This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAES.2019.2895709, IEEE Transactions on Aerospace and Electronic Systems

Single Space Based Sensor Bias Estimation using a Single Target of Opportunity

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Sensor registration refers to the estimation/compensation of systematic (bias) errors, in contrast to the random errors from sensor noise. Various methods have been proposed for bias estimation of multiple optical sensors using common targets of opportunity. However, the proposed solutions required the use of multiple (two or more) optical sensors and the need to solve the data association problem arising from the fusion of measurements from multiple sensors. In order to remove these constraints, we provide in this paper a new methodology using a single exoatmospheric target of opportunity seen in a single satellite borne sensor's field of view to estimate the sensor's biases simultaneously with the state of the target. The satellite based sensor sees the target from a changing direction as a function of its position, allowing the target in this nonlinear tracking system to be observable. The sensor provides the Line Of Sight (LOS) measurements of azimuth and elevation to the target. Sensor pointing calibration is the key precondition of accurate tracking of a target in a space based system. Statistical tests on the results of simulations, and the evaluation of the Cramér-Rao Lower Bound (CRLB) on the covariance of the bias estimates show that this method is statistically efficient.

Index Terms

Bias estimation, space-based tracking, observability, statistical efficiency, CRLB, maximum likelihood.

I. Introduction

Despite the efforts for precise alignment of a space based sensor before launch, on-orbit calibration is needed because additional errors may be introduced after launch. For angle-only sensors, imperfect registration leads to LOS angle measurement biases in azimuth and elevation. Methods to enhance sensor LOS accuracy will minimize track uncertainties and enhance track state estimation. It is difficult to obtain an accurate alignment because the alignment error (bias) is the sum of various error sources [14], including:

- Location bias; the sensor is not exactly where it is assumed to be. This is especially important if a high precision sensor location measurement such as GPS is unavailable.

Approved for Public Release 17-MDA-8982 (17 January 17)

- Sensor attitude bias: wheel assembly controller error, and coordinate system translation round-off error.
- Timing bias: errors in timing caused by imperfections in the clocks of the sensors.
- Sensor alignment bias: the pitch, yaw and roll angles of the sensor boresight yield 3 alignment biases. In this paper we will estimate these alignment biases.

The estimation of range, azimuth and location biases for active sensors was presented in [20]. In [2] bias estimation of a single radar was calculated using frequency-weighted linear quadratic Gaussian estimation. The observability of location biases has been investigated in [3], where it was found that the observability criterion can be met using fixed targets of opportunity. Using a least squares (LS) method, biases for two radars of unknown locations were estimated in [15]. This method gives information on the observability of the biases, and some of them are not observable. A statistical analysis of bias estimation of two radars with known locations using track reports was presented in [17].

In [12] the analysis of sensor bias and timing error effects on the tracking quality of an infrared (IR) tracking system using an Extended Kalman Filter (EKF) was presented. This was extended in [13] by proposing a method of using fixed stars to eliminate the sensor bias errors.

In [18] an approach using maximum a posteriori (MAP) data association for concurrent bias estimation and data association based on sensor-level track state estimates was proposed and extended in [19]. In [8] simultaneous sensor bias and target position estimation using fixed passive sensors was proposed. A solution to the related observability issues discussed in [8] is proposed in [9] and extended in [10] using space based sensors instead of fixed ones. In [11] an approach to bias estimation in the presence of measurement association uncertainty using common targets of opportunity was developed. Simultaneous target state and passive sensors bias estimation was proposed in [5] and [7], however, the tracking system considered in these works consisted of two optical sensors (space based) tracking a single target of opportunity.

This paper considers the problem of estimating the orientation biases of a single space based sensor while simultaneously estimating the state of a single target (position and velocity) of opportunity. The method in our previous work [6] is examined in more detail and is extended in this paper. The main contributions of the new paper beyond [6] are: definition of bias estimability requirements, discussion of the observability issues related to the bias estimation (it is shown that for a single space based sensor there is a subtle marginal observability), and providing a simple practical solution to the observability issue by using more than the strict minimum number of measurements. The sensor location is assumed known[1] and its orientation biases are assumed constant during the entire tracking time. The new method is validated using two hypothetical scenarios created using System Tool Kit (STK) [1]. In the first scenario, the tracking system consists of a single optical space based sensor tracking a single target of opportunity. In the second case, the tracking system consists of two space based optical sensors tracking a single target [7].

The evaluation of the Cramér-Rao Lower Bound (CRLB) on the covariance of the bias estimates, and the statistical tests on the results of simulations show that both the target state and the biases are observable and that this method

[1]The solution of this problem can be extended to include the location biases.

is statistically efficient, in the scenarios considered. Section II presents the problem formulation and Section III the detailed solution. Section IV describes the results of simulations performed. Finally, Section V gives the conclusions.

