

Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory

Harry L. Van Trees

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To Diane

For her continuing support and
encouragement during the many years
that this book was discussed, researched,
and finally written. More importantly,
for her loyalty, love, and
understanding during a sequence of
challenging periods,

and to

Professor Wilbur Davenport, whose
book introduced me to random
processes and who was a mentor, friend,
and supporter during my career at
Massachusetts Institute of
Technology.

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Preface

Array processing has played an important role in many diverse application areas. Most modern radar and sonar systems rely on antenna arrays or hydrophone arrays as an essential component of the system. Many communication systems utilize phased arrays or multiple beam antennas to achieve their performance objectives. Seismic arrays are widely used for oil exploration and detection of underground nuclear tests. Various medical diagnosis and treatment techniques exploit arrays. Radio astronomy utilizes very large antenna arrays to achieve resolution goals. It appears that the third generation of wireless systems will utilize adaptive array processing to achieve the desired system capacity. We discuss various applications in Chapter 1.

My interest in optimum array processing started in 1963 when I was an Assistant Professor at M.I.T. and consulting with Arthur D. Little on a sonar project for the U.S. Navy. I derived the optimum processor for detecting Gaussian plane-wave signals in Gaussian noise [VT66a], [VT66b]. It turned out that Bryn [Bry62] had published this result previously (see also Vanderkulk [Van63]). My work in array processing decreased as I spent more time in the general area of detection, estimation, and modulation theory.

In 1968, Part I of *Detection, Estimation, and Modulation Theory* [VT68] was published. It turned out to be a reasonably successful book that has been widely used by several generations of engineers. Parts II and III ([VT71a], [VT71b]) were published in 1971 and focused on specific application areas such as analog modulation, Gaussian signals and noise, and the radar-sonar problem. Part II had a short life span due to the shift from analog modulation to digital modulation. Part III is still widely used as a reference and as a supplementary text. In a moment of youthful optimism, I indicated in the Preface to Part III and in Chapter III-14 that a short monograph on optimum array processing would be published in 1971. The bibliography lists it as a reference, (*Optimum Array Processing*, Wiley, 1971), which has been subsequently cited by several authors. Unpublished class notes [VT69] contained much of the planned material. In a very loose sense, this text is

the extrapolation of that monograph.

Throughout the text, there are references to Parts I and III of *Detection, Estimation, and Modulation Theory*. The referenced material is available in several other books, but I am most familiar with my own work. Wiley has republished Parts I and III [VT01a], [VT01b] in paperback in conjunction with the publication of this book so the material will be readily available.

A few comments on my career may help explain the thirty-year delay. In 1972, M.I.T. loaned me to the Defense Communications Agency in Washington, D.C., where I spent three years as the Chief Scientist and the Associate Director for Technology. At the end of this tour, I decided for personal reasons to stay in the Washington, D.C., area. I spent three years as an Assistant Vice-President at COMSAT where my group did the advanced planning for the INTELSAT satellites. In 1978, I became the Chief Scientist of the United States Air Force. In 1979, Dr. Gerald Dinneen, the former director of Lincoln Laboratories, was serving as Assistant Secretary of Defense for C³I. He asked me to become his Principal Deputy and I spent two years in that position. In 1981, I joined M/A-COM Linkabit. Linkabit is the company that Irwin Jacobs and Andrew Viterbi started in 1969 and sold to M/A-COM in 1979. I started an Eastern operations, which grew to about 200 people in three years. After Irwin and Andy left M/A-COM and started Qualcomm, I was responsible for the government operations in San Diego as well as Washington, D.C. In 1988, M/A-COM sold the division. At that point I decided to return to the academic world.

I joined George Mason University in September of 1988. One of my priorities was to finish the book on optimum array processing. However, I found that I needed to build up a research center in order to attract young research-oriented faculty and doctoral students. This process took about six years. The C³I Center of Excellence in Command, Control, Communications, and Intelligence has been very successful and has generated over \$30 million in research funding during its existence. During this growth period, I spent some time on array processing, but a concentrated effort was not possible.

The basic problem in writing a text on optimum array processing is that, in the past three decades, enormous progress had been made in the array processing area by a number of outstanding researchers. In addition, increased computational power had resulted in many practical applications of optimum algorithms. Professor Arthur Baggeroer of M.I.T. is one of the leading contributors to array processing in the sonar area. I convinced Arthur, who had done his doctoral thesis with me in 1969, to co-author the optimum array processing book with me. We jointly developed a comprehensive out-

line. After several years it became apparent that the geographical distance and Arthur's significant other commitments would make a joint authorship difficult and we agreed that I would proceed by myself. Although the final outline has about a 0.25 correlation with the original outline, Arthur's collaboration in structuring the original outline and commenting on the results have played an important role in the process.

In 1995, I took a sabbatical leave and spent the year writing the first draft. I taught a one-year graduate course using the first draft in the 1996–1997 academic year. A second draft was used in the 1997–1998 academic year. A third draft was used by Professor Kristine Bell in the 1998–1999 academic year. Unlike the M.I.T. environment where I typically had 40–50 graduate students in my detection and estimation classes, our typical enrollment has been 8–10 students per class. However, many of these students were actively working in the array processing area and have offered constructive suggestions.

The book is designed to provide a comprehensive introduction to optimum array processing for students and practicing engineers. It will prepare the students to do research in the array processing area or to implement actual array processing systems. The book should also be useful to people doing current research in the field. We assume a background in probability theory and random processes. We assume that the reader is familiar with Part I of *Detection, Estimation, and Modulation Theory* [VT68], [VT01a] and parts of Part III [VT71b], [VT01b]. The first use of [VT68], [VT01a] is in Chapter 5, so that a detection theory course could be taken at the same time. We also assume some background in matrix theory and linear algebra. The book emphasizes the ability to work problems, and competency in MATLAB[®] is essential.

The final product has grown from a short monograph to a lengthy text. Our experience is that, if the students have the correct background and motivation, we can cover the book in two fifteen-week semesters.

In order to make the book more useful, Professor Kristine Bell has developed a Web site:

<http://ite.gmu.edu/DetectionandEstimationTheory/>

that contains material related to all four parts of the *Detection, Estimation, and Modulation Theory* series.

The *Optimum Array Processing* portion of the site contains:

- (i) MATLAB[®] scripts for most of the figures in the book. These scripts enable the reader to explore different signal and interference environments and are helpful in solving the problems. The disadvantage is

that a student can use them without trying to solve the problem independently. We hope that serious students will resist this temptation.

- (ii) Several demos that allow the reader to see the effect of parameter changes on beam patterns and other algorithm outputs. Some of the demos for later chapters allow the reader to view the adaptive behavior of the system dynamically. The development of demos is an ongoing process.
- (iii) An erratum and supplementary comments regarding the text will be updated periodically on the Web site. Errors and comments can be sent to either hlv@gmu.edu or kbell@gmu.edu.
- (iv) Solutions, including MATLAB[®] scripts where appropriate, to many of the problems and some of the exams we have used. This part is password protected and is only available to instructors. To obtain a password, send an e-mail request to either hlv@gmu.edu or kbell@gmu.edu.

In order to teach the course, we created a separate LATEX file containing only the equations. By using Ghostview, viewgraphs containing the equations can be generated. A CD-rom with the file is available to instructors who have adopted the text for a course by sending me an e-mail at hlv@gmu.edu.

The book has relied heavily on the results of a number of researchers. We have tried to acknowledge their contributions. The end-of-chapter bibliographies contain over 2,000 references. Certainly the book would not have been possible without this sequence of excellent research results.

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