The chromaticity coordinates in the XYZ color space are defined as-

$$x = \frac{X}{X+Y+Z}, \quad = \frac{Y}{X+Y+Z}$$

While $z = 1 - x - y$.

When we combine the two lights, C1 and C2, we simply add their XYZ components.

And so-

$$C = C_1 + C_2 = (X_1 + X_2, Y_1 + Y_2, Z_1 + Z_2)$$

We will mark the coordinates of C_1 as $c_1 = (x_1, y_1)$ and of C_2 as $c_2 = (x_2, y_2)$.

Now we shall find the coordinates of $c = (x_c, y_c)$ on the chromaticity diagram.

These coordinates can be calculated as follows:

$$x_c = \frac{X_1 + X_2}{\sum_{i=1}^{2} X_i + Y_i + Z_i} = \frac{X_1 + X_2}{E_1 + E_2}$$

$$y_c = \frac{Y_1 + Y_2}{\sum_{i=1}^{2} X_i + Y_i + Z_i} = \frac{Y_1 + Y_2}{E_1 + E_2}$$

Where $E_i = X_i + Y_i + Z_i$.

Let's begin by finding an expression for the first coordinate component, x_c-

$$x_c = \frac{X_1 + X_2}{E_1 + E_2} = \frac{x_1 E_1 + x_2 E_2}{E_1 + E_2} = \frac{E_1}{E_1 + E_2} \times x_1 + \left(1 - \frac{E_1}{E_1 + E_2}\right) \times x_2$$

Similarly, for y_c-

$$y_c = \frac{Y_1 + Y_2}{E_1 + E_2} = \frac{y_1 E_1 + y_2 E_2}{E_1 + E_2} = \frac{E_1}{E_1 + E_2} \times y_1 + \left(1 - \frac{E_1}{E_1 + E_2}\right) \times y_2$$

And so, if we choose $\lambda = \frac{E_1}{E_1 + E_2}$ we will find that-

$$x_c = \lambda \times x_1 + (1 - \lambda) \times x_2, \quad y_c = \lambda \times y_1 + (1 - \lambda) \times y_2$$

Which implies that-

$$c = (x_c, y_c) = \lambda \times c_1 + (1 - \lambda) \times c_2$$

As required.