Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory
Harry L. Van Trees

ISBNs: 0-471-09390-4 (Hardback); 0-471-22110-4 (Electronic)

Chapter 10

Detection and Other Topics

In this chapter we discuss several areas of interest that we have not considered previously. The optimum detection problem is an obvious topic that we have not discussed up to this point. In addition to its importance, it is the first word in the title of the "Detection, Estimation, and Modulation" series, so the reader may be misled. We discuss the detection problem briefly in Section 10.1.

In Section 10.2, we discuss five topics: target tracking, space-time processing for radar, space-time processing for wireless communications, matched field processing, and spatial spectrum estimation. These topics are closely related to the subjects in this book.

In Section 10.3 we make some concluding comments.

10.1 Optimum Detection

An important problem that we have not discussed is the detection problem. The detection problem was discussed in detail in DEMT I [VT68], [VT01a] and DEMT III [VT71], [VT01b]. In many array processing models of interest, the optimum detection consists of a beamformer designed using the techniques in Chapters 6 and 7, followed by a scalar detector designed using the techniques of DEMT I and DEMT III. Because of this overlap we restrict ourselves to a very brief discussion of the various models and issues involved.

In Section 10.1.1, we discuss the classic binary detection problem. In Section 10.1.2, we discuss matched subspace detectors. In Section 10.1.3, we discuss the detection of spatially spread Gaussian random processes. In Section 10.1.4, we discuss adaptive detection techniques. In Section 10.1.5, we summarize our comments.

Historically, the early results on optimum detection for arrays were derived by Bryn [Bry62], Vanderkulk [Van63], Middleton and Groginsky [MG65], and Van Trees ([VT66], [VT64]). Wolf [Wol59] derived results for multiple non-stationary processes. The detection of Gaussian signals in noise has been studied earlier by Price (e.g., [Pri53], [Pri54], and [Pri56]). Other early references include Stocklin [Sto63], Cox ([Cox64], [Cox69]), Mermoz [Mer64], Young and Howard ([YH70a], [YH70b]), Gaarder ([Gaa67], [Gaa66]), Nuttall and Hyde [NH69], and Lewis and Schultheiss [LS71].

10.1.1 Classic Binary Detection

In this section, we discuss the array processing version of the classic binary detection problem from DEMT I.[1] For simplicity, we restrict our attention to the narrowband case.

We use the frequency-domain snapshot model, but we suppress the k variable in our discussion. The signals on the two hypotheses are

$$\begin{aligned} \mathbf{X}(\omega) &= \mathbf{v}(\psi_s)F(\omega) + \mathbf{N}(\omega) \qquad &: H_1 \\ \mathbf{X}(\omega) &= \mathbf{N}(\omega) \qquad &: H_0. \end{aligned} \tag{10.1}$$

This corresponds to a single plane-wave signal impinging on the array from ψ_s, which is assumed to be known. The noise is a sample function from a zero-mean Gaussian random process whose spatial spectral matrix $\mathbf{S_n}(\omega)$ is known. The signal is one of the following types:

(i) A known signal,

(ii) A known signal contains unknown random parameters,

(iii) A sample function of a zero-mean Gaussian random process that is statistically independent of $\mathbf{N}(\omega)$.

This model is appropriate for the case in which we have steered the array in a specific direction and want to determine whether a signal is present or not. In all of these cases, the MVDR beamformer from Chapter 6 generates a sufficient statistic. The scalar output of the MVDR beamformer is processed by the appropriate optimum detector. All of the results in DEMT I and III apply directly. The effect of the beamformer is completely characterized by the array gain. Note that the beamformer is an MVDR beamformer so that knowledge of $\mathbf{S_n}$ or an ability to estimate it is necessary.

[1] The reader may want to review DEMT I [VT68], [VT01a].

If the signal direction is mismatched, we can use an LCMV beamformer to combat the uncertainty. This approach is generally not optimal, but provides almost optimum performance for small values of mismatch. Two approaches that develop optimal processors are described by Nolte and his colleagues (e.g., [HN78], [HN76b], [HN76a], [GN74]) and Bell et al. [BEV00].

In some applications, array perturbations must be considered. If the perturbations are modeled as complex Gaussian random vector, then the optimum detection contains a linear term and a quadratic term. A similar result is obtaained if there is imperfect spatial coherence of the wavefronts. Refrences that discuss theses issues include Morgan [Mor90], Gershman et al. [GMB97], Paulraj and Kailath [PK88], and Rao and Jones [RJ01]. The last paper derives a DFT approximation to the optimum receiver that is computationally feasible.

In some applications (e.g., multipath) the signal consists of multiple plane waves. If the $N \times D$ array manifold matrix is known, we construct a D-dimensional subspace and solve for the optimum binary detector in that subspace. The form of the optimum detector depends on the temporal signal model that is assumed. We discuss one of these models in the next section.

10.1.2 Matched Subspace Detector

In this section, we discuss a detector that is referred to as the matched subspace detector. Our discussion is based on Sections 4.11 and 4.12 of Scharf's book [Sch91]. The history of the development is given on pp.166-167 of [Sch91]. We will summarize the result.

