acoustic noise generated by various sources in the ocean such as ships, industrial activity, sea life, precipitation, ice, and explosions. Typically, it is spread in both frequency and space. Self noise is generated by the platform. Examples include cavitation noise, flow noise, and machinery noise. Self noise may be either frequency spread or tonal and is normally concentrated spatially. Reverberation noise is due to reflections of the transmitted signal and is analogous to clutter in a radar system.

Other factors such as interaction with the boundaries, spatial coherence of the acoustic waves, and the severity of the ocean environment must also be taken into account.

Passive sonar systems listen to incoming acoustic energy and use it to estimate the temporal and spatial characteristics of the observed signal field. The most important application is the detection and tracking of submarines. Some representative configurations are shown in Figure 1.3. All of the comments about propagation and noise also apply to the passive sonar case. We analyze several passive sonar examples in the text.

Other sonar references include Wagstaff and Baggeroer [WB83], Officer [Off58], Tolstoy and Clay [TC66], Dyer [Dye70], Griffiths et al. [GSS73], Baggeroer [Bag76], Cron and Sherman [CS62], Cox [Cox73], and Blahut et

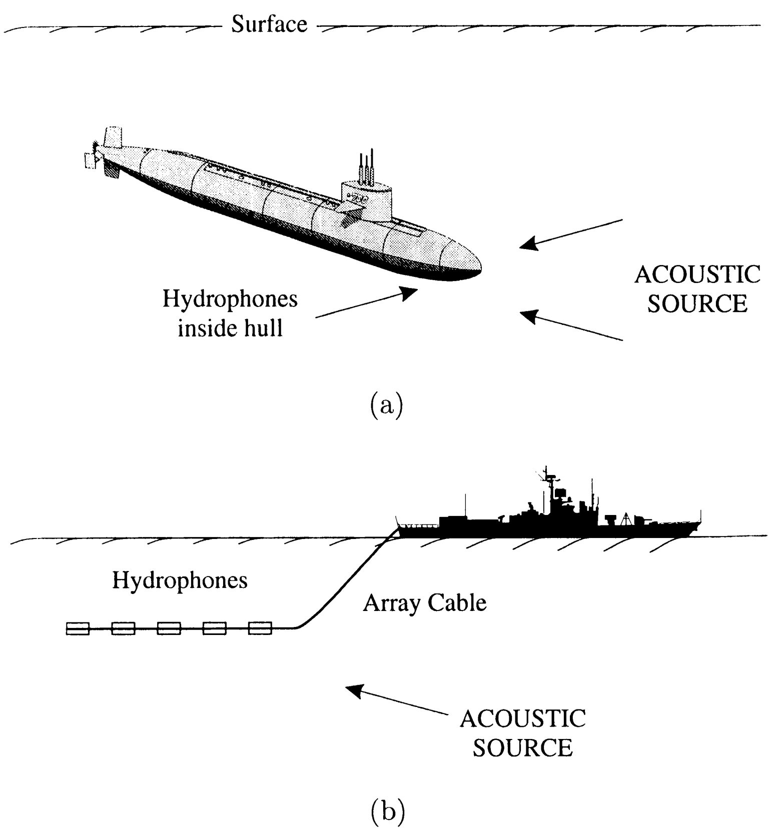

Figure 1.3 Examples of several passive sonar systems: (a) submarine mounted hydrophone arrays; (b) towed array.

al. [BMW91]. Current research is usually reported in the *Journal of the Acoustic Society of America* (JASA) and the *IEEE Transactions on Signal Processing.*

1.2.4 Communications

Antenna arrays are used in many communication systems. One of the first usages was for transatlantic shortwave communication in the 1930s [FF37]. Current usage spans the frequency bands from ELF to EHF and includes both terrestially based and satellite communications.

The communication signals are point sources but, due to the channel characteristics, they may arrive at the receiving array as a single plane wave, multiple plane waves due to multipath, or as a spatially spread signal. In addition to receiver noise, the interference may include other communications signals or intentional jamming signals.

