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Chapter 5

Side-Looking Sonar

In this chapter we shall discuss some of the fundamentals concerning the design and analysis of a *side-looking sonar* (SLS), also referred to as a *side-scan sonar*. A SLS is an active sonar system capable of producing high resolution underwater images of large areas on the ocean bottom. Side-looking sonars are commonly used to detect and identify objects lying on the ocean bottom. They are also used to perform bathymetric surveys (seafloor mapping).

A SLS generates underwater images by periodically transmitting a pulse and then receiving the scattered returns while the platform is in motion with a constant velocity vector (constant speed and direction) and at a constant height (altitude) above the ocean bottom. A SLS may be composed of a single planar aperture that is used alternately in the active (transmit) mode and passive (receive) mode, or two separate planar apertures – a transmit aperture and a receive aperture. Side-looking sonars are usually mounted on both sides of an underwater vehicle. Some SLS systems are *towed* while others are mounted on *remotely operated vehicles* (ROVs) or *autonomous underwater vehicles* (AUVs), also known as *unmanned underwater vehicles* (UUVs). Understanding the fundamentals of a SLS system provides a very good foundation for the understanding of the fundamentals of a *synthetic aperture sonar* (SAS) system.

5.1 Swath Width

We begin our discussion of side-looking sonars (SLSs) by deriving an equation for the *swath width* (SW), which is a measure of the width of the area on the ocean bottom ensonified by a SLS. We shall analyze a SLS modeled as a planar aperture lying in the YZ plane as shown in Fig. 5.1-1. The planar aperture is rectangular in shape with sides of length L_Y and L_Z meters in the Y and Z directions, respectively. The X axis is cross-range (the *cross-track direction*), the Y axis is depth, and the Z axis is down-range (the *along-track direction*). Since the planar aperture (SLS) lies in the YZ plane, its far-field beam pattern $D(f, f_Y, f_Z)$ depends on the spatial frequencies in the Y and Z directions given by

$$f_Y = v/\lambda \tag{5.1-1}$$

and

$$f_Z = w/\lambda \,, \tag{5.1-2}$$

respectively, with units of cycles per meter, where

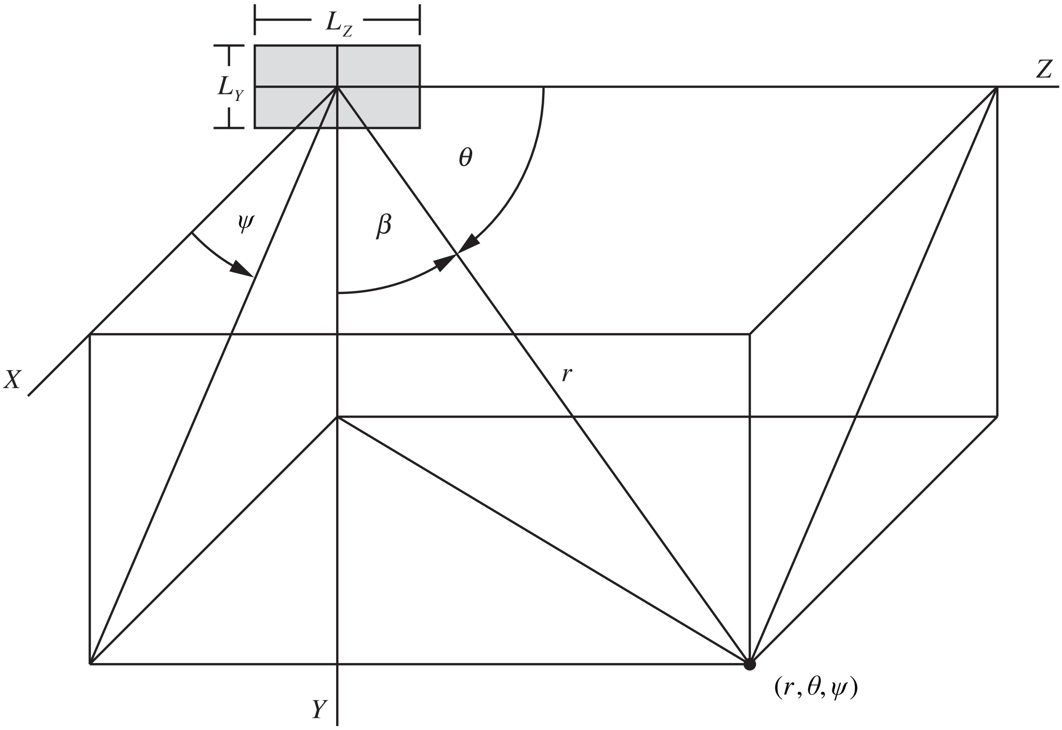


Figure 5.1-1 A SLS (planar aperture, rectangular in shape) lying in the YZ plane. Also shown is a field point in three-dimensional space with spherical coordinates (r,θ,ψ) as measured from the center of the aperture, and the angle β which is measured from the positive Y axis.

$$v = \sin\theta \sin\psi = \cos\beta \tag{5.1-3}$$

and

$$w = \cos\theta \tag{5.1-4}$$

are the dimensionless direction cosines in the Y and Z directions, respectively, λ is the wavelength in meters, and $c = f\lambda$.

In order to derive an equation for the SW, and to introduce some of the other parameters associated with a SLS and to show their interdependence, we first need to derive an equation for the 3-dB beamwidth (half-power beamwidth) of the vertical, far-field beam pattern in the XY plane. In the XY plane, $\theta = 90°$ (see Fig. 5.1-1), and as a result [see (5.1-3) and (5.1-4)],

$$v = \sin\psi = \cos\beta \tag{5.1-5}$$

and

$$w = 0\,. \tag{5.1-6}$$

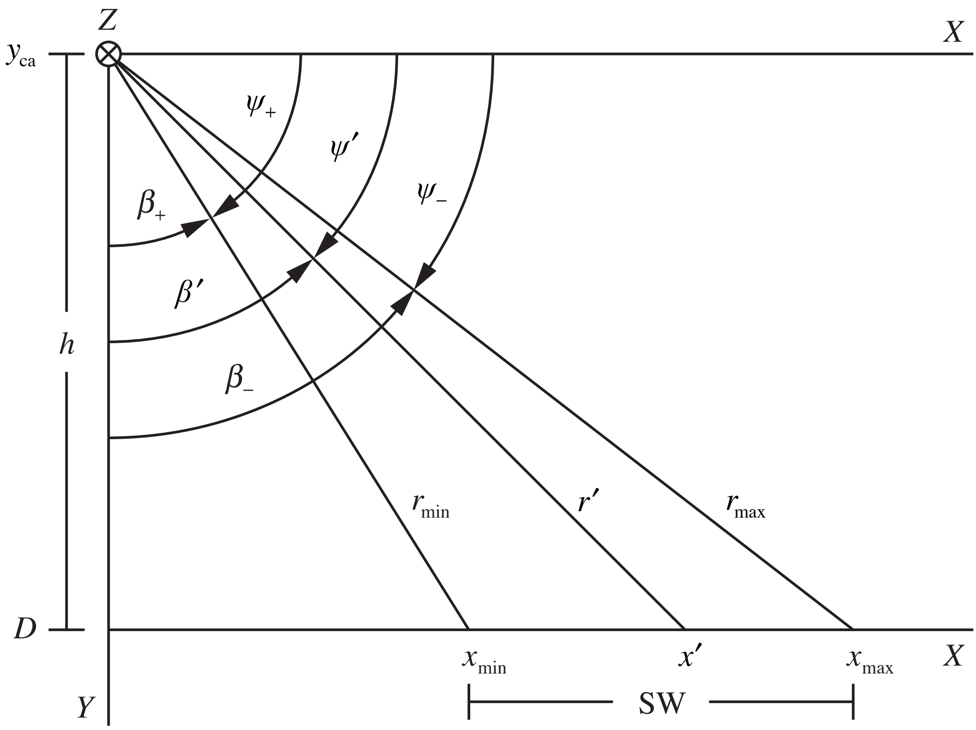


Figure 5.1-2 Angles involved in the derivation of the 3-dB beamwidth $\Delta\psi = \psi_+ - \psi_-$ (or $\Delta\beta = \beta_- - \beta_+$) of the vertical, far-field beam pattern in the XY plane. The parameters y_{ca} and D are the depths of the center of the aperture and ocean, respectively. Also shown is the swath width $\mathrm{SW} = x_{max} - x_{min}$.

From Fig. 5.1-2 it can be seen that the 3-dB beamwidth is given by either

$$\Delta\psi = \psi_+ - \psi_- \tag{5.1-7}$$

or

$$\Delta\beta = \beta_- - \beta_+ , \tag{5.1-8}$$

where ψ' (or β') is the beam-steer angle in the XY plane. For a SLS, $\psi' \neq 0°$ ($\beta' \neq 90°$) since this corresponds to steering the vertical beam pattern to broadside, and $\psi' \neq 90°$ ($\beta' \neq 0°$) since this corresponds to steering the vertical beam pattern to endfire. Note that the *half-beamwidth angles* $\psi_+ - \psi'$ and $\psi' - \psi_-$ are *not* equal for $0° < \psi' < 90°$, and that the *half-beamwidth angles* $\beta' - \beta_+$ and $\beta_- - \beta'$ are *not* equal for $0° < \beta' < 90°$. Also note that $\Delta\beta = \Delta\psi$, $\beta' - \beta_+ = \psi_+ - \psi'$, and $\beta_- - \beta' = \psi' - \psi_-$.

Following the procedure discussed in Section 2.5, let

$$v_+ = \sin\psi_+ > 0 \quad \text{or} \quad v_+ = \cos\beta_+ > 0 \,, \tag{5.1-9}$$

$$v_- = \sin\psi_- > 0 \quad \text{or} \quad v_- = \cos\beta_- > 0 \,, \tag{5.1-10}$$

and

$$v' = \sin\psi' > 0 \quad \text{or} \quad v' = \cos\beta' > 0 \,, \tag{5.1-11}$$

where

$$v_+ > v_- \,, \tag{5.1-12}$$

$$v_+ = v' + (\Delta v/2) \,, \tag{5.1-13}$$

$$v_- = v' - (\Delta v/2) \,, \tag{5.1-14}$$

and Δv is the dimensionless 3-dB beamwidth of the vertical, far-field beam pattern in direction-cosine space. Therefore,

$$\psi_+ = \sin^{-1} v_+ = \sin^{-1}\left(\sin\psi' + \frac{\Delta v}{2}\right), \qquad \left|\sin\psi' + \frac{\Delta v}{2}\right| \le 1 \,, \tag{5.1-15}$$

$$\psi_- = \sin^{-1} v_- = \sin^{-1}\left(\sin\psi' - \frac{\Delta v}{2}\right), \qquad \left|\sin\psi' - \frac{\Delta v}{2}\right| \le 1 \,, \tag{5.1-16}$$

$$\beta_+ = \cos^{-1} v_+ = \cos^{-1}\left(\cos\beta' + \frac{\Delta v}{2}\right), \qquad \left|\cos\beta' + \frac{\Delta v}{2}\right| \le 1 \,, \tag{5.1-17}$$

and

$$\beta_- = \cos^{-1} v_- = \cos^{-1}\left(\cos\beta' - \frac{\Delta v}{2}\right), \qquad \left|\cos\beta' - \frac{\Delta v}{2}\right| \le 1 \,. \tag{5.1-18}$$

Substituting (5.1-15) and (5.1-16) into (5.1-7) yields

$$\boxed{\Delta\psi = \sin^{-1}\left(\sin\psi' + \frac{\Delta v}{2}\right) - \sin^{-1}\left(\sin\psi' - \frac{\Delta v}{2}\right), \qquad \left|\sin\psi' \pm \frac{\Delta v}{2}\right| \le 1} \tag{5.1-19}$$

and substituting (5.1-17) and (5.1-18) into (5.1-8) yields

$$\boxed{\Delta\beta = \cos^{-1}\left(\cos\beta' - \frac{\Delta v}{2}\right) - \cos^{-1}\left(\cos\beta' + \frac{\Delta v}{2}\right), \qquad \left|\cos\beta' \pm \frac{\Delta v}{2}\right| \le 1} \tag{5.1-20}$$

The 3-dB beamwidth (in degrees) of the vertical, far-field beam pattern in the XY plane is given by either (5.1-19) or (5.1-20).