The hypothesis testing problem of interest is

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{V}\mathbf{f}(k) + \mathbf{w}(k)\,, \quad k = 1, \cdots, K : H_1 \\ \mathbf{x}(k) &= \mathbf{w}(k)\,, \quad k = 1, \cdots, K : H_0, \end{aligned} \tag{10.2}$$

where we have used a narrowband time-domain model. The matrix $\mathbf{V}$ is an $N \times D$ array manifold matrix and $\mathbf{f}(k)$ is a $D \times 1$ signal vector. We assume that the array manifold, and therefore the signal subspace, is known. The additive Gaussian noise $\mathbf{w}(k)$ is uncorrelated with variance $\sigma_w^2\mathbf{I}$. The new factor in this model is that the signal vector $\mathbf{f}(k)$ is unknown but nonrandom.

We know that the projection onto the signal subspace will generate a sufficient statistic. Thus,

$$\mathbf{y}_s(k) = \mathbf{P}_{\mathbf{V}}\mathbf{x}(k)\,, \tag{10.3}$$

where $\mathbf{P}_{\mathbf{V}}$ is the projection matrix onto the signal subspace,

$$\mathbf{P}_{\mathbf{V}} = \mathbf{V}\left[\mathbf{V}^H\mathbf{V}\right]^{-1}\mathbf{V}^H\,. \tag{10.4}$$

Because $\mathbf{f}(k)$ is unknown, but nonrandom, we use a generalized likelihood ratio test (GLRT) (see Section 2.5 of DEMT I [VT68], [VT01a]). The GLRT approach to this problem is due to Scharf and Friedlander [SF94].

The resulting test consists of constructing the test statistic,

$$t_e \triangleq \chi^2(k) = \mathbf{x}^H \mathbf{P_V} \mathbf{x}. \tag{10.5}$$

The statistic χ^2 is a quadratic form in the vector $\mathbf{P_V x}$, which is a Gaussian random vector with mean $\mathbf{Vf}(k)$ and covariance matrix $\sigma_w^2 \mathbf{P_V}$.

Then, χ^2/σ_w^2 is chi-squared distributed with N degrees of freedom and the noncentrality parameter E_s/σ_w^2 where

$$E_s \triangleq \sum_{k=1}^{K} \mathbf{f}^H(k)\mathbf{V}^H\mathbf{V}\mathbf{f}(k). \tag{10.6}$$

The chi-squared distribution has a monotone likelihood ratio so the test,

$$\sum_{k=1}^{K} \frac{\chi^2}{\sigma_w^2} \underset{H_0}{\overset{H_1}{\gtrless}} \chi_0^2, \tag{10.7}$$

is a uniformly most powerful (UMP) detector. This test is referred to as a **matched subspace detector** (see discussion of p. 166 on [Sch91]. Note that the detector is computing the energy in the signal subspace, so we can also refer to it as a **generalized energy detector**.

In many cases, the variance of the noise σ_w^2 is unknown. If possible, we would like to construct a test that has a constant false alarm rate. Scharf (pp. 148–152 of [Sch91]) also derives a constant false alarm rate (CFAR) version for the case of unknown σ_w^2.

The reader is referred to the above references for a further discussion of the matched subspace detector.

10.1.3 Spatially Spread Gaussian Signal Processes

In this section we consider the binary detection problem for the case in which the signal is a sample function of Gaussian random process. The plane-wave models in Section 10.1, in which the temporal signals were Gaussian processes are special cases of the model in this section. In this subsection, we focus on the case where the signal has significant spread in ψ-space.

In many physical situations the signals that are the object of the optimum array processing are not true plane waves. This may occur in an acoustic environment in which local spreading is due to scattering or in wireless communication scenarios.

The principal difference in analyzing spatially spread signals involves their representation across the receiving array. In the plane-wave situation, the statistical representation in terms of an eigenvector expansion is one-dimensional, that is, only one eigenvector is required in the representation. In the spatially spread situation, more than one eigenvector is required. The number of eigenvectors required is determined by the area of the array and the spatial extent of the spreading.

In many situations, we can model the spatial spreading with a channel or target-scattering function. The model is similar to the singly-spread and doubly-spread delay-Doppler models that we studied in Chapters 11-13 of DEMT III [VT71], [VT01b]. In the current situation, the scattering function can be represented in either frequency-wavenumber space or ω-ψ-space. An idealized scattering function in ω-ψ-space is shown in Figure 10.1. We limit our discussion in this section to the narrowband case so the scattering function is 1-D. Although we will only discuss 1-D (ψ) scattering functions, all of the results can be extended easily to 2-D functions. An idealized 1-D rectangular scattering function is shown in Figure 10.2. In Figure 10.2(a), the width of the scattering function is less than the width of the main lobe, so one eigenvector will be adequate. In Figure 10.2(b), the width of the scattering function is much wider than the main lobe, so multiple eigenvectors will be required.