II. Problem Formulation

A. Coordinate frames

The fundamental frame of reference used in this paper is a 3D Cartesian Common Coordinate System (CCS). The CCS frame has its origin at the center of a fixed non-rotating spherical earth of mass M_e (assumed) with uniform density. The CCS is defined by the orthogonal set of unit vectors (i_x, i_y, i_z). The xy plane coincides with the Earth's equatorial plane. The x axis is permanently fixed in a direction relative to the celestial sphere (which does not rotate like the Earth does). The z axis lies at a $90°$ angle to the equatorial plane and extends through the North Pole.

The sensor has its own local coordinate system (measurement frame of the sensor). The measurement frame of the sensor is defined by an orthogonal set of unit vectors (i_ξ, i_η, i_ζ). The origin of the sensor frame is a translation of the origin of the CCS coordinate frame from the center of the Earth to the center of the sensor, and its axes are rotated with respect to the CCS axes. This is the frame in which sensor measurements such as azimuth and elevation are obtained.

B. Coordinate Transformations

The operation to transform the target location expressed in CCS coordinates into the sensor coordinate system is a combination of a sequence of rotations about each of the three axes and a translation given by the difference between the vector position of the target and the vector position of the sensor [10]. The rotation between these frames can be described by a set of Euler angles $\varphi+\varphi^{\mathrm{n}}$, $\theta+\theta^{\mathrm{n}}$, $\psi+\psi^{\mathrm{n}}$ denoted as roll, pitch and yaw respectively, where φ^{n} is the nominal (true) roll angle, φ is the roll bias, θ^{n} is the nominal pitch angle, θ is the pitch bias, ψ^{n} is the nominal yaw angle and ψ is the yaw bias. The xyz Euler angle convention (rotation sequence) is used in this paper, which rotates first about the x axis by φ^{n}, then rotating about the y axis by θ^{n}, and finally rotating about the z axis by ψ^{n}. For each axis the rotation matrix is

$$
\begin{aligned}
T_x(\varphi^{\mathrm{n}}) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi^{\mathrm{n}} & \sin\varphi^{\mathrm{n}} \\ 0 & -\sin\varphi^{\mathrm{n}} & \cos\varphi^{\mathrm{n}} \end{bmatrix} \\
T_y(\theta^{\mathrm{n}}) &= \begin{bmatrix} \cos\theta^{\mathrm{n}} & 0 & -\sin\theta^{\mathrm{n}} \\ 0 & 1 & 0 \\ \sin\theta^{\mathrm{n}} & 0 & \cos\theta^{\mathrm{n}} \end{bmatrix} \\
T_z(\psi^{\mathrm{n}}) &= \begin{bmatrix} \cos\psi^{\mathrm{n}} & \sin\psi^{\mathrm{n}} & 0 \\ -\sin\psi^{\mathrm{n}} & \cos\psi^{\mathrm{n}} & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned} \tag{1}
$$

The overall rotation matrix of the successive xyz rotations is found by multiplying $T_z(\psi^{\mathrm{n}})$, $T_y(\theta^{\mathrm{n}})$, and $T_x(\varphi^{\mathrm{n}})$, which results in

$$\begin{aligned} T(\psi^{\mathrm{n}},\theta^{\mathrm{n}},\varphi^{\mathrm{n}}) &= T_z(\psi^{\mathrm{n}})T_y(\theta^{\mathrm{n}})T_x(\varphi^{\mathrm{n}}) \\ &\triangleq \begin{bmatrix} \cos(\phi^{\mathrm{n}})\cos(\psi^{\mathrm{n}}) & \cos(\phi^{\mathrm{n}})\sin(\gamma) & -\sin(\phi^{\mathrm{n}}) \\ \sin(\theta^{\mathrm{n}})\sin(\phi^{\mathrm{n}})\cos(\psi^{\mathrm{n}})-\cos(\theta^{\mathrm{n}})\sin(\psi^{\mathrm{n}}) & \sin(\theta^{\mathrm{n}})\sin(\phi^{\mathrm{n}})\sin(\psi^{\mathrm{n}})+\cos(\theta^{\mathrm{n}})\cos(\psi^{\mathrm{n}}) & \sin(\theta^{\mathrm{n}})\cos(\phi^{\mathrm{n}}) \\ \cos(\theta^{\mathrm{n}})\sin(\phi^{\mathrm{n}})\cos(\psi^{\mathrm{n}})+\sin(\theta^{\mathrm{n}})\sin(\psi^{\mathrm{n}}) & \cos(\theta^{\mathrm{n}})\sin(\phi^{\mathrm{n}})\sin(\psi^{\mathrm{n}})-\sin(\theta^{\mathrm{n}})\cos(\psi^{\mathrm{n}}) & \cos(\theta^{\mathrm{n}})\cos(\phi^{\mathrm{n}}) \end{bmatrix} \end{aligned} \tag{2}$$