If we let the *height* (*altitude*) h in meters of the center of the aperture above the ocean bottom be given by

$$h = D - y_{\text{ca}} , \tag{5.1-21}$$

where D and y_{ca} are the depths of the ocean and center of the aperture in meters, respectively, then from Fig. 5.1-2 it can be seen that

$$x_{\min} = h \cot \psi_+ = h \tan \beta_+ , \tag{5.1-22}$$

$$x' = h \cot \psi' = h \tan \beta' , \tag{5.1-23}$$

$$x_{\max} = h \cot \psi_- = h \tan \beta_- , \tag{5.1-24}$$

$$r_{\min} = \sqrt{h^2 + x_{\min}^2} = \frac{h}{\sin \psi_+} = \frac{h}{\cos \beta_+} , \tag{5.1-25}$$

$$r' = \sqrt{h^2 + (x')^2} = \frac{h}{\sin \psi'} = \frac{h}{\cos \beta'} , \tag{5.1-26}$$

and

$$r_{\max} = \sqrt{h^2 + x_{\max}^2} = \frac{h}{\sin \psi_-} = \frac{h}{\cos \beta_-} , \tag{5.1-27}$$

where the angles ψ_+, ψ_-, β_+, and β_- are given by (5.1-15) through (5.1-18), respectively. Equations (5.1-25) through (5.1-27) are the *slant-ranges* corresponding to the *cross-ranges* given by (5.1-22) through (5.1-24), respectively. The cross-range $x_{\min}$ given by (5.1-22) is referred to as the *width of the blind zone* in meters because it corresponds to the distance on the ocean bottom from $x = 0$ to $x = x_{\min}$ that is *not* ensonified by the 3-dB beamwidth of the vertical, far-field beam pattern (see Fig. 5.1-2). The parameter $x_{\min}$ is also known as the width of the *one-sided* blind zone. The width of the *two-sided* blind zone is simply $2x_{\min}$.

In order to guarantee that the far-field beam pattern of the aperture (vis-à-vis its near-field beam pattern) will ensonify a point on the ocean bottom at the minimum slant-range $r_{\min}$, $r_{\min}$ must satisfy the far-field range criterion (e.g., see Section 3.1):

$$r_{\min} > \pi R_A^2 / \lambda > 2.414 R_A , \tag{5.1-28}$$

where R_A is the maximum radial extent of the aperture. Since the planar aperture is rectangular in shape with sides of length L_Y and L_Z meters,

$$R_A = \sqrt{\left(\frac{L_Y}{2}\right)^2 + \left(\frac{L_Z}{2}\right)^2} \, . \tag{5.1-29}$$

Therefore, substituting (5.1-29) into (5.1-28) yields

$$\boxed{r_{\min} > \pi \frac{L_Y^2 + L_Z^2}{4\lambda} > 1.207 \sqrt{L_Y^2 + L_Z^2}} \tag{5.1-30}$$

If (5.1-30) is satisfied, then *all* points on the ocean bottom within the swath width will be ensonified by the far-field beam pattern of the aperture vis-à-vis its near-field beam pattern. This is important because most of the parameters used to design and analyze a SLS are based on the far-field beam pattern of the SLS.

The *swath width* (SW) in meters, also referred to as the *ground-plane swath width*, is given by (see Fig. 5.1-2)

$$\boxed{\mathrm{SW} = x_{\max} - x_{\min}} \tag{5.1-31}$$

where $x_{\min}$ and $x_{\max}$ are given by (5.1-22) and (5.1-24), respectively. The parameter SW is also known as the *one-sided* swath width. The *two-sided* swath width is simply $2\mathrm{SW}$. Note that $x' - x_{\min} \neq x_{\max} - x'$ (see Fig. 5.1-2).

An alternative expression for SW can be obtained by substituting (5.1-22) and (5.1-24) into (5.1-31). Doing so yields

$$\begin{aligned} \mathrm{SW} &= h\left(\cot\psi_- - \cot\psi_+\right) \\ &= h\frac{\sin(\psi_+ - \psi_-)}{\sin\psi_- \sin\psi_+} = h\frac{\sin(\Delta\psi)}{v_- v_+} \\ &= h\frac{\sin(\Delta\psi)}{\left(v' - \frac{\Delta v}{2}\right)\left(v' + \frac{\Delta v}{2}\right)} = h\frac{\sin(\Delta\psi)}{(v')^2 - \left(\frac{\Delta v}{2}\right)^2} \end{aligned} \tag{5.1-32}$$

and finally,

$$\boxed{\mathrm{SW} = h\frac{\sin\left(\Delta\psi(\psi', \Delta v)\right)}{\sin^2\psi' - \left(\frac{\Delta v}{2}\right)^2}, \qquad 0° < \psi' < 90°, \quad \left|\sin\psi' \pm \frac{\Delta v}{2}\right| \le 1, \quad \sin\psi' > \frac{\Delta v}{2}}$$

(5.1-33)

where h is the height (altitude) in meters of the center of the aperture above the ocean bottom, $\Delta\psi(\psi', \Delta v)$ is the 3-dB beamwidth in degrees of the vertical, far-field beam pattern in the XY plane shown explicitly as a function of the independent variables ψ' and Δv [see (5.1-19)], ψ' is the beam-steer angle in degrees, and Δv is the dimensionless 3-dB beamwidth of the vertical, far-field beam pattern in direction-cosine space. Since $\Delta\psi = \Delta\beta$ and $\psi' = 90° - \beta'$, SW can also be expressed as

$$\boxed{\mathrm{SW} = h\frac{\sin\left(\Delta\beta(\beta', \Delta v)\right)}{\cos^2\beta' - \left(\dfrac{\Delta v}{2}\right)^2}, \qquad 0° < \beta' < 90°, \quad \left|\cos\beta' \pm \frac{\Delta v}{2}\right| \leq 1, \quad \cos\beta' > \frac{\Delta v}{2}} \tag{5.1-34}$$

where the 3-dB beamwidth $\Delta\beta(\beta', \Delta v)$ is given by (5.1-20).

The *area coverage rate* (ACR) in squared meters per second, also referred to as the *survey coverage rate*, is given by

$$\boxed{\mathrm{ACR} = \mathrm{SW} \times V} \tag{5.1-35}$$

where SW is the swath width given by either (5.1-31), (5.1-33), or (5.1-34), and V is the constant speed of the platform in meters per second. The parameter ACR is also known as the *one-sided* area coverage rate. The *two-sided* area coverage rate is simply $2\,\mathrm{ACR}$. As can be seen from (5.1-35), the ACR can be increased by increasing either the SW or V or both. In general, one would like the ACR to be as big as possible.

5.2 Cross-Track (Slant-Range) Resolution

The ability of a SLS to estimate the slant-range to different objects on the ocean bottom within the SW – at the same down-range (along-track) position – is referred to as the *cross-track (slant-range) resolution*. In order to discuss this topic, we need to know how to design rectangular-envelope, CW (continuous-wave) and LFM (linear-frequency-modulated) pulses in order to provide a desired range resolution, since these two kinds of waveforms are typically transmitted by a SLS. We also need to know how to determine a minimum value for the carrier frequency. The carrier frequency of the waveform transmitted by a SLS is commonly referred to as the operating frequency of the SLS. These two waveforms are discussed in detail in Chapter 13. In this section we shall only summarize those equations from Chapter 13 that are needed to continue our discussion.

The first kind of waveform that is typically transmitted by a SLS is known as a *rectangular-envelope*, *CW* (continuous-wave) *pulse*. Specifying a value for range resolution is equivalent to specifying a desired value for the magnitude of the range estimation error of the target $\left|e_{r,Trgt}\right|$. The pulse length T (in seconds) that is required for a rectangular-envelope, CW pulse to provide a range resolution equal to $\left|e_{r,Trgt}\right|$ meters is given by

$$\boxed{T = \frac{2\left|e_{r,Trgt}\right|}{c}} \tag{5.2-1}$$

where c is the constant speed of sound in meters per second. A conservative estimate for the minimum allowed value for the carrier frequency f_c (in hertz) for a rectangular-envelope, CW pulse is given by

$$\boxed{\min f_c = 5/T} \tag{5.2-2}$$

Therefore, for a rectangular-envelope, CW pulse, f_c is chosen to satisfy the following inequality:

$$\boxed{f_c > 5/T} \tag{5.2-3}$$

The second kind of waveform that is typically transmitted by a SLS is known as a *rectangular-envelope*, *linear-frequency-modulated* (LFM) *pulse*. The *swept-bandwidth* $\mathrm{BW}_{\mathrm{swept}}$ (in hertz) that is required for a rectangular-envelope, LFM pulse to provide a range resolution equal to $\left|e_{r,Trgt}\right|$ meters is given by

$$\boxed{\mathrm{BW}_{\mathrm{swept}} \approx \frac{c}{2\left|e_{r,Trgt}\right|}} \tag{5.2-4}$$

The pulse length T must then satisfy the inequality

$$\boxed{T \geq T_{\mathrm{min}}} \tag{5.2-5}$$

where

$$\boxed{T_{\mathrm{min}} = \frac{40}{\mathrm{BW}_{\mathrm{swept}}}} \tag{5.2-6}$$

is the minimum allowed value. A conservative estimate for the minimum allowed value for the carrier frequency f_c for a rectangular-envelope, LFM pulse is given by

$$\boxed{\min f_c = \mathrm{BW}_{\text{swept}} + \frac{5}{T}} \tag{5.2-7}$$

where T must satisfy (5.2-5). Therefore, for a rectangular-envelope, LFM pulse, f_c is chosen to satisfy the following inequality:

$$\boxed{f_c > \mathrm{BW}_{\text{swept}} + \frac{5}{T}} \tag{5.2-8}$$

As can be seen from the above equations, the minimum allowed value for the carrier frequency not only depends on the desired range resolution, but also on what kind of waveform is transmitted. In addition, the carrier frequency may need to be made much larger than the minimum allowed value in order to decrease the 3-dB beamwidth of the far-field beam pattern of the planar aperture (SLS) if the size of the aperture is small. However, keep in mind that as frequency increases, attenuation of sound in the ocean also increases. Therefore, both low and high carrier frequencies are used for short-range applications, whereas low carrier frequencies are used for long-range applications. Note that in the SLS and SAS literature, the terminology *operating frequency* f is commonly used instead of *carrier frequency* f_c. Table 5.2-1 summarizes the parameter values for both waveforms in order to obtain a cross-track (slant-range) resolution of 2.54 cm ($1 \text{ in} = 2.54$ cm) for $c = 1500$ m/sec. For example purposes, $T = 25$ msec was chosen for the rectangular-envelope, LFM pulse.