This model is a Gaussian signal in Gaussian noise problem that we studied in Chapters 2–5 of DEMT III [VT71], [VT01b]. The resulting processor is the spatial analog to the diversity receiver that we encountered in our analysis of doubly-spread channels in Chapter 13 of DEMT III [VT71], [VT01b]. The detector first forms a set of eigenbeams determined from the signal spectral matrix (a spatially "coherent" operation), then weights them, squares the signals, and sums the squares.

The cases of non-white noise (with a known spatial spectral matrix), the generalized binary detection problem, and the broadband signal model can be all be solved in a similar manner.

In order to evaluate the performance of these detectors, we utilize bounds and approximate error expressions that depend on a function $\mu(s)$ which we encountered previously in Section 2.7 of DEMT I [VT68], [VT01a]. The derivation of the necessary approximate expressions is given of pp. 38–42 on DEMT III [VT71], [VT01b]. We can also derive a closed-form expression for $\mu(s)$ for the spatially-spread signal case. Some of the expressions of interest are contained in the problems.

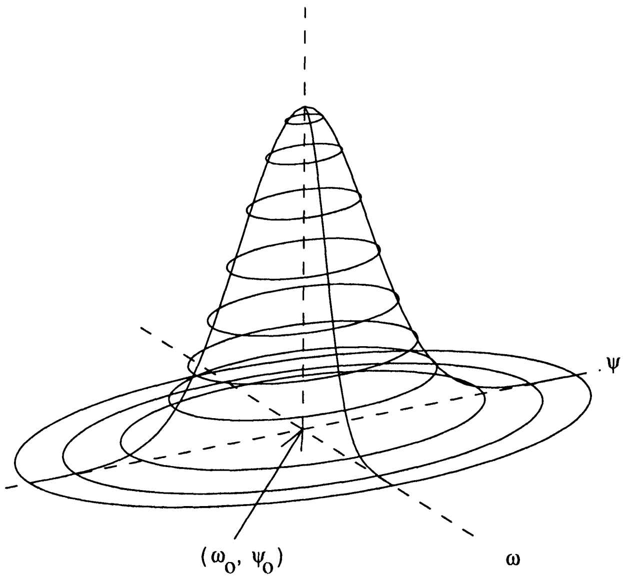

Figure 10.1 Scattering functions in ω-ψ-space.

The discussion in this section has assumed that the required spatial spectral matrices are known. In the next section, we consider the adaptive detection problem in which the statistics are estimated from the input data.

10.1.4 Adaptive Detection

In this section we introduce the problem of adaptive detection. We restrict our attention to the case of a narrowband signal propagating along a single plane wave. We assume that the wavenumber of the plane wave is known.

We use a time-domain snapshot model

$$\begin{aligned} \mathbf{x}(k) &= \mathbf{V}\mathbf{f}(k) + \mathbf{n}(k), \quad k = 1, \cdots, K : H_1 & (10.8) \\ \mathbf{x}(k) &= \mathbf{n}(k), \quad k = 1, \cdots, K : H_0 & (10.9) \end{aligned}$$