Several satellite systems utilize phased arrays in either the earth terminal or space segment. For example, the tracking and data relay satellite (TDRSS) uses a 30-element array at S-band. Other satellite systems use multiple beam antennas (MBAs) to achieve similar results. For example, the Defense Satellite Communication System (DSCS III) has a 61-beam receive MBA and 19-beam transmit MBA. Many of the low earth orbit (LEO) satellite systems utilize phased arrays.

The second generation of wireless cellular systems utilize various types of multiple access techniques such as TDMA, CDMA, and GSM (global system for mobile communication). Antenna array processing can provide significant performance in all of these systems and the planned third-generation systems. The term "smart antennas" come into usage to describe various types of adaptive arrays for wireless systems. Several references, [Win98], [God97a], [God97b], [PP97] and [Rap98], discuss this application area in detail.

1.2.5 Direction Finding

Antenna arrays are widely used to solve the direction finding (DF) problem. The objective is to locate the source of transmitted communication or radar signal. A common approach is to find the direction of arrival (DOA) of the signal at two separated antenna arrays and locate the source at the intersection of the two lines of bearing. The estimation of DOAs is the central focus of Chapters 8 and 9, and we discuss the problem further at that point.

1.2.6 Seismology

There are two areas of seismology in which array processing plays an important role. The first area is the detection and location of underground nuclear explosions. The area received significant attention in the 1960s and 1970s and a number of results that were obtained for that area, such as Capon's minimum variance distortionless response (MVDR) beamformer, are used in many other areas.

The second area is exploration seismology and is the most important at the present time. Justice in Chapter 2 of [Hay85] has a detailed discussion of array processing in exploration seismology, and his chapter has

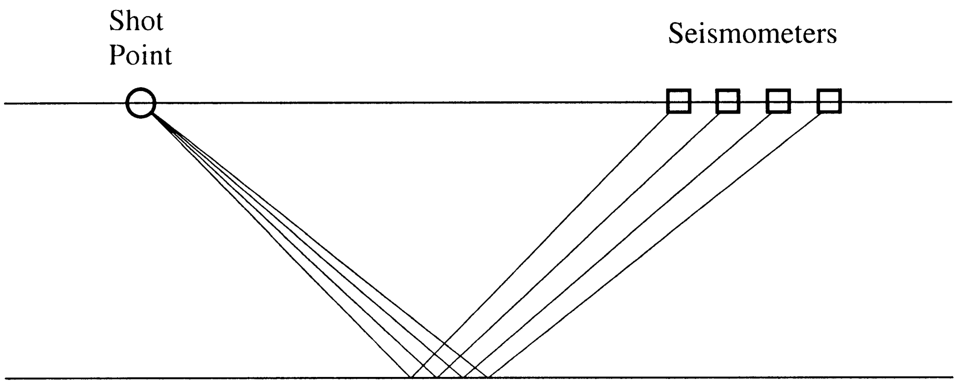

Figure 1.4 Seismic experiment.

322 references. Robinson and Treitel in Chapter 7 of [Opp78] also have a good discussion. The objective of the exploration process is to construct an image of the subsurface in which the structure and physical properties are described. As in the sonar case, the propagation characteristics of the acoustic signal in an inhomogeneous elastic medium has a dominant influence on the designs of the system.

A typical seismic experiment is shown in Figure 1.4. Acoustic energy is transmitted into the earth by a shot and reflected energy is received by a set of geophones arranged in linear array. Normally the earth is modeled as a stack of homogeneous layers and the array measures reflections from various layers. In the text we encounter various examples of seismic signal processing.

The references in Justice [Hay85] provide further discussions of seismic signal processing. Current research is reported in *Geophysics* and *Geophysical Prospecting.*

1.2.7 Tomography

Tomography is the cross-sectional imaging of objects from transmitted or reflected data. The object is illuminated from a number of different directions and data are collected at a receiving array. We then try to reconstruct the cross-sectional image from the data. Kak in Chapter 6 of [Hay85] has a good discussion of tomography. Tomography has had great success in the medical diagnosis area.

The processing algorithms used in tomography are different from those that we develop in the text, so we will not discuss them further. The interested reader is referred to [Hay85] and the references contained in that chapter for further discussion.