Table 5.2-1 Parameter Values for a Rectangular-Envelope, CW and LFM Pulse in Order to Obtain a Cross-Track (Slant-Range) Resolution of 2.54 cm (1 in) for $c = 1500$ m/sec

Pulse	$\lvert e_{r,Trgt} \rvert$	$\mathrm{BW}_{\text{swept}}$	$T_{\min}$	T	$\min f_c$
RE CW	2.54 cm	NA	NA	33.9 μsec	147,637.8 Hz
RE LFM	2.54 cm	29,527.6 Hz	1.355 msec	25 msec	29,727.6 Hz

5.3 Along-Track (Azimuthal) Resolution

In order to derive an equation for the *along-track (azimuthal) resolution* (see Fig. 5.3-1), we first need to derive an equation for the 3-dB beamwidth (half-power beamwidth) $\Delta\theta$ of the horizontal, far-field beam pattern in the XZ plane (see Fig. 5.3-2). In the XZ plane, $\psi = 0°$ (positive X axis) and $\beta = 90°$ (see Fig. 5.1-1), and as a result [see (5.1-3)],

$$v = 0 \,. \tag{5.3-1}$$

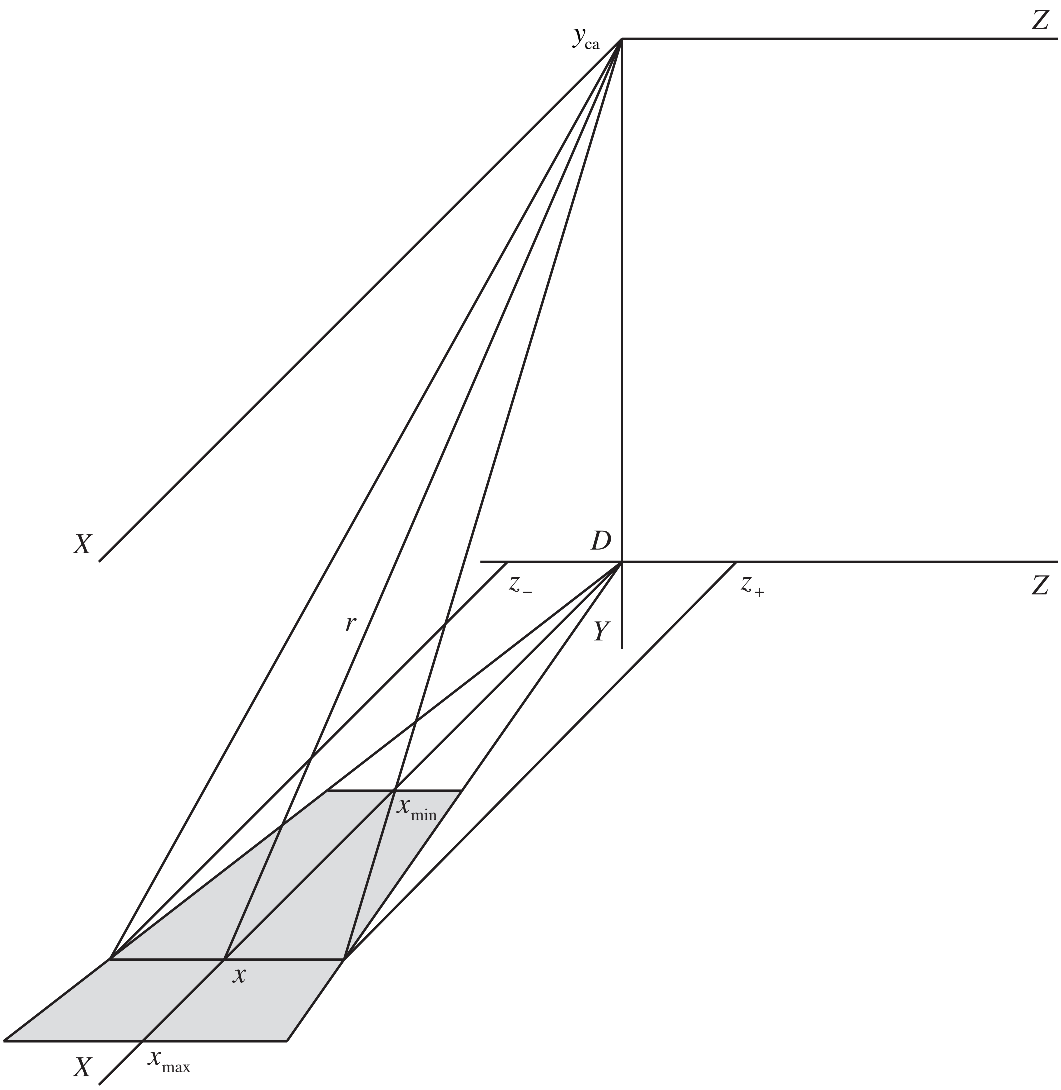


Figure 5.3-1 Along-track (azimuthal) resolution $\Delta z = z_+ - z_-$ at slant-range r and cross-range x. The shaded area represents the area on the ocean bottom within the swath width $\text{SW} = x_{max} - x_{min}$ ensonified by the 3-dB beamwidth of the horizontal, far-field beam pattern in the XZ plane.

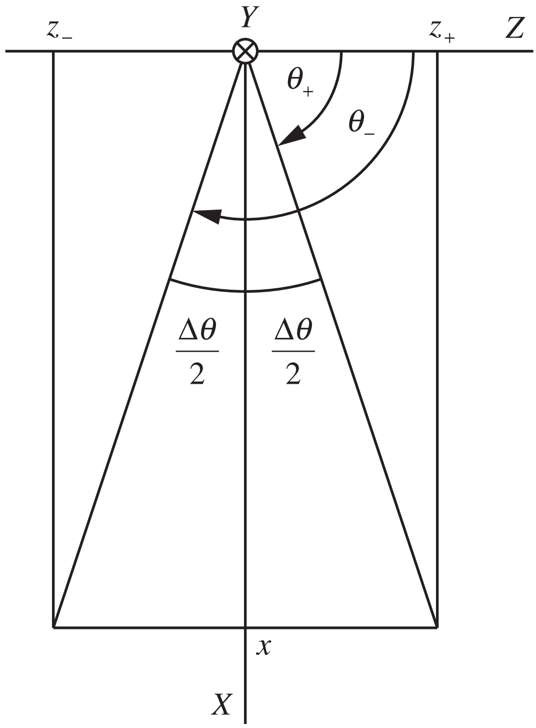


Figure 5.3-2 Angles involved in the derivation of the 3-dB beamwidth $\Delta\theta$ of the horizontal, far-field beam pattern in the XZ plane.

Following the procedure discussed in Example 2.3-2, let

$$w_+ = \cos\theta_+ > 0 \tag{5.3-2}$$

and

$$w_- = \cos\theta_- < 0\,, \tag{5.3-3}$$

where from Fig. 5.3-2 it can be seen that

$$\theta_+ = \frac{\pi}{2} - \frac{\Delta\theta}{2} \tag{5.3-4}$$

and

$$\theta_- = \frac{\pi}{2} + \frac{\Delta\theta}{2}\,. \tag{5.3-5}$$

Substituting (5.3-4) and (5.3-5) into (5.3-2) and (5.3-3), respectively, yields

$$w_+ = \cos\left(\frac{\pi}{2} - \frac{\Delta\theta}{2}\right) = \sin\left(\frac{\Delta\theta}{2}\right) > 0 \tag{5.3-6}$$

and

$$w_- = \cos\left(\frac{\pi}{2} + \frac{\Delta\theta}{2}\right) = -\sin\left(\frac{\Delta\theta}{2}\right) < 0 . \tag{5.3-7}$$

Since the dimensionless 3-dB beamwidth Δw of the horizontal, far-field beam pattern in direction-cosine space is given by

$$\Delta w = w_+ - w_- , \tag{5.3-8}$$

substituting (5.3-6) and (5.3-7) into (5.3-8) yields

$$\Delta w = 2\sin(\Delta\theta/2) > 0 , \tag{5.3-9}$$

or

$$\boxed{\Delta\theta = 2\sin^{-1}(\Delta w/2), \qquad \Delta w/2 \le 1} \tag{5.3-10}$$

where $\Delta\theta$ is the 3-dB beamwidth (in degrees) of the horizontal, far-field beam pattern in the XZ plane. We are now in a position to derive equations for the along-track resolutions at the three different cross-ranges $x_{\min}$, x', and $x_{\max}$.

From Fig. 5.3-2 it can be seen that

$$z_+ = x\tan(\Delta\theta/2) , \tag{5.3-11}$$

and since

$$\Delta z = z_+ - z_- = 2z_+ , \tag{5.3-12}$$

substituting (5.3-11) into (5.3-12) yields the following expression for the along-track resolution Δz in meters at cross-range x meters:

$$\boxed{\Delta z = 2x\tan(\Delta\theta/2)} \tag{5.3-13}$$

As can be seen from (5.3-13), for a given value of horizontal, 3-dB beamwidth $\Delta\theta$, the along-track resolution Δz increases as the cross-range x increases. As Δz *increases*, the ability of a SLS to resolve closely-spaced objects on the ocean bottom *decreases* (see Fig. 5.3-3). Therefore, in order to obtain small values for Δz for large values of x, $\Delta\theta$ must be small. The along-track (azimuthal) resolutions at cross-ranges $x_{\min}$ (the beginning or near-edge of the SW), x', and $x_{\max}$ (the end or far-edge of the SW) are given by

$$\boxed{\Delta z_{\min} = 2x_{\min}\tan(\Delta\theta/2)} \tag{5.3-14}$$

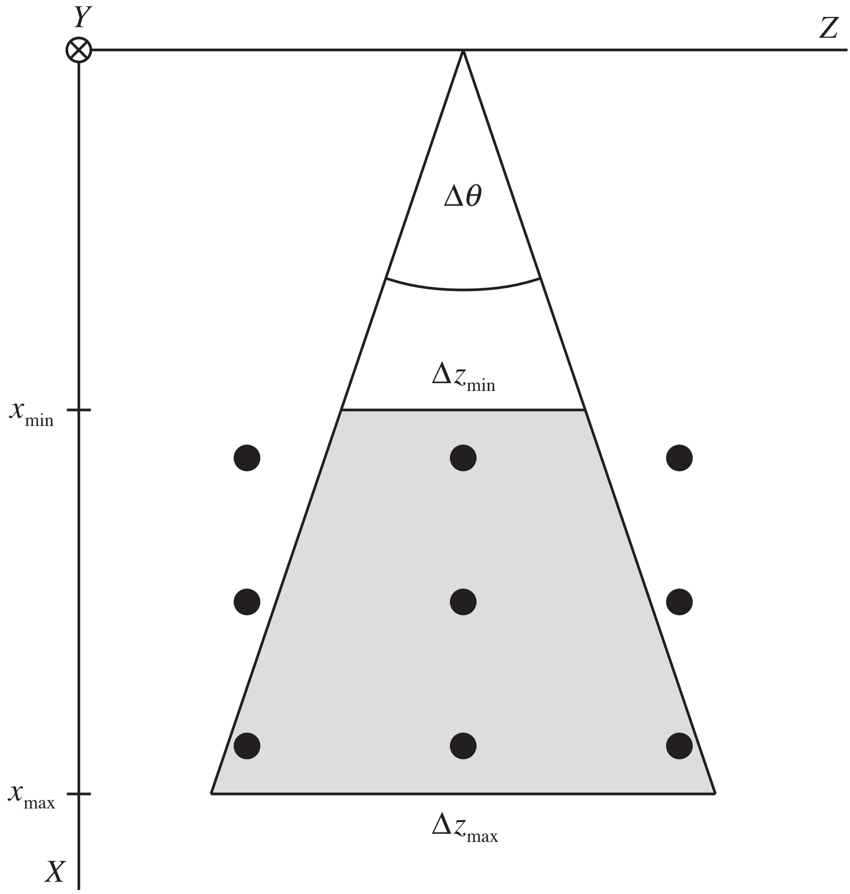


Figure 5.3-3 For a given value of horizontal, 3-dB beamwidth $\Delta\theta$, the ability of a SLS to resolve closely-spaced objects on the ocean bottom decreases as the cross-range increases.

$$\boxed{\Delta z' = 2x' \tan(\Delta\theta/2)} \tag{5.3-15}$$

and

$$\boxed{\Delta z_{max} = 2x_{max} \tan(\Delta\theta/2)} \tag{5.3-16}$$

respectively, where x_{min}, x', and x_{max} are given by (5.1-22) through (5.1-24), respectively, and $\Delta\theta$ is the 3-dB beamwidth (in degrees) of the horizontal, far-field beam pattern in the XZ plane given by (5.3-10).

5.4 Slant-Range Ambiguity

A SLS transmits a pulse with *pulse length* T seconds every PRI seconds, where PRI is the abbreviation for *pulse repetition interval*. Sometimes the abbreviation PRP for *pulse repetition period* is used instead of PRI. The minimum and maximum *round-trip time delays* (in seconds) corresponding to the SW are given by

$$\tau_{\min} = 2 r_{\min} / c \tag{5.4-1}$$

and

$$\tau_{\max} = 2 r_{\max} / c \,, \tag{5.4-2}$$

where $r_{\min}$ and $r_{\max}$ are the slant-ranges corresponding to the cross-ranges $x_{\min}$ and $x_{\max}$, respectively (see Fig. 5.1-2), and c is the constant speed of sound in meters per second. If the relativistic effects of platform motion are ignored (i.e., time compression or time expansion), then the total duration of the received signal from the SW is $\tau_{\max} - \tau_{\min} + T$ seconds (see Fig. 5.4-1).