where $\mathbf{f}(k)$ is a known vector,

$$\mathbf{f}(k) = \mathbf{v}(\psi_s) f(k), \tag{10.10}$$

and b is an unknown complex scalar. For simplicity, we let

$$f(k) = 1, \quad k = 1, \cdots, K, \tag{10.11}$$

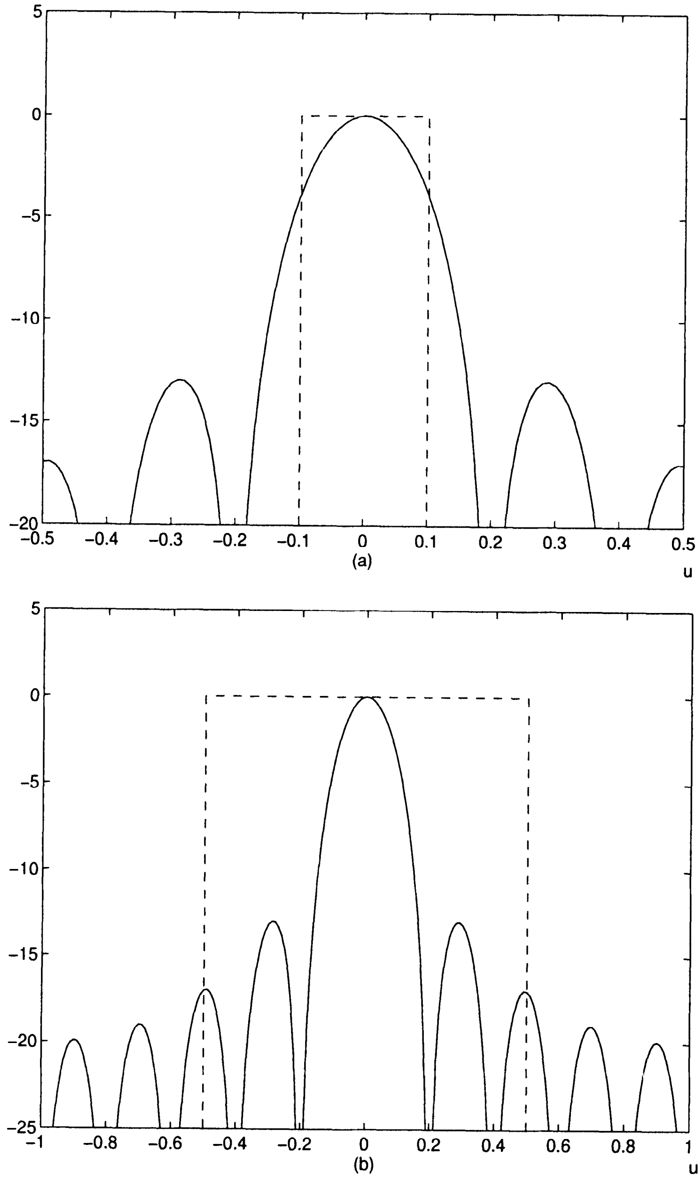

Figure 10.2 Spatial spreading versus array resolution: (a) spatially spread; (b) minimal spread.

so

$$\mathbf{f}(k) = \mathbf{v}(\psi_s)\,. \tag{10.12}$$

The noise $\mathbf{n}(k)$ is a zero-mean complex Gaussian random vector whose covariance is $\mathbf{R_n}$. The snapshots are statistically independent. We denote the collection of snapshots by the $NK \times 1$ vector $\mathbf{X}$. Thus,

$$p_{\mathbf{X}|H_0}(\mathbf{X}) = \prod_{\mathbf{k=1}}^{\mathbf{K}} \frac{\mathbf{1}}{|\pi\mathbf{R_n}|} \exp\left\{-\mathbf{x^H(k)R_n^{-1}x(k)}\right\}\,, \tag{10.13}$$

and

$$p_{\mathbf{X}|H_1}(\mathbf{X}) = \prod_{\mathbf{k=1}}^{\mathbf{K}} \frac{\mathbf{1}}{|\pi\mathbf{R_n}|} \exp\left\{-\left[\mathbf{x^H(k)} - \mathbf{b^*v^H}\right]\mathbf{R_n^{-1}}\left[\mathbf{x(k)} - \mathbf{bv}\right]\right\}\,, \tag{10.14}$$

where we have suppressed ψ_s in the argument of the array manifold vector.

If $\mathbf{R_n}$ were known, we would use a generalized likelihood ratio test (GRLT). From Section 2.5 of DEMT I [VT68] [VT01a],

$$\Lambda(\mathbf{X}) = \frac{\max\limits_{b}\; p_{\mathbf{X}|H_1}(\mathbf{X}, b|H_1)}{p_{\mathbf{X}|H_0}(\mathbf{X}|H_1)} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma\,. \tag{10.15}$$

Substituting (10.13) and (10.14) into (10.15), cancelling common terms, and taking the logarithm gives

$$\ln\Lambda(\mathbf{X}) = 2b^* Re\left\{\sum_{k=1}^{K} \mathbf{v}^H\mathbf{R_n^{-1}}\mathbf{x}(k)\right] - K|b|^2\mathbf{v}^H\mathbf{R_n^{-1}}\mathbf{v}\,. \tag{10.16}$$

Differentiating with respect to b^* and setting the result to zero gives

$$\hat{b} = \frac{\mathbf{v}^H\mathbf{R_n^{-1}}\hat{\mathbf{x}}}{\mathbf{v}^H\mathbf{R_n^{-1}}\mathbf{v}}\,, \tag{10.17}$$

where

$$\hat{\mathbf{x}} \triangleq \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\,, \tag{10.18}$$

which is implemented using an MVDR beamformer.

Substituting (10.17) into (10.16) and (10.15) gives the GLRT for known $\mathbf{R_n}$,

$$\frac{\left|\mathbf{v}^H\mathbf{R_n^{-1}}\hat{\mathbf{x}}\right|^2}{\mathbf{v}^H\mathbf{R_n^{-1}}\mathbf{v}} \underset{H_0}{\overset{H_1}{\gtrless}} \alpha\,. \tag{10.19}$$

We now consider the case in which $\mathbf{R_n}$ is unknown and must be estimated. This problem has been studied in a number of application areas. An early solution in the sonar area was derived by Liggett [Lig72] (see also Chang and Tuteur [CT71], Bryn [Bry62], Middleton and Groginsky [MG65], Edelblute et al. [EFK67], Cox [Cox69] and McDonough [McD71], Lewis and Schultheiss [LS71], and Vanderkulk [Van63]). In the radar area, an early solution was given by Brennan and Reed [BR73] (see also Reed et al. [RMB74]).

The model that is normally used in the literature assumes that, in order to estimate $\mathbf{R_n}$, we receive K independent vector snapshots, $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_K$, that do not contain a signal component. We refer to these vectors as secondary vectors. The subsequent snapshots that may contain the signal are referred to as the primary vectors.

