1 The Extended Kalman Filter

Simulation Model

For this simulation, I used a two dimensional model consisting of linear and non-linear equations of motion and non-linear measurement equations. The simulated vehicle operated for three hours at .5 m/s on three different courses. A constant current of .2 m/s at 30 degrees affected the vehicle. The simulation includes the following inputs:

Speed through the water(stw): the speed in the vehicle reference frame as estimated by the onboard dynamic vehicle model.

Heading(hdg): the roll-stabilized heading reported by the Altitude and Heading Reference System.

Range($range$): the horizontal range from the acoustic beacon to the vehicle as calculated by the OWTT-iUSBL system.

Azimuth(azi): the angle of azimuth between the acoustic beacon and the vehicle as calculated by the OWTT-iUSBL system.

Components

The Extended Kalman Filter is an algorithm that estimates the current state of a system based on the previous state and augments that prediction using a measurement. The Extended Kalman filter differs from the standard Kalman filter in that non-linear state update and measurement mapping equations can be used. The Extended Kalman filter has the following parts:

State vector ($\hat{x}$): a vector describing the state of the system at a given time. For this filter, the state vector is composed of the x and y position and velocity of the system in the world frame and the x and y velocity of the current affecting the vehicle.

$$\hat{x}_k = \begin{bmatrix} x_w \\ y_w \\ \dot{x}_w \\ \dot{y}_w \\ \dot{x}_c \\ \dot{y}_c \end{bmatrix}$$

State Transition Equations (F, G, u, w): a matrix describing the physical model of the system. In this case, the model combines simple linear motion, an input vector as a function of speed through the water (stw) and heading (hdg), and white noise:

$$x_{w_{k+1}} = x_{w_k} + \dot{x_{c_k}}\Delta t + stw_{k+1}\cos(hdg_{k+1})\Delta t + .5a_{w_k}\Delta t^2 + w_k$$

$$y_{w_{k+1}} = y_{w_k} + \dot{y_{c_k}}\Delta t + stw_{k+1}\sin(hdg_{k+1})\Delta t + .5a_{w_k}\Delta t^2 + w_k$$

$$\dot{x_{w_{k+1}}} = \dot{x_{c_k}} + stw_{k+1}\cos(hdg_{k+1}) + a_{w_k}\Delta t + w_k$$

$$\dot{y_{w_{k+1}}} = \dot{y_{c_k}} + stw_{k+1}\sin(hdg_{k+1}) + a_{w_k}\Delta t + w_k$$

$$\dot{x_{c_{k+1}}} = \dot{x_{c_k}} + a_{c_k}\Delta t + w_k$$

$$\dot{y_{c_{k+1}}} = \dot{y_{c_k}} + a_{c_k}\Delta t + w_k$$

The model is assumed to be constant velocity, therefore the acceleration terms are zero.

Rearranging into a matrix format results in:

$$\hat{x_{k+1}} = F\hat{x_k} + u_k + Ga_k + w_k$$

$$F = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, u_k = \begin{bmatrix} stw_{k+1}\cos(hdg_{k+1})\Delta t \\ stw_{k+1}\sin(hdg_{k+1})\Delta t \\ stw_{k+1}\cos(hdg_{k+1}) \\ stw_{k+1}\sin(hdg_{k+1}) \\ 0 \\ 0 \end{bmatrix}, G = \begin{bmatrix} .5\Delta t^2 \\ .5\Delta t^2 \\ \Delta t \\ \Delta t \\ \Delta t \\ \Delta t \end{bmatrix}$$

Process Covariance Matrix (Q): a matrix containing the variances and covariances for each state vector variable. This matrix characterizes the assumed white noise present in the environment.

$$Q = G\sigma_q^2 G^T$$

Measurement Covariance Matrix (R): a square, diagonal matrix containing the variances and covariances for each measurement vector variable. This matrix represents the confidence in the accuracy of the measurements.

$$R = \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \sigma_r^2 \end{bmatrix}$$