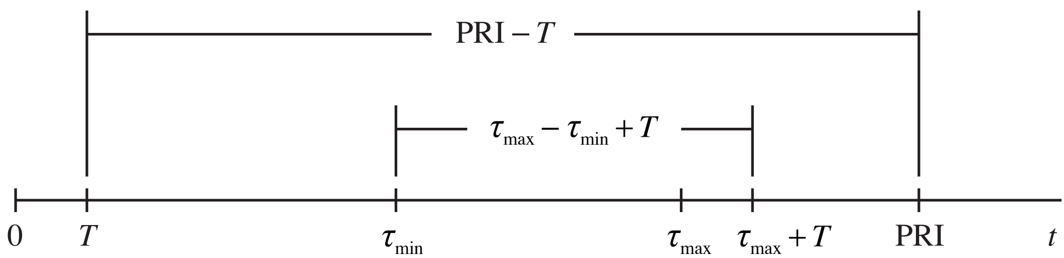


Figure 5.4-1 Time line showing the relationship between the pulse length T of the transmitted signal, the duration of the received signal $\tau_{\max} - \tau_{\min} + T$, where $\tau_{\min}$ and $\tau_{\max}$ are the minimum and maximum round-trip time delays corresponding to the swath width, and the pulse repetition interval (PRI).

In order to allow enough time for the entire received signal to arrive during the transmitter's down time before the next transmission, the following inequality must be satisfied:

$$\tau_{\max} - \tau_{\min} + T \le \mathrm{PRI} - T \,, \tag{5.4-3}$$

or

$$\mathrm{PRI} \ge \mathrm{PRI}_{\min} \,, \tag{5.4-4}$$

where

$$\boxed{\mathrm{PRI}_{\min} = \tau_{\max} - \tau_{\min} + 2T = \frac{2\,\mathrm{SRSW}}{c} + 2T} \tag{5.4-5}$$

and

$$\boxed{\mathrm{SRSW} = r_{\max} - r_{\min}} \tag{5.4-6}$$

is the *slant-range swath width* (SRSW) in meters, where $r_{\min}$ and $r_{\max}$ are given by (5.1-25) and (5.1-27), respectively. Therefore, in order to avoid *slant-range ambiguities*, the PRI must be greater than or equal to the minimum value of PRI given by (5.4-5).

The minimum value of PRI given by (5.4-5) is the absolute smallest allowed value, and appears often in the SLS and SAS literature. However, from Fig. 5.4-1, it can be seen that an *operational* minimum value of PRI is given by

$$\boxed{\mathrm{PRI}_{\min,\,\mathrm{op}} = \tau_{\max} + T = \frac{2\,r_{\max}}{c} + T} \tag{5.4-7}$$

Note that

$$\mathrm{PRI}_{\min,\,\mathrm{op}} \geq \mathrm{PRI}_{\min}\,, \tag{5.4-8}$$

and that $\mathrm{PRI}_{\min,\,\mathrm{op}} = \mathrm{PRI}_{\min}$ when $\tau_{\min} = T$ [see (5.4-3)].

Since the *pulse repetition frequency* (PRF) in pulses per second (pps) – also known as the *pulse repetition rate* (PRR) – is equal to the reciprocal of the pulse repetition interval (PRI), that is, since

$$\mathrm{PRF} = 1/\mathrm{PRI}\,, \tag{5.4-9}$$

then

$$\mathrm{PRF} \leq \mathrm{PRF}_{\max}\,, \tag{5.4-10}$$

where

$$\boxed{\mathrm{PRF}_{\max} = \frac{1}{\mathrm{PRI}_{\min}} = \frac{1}{\tau_{\max} - \tau_{\min} + 2T} = \frac{1}{\dfrac{2\,\mathrm{SRSW}}{c} + 2T}} \tag{5.4-11}$$

Therefore, in order to avoid slant-range ambiguities, the PRF must be less than or equal to the maximum value of PRF given by (5.4-11). The maximum value of PRF given by (5.4-11) is the absolute largest allowed value, and also appears often in the SLS and SAS literature along with $\mathrm{PRI}_{\min}$. However, an *operational* maximum value of PRF is given by

$$\boxed{\mathrm{PRF}_{\max,\,\mathrm{op}} = \frac{1}{\mathrm{PRI}_{\min,\,\mathrm{op}}} = \frac{1}{\tau_{\max} + T} = \frac{1}{\dfrac{2\,r_{\max}}{c} + T}} \tag{5.4-12}$$

Note that

$$\mathrm{PRF}_{\mathrm{max,\,op}} \leq \mathrm{PRF}_{\mathrm{max}} \tag{5.4-13}$$

since $\mathrm{PRI}_{\mathrm{min,\,op}} \geq \mathrm{PRI}_{\mathrm{min}}$, and that $\mathrm{PRF}_{\mathrm{max,\,op}} = \mathrm{PRF}_{\mathrm{max}}$ when $\mathrm{PRI}_{\mathrm{min,\,op}} = \mathrm{PRI}_{\mathrm{min}}$.

5.5 Azimuthal Ambiguity

The constant velocity vector of the SLS shown in Fig. 5.1-1 is given by

$$\mathbf{V} = V\hat{z}\,, \tag{5.5-1}$$

where V is the constant speed of the platform in meters per second and $\hat{z}$ is the unit vector in the positive Z direction. Therefore, according to (5.5-1), the SLS is traveling in the positive Z direction with a constant speed of V meters per second. A typical range of values for the speed of a SLS is from 3 to 7 knots (approximately 1.5 to 3.6 meters per second or 3.5 to 8.1 miles per hour) where $1\,\text{knot} \approx 0.514\,\text{m/sec}$ or $1\,\text{knot} \approx 1.151\,\text{mph}$. As can be seen from Fig. 5.5-1, in order to fully ensonify the ocean bottom within the SW (without leaving *gaps* in the coverage) as the SLS moves in the positive Z direction, the SLS must transmit (ping) every Δz_{min} meters or *less*. In other words,

$$\mathrm{PRI} \leq \mathrm{PRI}_{\mathrm{max}}\,, \tag{5.5-2}$$

where

$$\boxed{\mathrm{PRI}_{\mathrm{max}} = \Delta z_{\mathrm{min}}/V} \tag{5.5-3}$$

and Δz_{min} is the minimum value of along-track (azimuthal) resolution at cross-range x_{min} given by (5.3-14). Therefore, in order to avoid *azimuthal ambiguities*, the PRI must be less than or equal to the maximum value of PRI given by (5.5-3).

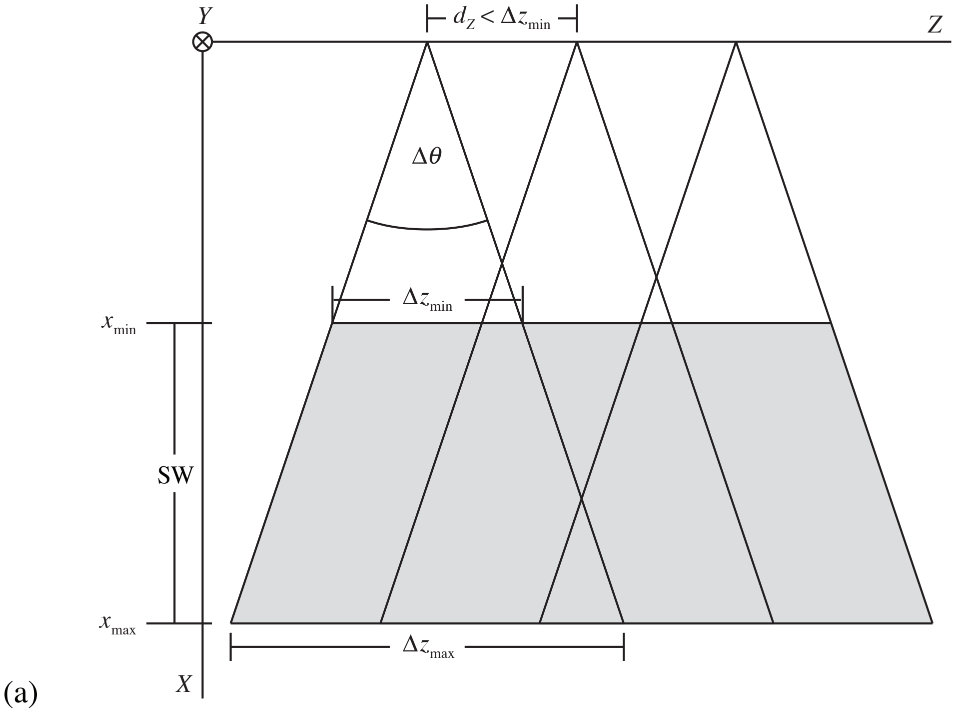
The reciprocal of (5.5-2) is

$$\mathrm{PRF} \geq \mathrm{PRF}_{\mathrm{min}}\,, \tag{5.5-4}$$

where

$$\boxed{\mathrm{PRF}_{\mathrm{min}} = \frac{1}{\mathrm{PRI}_{\mathrm{max}}} = \frac{V}{\Delta z_{\mathrm{min}}}} \tag{5.5-5}$$

Therefore, in order to avoid azimuthal ambiguities, the PRF must be greater than or equal to the minimum value of PRF given by (5.5-5).



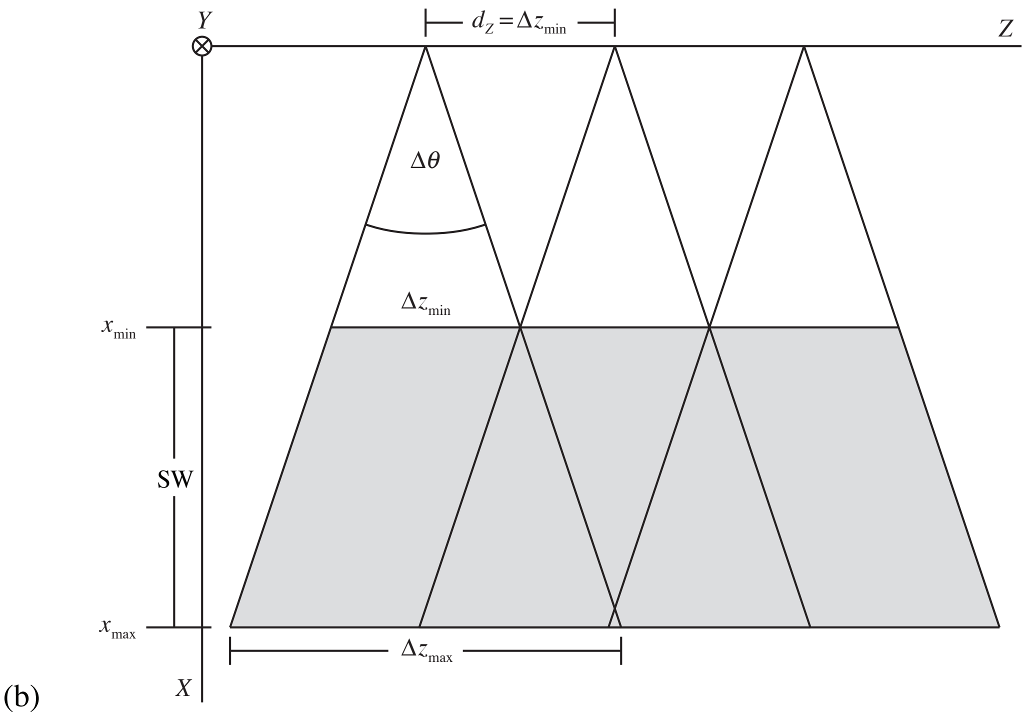
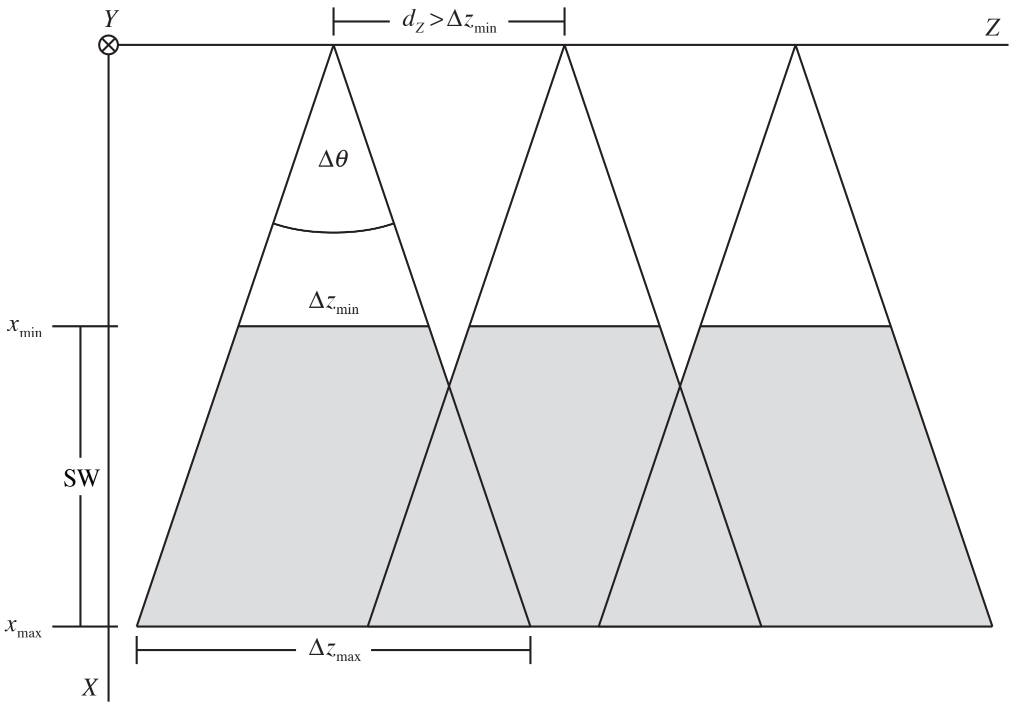