This model can be used to derive several adaptive detectors. We briefly discuss two of these detectors, the adaptive matched filter (AMF) and the GLRT detector.

The adaptive matched filter is due to Robey [Rob91] (see Chapter 4 of Robey [Rob91], which utilizes techniques from Kelly [Kel86]) (see [RFKN92] also). The AMF forms a ML estimate of $\mathbf{R_n}$ using the K secondary vectors,

$$\hat{\mathbf{R}}_{\mathbf{n}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k)\mathbf{x}^H(k) \,. \tag{10.20}$$

The test is obtained by substituting $\hat{\mathbf{R}}_{\mathbf{n}}$ into (10.19). The result is

$$t_{AMF} = \frac{\left|\mathbf{v}^H \hat{\mathbf{R}}_{\mathbf{n}}^{-1} \hat{\mathbf{x}}\right|^2}{\mathbf{v}^H \hat{\mathbf{R}}_{\mathbf{n}}^{-1} \mathbf{v}} \underset{H_0}{\overset{H_1}{\gtrless}} \alpha \,. \tag{10.21}$$

Robey [Rob91] shows that the AMF is a CFAR test and derives expressions for $P_D(AMF)$ and $P_F(AMF)$. (see pp. 37–42 of [Rob91]) The reader is referred to this reference for further discussion.

The GLRT adaptive detector is obtained by utilizing both the primary and secondary vectors and jointly estimating b and $\mathbf{R_n}$. This detector was derived and analyzed by Kelly [Kel86]. (see also [Kel81], [Kel85], [Kel87a], [Kel87b]) The reader is referred to these references for further discussion.

In this section we have introduced the topic of adaptive detection. Our discussion is so brief that it requires the reader to explore the literature to understand the issues.

In addition to the references list above, relevant references include Haykin and Steinhardt [HS92], Kelly and Forsythe [KF89], Monticciolo [Mon94], Tufts and Kristeins [TK85], and Fuhrmann [Fuh91].

10.2 Related Topics

In this section we identify several array processing problems that are of interest.

Target Tracking

In many applications the signals originate from moving sources. As the array receives data, we want to track the location of these sources. The two components of the problem are:

(i) A tracking component that incorporates the target dynamics into the algorithm;

(ii) A data association component that assigns new data to the appropriate target

There are a large number of books and papers that address this problem. Books include Blackman [Bla86], Bar-Shalom and Fortmann [B-SF88], Bar-Shalom and Li [B-SL98], Bar-Shalom [B-S98a], [B-S98b], Stone et al. [SCB99]. Papers that deal with the tracking problem include Reid [Rei79], Sword et al. [SSK93], Rao et al. [RZZ93], [RSZ94], and Zhou et al. [ZYL99a], [ZYL99b].

Most of these discussions treat the problem as a detection problem followed by a tracking problem. A Ph.D. thesis by Zarnich [Zar00] (Zarnich et al. [ZBV01]) treats the problem as a joint problem and obtains interesting results.

Space-Time Processing for Radar

Airborne radars detect targets in an environment that is dominated by clutter and jammers. Space-time adaptive processing (STAP) uses algorithms, which combine the outputs from an antenna array and multiple pulses of a coherent radar waveform to achieve target detection. This approach leads to a 2-D problem in the Doppler-azimuth domain. Many of our results can be extended to this problem. Ward [War94] provides a comprehensive discussion in his Lincoln Laboratory report. The book by Klemm [Kle98] discusses the space-time adaptive processing in detail and has an extensive list of references. There is a collection of papers on STAP in the *April 2000 IEEE Transactions on Aerospace and Electronic Systems* [Mel00].

Space-Time Processing for Wireless Communications

There has been a significant amount of research on space-time processing for wireless communications. It is predicted that many of the third-generation systems will implement some type of adaptive spatial processing in addition to the adaptive temporal processing already in use. These space-time processors are often referred to in the literature as smart antennas.

The biggest challenges faced by these systems are intersymbol interference (ISI) and signal fading due to multipath propagation, and multiuser interference (MUI). The spatial processing at the receiver can reduce MUI by separating user signals arriving at the antenna from different directions. Combined space-time processing can reduce channel fading by suppressing multipath components of the desired signal, or by combining the multipath components in a constructive rather than destructive manner. Performance improvements can also be achieved by using space-time coding with a transmit antenna array. In fixed wireless systems, it is possible to employ antennas at both the transmitter and receiver. Recent research has shown that significant capacity increases can be achieved in these systems by exploiting the diversity gain in rich multipath channels (Foschini et al. [FGVW99]).

Space-time processing for wireless communications has grown to be a significant research area in both the signal processing and communications communities, and promises to be a compelling and challenging problem well into the future as demand for wireless services continues to increase dramatically.

There are a number of papers and books in the area. Books include Liberti and Rappaport [LR99] and Rappaport (ed.) [Rap98]. Papers include Paulraj and Papadias [PP97] and the special issue of the *IEEE Personal Communications magazine on Smart Antennas* [Gol98].