Assuming there is one space based sensor, with known positions in CCS coordinates,

$$\mathbf{s}_{\mathrm{p}}(k) = [\xi(k),\eta(k),\zeta(k)]', \quad k=1,2,...,K \tag{3}$$

tracking a single target, in the same coordinate system, at unknown positions

$$\mathbf{x}_{\mathrm{p}}(k) = [x(k),y(k),z(k)]', \quad k=1,2,...,K \tag{4}$$

With the previous convention, the operations needed to transform the position of the target expressed in CCS coordinates into the sensor coordinate system is

$$\begin{aligned} \mathbf{x}_{\mathrm{s}}^{\mathrm{n}}(k) &= T(\boldsymbol{\omega}(k))(\mathbf{x}_{\mathrm{p}}(k)-\mathbf{s}_{\mathrm{p}}(k)) \\ &= [x_{\mathrm{s}}^{\mathrm{n}}(k),y_{\mathrm{s}}^{\mathrm{n}}(k),z_{\mathrm{s}}^{\mathrm{n}}(k)] \quad k=1,2,...,K \end{aligned} \tag{5}$$

where

$$\boldsymbol{\omega}(k) = [\varphi^{\mathrm{n}}(k),\theta^{\mathrm{n}}(k),\psi^{\mathrm{n}}(k)]' \tag{6}$$

is the nominal orientation of the sensor, $T(\boldsymbol{\omega}(k))$ is the appropriate rotation matrix as in (2), and $(\mathbf{x}_{\mathrm{p}}(k)-\mathbf{s}_{\mathrm{p}}(k))$ is the difference between the vector position of the target and the vector position of the sensor, both expressed in CCS coordinates.

C. Target Dynamics

The state space model for a noiseless discrete-time system is of the general form

$$\mathbf{x}(k+1) = f[\mathbf{x}(k),\mathbf{u}(k)] \quad k=1,2,...,K \tag{7}$$

The motion model (with small time steps) can be described with the discrete-time dynamic equation

$$\mathbf{x}(k+1) = F\mathbf{x}(k)+G\mathbf{u}(k) \tag{8}$$

where

$$\mathbf{x}(k) = [x(k),y(k),z(k),\dot{x}(k),\dot{y}(k),\dot{z}(k)]', \quad k=1,2,...,K \tag{9}$$

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is the 6 dimensional state vector at time k, F is the state transition matrix, and $\mathbf{u}$ is a known input representing the gravitational effects (given in (12)). For a simplified space target with acceleration due to the effect of gravity, the state transition matrix is

$$F = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{10}$$

and the gain matrix is

$$G = \begin{bmatrix} \Delta t^2/2 & 0 & 0 \\ 0 & \Delta t^2/2 & 0 \\ 0 & 0 & \Delta t^2/2 \\ \Delta t & 0 & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \Delta t \end{bmatrix} \tag{11}$$

where Δt is the sampling interval. The gravity term [21] is given by

$$\mathbf{u}(k) = -\mu_e \frac{\mathbf{x}_{\mathrm{p}}(k)}{||\mathbf{x}_{\mathrm{p}}(k)||^3} \tag{12}$$

where $\mathbf{x}_{\mathrm{p}}$ is the position part of the state $\mathbf{x}$ in (7), $\mu_e = GM_e$ is the gravitational constant, M_e is the mass of the fixed non-rotating spherical earth, with uniform density. G is the universal gravitational constant, and

$$||\mathbf{x}_{\mathrm{p}}(k)|| = \sqrt{x(k)^2 + y(k)^2 + z(k)^2} \tag{13}$$

is the distance from the target to the origin of the coordinates system. The ratio $\mathbf{x}_{\mathrm{p}}/||\mathbf{x}_{\mathrm{p}}(k)||^3$ yields the components of the gravity acting on the target and provides the scaling factor for the gravity term. Note that the state model (8) is nonlinear due to the gravity term (12),

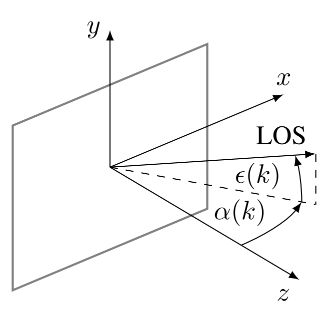

Fig. 1. Optical sensor coordinate system with the origin in the center of the focal plane.

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D. The measurement model

The optical sensor provides LOS measurements of the target position. As shown in Figure 1, the azimuth angle $\beta(k)$ is the angle in the sensor xz plane between the sensor z axis and the line of sight to the target, while the elevation angle $\gamma(k)$ is the angle between the line of sight to the target and its projection onto the xz plane, i.e.,

$$\begin{bmatrix} \beta_s(k) \\ \gamma_s(k) \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{x_s(k)}{z_s(k)}\right) \\ \tan^{-1}\left(\frac{y_s(k)}{\sqrt{x_s^2(k)+z_s^2(k)}}\right) \end{bmatrix} \tag{14}$$