Figure 5.5-1 As the SLS moves in the positive Z direction, it transmits every d_Z meters, where (a) $d_Z < \Delta z_{\min}$, (b) $d_Z = \Delta z_{\min}$, and (c) $d_Z > \Delta z_{\min}$. If $d_Z > \Delta z_{\min}$, then there are *gaps* in the coverage within the SW as shown in (c).



(c)

Figure 5.5-1 *continued*.

In summary, in order to avoid both slant-range and azimuthal ambiguities,

$$\boxed{\mathrm{PRI}_{\min,\,\mathrm{op}} \le \mathrm{PRI} \le \mathrm{PRI}_{\max}} \tag{5.5-6}$$

or

$$\boxed{\tau_{\max} + T \le \mathrm{PRI} \le \Delta z_{\min} / V} \tag{5.5-7}$$

where $\tau_{\max}$ is the maximum round-trip time delay corresponding to the end (far-edge) of the SW given by (5.4-2) and T is the pulse length of the transmitted signal. Equivalently,

$$\boxed{\mathrm{PRF}_{\min} \le \mathrm{PRF} \le \mathrm{PRF}_{\max,\,\mathrm{op}}} \tag{5.5-8}$$

or

$$\boxed{\frac{V}{\Delta z_{\min}} \le \mathrm{PRF} \le \frac{1}{\tau_{\max} + T}} \tag{5.5-9}$$

5.6 A Rectangular-Piston Model for a Side-Looking Sonar

In this section we shall derive approximate expressions for the dimensionless 3-dB beamwidths Δv and Δw of the vertical and horizontal, far-field beam patterns in direction-cosine space, respectively, of a SLS (a planar aperture) lying in the YZ plane as shown in Fig. 5.1-1. Having equations for Δv and Δw in terms of system parameters such as the operating (carrier) frequency and the physical dimensions of the aperture is very important because many of the SLS parameters depend on knowing the values of Δv and Δw.

As was mentioned at the beginning of this chapter, a SLS may be composed of a single planar aperture that is used alternately in the active (transmit) mode and passive (receive) mode, or two separate planar apertures – a transmit aperture and a receive aperture. A realistic SLS (planar aperture) is a *rectangular array* of identical, equally-spaced, omnidirectional, electroacoustic transducers with *rectangular amplitude weights*. A very good approximation of this kind of array is a *rectangular piston* as discussed in Section 3.2. A rectangular piston model is often used for a SLS because it is very easy to derive equations for Δv and Δw. Therefore, if the SLS (planar aperture) lying in the YZ plane is modeled as a rectangular piston with sides of length L_Y and L_Z meters in the Y and Z directions, respectively, then its normalized, far-field beam pattern is given by

$$D_N(f, f_Y, f_Z) = \operatorname{sinc}(f_Y L_Y)\operatorname{sinc}(f_Z L_Z)\,, \tag{5.6-1}$$

or, in direction-cosine space,

$$D_N(f, v, w) = \operatorname{sinc}\left(\frac{L_Y}{\lambda} v\right)\operatorname{sinc}\left(\frac{L_Z}{\lambda} w\right), \tag{5.6-2}$$

where $c = f\lambda$.

Since direction cosine $w = 0$ in the XY plane [see (5.1-6)] and $\operatorname{sinc}(0) = 1$, the normalized, vertical, far-field beam pattern in the XY plane is given by

$$D_N(f, v, 0) = \operatorname{sinc}\left(\frac{L_Y}{\lambda} v\right), \tag{5.6-3}$$

and

$$\boxed{\Delta v/2 \approx 0.443\,\lambda/L_Y} \tag{5.6-4}$$

where Δv is the dimensionless 3-dB beamwidth of the vertical, far-field beam pattern in direction-cosine space (see Example 2.3-3).

Since direction cosine $v = 0$ in the XZ plane [see (5.3-1)] and $\text{sinc}(0) = 1$, the normalized, horizontal, far-field beam pattern in the XZ plane is given by

$$D_N(f, 0, w) = \text{sinc}\left(\frac{L_Z}{\lambda} w\right), \tag{5.6-5}$$

and

$$\boxed{\Delta w/2 \approx 0.443\lambda/L_Z} \tag{5.6-6}$$

where Δw is the dimensionless 3-dB beamwidth of the horizontal, far-field beam pattern in direction-cosine space (see Example 2.3-3).

5.7 Design and Analysis of a Side-Looking Sonar Mission

5.7.1 Deep Water

If a SLS is going to operate in deep water, then one can choose its height (altitude) of operation h above the ocean bottom to satisfy the far-field range criterion [see (5.1-30)]. In the case of a SLS, the far-field range criterion can be stated as follows: if $h > h_{\min}$, then *all* points on the ocean bottom within the swath width (SW) will be ensonified by the far-field beam pattern of the SLS vis-à-vis its near-field beam pattern, where $h_{\min}$ is the minimum allowed height (altitude) of operation. As was mentioned previously, this is important because most of the parameters used to design and analyze a SLS are based on the far-field beam pattern of the SLS. However, in a shallow-water environment, it may not always be possible to choose $h > h_{\min}$ (see Subsection 5.7.2). In this subsection we shall summarize those equations that need to be solved in order to determine $h_{\min}$. Solving this set of equations corresponds to following a design procedure for choosing the altitude of operation h so that $h > h_{\min}$ (see Table 5.7-1). We shall also summarize those equations that are needed to solve for the set of angles $\{\beta_+, \beta', \beta_-\}$. Solving for the required beam-steer angle β' is also part of designing the mission. Once this set of angles is known, the other parameters that describe the performance of a SLS can be evaluated in a straightforward fashion. This constitutes the analysis portion of a SLS mission.

In most SLS problems, the dimensions L_Y and L_Z, and the operating (carrier) frequency f of a SLS are given (specified). Recall from Section 5.2 that the operating frequency is determined by the waveform to be transmitted, the desired cross-track (slant-range) resolution, and the size of the aperture. The desired value for the along-track (azimuthal) resolution Δz_{min} at cross-range x_{min} (the beginning or near-edge of the SW) is also usually given. Since a SLS is an imaging sonar, it is not surprising that the desired cross-track and along-track resolutions are specified.

Table 5.7-1 Design Procedure for Choosing the Altitude of Operation for a SLS in Order to Satisfy the Far-Field Range Criterion in Deep Water

Given	Resulting Fixed Parameter Values
L_Y , L_Z , and f	$r_{\text{NF/FF}}$, $\Delta\psi$ ($\psi' = 0°$), and $\Delta\theta$
Δz_{min} and $\Delta\theta$	x_{min}
$r_{\text{NF/FF}}$ and x_{min} where $r_{\text{NF/FF}} > x_{\text{min}}$	h_{min} $h > h_{\text{min}}$ satisfies far-field range criterion

If the dimensions L_Y and L_Z, and the operating (carrier) frequency f of a SLS are given, then the values of the range to the near-field/far-field boundary $r_{\text{NF/FF}}$, the 3-dB beamwidth $\Delta\psi$ of the vertical, far-field beam pattern in the XY plane when *no* beam steering is done ($\psi' = 0°$), and the 3-dB beamwidth $\Delta\theta$ of the horizontal, far-field beam pattern in the XZ plane of the SLS are *fixed*. By referring to (5.1-30), it can be seen that the range to the near-field/far-field boundary is given by

$$\boxed{r_{\text{NF/FF}} = \pi \frac{L_Y^2 + L_Z^2}{4\lambda}} \tag{5.7-1}$$

where $c = f\lambda$. If we let the beam-steer angles $\psi' = 0°$ and $\beta' = 90°$, then the 3-dB beamwidths $\Delta\psi$ and $\Delta\beta$ given by (5.1-19) and (5.1-20), respectively, reduce to

$$\Delta\psi = \Delta\beta = 2\sin^{-1}\left(\frac{\Delta v}{2}\right), \qquad \frac{\Delta v}{2} \le 1, \quad \psi' = 0° \ (\beta' = 90°), \tag{5.7-2}$$

since

$$\sin^{-1}(-x) = -\sin^{-1}(x) \tag{5.7-3}$$

and

$$\cos^{-1}(-x) - \cos^{-1}(x) = 2\sin^{-1}(x). \tag{5.7-4}$$

And by substituting (5.6-4) into (5.7-2), we obtain

$$\boxed{\Delta\psi = \Delta\beta \approx 2\sin^{-1}\left(0.443\frac{\lambda}{L_Y}\right), \qquad 0.443\frac{\lambda}{L_Y} \le 1, \quad \psi' = 0° \, (\beta' = 90°)}$$

(5.7-5)

Although $\Delta\psi$ is not needed to compute $h_{\min}$, since L_Y and f are given, the 3-dB beamwidth of the vertical, far-field beam pattern in the XY plane when *no* beam steering is done ($\psi' = 0°$) is fixed and its value can be computed from (5.7-5). And if (5.6-6) is substituted into (5.3-10), then

$$\boxed{\Delta\theta \approx 2\sin^{-1}\left(0.443\frac{\lambda}{L_Z}\right), \qquad 0.443\frac{\lambda}{L_Z} \le 1} \tag{5.7-6}$$

where $\Delta\theta$ is the 3-dB beamwidth of the horizontal, far-field beam pattern in the XZ plane.

Since $\Delta z_{\min}$ is given and $\Delta\theta$ is now known, we can use (5.3-14) to solve for $x_{\min}$. Doing so yields

$$\boxed{x_{\min} = \frac{\Delta z_{\min}}{2\tan(\Delta\theta/2)}} \tag{5.7-7}$$

where $x_{\min}$ is the width of the one-sided blind zone, also referred to as the cross-range coordinate of the beginning (near-edge) of the SW. Now that $r_{\text{NF/FF}}$ and $x_{\min}$ are known, we are finally in a position to solve for $h_{\min}$.