Matched Field Processing

In the sonar environment, we can construct a reasonably accurate model of the propagation of the signal. Therefore, instead of matching the array processor to a plane wave, we match it to the propagation model. This technique is referred to as matched field processing. The original work is by Baggeroer et al. [BKS88]. Later papers include Schmidt et al. [SBKS90] and Preisig [Pre94].

Spatial Spectral Estimation

In Chapter 5, we discussed space-time processes. In some applications, we would like to estimate the spatial correlation function at a particular frequency, $S_f(\omega : \Delta\mathbf{p})$ (5.91). A more general problem is to estimate the frequency-wavenumber spectrum, (5.93).

For a standard linear array, the problem is identical to the temporal spectrum estimation problem (e.g., Kay [Kay88] or Marple [Mar87]). We can use the parametric wavenumber models in Section 5.6. Most of the estimation techniques carry over to the spatial problem.

In each of these areas, the techniques that we have developed in this book will provide the necessary background to explore these areas.

10.3 Epilogue

This chapter concludes our development of optimum array processing and the set of books on detection, estimation, and modulation theory.

We hope that, in spite of the thirty-year gap between volumes, that the collection will prove useful to practicing engineers, researchers, and new students in the area.

In the thirty-year period, there has been a dramatic change in the array processing and signal processing areas. Advances in computational capability have allowed the implementation of complex algorithms that were only of theoretical interest in the past. In many applications, algorithms can be implemented that reach the theoretical bounds. In spite of these advances, there are still a number of challenging problems that these books should help the reader solve.

The advances in computational capability have also changed how the material is taught. In Parts I and III, there was an emphasis on compact analytical solutions to problems. In Part IV, there is a much greater emphasis on efficient iterative solutions and simulations. We have tried to achieve the correct balance between theory and experiment (simulation) in our presentation.

10.4 Problems

P10.1 Optimum Detection[2]

Problem 10.1.1

Consider the following detection problem:

$$\begin{aligned} \mathbf{X}(\omega) &= \mathbf{v}(\psi_s)\mathbf{F}_1(\omega) + \mathbf{W}(\omega) \; : H_1, \\ \mathbf{X}(\omega) &= \mathbf{v}(\psi_s)\mathbf{F}_0(\omega) + \mathbf{W}(\omega) \; : H_0. \end{aligned} \tag{10.22}$$

The array is a standard 10-element linear array. The sensor outputs have been processed by a quadrature demodulator. The two signals, $\mathbf{F}_0(\omega)$ and $\mathbf{F}_1(\omega)$, are known complex signals with energy E_s, and $\mathbf{F}_0(\omega) = -\mathbf{F}_1(\omega)$. The additive noise is a sample function

[2]Most of the problems will require a review of appropriate material from DEMT I, [VT68], [VT01a] or DEMT III, [VT71], [VT01b].

from a complex zero-mean Gaussian process with spectral matrix,

$$S_n(\omega) = \sigma_w^2 \mathbf{I}. \tag{10.23}$$

(a) Find the optimum detector to minimize the probability of error. The two hypotheses are equally likely.

(b) Plot $Pr(\epsilon)$ versus E_s/N_o.

(c) Repeat parts (a) and (b) for the case in which $\mathbf{F}_0(\omega)$ and $\mathbf{F}_1(\omega)$ are orthogonal.

Problem 10.1.2 (continuation)

Repeat Problem 10.2.1 for the case in which

(a) $S_n(\omega) = \sigma_w^2 \mathbf{I} + M_1(\omega)\mathbf{v}(\psi_I)\mathbf{v}^H(\psi_I)$.

(b) Specialize your results in part (a) to the case in which $M_1(\omega)$ is constant over the frequency range of $\mathbf{F}_0(\omega)$ and $\mathbf{F}_1(\omega)$.

(c) Specialize your results in part (a) to the case in which $M_1(\omega)$ corresponds to sine wave with random phase at carrier frequency.

(d) Specialize your results in part (a) to the case in which the interfering signal propagating along $\mathbf{v}(\psi_I)$ consists of either $\mathbf{F}_0(\omega)$ or $\mathbf{F}_1(\omega)$ with equal probability.

Problem 10.1.3

Consider the following detection problem:

$$\begin{aligned} \mathbf{X}(\omega) &= \mathbf{v}(\psi_s)\mathbf{F}(\omega) + \mathbf{W}(\omega) : \quad H_1, \\ \mathbf{X}(\omega) &= \mathbf{W}(\omega) : \quad H_0. \end{aligned} \tag{10.24}$$

The array is a standard 10-element linear array. The signal $f(t)$ is a known complex signal with energy E_s and a uniform phase angle,

$$f(t) = f_1(t)e^{j\theta}, \tag{10.25}$$

and θ has uniform density over $(0, 2\pi)$. The additive noise is a sample function from a complex zero-mean Gaussian process with spectral matrix,

$$S_n(\omega) = \sigma_w^2 \mathbf{I}. \tag{10.26}$$

(a) Find the optimum Neyman-Pearson detector.