1.2.8 Array Processing Literature

Due to the wide variety of applications of array processing, the literature is spread across a number of different journals and conferences. In Appendix B, we have included a representative list of sources.

1.3 Organization of the Book

The book is organized into five parts. The first part consists of Chapters 2, 3, and 4. These three chapters discuss classical array analysis and synthesis techniques. We use the adjective "classical" because the techniques rely primarily on deterministic models and the theory was reasonably mature by the early 1970s. The techniques are important for two reasons:

(i) These techniques are still widely used in practical array applications.

(ii) They form the foundation for the statistical approaches that we use later in the book.

Chapter 2 introduces the basic definitions and relationships that are used to analyze and synthesize arrays. Our approach is to introduce the concept for an arbitrary array geometry. We then specialize the result to a uniform linear array and then further specialize the result to a uniform linear array with uniform weighting.

In Chapter 3, we return to linear arrays and provide a detailed discussion of the analysis and synthesis of linear arrays. In Chapter 4, we study the analysis and synthesis of planar and volumetric arrays.

The second part of the book consists of Chapter 5 and studies the characterization of space-time random processes. We develop second-moment theory for arbitrary processes and a complete characterization for Gaussian random space-time processes. We develop orthogonal expansions and review the concept of signal subspaces and noise subspaces that we utilized for the temporal problem in Chapter I-4 of DEMT I [VT68], [VT01a]. We introduce parametric spatial models and discuss their usage in array processing problems. The chapter provides the statistical models that will be used in the remainder of the text.

The third part of the book consists of Chapters 6 and 7 and studies waveform estimation. In Chapter 6, we derive optimum beamformers under the assumption of known statistics and analyze their performance. We investigate the sensitivity of the optimum beamformers to perturbations in the signal and noise model and the array description. These sensitivity results motivate the development of constrained processors that are more robust to model perturbations.

In Chapter 7, we consider the case in which the statistics must be determined from the data. This problem leads us into adaptive beamforming. We develop various adaptive algorithms and analyze their behavior.

The fourth part of the book consists of Chapters 8 and 9. These chapters consider the parameter estimation problem with emphasis on estimating the direction of arrival of incoming plane-wave signals. We first develop maximum likelihood estimators and compute bounds on the performance of any estimator. We then study a large variety of estimation algorithms and compare their performance to the maximum likelihood estimators and the bounds.

The fifth part of the book consists of Chapter 10 and contains a brief discussion of the optimum detection problem. Chapter 10 also contains a discussion of some of the areas that the book has not covered. There is an appendix that summarizes some of the matrix algebra results that we use in the text.

There are problems at the end of each chapter. As in DEMT I – III [VT68], [VT01a], [VT71a], [VT71b], [VT01b], it is necessary for the reader to solve problems in order to understand the material fully. Throughout the course and the book we emphasize the development of an ability to work problems. The problems range from routine manipulations to significant extensions of the material in the text. In many cases they are equivalent to journal articles currently being published. Only by working a fair number of them is it possible to appreciate the significance and generality of the results. Many of the problems require the use of mathematical computation package such as MATLAB® , Mathematica, or MAPLE. We assume that the student is experienced with one of these packages. The solutions on the Web site use MATLAB.®

The book can be covered in a two-semester graduate course for students with the appropriate background. Chapters 1 through 6 can be covered in the first semester. The common theme in these chapters is that the design is either deterministic or assumes that the necessary statistics are known. Chapters 7 through 10 can be covered in the second semester. The common theme in these chapters is that the algorithms obtain the necessary statistics from the data. This results in adaptive beamformers, adaptive detectors, and DOA estimators.

1.4 Interactive Study

The book is designed to be used in two different modes. The first mode is as a stand-alone text. By reading the text and doing a representative set of homework problems, one can understand the material. The difficulty with this mode is that we rely on a large number of examples to develop the material. By necessity, many of the examples will choose specific parameter values to demonstrate the point. It would be more desirable to be able to explore a family of parameters.