In order to satisfy the far-field range criterion, we require $r_{\min} > r_{\text{NF/FF}}$. Therefore, with the use of (5.1-25), we can write that

$$r_{\min} = \sqrt{h^2 + x_{\min}^2} > r_{\text{NF/FF}}, \tag{5.7-8}$$

and by solving for h, we obtain

$$\boxed{h > h_{\min}} \tag{5.7-9}$$

where

$$\boxed{h_{\min} = \sqrt{r_{\text{NF/FF}}^2 - x_{\min}^2}, \qquad r_{\text{NF/FF}} > x_{\min}} \tag{5.7-10}$$

is the minimum allowed height (altitude) of operation of the SLS above the ocean bottom. If $h > h_{\min}$, then *all* points on the ocean bottom within the SW will be ensonified by the far-field beam pattern of the SLS vis-à-vis its near-field beam pattern.

Since we now have solutions for $x_{\min}$ and h, we can use them to compute the angle β_+, which will allow us to solve for the required beam-steer angle β', and then the angle β_-. Solving for β_+ from (5.1-22) yields

$$\boxed{\beta_+ = \tan^{-1}(x_{\min}/h)} \tag{5.7-11}$$

The angle β_+ must be greater than $0°$ (see Fig. 5.1-2). With the use of h and $x_{\min}$, or h and β_+, the slant-range $r_{\min}$ to the beginning (near-edge) of the SW can be computed from (5.1-25).

Next, solve for the beam-steer angle β' from (5.1-17). Doing so yields

$$\beta' = \cos^{-1}\left(\cos\beta_+ - \frac{\Delta v}{2}\right), \qquad \left|\cos\beta_+ - \frac{\Delta v}{2}\right| \le 1, \tag{5.7-12}$$

and by substituting (5.6-4) into (5.7-12), we obtain

$$\boxed{\beta' \approx \cos^{-1}\left(\cos\beta_+ - 0.443\frac{\lambda}{L_Y}\right), \qquad \left|\cos\beta_+ - 0.443\frac{\lambda}{L_Y}\right| \le 1} \tag{5.7-13}$$

Now that h and β' are known, the cross-range coordinate x' can be computed from (5.1-23), and with the use of h and x', or h and β', the corresponding slant-range r' can be computed from (5.1-26). And finally, substituting (5.6-4) into (5.1-18) yields

$$\boxed{\beta_- \approx \cos^{-1}\left(\cos\beta' - 0.443\frac{\lambda}{L_Y}\right), \qquad \left|\cos\beta' - 0.443\frac{\lambda}{L_Y}\right| \le 1} \tag{5.7-14}$$

The angle β_- must be less than $90°$ (see Fig. 5.1-2). Now that h and β_- are known, the cross-range coordinate $x_{\max}$ of the end (far-edge) of the SW can be computed from (5.1-24), and with the use of h and $x_{\max}$, or h and β_-, the corresponding slant-range $r_{\max}$ can be computed from (5.1-27).

Since $x_{\min}$ and $x_{\max}$, and $r_{\min}$ and $r_{\max}$ are now known, the SW and the slant-range swath width (SRSW) can be computed from (5.1-31) and (5.4-6), respectively. Note that the angles ψ_+, ψ', and ψ_- can be obtained from β_+, β', and β_- as follows (see Fig. 5.1-2):

$$\psi_+ = 90° - \beta_+ , \tag{5.7-15}$$

$$\psi' = 90° - \beta' , \tag{5.7-16}$$

$$\psi_- = 90° - \beta_- . \tag{5.7-17}$$

Recall that the 3-dB beamwidth of the vertical, far-field beam pattern in the XY plane is given by either [see (5.1-7) and (5.1-8)]

$$\Delta\psi = \psi_+ - \psi_- \tag{5.7-18}$$

or

$$\Delta\beta = \beta_- - \beta_+ , \tag{5.7-19}$$

where $\Delta\beta = \Delta\psi$. Using the solutions of the parameters discussed above, the remaining parameters that describe the performance of a SLS – such as the area coverage rate (ACR) and the lower and upper bounds on the pulse repetition interval (PRI) and pulse repetition frequency (PRF) that are required to avoid both slant-range and azimuthal ambiguities – can easily be evaluated.

Example 5.7-1 Deep Water Mission

The SLS (planar aperture) shown in Fig. 5.1-1 has the following specifications:

Dimensions:	$L_Y = 0.1$ m , $L_Z = 2.5$ m
Transmit Waveform:	rectangular-envelope, LFM pulse
Operating (Carrier) Frequency:	$f = 30$ kHz
Pulse Length:	$T = 25$ msec (see Table 5.2-1)
Cross-Track (Slant-Range) Resolution:	2.54 cm (1 in) (see Table 5.2-1)
Along-Track (Azimuthal) Resolution:	$\Delta z_{\min} = 1$ m at cross-range $x_{\min}$
Speed:	3 to 7 knots

In this example the SLS is meant to operate in deep water at a speed $V = 2$ m/sec (≈ 4 knots). Therefore, following the design procedure in Table 5.7-1 and using $c = 1500$ m/sec, a design and analysis of this SLS mission is summarized in Table 5.7-2. From Table 5.7-2 it can be seen that the width of the one-sided blind zone $x_{\min} = 56.4$ m (this value is fixed) and the minimum altitude of operation for the SLS above the ocean bottom required to satisfy the far-field range criterion is $h_{\min} = 80.5\text{ m}$. Since the SLS is operating in deep water, we have the ability to choose $h > h_{\min}$. As can be seen from Table 5.7-2, $h = 85\text{ m}$ was chosen, resulting in $r_{\min} = 102\text{ m}$ (which is greater than $r_{\text{NF/FF}} = 98.3\text{ m}$), a required beam-steer angle $\psi' = 37.71°$, a swath width $\text{SW} = 144.2\text{ m}$, and an area coverage rate $\text{ACR} = 288.4\text{ m}^2/\text{sec}$. As was mentioned in Section 5.1, the ACR can be increased by increasing either the SW or V or both.

Since the SLS is operating in deep water, the SW can be increased while keeping $x_{\min}$ fixed by increasing h. Note that $x_{\min}/h_{\min} = 0.701$ and $x_{\min}/h = 0.664$ for $h = 85\text{ m}$ (see Table 5.7-2). Increasing h while keeping $x_{\min}$ fixed will increase the required beam-steer angle ψ' (see Fig. 5.7-1), which will increase the 3-dB beamwidth $\Delta\psi$ of the vertical, far-field beam pattern (see Fig. 5.7-2) and, hence, the SW (see Fig. 5.7-3). As $x_{\min}/h \to 0$, $\beta_+ \to 0$ [see (5.7-11) and Fig. 5.1-2]. If a bigger value for h is chosen, then the design and analysis procedure must begin again starting with (5.7-11). Figures 5.7-1 through 5.7-3 are based on $\lambda/L_Y = 0.5$ because this is the value of the ratio in this example. ■

Table 5.7-2 Design and Analysis Parameter Values for Example 5.7-1

$c = 1500\text{ m/sec}$	$f = 30\text{ kHz}$	$\lambda = 0.05\text{ m}$	$T = 25\text{ msec}$	$V = 2\text{ m/sec}$
$L_Y = 0.1\text{ m}$ $\lambda/L_Y = 0.5$	$\Delta\psi = 25.59°\ (\psi' = 0°)$			
$L_Z = 2.5\text{ m}$ $\lambda/L_Z = 0.02$	$\Delta\theta = 1.02°$			
$r_{\text{NF/FF}} = 98.3\text{ m}$				
$h_{\min} = 80.5\text{ m}$ $h = 85\text{ m}$				
$r_{\min} = 102\text{ m}$ $r' = 139\text{ m}$ $r_{\max} = 217.9\text{ m}$	$\psi_+ = 56.42°$ $\psi' = 37.71°$ $\psi_- = 22.96°$	$\beta_+ = 33.58°$ $\beta' = 52.29°$ $\beta_- = 67.04°$	$x_{\min} = 56.4\text{ m}$ $x' = 110\text{ m}$ $x_{\max} = 200.6\text{ m}$	$\Delta z_{\min} = 1\text{ m}$ $\Delta z' = 1.9\text{ m}$ $\Delta z_{\max} = 3.6\text{ m}$
$\text{SRSW} = 115.9\text{ m}$ $\tau_{\min} = 136\text{ msec}$ $\tau_{\max} = 290.5\text{ msec}$	$\psi_+ - \psi' = 18.71°$ $\psi' - \psi_- = 14.75°$ $\Delta\psi = 33.46°$	$\beta' - \beta_+ = 18.71°$ $\beta_- - \beta' = 14.75°$ $\Delta\beta = 33.46°$	$\text{SW} = 144.2\text{ m}$ $\text{ACR} = 288.4\text{ m}^2/\text{sec}$	
			$x_{\min}/h_{\min} = 0.701$ $x_{\min}/h = 0.664$	
$\text{PRI}_{\min} = 204.5\text{ msec}$ $\text{PRI}_{\min,\text{op}} = 315.5\text{ msec}$ $\text{PRI}_{\max} = 500\text{ msec}$	$\text{PRF}_{\min} = 2\text{ pps}$ $\text{PRF}_{\max,\text{op}} = 3.2\text{ pps}$ $\text{PRF}_{\max} = 4.9\text{ pps}$			

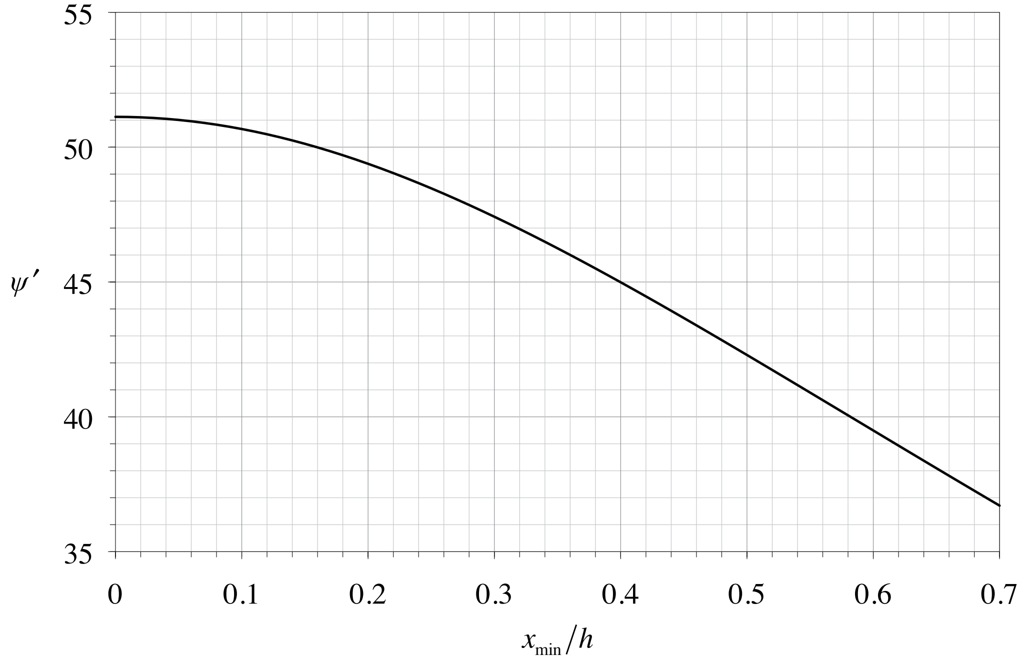


Figure 5.7-1 Beam-steer angle ψ' in degrees versus $x_{\min}/h$ for $\lambda/L_Y = 0.5$ where $h > h_{\min}$ and $x_{\min}/h_{\min} = 0.701$.