(b) Plot $\mathbf{P}_D$ versus $\mathbf{P}_F$ for various E_s/N_o.

(c) Repeat parts (a) and (b) for the case in which

$$S_n(\omega) = \sigma_w^2 \mathbf{I} + M_0\mathbf{v}(\psi_I)\mathbf{v}^H(\psi_I). \tag{10.27}$$

Problem 10.1.4

Consider the model in Problem 10.2.1 and assume $\mathbf{F}_0(\omega) = \mathbf{0}$.

(a) Find the optimum Neyman-Pearson detector.

(b) Plot $\mathbf{P}_D$ versus $\mathbf{P}_F$ for various E_s/σ_w^2.

Problem 10.1.5

Consider a standard 10-element linear array. The signals on the two hypotheses can be written in the frequency domain as

$$\begin{aligned} \mathbf{X}(k, m\omega_0) &= \mathbf{v}(\psi_s)F(k, m\omega_0) + \mathbf{W}(m\omega_0) : \quad H_1, \\ \mathbf{X}(k, m\omega_0) &= \mathbf{W}(m\omega_0) : \quad H_0. \end{aligned} \tag{10.28}$$

The signal $f(k)$ is a zero-mean complex Gaussian AR(1) temporal random process (see (5.308)). The additive noise is a sample function from a white Gaussian random process with spectral height σ_w^2.

(a) Find the optimum Neyman-Pearson detector. Assume $\sigma_s^2/\sigma_w^2 = 10$ dB.

(b) Simulate its performance for $a(1) = 0.9, \phi_a = 0$, and $\mathbf{P}_F = 10^{-3}$.

Problem 10.1.6

Consider a standard 10-element linear array. The signals on the two hypotheses can be written in the frequency domain as

$$\begin{aligned} \mathbf{X}(k, m\omega_0) &= \mathbf{v}(\psi_s)F_1(k, m\omega_0) + \mathbf{W}(m\omega_0) : \quad H_1, \\ \mathbf{X}(k, m\omega_0) &= \mathbf{v}(\psi_s)F_0(k, m\omega_0) + \mathbf{W}(m\omega_0) : \quad H_0. \end{aligned} \tag{10.29}$$

The signals $f_1(k)$ and $f_0(k)$ are zero-mean complex Gaussian AR(1) temporal random processes. The difference is that $\phi_{a_1} = 0.1$ and $\phi_{a_0} = -0.1$. For both signals $|a(1)| = 0.9$. The additive noise is a sample function from a white Gaussian random process with spectral height σ_w^2.

(a) Find the minimum probability of error receiver. Assume the two hypotheses are equally likely. Assume $\sigma_{s_1}^2/\sigma_w^2 = \sigma_{s_0}^2/\sigma_w^2 = 10$ dB.

(b) Simulate its performance.

Problem 10.1.7

Consider a standard 10-element linear array. The nominal detection problem of interest is given by the model in Problem 10.2.4 with $\psi_s = 0$. However, the actual arrival angle θ of the plane wave is a random variable whose probability density is

$$p_\theta(\theta : \Lambda_m) = \frac{\exp\left[\Lambda_m \cos\theta\right]}{2\pi I_0(\Lambda_m)}, \quad -\pi \le \theta \le \pi, \tag{10.30}$$

where θ is measured from broadside.

(a) Choose Λ_m so that the probability is 0.9, that θ, the actual arrival angle, is within the main lobe. Design a constrained beamformer followed by an optimum scalar detector. Compute the resulting performance.

(b) Compare your result to a detector that treats θ as a nuisance parameter and averages over its probability (e.g., p. 335 of DEMT I [VT68], [VT01a], or [HN78]).

Problem 10.1.8

Consider the following detection problem

$$\mathbf{r}(t) = \mathbf{f}_1(t) + \mathbf{n}(t), \qquad T_i \le t \le T_f \quad : H_1, \tag{10.31}$$

$$\mathbf{r}(t) = \mathbf{f}_0(t) + \mathbf{n}(t), \qquad T_i \le t \le T_f \quad : H_0, \tag{10.32}$$

where

$$\begin{aligned} \mathbf{F}_1(\omega) &= \mathbf{v}(\psi_1)F(\omega)\,, & (10.33)\\ \mathbf{F}_2(\omega) &= \mathbf{v}(\psi_2)F(\omega)\,, & (10.34)\end{aligned}$$

and $F(\omega)$ is a rectangular pulse with energy E_s and duration T.

The noise process is stationary

$$\mathbf{S_n}(\omega) = \mathbf{S_c}(\omega) + \sigma_w^2\mathbf{I}\,, \tag{10.35}$$

and $T_i = -\infty$ and $T_f = \infty$.

(a) Find the optimum receiver to minimize the $Pr(\epsilon)$.

(b) Find an expression for d^2.

