# Spin glasses

Santiago Sanz Wuhl

February 18, 2025

# 1 Introduction

A (classical) spin system with n particles is governed by the Hamiltonian

$$H = \sum_{i=1}^{N} \sum_{1 \leq j < i} A_{ij} s_i s_j, \tag{1}$$

where $A_{ij}$ are the matrix elements of a 2-dimensional symmetric matrix, measuring the strength of the interactions between the particle at the site $i$ and the particle at the site $j$, and $s_i$ is the spin of the particle at the site $i$. When only considering long-range interactions,

$$A_{ij} = r_{ij}^{-\sigma}. \tag{2}$$

This interaction is however very computationaly expensive. These systems are usually approximated (for example in the Ising model) via the nearest-neighbors approximation[1], i.e.

$$A_{ij} = \begin{cases} 1 & j = i \pm 1 \\ 0 & \text{else} \end{cases}. \tag{3}$$

To avoid the computational expense of calculating the $\frac{N(N-1)}{2}$ interactions per iteration we state the following approximation: the interaction strength of any two particles is always 1, independently of the distance separating them, but not every particle interacts with every particle. We distribute a fixed $N_l$ amount of interactions among the particles, and the probability that two particles are interacting is governed by $r_{ij}^{-\sigma}$.

# 2 Methods

## 2.1 Random number generation

The core of these calculations is the Walker's Alias random number generating algorithm. This algorithm rolls an unfair $m$ sided die with complexity $\mathcal{O}(m)$.

[1]This expression is valid only for 1-dimensional setups. The expression for higher dimensions is similar.

## 2.2 Two different algorithms

We discussed two different algorithms to generate the matrix $A_{ij}$ (which is not anymore a matrix, it is only the set of 1 valued elements)

**Set algorithm** This algorithm draws random bonds between any two particles and when the number of bonds reaches $N_l$, it stops. Each draw of a bond rolls two dice: The first die draws a uniformly distributed integer $n$ from 1 to N and chooses the starting particle. The second die draws a random integer from 1 to $\lfloor \frac{N}{2} \rfloor$ where the probability of drawing a number number $m \in [1, \lfloor \frac{N}{2} \rfloor]$ is

$$P_m = m^{-\sigma}. \tag{4}$$

The drawn bond is then $\{n, \mathrm{mod}\,(n+m, N)\}$.

Precisely, it goes as follows.

```
uniformAliasTable = AliasTable(ones(N)) # To chose the first particle
```