The second mode uses the Web site that was discussed in the Preface:

http://ite.gmu.edu/DetectionandEstimationTheory/

The contents of the Web site is described in the Preface. We anticipate that this second mode will be used by most serious readers and instructors.

Bibliography

[Alb60] V. O. Albers. *Underwater Acoustics Handbook.* Pennsylvania State Univesity Press, University Park, Pennsylvania, 1960.

[All63] J. Allen. The theory of arrays antennas with emphasis on radar applications. Technical Report 323, M.I.T. Lincoln Laboratory, Lexington, Massachusetts, July 1963.

[Bag76] A. B. Baggeroer. Space-time processes and optimal array processing. Technical Report 506, Navy Undersea Center, San Diego, California, December 1976.

[Bar65] D. K. Barton. *Radar System Analysis.* Prentice-Hall, Englewood Cliffs, New Jersey, 1965.

[Ber65] R. S. Berkowitz. *Modern Radar: Analysis, Evaluation, and System Design.* Wiley, New York, 1965.

[BMW91] R. E. Blahut, W. Miller, Jr., and C. H. Wilcox. *Radar and Sonar, Part I.* Springer-Verlag, New York, 1991.

[Bro85] E. Brookner. Phased array radars. *Sci. Amer.*, pp. 94–102, February 1985.

[Bro91] E. Brookner, editor. *Practical Phased-Array Antenna Systems.* Artech House, Boston, 1991.

[Bur91] W. S. Burdic. *Underwater Acoustic System Analysis.* Prentice-Hall, Englewood Cliffs, New Jersey, 2nd edition, 1991.

[BW69] D. K. Barton and H. R. Ward. *Handbook of Radar Measurement.* Prentice-Hall, New Jersey, 1969.

[CB67] C. E. Cook and M. Bernfeld. *Radar Signals, An Introduction to Theory and Application.* Academic Press, New York, 1967.

[CH69] W. N. Christiansen and J. A. Högbom. *Radiotelescopes.* Cambridge University Press, Cambridge, 1969.

[Cox73] H. Cox. Resolving power and sensitivity to mismatch of optimum array processors. *J. Acoust. Soc. of Amer.*, vol.54(3), pp. 771–785, September 1973.

[CS62] B. F. Cron and C. H. Sherman. Spatial-correlation functions for various noise models. *J. Acoust. Soc. Amer.*, vol.34, pp. 1732–1737, November 1962.

[Dye70] I. Dyer. Statistics of sound propagation in the ocean. *J. Acoust. Soc. Amer.*, vol.48, pp. 337–345, May 1970.

[EH68] J. V. Evans and T. Hagfors, editors. *Radar Astronomy.* McGraw-Hill, New York, 1968.

[EJS82] J. E. Evans, J. R. Johnson, and D.F. Sun. Application of advanced signal processing techniques to angle of arrival estimation in ATC navigation and surveillance systems. Technical Report, M.I.T. Lincoln Laboratory, Lexington, Massachusetts, June 1982.

[FF37] H. T. Friis and C. B. Feldman. A multiple unit steerable antenna for short-wave reception. *Bell Syst. Tech. J.*, vol.16, pp. 337–419, July 1937.

[FR68] J. V. Di Franco and W. L. Rubin. *Radar Detection.* Prentice-Hall, Englewood Cliffs, New Jersey, 1968.

[God97a] L. C. Godara. Application of antenna arrays to mobile communications, Part I: Performance improvement, feasibility, and system considerations. *Proc. IEEE*, vol.85(7), July 1997.

[God97b] L. C. Godara. Application of antenna arrays to mobile communications, Part II: Beamforming and direction-of-arrival considerations. *Proc. IEEE*, vol.85(8), August 1997.

[GSS73] J. W. R. Griffiths, P. Stocklin, and C. Van Schoonveld, editors. *Signal Processing.* Academic Press, New York, 1973.

[Has89] J. C. Hassab. *Underwater Signal and Data Processing.* CRC Press, Inc., Boca Raton, Florida, 1989.

[Hay80] S. Haykin, editor. *Array Processing: Applications to Radar.* Dowden, Hutchinson, & Ross, Stroudsburg, Pennsylvania, 1980.