Error Covariance Matrix (P): a matrix containing the variances and covariances of the error between the estimated state vector and the calculated output vector. Initially, the vector components are assumed to be uncorrelated and the off-diagonal entries are zero. As the filter evolves, the covariances will change depending on the calculated correlation of the components

$$P = \begin{bmatrix} \sigma^2_{x_w} & \sigma_{x_w y_w} & \sigma_{x_w \dot{x_w}} & \sigma_{x_w \dot{y_w}} & \sigma_{x_w \dot{x_c}} & \sigma_{x_w \dot{y_c}} \\ \sigma_{y_w x_w} & \sigma^2_{y_w} & \sigma_{y_w \dot{x_w}} & \sigma_{y_w \dot{y_w}} & \sigma_{y_w \dot{x_c}} & \sigma_{y_w \dot{y_c}} \\ \sigma_{\dot{x_w} x_w} & \sigma_{\dot{x_w} y_w} & \sigma^2_{\dot{x_w}} & \sigma_{\dot{x_w} \dot{y_w}} & \sigma_{\dot{x_w} \dot{x_c}} & \sigma_{\dot{x_w} \dot{y_c}} \\ \sigma_{\dot{y_w} x_w} & \sigma_{\dot{y_w} y_w} & \sigma_{\dot{y_w} \dot{x_w}} & \sigma^2_{\dot{y_w}} & \sigma_{\dot{y_w} \dot{x_c}} & \sigma_{\dot{y_w} \dot{y_c}} \\ \sigma_{\dot{x_c} x_w} & \sigma_{\dot{x_c} y_w} & \sigma_{\dot{x_c} \dot{x_w}} & \sigma_{\dot{x_c} \dot{y_w}} & \sigma^2_{\dot{x_c}} & \sigma_{\dot{x_c} \dot{y_c}} \\ \sigma_{\dot{y_c} x_w} & \sigma_{\dot{y_c} y_w} & \sigma_{\dot{y_c} \dot{x_w}} & \sigma_{\dot{y_c} \dot{y_w}} & \sigma_{\dot{y_c} \dot{x_c}} & \sigma^2_{\dot{y_c}} \end{bmatrix}$$

Measurement Mapping Equation ($h(k, x_w, y_w)$: a nonlinear equation to map the state vector to the measurements of range and angle of azimuth.

$$h = \begin{bmatrix} \sqrt{x^2_{w_k} + y^2_{w_k}} & \arctan(\frac{y_{w_k}}{x_{w_k}}) \end{bmatrix}$$

Measurement Mapping Jacobian (H): a matrix containing the partial derivatives of $h(k, x_w, y_w)$ with respect to the state vector components.

$$H = \begin{bmatrix} \frac{x_k}{\sqrt{x_k^2 + y_k^2}} & \frac{y_k}{\sqrt{x_k^2 + y_k^2}} & 0 & 0 & 0 & 0 \\ \frac{-y_k}{x_k^2 + y_k^2} & \frac{1}{x_k + \frac{y_k^2}{x_k}} & 0 & 0 & 0 & 0 \end{bmatrix}$$

Kalman Gain Matrix (K): a matrix used to weight the contributions of the state estimate and measurement to produce the most accurate calculated state of the system.

$$K = \frac{PH^T}{HPH^T + R}$$

Measurement vector(z_k): a simulated measurement composed of the actual state of the system with Gaussian distributed noise added.

$$z_k = \begin{bmatrix} range \\ azi \end{bmatrix} + v_k$$

Execution

The filter is first initialized with the initial condition of the state vector. For the simulation, I chose a speed of .5 m/s and an initial heading of 30 degrees with a .2 m/s current at an angle of 30 degrees which resulted in:

$$\hat{x_0} = \begin{bmatrix} 0 \\ 0 \\ .61 \\ .35 \\ .17 \\ .10 \end{bmatrix}$$

Next, the covariance matrices are initialized. For P, I chose an initial covariance value of 10m squared. For R, I chose a an error of seven meters for range measurement and an error of two degrees for the heading measurement. For Q, I chose an error of .02. The resulting matrices:

$$P_0 = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 100 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix}, R = \begin{bmatrix} 49 & 0 \\ 0 & 4 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1\times10^{-4} & 1\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} \\ 1\times10^{-4} & 1\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} & 2\times10^{-4} \\ 2\times10^{-4} & 2\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} \\ 2\times10^{-4} & 2\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} \\ 2\times10^{-4} & 2\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} \\ 2\times10^{-4} & 2\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} & 4\times10^{-4} \end{bmatrix}$$

Once the initial conditions are established, the state vector and error covariance matrix are estimated for the next step:

$$\hat{x_k}^- = F\hat{x_{k-1}}^+ + u_k + w_k$$

$$P_k^- = FP_{k-1}^+F^T + Q$$

Next, H is calculated using the previously determined variables and used to compute K. Then, the state vector is updated using the measurement.