Denote the biased LOS measurement vector at time k as

$$\begin{aligned} \begin{bmatrix} \beta_s^b(k) \\ \gamma_s^b(k) \end{bmatrix} &= \begin{bmatrix} h_1(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) \\ h_2(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) \end{bmatrix} \\ &\triangleq \boldsymbol{h}(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) \end{aligned} \tag{15}$$

where

$$\begin{bmatrix} h_1(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) \\ h_2(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) \end{bmatrix} = \begin{bmatrix} \arctan\left(\frac{x_s^\mathrm{b}(k)}{z_s^\mathrm{b}(k)}\right) \\ \arctan\left(\frac{y_s^\mathrm{b}(k)}{\sqrt{(x_s^\mathrm{b}(k))^2+(z_s^\mathrm{b}(k))^2}}\right) \end{bmatrix} \tag{16}$$

and

$$\begin{aligned} \mathbf{x}_\mathrm{s}^\mathrm{b}(k) &= [x_s^b(k), y_s^b(k), z_s^b(k)]' \quad k = 1, 2, ..., K \\ &= T(\boldsymbol{\omega}^b(k))(\mathbf{x}_\mathrm{p}(k) - \mathbf{s}_\mathrm{p}(k)) \end{aligned} \tag{17}$$

where the sensor bias vector is

$$\mathbf{b} = [\varphi, \theta, \psi]' \tag{18}$$

and the orientation of the sensor is

$$\boldsymbol{\omega}^b(k) = [\varphi^\mathrm{n}(k) + \varphi, \theta^\mathrm{n}(k) + \theta, \psi^\mathrm{n}(k) + \psi]' \tag{19}$$

The biased noisy measurement model is

$$\mathbf{z}(k) = \boldsymbol{h}(\mathbf{x}_\mathrm{p}(k), \mathbf{s}_\mathrm{p}(k), \boldsymbol{\omega}(k), \mathbf{b}) + \mathbf{w}(k) \tag{20}$$

where

$$\mathbf{w}(k) = [w_\beta(k), w_\gamma(k)]' \tag{21}$$

The measurement noises $\mathbf{w}(k)$ are assumed to be a zero-mean Gaussian white sequence.

The cumulative set of measurements during the entire period is denoted as $\mathbf{Z}$ which consists of the batch stacked measurements from the sensor up to time K

$$\mathbf{Z} = [\mathbf{z}(1)', ..., \mathbf{z}(K)']' \tag{22}$$

The focus of this work is to estimate the bias vector for the sensor and the single target of opportunity state vector (position and velocity)

$$\mathbf{\Theta} = [x(1), y(1), z(1), \dot{x}(1), \dot{y}(1), \dot{z}(1), \varphi, \theta, \psi]' \tag{23}$$

from the batch measurement vector

$$\mathbf{Z} = \mathbf{H}(\mathbf{\Theta}) + \mathbf{w} \tag{24}$$

where

$$\mathbf{H}(\mathbf{\Theta}) = [H_1(\mathbf{\Theta})', ..., H_K(\mathbf{\Theta})']' \tag{25}$$

and, based on (15), for $k = 1, ..., K$

$$H_k(\mathbf{\Theta}) = \boldsymbol{h}(\mathbf{x}_{\mathrm{p}}(k), \mathbf{s}_{\mathrm{p}}(k), \boldsymbol{\omega}(k), \mathbf{b}) \tag{26}$$

The position $\mathbf{x}_{\mathrm{p}}(k)$ is obtained from the deterministic forward iteration of (8) starting from $k = 1$ to K. The covariance of the stacked measurement noise $\mathbf{w} = [\mathbf{w}(1)', ..., \mathbf{w}(K)']'$ is the $(2K \times 2K)$ block-diagonal matrix

$$\mathbf{R} = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & R_K \end{bmatrix} \tag{27}$$

where R_k is the measurement noise covariance matrix of the sensor at time k.

III. The Solution of the Estimation Problem

While there are many methods to obtain $\hat{\mathbf{\Theta}}$, the iterative least squares (ILS) technique [4] is preferred since it is easy to implement (no Hessian involved) and provides a (approximate) covariance matrix for its estimate at the same time.

The maximum likelihood (ML) estimate of the augmented parameter vector (23) consisting of the (unknown) target position, velocity and sensor biases, will be obtained, by maximizing the likelihood function (LF) of $\mathbf{\Theta}$ based on $\mathbf{Z}$

$$\Lambda(\mathbf{\Theta}; \mathbf{Z}) = p\left(\mathbf{Z}|\mathbf{\Theta}\right) \tag{28}$$

The ML estimate (MLE) is then

$$\hat{\mathbf{\Theta}}^{\mathrm{ML}}(\mathbf{Z}) = \arg\max_{\mathbf{\Theta}} \Lambda(\mathbf{\Theta}; \mathbf{Z}) \tag{29}$$

where

$$\Lambda(\mathbf{\Theta}; \mathbf{Z}) = |2\pi R|^{-1/2} \exp\left(-\frac{1}{2}\left[\mathbf{Z} - \mathbf{H}\left(\mathbf{\Theta}\right)\right]' R^{-1} \left[\mathbf{Z} - \mathbf{H}\left(\mathbf{\Theta}\right)\right]\right) \tag{30}$$

To obtain the MLE, a numerical search via the Batch Iterated Least Squares (ILS) technique will be used to solve the nonlinear least squares problem.

