We will begin by calculating .

We will now calculate the covariance of y.

If the rows of A are chosen to be the eigenvectors of , that means that A diagonalizes , meaning that for some diagonal matrix D.

We’ve shown that is a diagonal matrix, meaning that y is uncorrelated.

The Gaussian filter is a linear filter, applying a weighted average to the points of an image(or a signal).

The Gaussian filter is a low-pass filter, smoothing the image(or signal) by averaging neighboring points based on the filter’s weights.

Each application of the filter reduces high frequency components and smooths out sharp transitions. After many applications, the result tends towards a uniform distribution, as any higher-frequency component is later reduced by another application of the filter.

Therefore, applying the gaussian filter infinitely many times results in a uniform image, where every point is equal to the mean of the original image. As the high-frequency details are eliminated, only the mean value remains.

And so we’ve shown that applying the gaussian filter infinitely many times results in a uniform image with its coloring being the average of the original image.