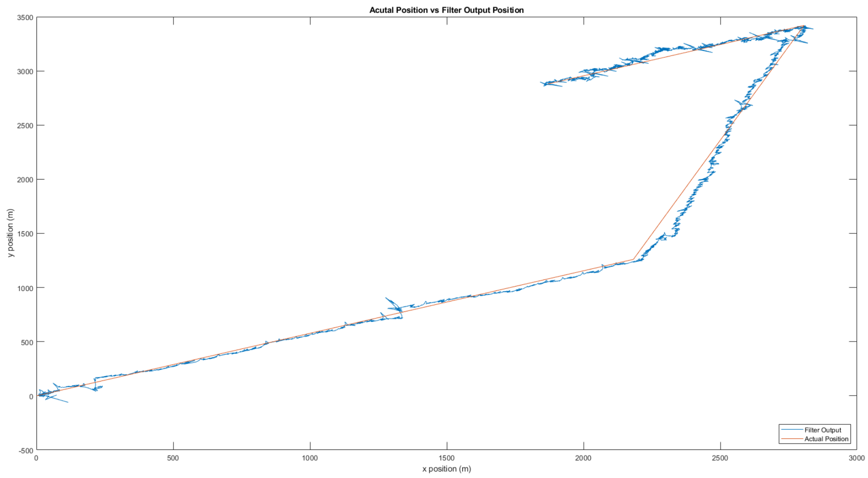
$$\hat{x_k}^+ = \hat{x_k}^- + K(z_k - H\hat{x_k}^-)$$

Now, compute the updated state vector covariance. I chose this particular version, the Joesph stabilized equation, due to the added stability provided by the fact that the equation always results in a positive definite matrix:

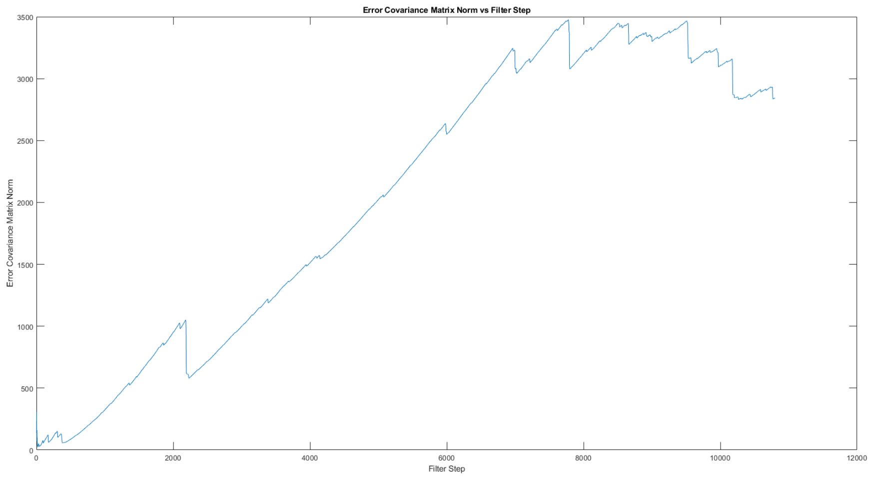
$$P_k^+ = (I - KH)P_k^-(I - KH)^T + KRK^T$$

Results

In the plot of filter position vs actual position, the filter position shows jumps that are not possible in reality. I am researching methods that use equality or inequality constraints to bound those jumps. Positively, the plot also shows that when filter position is significantly deviated from actual, the filter recovers and will resume plotting consistent with actual.



The plot of the norm of the Filter Error Covariance matrix does not show convergence, but it does also not monotonically increase towards an infinite value, which is promising.



Future Development

Future simulations plans include having multiple current values, explicitly evaluating and comparing whether the geometry of the track legs affects the accuracy of the calculated current and post-processing actual dive data to determine the effectiveness of the filter. Also, the filter will be increased in scope to cover all three dimensions.

In addition to the simulations, I will be designing an in-water experiment for the Charles River to gather data to compare to the simulation results.