2.1

We will mark the Fourier transform of as .

From the convolution theorem we get that

So, our process will be like so-

1. Calculate
2. Calculate
3. Using the inverse Fourier transform, calculate

Step 1

We’ll define the box function .

So

is even, meaning . So, we know that

From the duality property of Fourier transformations, we know that

Combining (1) and (2), we get that .

Step 2

We know that the Fourier transformation of a convolution is the product of the Fourier transformations.

So,

Step 3

Since we know that , by applying , we get that .

And so, we got that

So .

2.2

Zooming means inserting new pixels to the image with some method to define the values of the new pixels.

For example, zooming a image to a image involves adding 3 new pixels for every existing pixel.

We do it in four steps:

1. Take the original image and apply a Fourier Transform to it.
2. Zero-pad the Fourier Transformation with the number of pixels we want to add.
3. Apply the Inverse-Fourier Transformation.
4. Normalize the pixel values again in the spatial domain.

The resolution of the image increased when we zero-padded the Fourier Transformation, increasing the sampling rate.

This way, we didn’t introduce new higher frequencies, and so maintained the frequencies within the Nyquist limit.

We did however add ‘artificial’ pixels that are an interpolation of the initial data, thanks to the zero-padding and the effects it had on the Inverse-Fourier Transformation.

It is worth noting that normalizing didn’t introduce new, potentially higher frequencies because it is a linear transformation.

2.3

Need to prove-

From the definition of impulse, we get that

We also know that

And from the duality property we get that

Meaning that .

From the definition of Fourier transformations, we know that

And so,

Substituting this result back, we get that

As required.