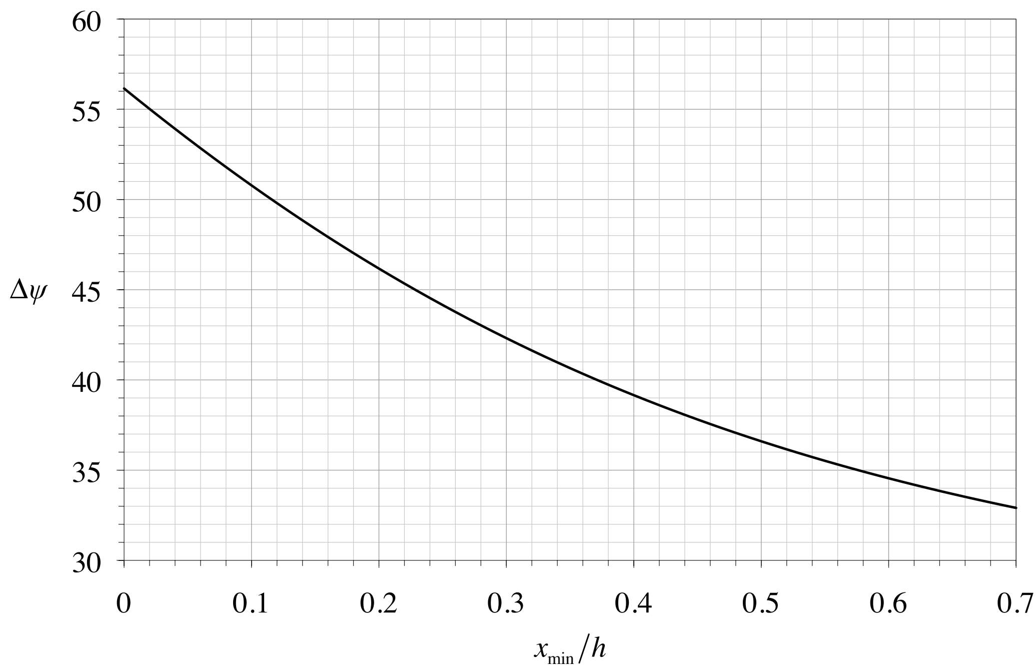


Figure 5.7-2 Three-dB beamwidth $\Delta\psi$ in degrees of the vertical, far-field beam pattern versus $x_{\min}/h$ for $\lambda/L_Y = 0.5$ where $h > h_{\min}$ and $x_{\min}/h_{\min} = 0.701$.

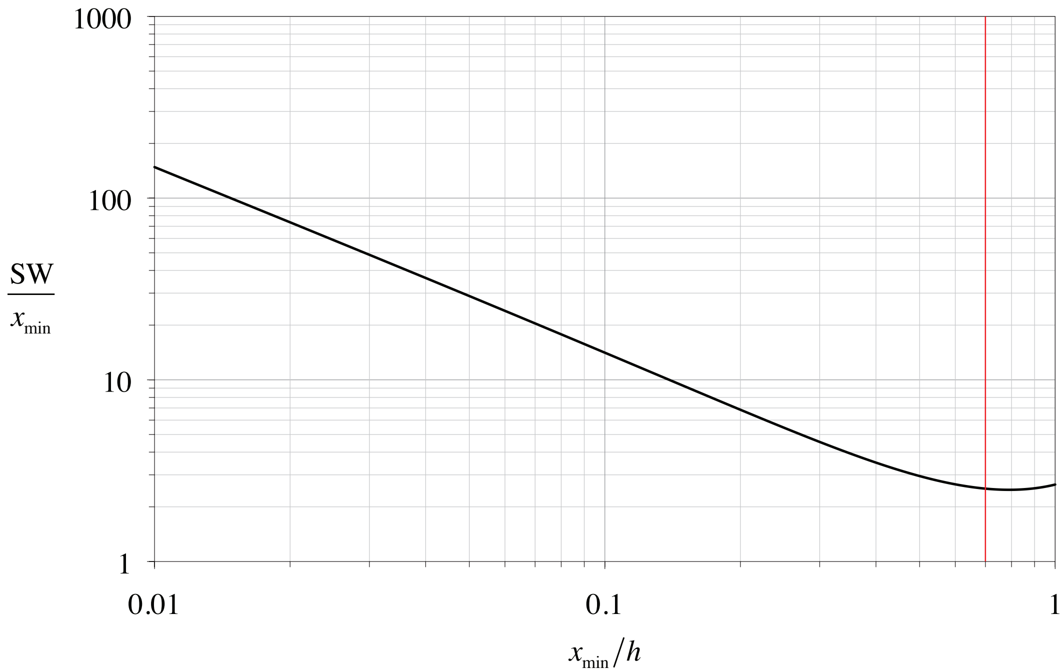


Figure 5.7-3 Ratio $\text{SW}/x_{\min}$ versus $x_{\min}/h$ for $\lambda/L_Y = 0.5$ where $h > h_{\min}$ and $x_{\min}/h_{\min} = 0.701$.

5.7.2 Shallow Water

In shallow water, severe constraints are placed on the allowed range of values for the altitude of operation h of a SLS. In fact, the value for h may be given (specified). In this subsection, values for the following parameters are given: the operating (carrier) frequency f, the height (altitude) of operation h, the width of the one-sided blind zone $x_{\min}$ (a.k.a. the beginning or near-edge of the SW), and the along-track (azimuthal) resolution $\Delta z_{\min}$ at cross-range $x_{\min}$. With the parameter values that are given, we shall determine the dimensions L_Y and L_Z of a SLS in order to guarantee that the ocean bottom will be in the far-field region of the SLS. Table 5.7-3 summarizes the design procedure for this shallow-water problem.

Since h and $x_{\min}$ are given, the slant-range $r_{\min}$ to the beginning (near-edge) of the SW can be computed from [see (5.1-25) and Fig. 5.1-2]

$$r_{\min} = \sqrt{h^2 + x_{\min}^2} \,. \tag{5.7-20}$$

And since $x_{\min}$ and $\Delta z_{\min}$ are given, using (5.3-14) to solve for $\Delta\theta$ yields

Table 5.7-3 Design Procedure for Determining the Dimensions of a SLS in Order to Satisfy the Far-Field Range Criterion in Shallow Water

Given	Resulting Fixed Parameter Values
h, $x_{\min}$, and $\Delta z_{\min}$	$r_{\min}$ and $\Delta\theta$
f and $\Delta\theta$	L_Z
f, L_Z, and $r_{\min}$	L_Y so that $r_{\min} > r_{\mathrm{NF/FF}}$
f and L_Y	$\Delta\psi$ ($\psi' = 0°$)

$$\boxed{\Delta\theta = 2\tan^{-1}\left(\frac{\Delta z_{\min}}{2x_{\min}}\right)} \tag{5.7-21}$$

where $\Delta\theta$ is the 3-dB beamwidth of the horizontal, far-field beam pattern in the XZ plane (see Fig. 5.7-4). Now that $\Delta\theta$ is known and f is given, we can solve for L_Z by substituting (5.6-6) into (5.3-9). Doing so yields

$$\boxed{L_Z \approx 0.443\frac{\lambda}{\sin(\Delta\theta/2)}} \tag{5.7-22}$$

where L_Z is the length of the planar aperture (SLS) in the Z direction (see Fig. 5.1-1) and $c = f\lambda$ (see Fig. 5.7-5). Since f, L_Z, and $r_{\min}$ are known, we shall solve for L_Y next in order to guarantee that

$$r_{\min} > r_{\mathrm{NF/FF}}\,, \tag{5.7-23}$$

where $r_{\mathrm{NF/FF}}$ is the range to the near-field/far-field boundary given by (5.7-1). Substituting (5.7-1) into (5.7-23) yields

$$r_{\min} > \pi\frac{L_Y^2 + L_Z^2}{4\lambda}\,, \tag{5.7-24}$$

and by solving for L_Y, we obtain

$$\boxed{L_Y < \max L_Y} \tag{5.7-25}$$

where

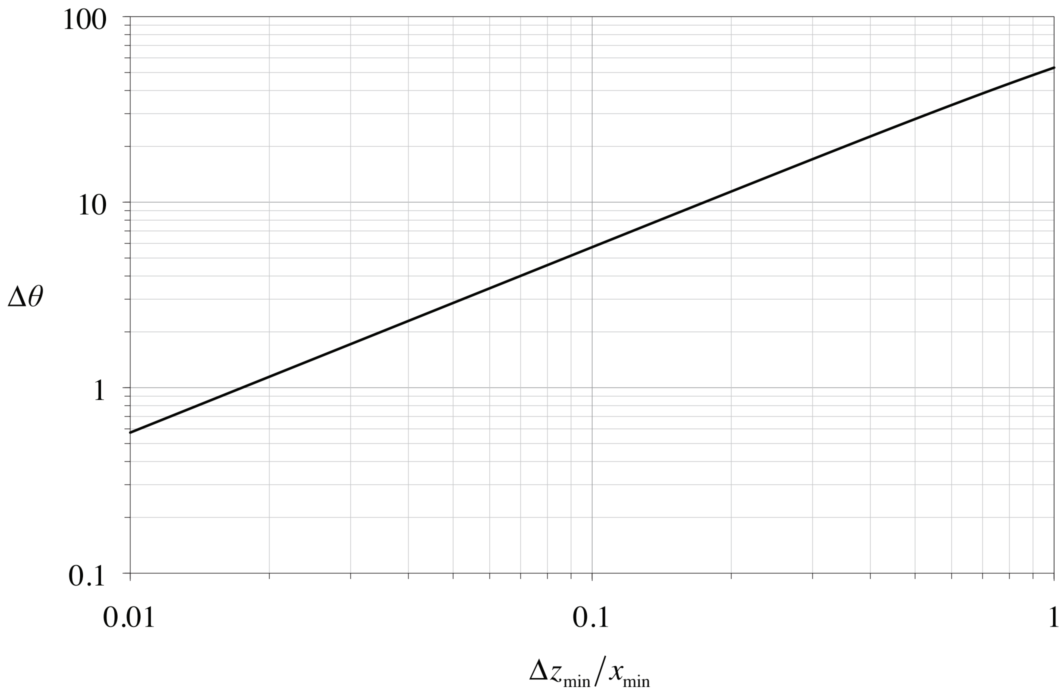


Figure 5.7-4 Three-dB beamwidth $\Delta\theta$ in degrees of the horizontal, far-field beam pattern versus $\Delta z_{\min}/x_{\min}$.