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A. Iterated Least Squares for Maximization of the LF of $\mathbf{\Theta}$

The batch ILS estimate after the $(j+1)$th iteration will be

$$\hat{\mathbf{\Theta}}^{j+1} = \hat{\mathbf{\Theta}}^{j} + \left[(\Delta^j)' R^{-1} \Delta^j\right]^{-1} (\Delta^j)' R^{-1} [\mathbf{Z} - \mathbf{H}(\hat{\mathbf{\Theta}}^j)] \tag{31}$$

where

$$\mathbf{H}(\hat{\mathbf{\Theta}}^j) = [H_1(\hat{\mathbf{\Theta}}^j)', ..., H_K(\hat{\mathbf{\Theta}}^j)']' \tag{32}$$

and

$$\Delta^j = \left. \frac{\partial \mathbf{H}\left(\mathbf{\Theta}\right)}{\partial \mathbf{\Theta}} \right|_{\mathbf{\Theta}=\hat{\mathbf{\Theta}}^j} \tag{33}$$

is the Jacobian matrix of the vector consisting of the stacked measurement functions (32) w.r.t. (23). With the iteration index omitted for simplicity, the Jacobian matrix is then

$$\Delta = \begin{bmatrix} \Delta_1' & \Delta_2' & \cdots & \Delta_K' \end{bmatrix}' \tag{34}$$

where, for $k = 1, ..., K$

$$\Delta_k = \begin{bmatrix} \frac{\partial h_1(k)}{\partial x(1)} & \frac{\partial h_1(k)}{\partial y(1)} & \frac{\partial h_1(k)}{\partial z(1)} & \frac{\partial h_1(k)}{\partial \dot{x}(1)} & \frac{\partial h_1(k)}{\partial \dot{y}(1)} & \frac{\partial h_1(k)}{\partial \dot{z}(1)} & \frac{\partial h_1(k)}{\partial \varphi} & \frac{\partial h_1(k)}{\partial \theta} & \frac{\partial h_1(k)}{\partial \psi} \\ \frac{\partial h_2(k)}{\partial x(1)} & \frac{\partial h_2(k)}{\partial y(1)} & \frac{\partial h_2(k)}{\partial z(1)} & \frac{\partial h_2(k)}{\partial \dot{x}(1)} & \frac{\partial h_2(k)}{\partial \dot{y}(1)} & \frac{\partial h_2(k)}{\partial \dot{z}(1)} & \frac{\partial h_2(k)}{\partial \varphi} & \frac{\partial h_2(k)}{\partial \theta} & \frac{\partial h_2(k)}{\partial \psi} \end{bmatrix} \tag{35}$$

The Matlab built-in function “lsqnonlin” will be used in the simulation to calculate the estimate (31).

B. Initialization

To start the ILS recursion an initial estimate $\hat{\mathbf{\Theta}}^0$ is required and the biases are assumed to be zero. The first two LOS measurements from the sensor are used to solve for an initial Cartesian target position, as discussed in [10], using (36)–(38)

$$x^0(2) = \frac{\xi(2) - \xi(1) + \zeta(1)\tan\beta(1) - \zeta(2)\tan\beta(2)}{\tan\beta(1) - \tan\beta(2)} \tag{36}$$

$$y^0(2) = \frac{\tan\beta(1)\left(\xi(2) + \tan\beta(2)\left(\zeta(1) - \zeta(2)\right)\right) - \xi(1)\tan\beta(2)}{\tan\beta(1) - \tan\beta(2)} \tag{37}$$

$$z^0(2) = \eta(1) + \tan\gamma(1)\left|\frac{(\xi(1) - \xi(2))\cos\beta(2) + (\zeta(2) - \zeta(1))\sin\beta(2)}{\sin\left(\beta(1) - \beta(2)\right)}\right| \tag{38}$$

Two consecutive Cartesian positions formed using (36)–(38) [with, e.g., $\beta(3)$, $\beta(4)$ and $\gamma(2)$] can then be differenced to provide an approximate velocity and mapped to time K via (7). This procedure is analogous to two-point differencing [4] and will provide a full six-dimensional state to initialize the ILS algorithm.

C. Bias Estimability

Passive sensor tracking can be a very ill-conditioned estimation problem. In order to establish observability the sensor needs to have an acceleration with respect to a target that is moving with a constant velocity [16]. For a passive target tracking problem in a gravitational field, the known acceleration in the vertical direction makes the target observable. Intuitively, the observability of a system guarantees that the sensor measurements provide sufficient information for estimating the unknown parameters. As discussed in [5] the two necessary requirements for bias estimability are:

- The number of equations (size of the stacked measurement vector) has to be at least equal to the number of parameters to be estimated (6 target state components and 3 biases), meaning that we must have

$$2K \geq \ 9 \tag{39}$$

  If (39) is not satisfied one cannot solve for the parameter estimates. If K is at the minimum or slightly above it then in our experience, the condition number of the FIM is, typically, very large, i.e., "marginal observability" and the search for the MLE will have difficulty finding the global maximum and even if it finds it, the accuracy of the bias estimates is poor (the s.d. exceeds by far the s.d. of the measurement noises). Typically, K should be at least an order of magnitude larger than the minimum based on (39) to get estimates with reasonable s.d. (see table IV).