[Hay85] S. Haykin. *Array Signal Processing.* Prentice-Hall, Englewood Cliffs, New Jersey, 1985.

[Hor57] J. W. Horton. *Fundamentals of Sonar.* U. S. Naval Institute, Washington, D. C., 1957.

[KPK81] W. C. Knight, R. G. Pridham, and S. M. Kay. Digital signal processing for sonar. *Proc. IEEE*, vol.69(11), pp. 1451–1507, November 1981.

[Kra66] J. D. Kraus. *Radio Astronomy.* McGraw-Hill, New York, 1966.

[KV73] K. I. Kellermann and G. L. Verschuur, editors. *Galactic and Extra-Galactic Radio Astronomy.* Springer-Verlag, New York, 1973.

[Off58] C. B. Officer. *Introduction to the Theory of Sound Transmission.* McGraw-Hill, New York, 1958.

[OK72] A. A. Oliner and G. H. Knittel, editors. *Phased Array Antennas.* Artech House, Boston, Massachusetts, 1972.

[Opp78] A. V. Oppenheim, editor. *Applications of Digital Signal Processing.* Prentice-Hall, Englewood Cliffs, New Jersey, 1978.

[PP97] A. J. Paulraj and C. B. Papadias. Space-time processing for wireless communications. *IEEE Signal Processing Mag.*, vol.14(6), November 1997.

[Rap98] T. S. Rappaport, editor. *Smart Antennas.* IEEE Press, New York, 1998.

[RC52] J. F. Reintjes and G. T. Coate. *Principles of Radar.* McGraw-Hill, New York, 1952.

[Ree69] J. E. Reed. The AN/FPS-85 radar system. *Proc. IEEE,* vol.57(3), pp. 324–335, March 1969.

[Rid47] L. N. Ridenour. *Radar System Engineering.* McGraw-Hill, New York, 1947.

[Ryl73] M. Ryle. The 5-km radio telescope at Cambridge. *Nature,* vol.239, pp. 435–438, October 1973.

[SH92] T. J. Shepherd, S. Haykin, and J. Litva, editors. *Radar Array Processing.* Springer-Verlag, New York, 1992.

[Sko70] M. I. Skolnik. *Radar Handbook.* McGraw-Hill, New York, 1970.

[Sko80] M. I. Skolnik. *Introduction to Radar Systems.* McGraw-Hill, New York, 1980.

[Smi49] R. A. Smith. *Aerials for Metre and Decimetre Wave-lengths.* Cambridge University Press, London, 1949.

[Sou62] G. C. Southworth. *Forty Years of Radio Research.* Gordon and Breach, New York, 1962.

[TC66] I. Tolstoy and C. S. Clay. *Ocean Acoustics: Theory and Experiment in Underwater Sound.* McGraw-Hill, New York, 1966.

[TCWN80] A. R. Thompson, B. G. Clark, C. M. Wade, and P. J. Napier. The very large array. *Astrophys. J. Suppl.,* vol.44, pp. 151–167, April 1980.

[Uri67] R. J. Urick. *Principles of Underwater Sound for Engineers.* McGraw-Hill, New York, 1967.

[VT68] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part I.* Wiley, New York, 1968.

[VT01a] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part I.* Wiley Interscience, New York, 2001.

[VT01b] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part III.* Wiley Interscience, New York, 2001.

[VT71a] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part II.* Wiley, New York, 1971.

[VT71b] H. L. Van Trees. *Detection, Estimation, and Modulation Theory, Part III.* Wiley, New York, 1971.

[War94] J. Ward. Space-time adaptive processing for airborne radar. Technical Report 1015, M.I.T Lincoln Laboratory, Lexington, Massachusetts, December 1994.

[WB83] R. A. Wagstaff and A. B. Baggeroer, editors. *High-Resolution Spatial Processing in Underwater Acoustics.* Naval Ocean Research and Development Activity, NSTL, Gulf Park, Mississippi, 1983.

[Win98] J. H. Winters. Smart Antennas for Wireless Systems. *IEEE Personal Commun.,* vol.1, pp. 23–27, February 1998.