(c) Assume $\mathbf{S_c}(\omega) = 0$. Plot d^2 as a function of ρ_{12} and T for a standard linear array.

Problem 10.1.9

Consider the problem of detecting a Gaussian random process in non-white Gaussian noise.

$$\begin{aligned} \mathbf{r}(t) &= \mathbf{f}(t) + \mathbf{n}_c(t) + \mathbf{w}(t), \quad T_i \le t \le T_f \;:\; H_1\,, & (10.36)\\ \mathbf{r}(t) &= \mathbf{n}_c(t) + \mathbf{w}(t), \quad T_i \le t \le T_f \;:\; H_0\,, & (10.37)\end{aligned}$$

where

$$\mathbf{F}(\omega) = \mathbf{v}(\psi_s)F(\omega)\,, \tag{10.38}$$

and the spectrum of $f(t)$ is

$$S_f(\omega) = \begin{cases} \frac{P_s}{2W}\,, & -2\pi W \le \omega \le 2\pi W\,,\\ 0\,, & \text{elsewhere}\,. \end{cases} \tag{10.39}$$

The colored noise is a single plane-wave signal,

$$\mathbf{N}_c(\omega) = \mathbf{v}(\psi_n)N_c(\omega) \tag{10.40}$$

where

$$S_{n_c}(\omega) = \begin{cases} \frac{P_n}{2W}\,, & -2\pi W \le \omega \le 2\pi W\,,\\ 0\,, & \text{elsewhere}\,. \end{cases} \tag{10.41}$$

Assume $T = T_f - T_i$ is large.

(a) Find the optimum receiver.

(b) Compute $\mu_\infty(s)$.

(c) Assume the criterion is minimum $Pr(\epsilon)$ and the hypotheses are equally likely. Find an approximate expression for $Pr(\epsilon)$.

Problem 10.1.10

Consider the problem of deciding which of two Gaussian random processes are present.

$$\mathbf{r}(t) = \mathbf{f}_1(t) + \mathbf{w}(t), \quad T_i \le t \le T_f \;:\; H_1\,, \tag{10.42}$$

$$\mathbf{r}(t) = \mathbf{f}_0(t) + \mathbf{w}(t), \quad T_i \le t \le T_f \;:\; H_0\,, \tag{10.43}$$

where

$$\mathbf{F}_1(\omega) = \mathbf{v}(\psi_1)F_1(\omega), \tag{10.44}$$

$$\mathbf{F}_0(\omega) = \mathbf{v}(\psi_0)F_0(\omega). \tag{10.45}$$

The signals $f_1(t)$ and $f_0(t)$ are independent with identical spectrum,

$$S_{f_1}(\omega) = S_{f_0}(\omega) = \begin{cases} \frac{P}{2W}, & -2\pi W \le \omega \le 2\pi W, \\ 0, & \text{elsewhere}. \end{cases} \tag{10.46}$$

Assume $T = T_f - T_i$ is large, the hypotheses are equally likely, and the criterion is min $Pr(\epsilon)$.

(a) Find the optimum receiver.

(b) Find $\mu_\infty(s)$.

(c) Find an approximate expression for the $Pr(\epsilon)$.

(d) Consider a standard 10-element linear array. Plot $Pr(\epsilon)$ as a function of

$$\Delta\psi = \psi_1 - \psi_0. \tag{10.47}$$

Problem 10.1.11

The received waveforms on the two hypotheses are:

$$\mathbf{r}(t) = \mathbf{f}_1(t) + \mathbf{f}_2(t) + \mathbf{w}(t), \quad 0 \le t \le T : \quad H_1, \tag{10.48}$$

$$\mathbf{r}(t) = \mathbf{w}(t), \quad 0 \le t \le T : \quad H_0, \tag{10.49}$$

where the $N \times 1$ signal vectors can be written in the transform domain as,

$$\mathbf{F}_1(n\omega_o) = \mathbf{v}(n\omega_o, \psi_1)F_1(n\omega_o) \tag{10.50}$$

$$\mathbf{F}_2(n\omega_o) = \mathbf{v}(n\omega_o, \psi_2)F_2(n\omega_o) \tag{10.51}$$

where $\omega_o = \frac{2\pi}{T}$ and $n = 1, \cdots, M$.

The source signals $f_1(t)$ and $f_2(t)$ are statistically independent Gaussian random processes with spectra,

$$S_i(\omega) = \begin{cases} \frac{\sigma_i^2}{2W}, & |\omega| \le 2\pi W, \\ 0, & \text{elsewhere}, \quad i = 1, 2. \end{cases} \tag{10.52}$$

The noise process $\mathbf{w}(t)$ is a Gaussian random process that is temporally and spatially white,

$$E\left[\mathbf{w}(t)\mathbf{w}^T(u)\right] = \sigma_w^2 \delta(t-u)\mathbf{I}. \tag{10.53}$$

The hypotheses are equally likely and T is large.

(a) Find the minimum $Pr(\epsilon)$ test.

(b) Evaluate the performance.

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