- The Fisher Information Matrix (FIM) must be invertible. If the FIM is not invertible (i.e., it is singular), then the CRLB (the inverse of the FIM) will not exist [2].

For the single sensor single target tracking system discussed in this paper, the search is in a 9-dimensional space (3 biases of the single sensor and 6 target state components). As stated previously, the FIM must be invertible, so the rank of the FIM has to be equal to the number of parameters to be estimated (9). However, with the minimum number of measurements, the observability for this system is marginal, and necessitates the use of more measurements than the strict minimum number given by (39).

D. Cramér-Rao Lower Bound

The CRLB provides a lower bound on the covariance matrix of an unbiased estimator [4] as

$$E\{(\boldsymbol{\Theta} - \hat{\boldsymbol{\Theta}})(\boldsymbol{\Theta} - \hat{\boldsymbol{\Theta}})'\} \geq J(\boldsymbol{\Theta})^{-1} \tag{40}$$

where $\boldsymbol{\Theta}$ is the true parameter vector to be estimated, $\hat{\boldsymbol{\Theta}}$ is the estimate, and J is the FIM given as

$$\begin{aligned} J(\boldsymbol{\Theta}) &= E\left\{[\nabla_{\boldsymbol{\Theta}} \ln \Lambda(\boldsymbol{\Theta})]\,[\nabla_{\boldsymbol{\Theta}} \ln \Lambda(\boldsymbol{\Theta})]'\right\}\big|_{\boldsymbol{\Theta}=\boldsymbol{\Theta}_{\text{true}}} \\ &= \Delta' \left(R^{-1}\right) \Delta\big|_{\boldsymbol{\Theta}=\boldsymbol{\Theta}_{\text{true}}} \end{aligned} \tag{41}$$

where Δ is given by (34) and R given by (27).

E. Statistical Test for Efficiency with Monte Carlo Runs

As discussed in [4], the normalized estimation error squared (NEES) for the parameter $\mathbf{\Theta}$ (under the hypothesis of efficiency), defined as

$$\gamma_{\mathbf{\Theta}} = (\mathbf{\Theta} - \hat{\mathbf{\Theta}})' P^{-1} (\mathbf{\Theta} - \hat{\mathbf{\Theta}}) = (\mathbf{\Theta} - \hat{\mathbf{\Theta}})' J(\mathbf{\Theta}) (\mathbf{\Theta} - \hat{\mathbf{\Theta}}) \tag{42}$$

is chi-square distributed with $n_{\mathbf{\Theta}}$ (the dimension of $\mathbf{\Theta}$) degrees of freedom, assuming that estimation errors are Gaussian, that is,

$$\gamma_{\mathbf{\Theta}} \sim \chi^2_{n_{\mathbf{\Theta}}} \tag{43}$$

The hypothesis test whether efficiency can be accepted, i.e., that $P = J^{-1}$, is discussed in [4] and outlined next. The NEES is used in simulations to check whether the estimator is efficient. In practice, to check the estimator efficiency we use the sample average NEES from N independent Monte Carlo runs, defined as

$$\bar{\gamma}_{\mathbf{\Theta}} = \frac{1}{N} \sum_{i=1}^{N} \gamma^i_{\mathbf{\Theta}} \tag{44}$$

The quantity $N\bar{\gamma}_{\mathbf{\Theta}}$ is chi-square distributed with $Nn_{\mathbf{\Theta}}$ degrees of freedom.

IV. Simulations

A. Single-Sensor Single-Target Case

In this paper we used a hypothetical single sensor single target scenario to test our method. The sensor satellite is in a circular orbit of 689 km altitude with $58°$ degrees inclination. As discussed previously, the three sensor biases were roll, pitch and yaw angle offsets (φ, θ, ψ). Table I summarizes the bias values (in mrad). The measurement noise standard deviation $\sigma_\beta = \sigma_\gamma = \sigma$ was assumed to be 30 μrad.

The target represents a space object under the influence of gravity with a flight time of about 20 minutes. As shown in Figure 2, the satellite orbit enabled maximum visibility of the target trajectory from multiple angles. The sensor reports measurements at 1 s intervals. The target and satellite trajectory displayed in Figure 2 represents 5 minutes (K= 300) of the target's flight time. In order to establish a (clairvoyant) baseline for evaluating the performance of our method, we also ran the simulations without biases and also with biases, but without bias estimation.

See figure 2. See figure 3.

TABLE I

Sensor Biases (mrad) (single-sensor case).