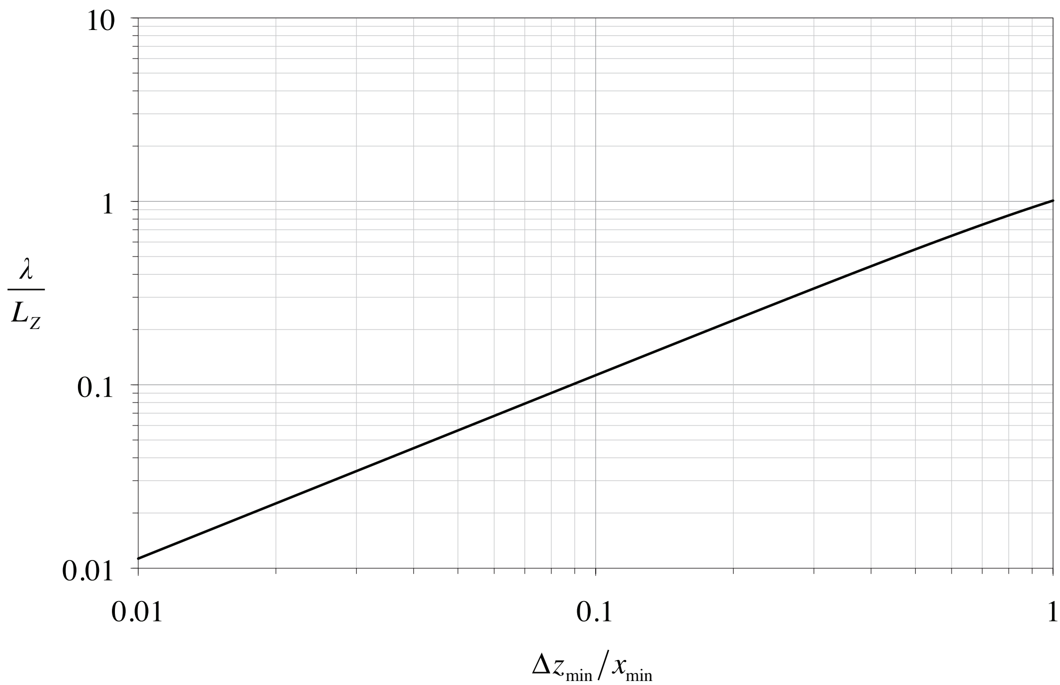
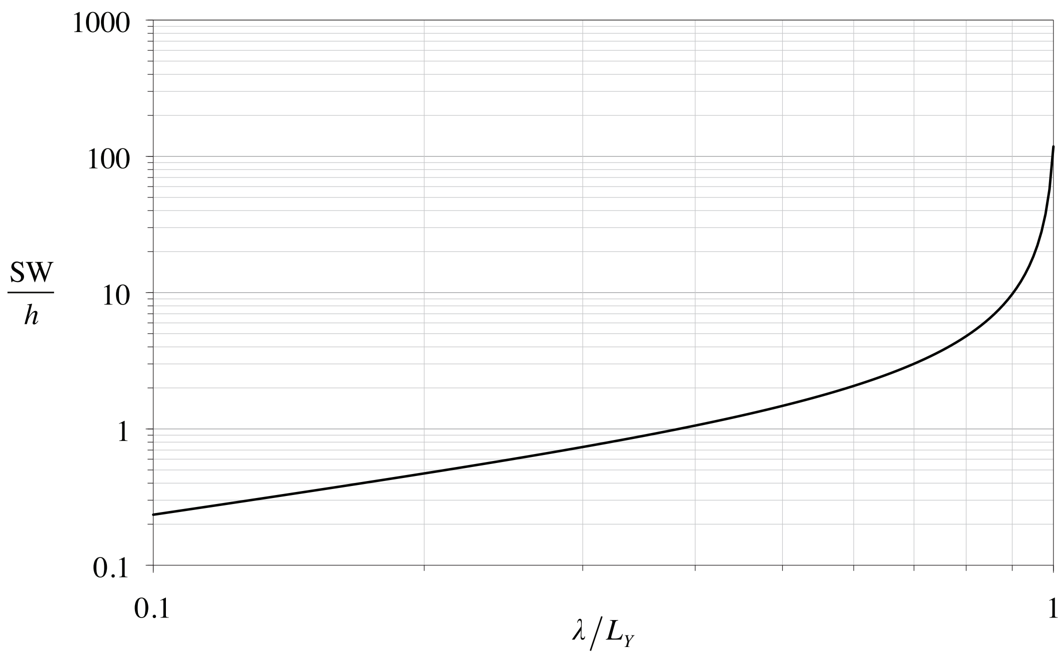


Figure 5.7-5 Ratio λ/L_Z versus $\Delta z_{\min}/x_{\min}$.

$$\boxed{\max L_Y = \sqrt{\frac{4\lambda}{\pi} r_{\min} - L_Z^2}\,, \qquad \frac{4\lambda}{\pi} r_{\min} > L_Z^2} \tag{5.7-26}$$

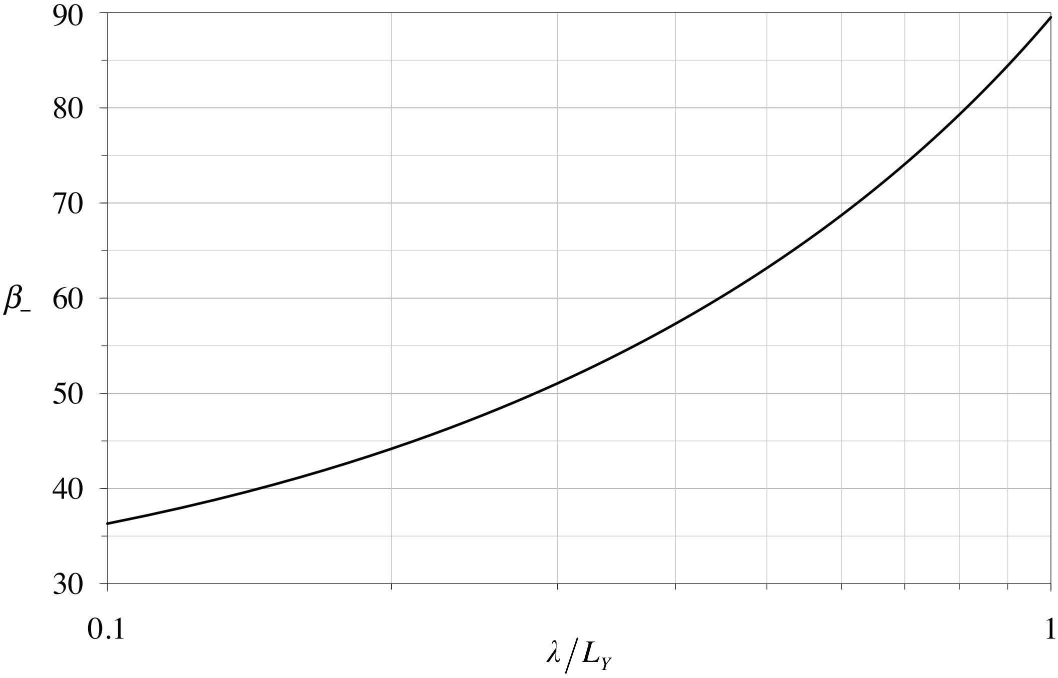
is the maximum allowed value for the length L_Y of the planar aperture (SLS) in the Y direction (see Fig. 5.1-1) in order to satisfy (5.7-23). For this shallow-water problem, the SW can be increased for a fixed value of the ratio $x_{\min}/h$ by decreasing L_Y since f is fixed (see Fig. 5.7-6). The SW can also be increased by increasing $x_{\min}/h$. However, if $x_{\min}/h$ is too big, then it is possible for $\beta_- \geq 90°$ ($\psi_- \leq 0°$) for a particular value of λ/L_Y, which is not allowed (see Fig. 5.7-7). In this case, the ratio $x_{\min}/h$ must be decreased.

Now that L_Y is known and f is given, we can solve for the 3-dB beamwidth $\Delta\psi$ of the vertical, far-field beam pattern in the XY plane when *no* beam steering is done ($\psi' = 0°$) by using (5.7-5). After the dimensions of the SLS have been determined, the design and analysis procedure continues with (5.7-11), as discussed in Subsection 5.7.1.



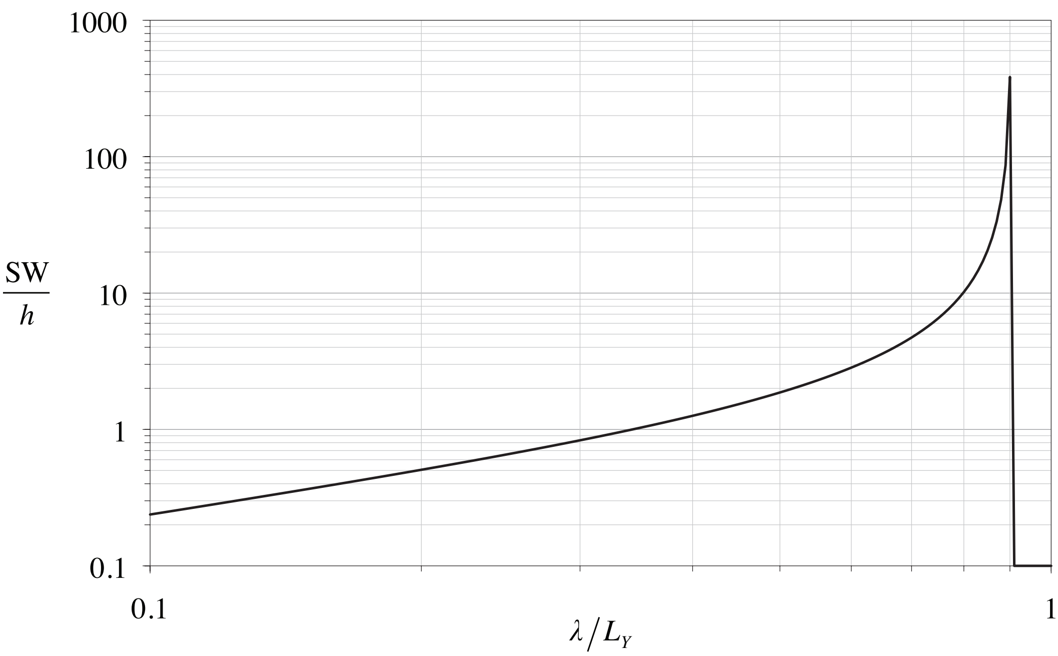
(a)

Figure 5.7-6 (a) Ratio SW/h versus λ/L_Y and (b) β_- in degrees versus λ/L_Y for $x_{\min}/h = 0.5$ where $L_Y < \max L_Y$.



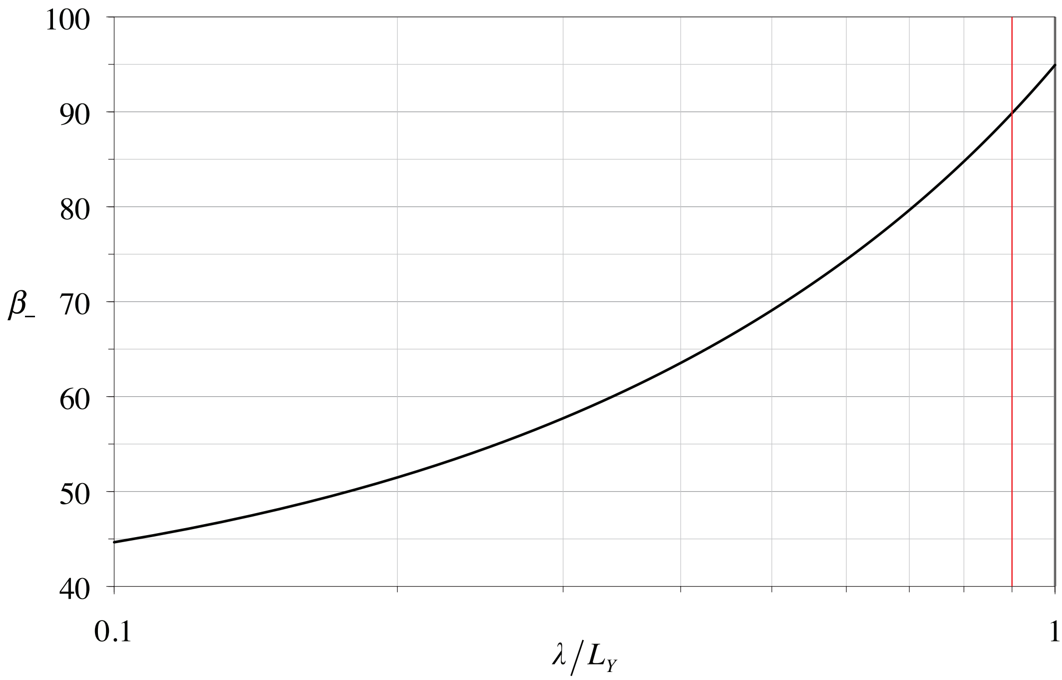
(b)

Figure 5.7-6 *continued*.



(a)

Figure 5.7-7 (a) Ratio SW/h versus λ/L_Y and (b) β_- in degrees versus λ/L_Y for $x_{min}/h = 0.75$ where $L_Y < \max L_Y$.



(b)

Figure 5.7-7 *continued*.

Problems

Section 5.2

5-1 A SLS transmits a rectangular-envelope, CW pulse, with a pulse length of $40\ \mu\text{sec}$. Find the cross-track (slant-range) resolution and the minimum allowed value for the operating (carrier) frequency. Use $c = 1500\ \text{m/sec}$.

Section 5.3

5-2 A SLS is modeled as a rectangular piston lying in the YZ plane. If the far-edge of the swath width is at a cross-range of $200\ \text{m}$ and the 3-dB beamwidth of the horizontal, far-field beam pattern in the XZ plane is $1°$, then what is the along-track (azimuthal) resolution in meters at the far-edge of the swath width?

Section 5.4

5-3 Show that $\text{PRI}_{\text{min, op}} \geq \text{PRI}_{\text{min}}$.