ψ	θ	φ
0.5934	1.6057	-4.1364

Statistical Efficiency of the Estimates (Single-Sensor Case)

The NEES is used to test for the statistical efficiency of the estimate (the 9 dimensional vector (23)), with the CRLB

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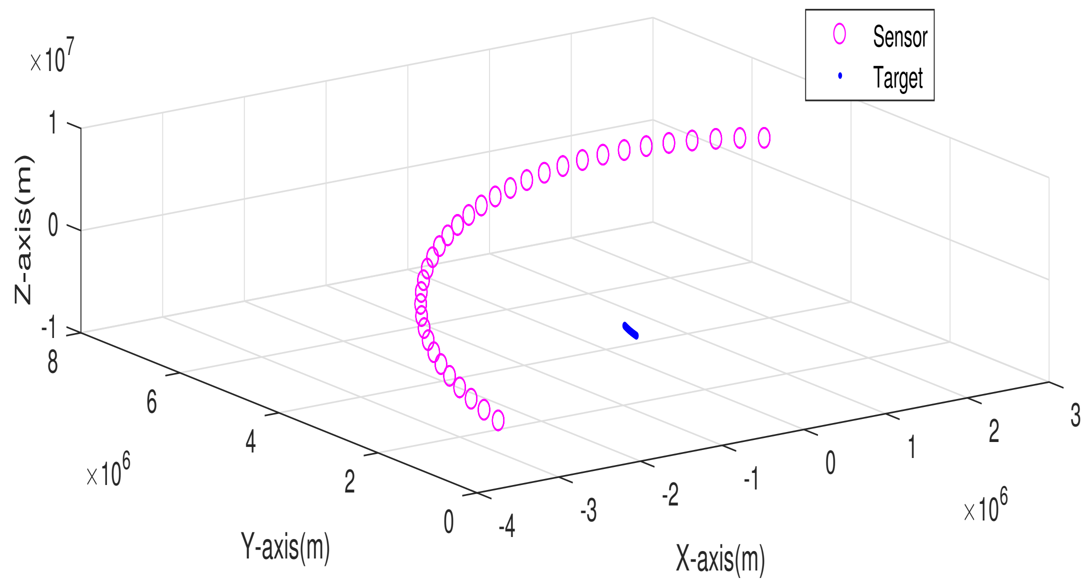

Fig. 2. Target and satellite trajectories for the single sensor single target case

as the covariance matrix. The simulation results are based on 100 Monte Carlo runs. The 95% probability region for the 100 sample average NEES of a 9 dimensional parameter vector is [8.18, 9.85]. The sample average NEES calculated using the FIM evaluated at the true parameters values (bias, target position, and velocity) is 8.52, and the sample average NEES calculated using the FIM evaluated at the estimated parameters values is 8.45 and both fall in the acceptance region, which means the MLE is statistically efficient. The 95% probability region for the 100 sample average single component NEES is [0.74, 1.29], the individual bias component NEES are summarized in Table II and it can be seen that these NEES are found to be within the acceptance interval as well.

TABLE II

SAMPLE AVERAGE BIAS NEES (CRLB EVALUATED AT THE ESTIMATE), FOR EACH OF THE 3 BIASES, OVER 100 MONTE CARLO RUNS (SINGLE-SENSOR CASE).

	ψ	θ	φ
NEES $\|_{\Theta=\hat{\Theta}}$	0.8934	0.9057	1.0364

The RMS errors for the target position and velocity are summarized in Table III. In this table, estimation scheme A was established as a (clairvoyant) baseline using bias-free LOS measurements to estimate the target position and velocity. For scheme B, we used biased LOS measurements but we only estimated target position and velocity that is, we ignored the fact that there was bias. In scheme C, we used biased LOS measurements and we simultaneously estimated the target position, velocity, and sensor biases. Once again, bias estimation yields significantly improved target RMS position and velocity errors in the presence of biases.

Table IV shows that each individual bias RMSE component is within 15% of its corresponding bias standard

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAES.2019.2895709, IEEE Transactions on Aerospace and Electronic Systems

TABLE III

SAMPLE AVERAGE RMSE FOR THE FINAL TARGET POSITION (m) AND VELOCITY (ms^{-1}), FROM 100 MONTE CARLO RUNS, FOR THE 3 ESTIMATION SCHEMES (SINGLE-SENSOR CASE).

Scheme	Position RMSE	Velocity RMSE
A (bias free)	378	15
B (biases ignored)	53,266	2,029
C (biases estimated)	2861	111

TABLE IV

SAMPLE AVERAGE BIAS RMSE OVER 100 MONTE CARLO RUNS AND THE CORRESPONDING BIAS STANDARD DEVIATION FROM THE CRLB (σ_{CRLB}) (MRAD) FOR THE SINGLE-SENSOR CASE.

	ψ	θ	φ
RMSE	0.1318	0.1102	0.1686
σ_{CRLB}	0.1229	0.1237	0.1788

deviation[2] from the CRLB (σ_{CRLB}) with 95% probability, therefore each bias component is individually consistent with its corresponding σ_{CRLB}.

B. Two-Sensor Single-Target Case

To compare the results of the new approach with the previously proposed method [7], the case of two sensors is considered with the same parameter settings as in [7]. Similarly to the single-sensor case, the target modeled represents a long range space object with 20 minutes flight time. The bias values (in mrad) for both sensors is summarized in Table V. The two satellite orbits enabled good observability of the target trajectory as shown in Figure 2.

Statistical Efficiency of the Estimates (Two-Sensor Case)

In this case, to estimate the biases of 2 sensors (6 bias components) and 6 target components (3 position and 3 velocity components), the search is in a 12-dimensional space. Similarly to the single-sensor case, the NEES is used to test for the statistical efficiency of the estimate (the 12 dimensional vector), with the CRLB as the covariance matrix. The sample average NEES over 100 Monte Carlo runs is 11.52, which is inside the 95% probability region

[2]The 15% follows from the square root of the deviations from unity of the interval [0.74, 1.29] endpoints.

TABLE V

SENSOR BIASES (MRAD).

	ψ	θ	φ
Sensor 1	5.7596	4.3633	-3.8397
Sensor 2	4.8869	5.4105	-5.0615

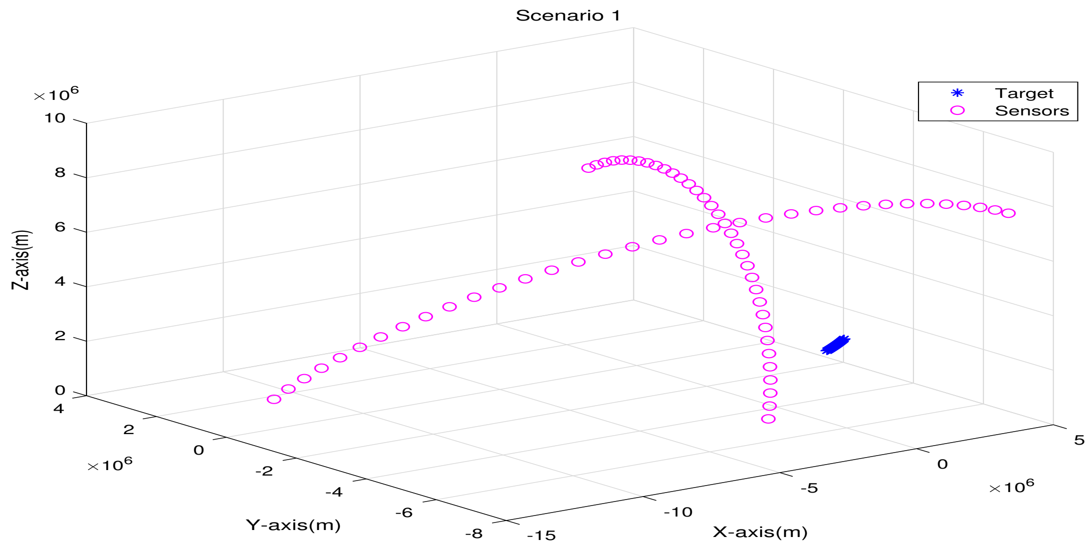

Fig. 3. Target and satellite trajectories for the two-sensor case

[11.20, 12.81] of 100 sample average NEES of a 12 dimensional parameter vector, therefore the MLE is statistically efficient.

Table VI shows the individual bias component NEES. The 95% probability region for the 100 sample average single component NEES is [0.74, 1.29]. These individual components NEES are found to be within this interval.

TABLE VI
SAMPLE AVERAGE BIAS NEES (CRLB EVALUATED AT THE ESTIMATE), FOR EACH OF THE 6 BIASES, OVER 100 MONTE CARLO RUNS (TWO-SENSOR CASE).

Biases	ψ_1	θ_1	φ_1	φ_2	ψ_2	θ_2
NEES	1.0336	0.9623	1.0139	1.0278	1.2409	0.8722

Similarly to the single-sensor case, we evaluated the RMS errors for the target position and velocity using the three estimation schemes discussed previously. The results are summarized in Table VII.

As expected, bias estimation yields significantly improved target RMS position and velocity errors in the presence of biases.

From Tables III and VII, it can be seen that the RMS errors for the target position and velocity are significantly improved with bias estimation compared with the target position and velocity estimation using the original biased measurements, even for a single passive sensor single target tracking system. The two-sensor case yields, for the scenario considered, better results.

TABLE VII

SAMPLE AVERAGE RMSE FOR THE FINAL TARGET POSITION (m) AND VELOCITY (ms^{-1}), OVER 100 MONTE CARLO RUNS, FOR THE 3 ESTIMATION SCHEMES (TWO-SENSOR CASE).

Scheme	Position RMSE	Velocity RMSE
A (bias free)	106	5
B (biases ignored)	47,253	25,237
C (biases estimated)	483	19

V. CONCLUSIONS

In this paper we presented a new method that uses a single target of opportunity for estimation of measurement biases of a space based single sensor together with the target's state. The first step was formulating a general bias model for a single space-based sensor at known location tracking a single target of opportunity at unknown locations. Based on this, we used the ML approach that led to a batch nonlinear least-squares estimation problem for simultaneous estimation of the 3D Cartesian position and velocity components of the target of opportunity and the measurement biases of the sensor. The bias estimates, obtained via ILS, were shown to be statistically efficient.

ACKNOWLEDGEMENT

The authors gratefully acknowledge Dr. Mallick Mahendra for his constructive comments and insightful feedback.

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TAES.2019.2895709, IEEE Transactions on Aerospace and Electronic Systems

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