

# Analysis of High-Dimensional Data

Dolev Dublon  
Moria Grohar  
Shirel Zecharia  
Roy Harel  
Hadas Evers

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## Abstract

This Survey explores the fascinating world of high-dimensional data analysis, focusing on the properties of random matrices. These matrices play a crucial role in various fields such as computer science, statistics, and machine learning, especially concerning their singularity and the analysis of their smallest singular values.

## 1 Introduction

In the realm of high-dimensional data analysis, random matrices have captivated the attention of mathematicians and scientists due to their ubiquitous presence in various domains, including computer science, statistics, and machine learning. Understanding their behavior is paramount, particularly regarding their singularity (non-invertibility) and the analysis of their smallest singular values. This article delves into ten research papers that shed light on these aspects.

## 2 Mathematical Context - Main

The article "Random Symmetric Matrices Are Almost Surely Non-Singular" by K. Costello, T. Tao, and V. Vu presents a significant result in the field of random matrices. The main finding of the article is the proof that a random symmetric matrix  $Q_n$  with independent and identically distributed (with the same distribution) Bernoulli variables as its upper diagonal entries is almost surely non-singular, with a probability of  $1 - O(n^{-1/8+\delta})$  for any  $\delta > 0$ . This result extends previous results for random matrices to more general models of random matrices. The article presents the history of the non-singularity problem in random matrices, namely whether it is true that a random matrix  $A_n$  with independent Bernoulli variables is almost surely non-singular. This question was positively answered by Komlós in 1967, and later he generalized the result to more general models of random matrices. In a recent paper, Tao and Vu found a different proof for random matrices that provides a precise estimate for the absolute value of the determinant of the matrix  $A_n$ . Building upon these previous proofs, the authors develop a quadratic version of Littlewood-Offord type results concerning the concentration of random variables to prove

the non-singularity of  $Q_n$ - a random symmetric matrix. This method allows researchers to overcome the challenge of the row and column transpose, which was a hurdle in previous proofs for random matrices due to the dependence between the row vectors of the matrix  $Q_n$ . The article raises open questions for future research in the field of random matrices:

**Determinant Estimation:** The article raises the question of estimating the determinant of random matrices. The estimation provided in the article is:  $|\det Q_n| = n^{(1/2-o(1))n}$ .

**Singularity Probability:** Another open question raised in the article relates to estimating the probability that a random matrix is singular. The authors estimate that the probability of  $Q_n$  being a singular matrix is  $(1/2 + o(1))^n$ .

**The quadratic variant of the Littlewood-Offord :** Let  $Q$  be a quadratic random variable defined as:  $Q = \sum_{1 \leq i, j \leq n} c_{ij} z_i z_j$  where  $z_i$  are random variables,  $1, \dots, n = U_1 \cup U_2$  is a non-trivial partition, and  $S$  is a non-empty subset of  $U_1$ . For each  $i \in S$ , let  $d_i$  be the number of indices  $j \in U_2$  such that  $|c_{ij}| \geq 1$ . If  $d_i \geq 1$  for each  $i \in S$ , and  $I$  is an interval of length 1, then:  $P(Q \in I) = O(|S|^{-1/2} + |S|^{-1} \sum_{i \in S} d_i^{-1/2})^{1/4}$

### 3 roy article1 summary

### 4 roy article2 summary

### 5 shirel Singularity of random symmetric matrices revisited summery

### 6 moria Singularity of random symmetric matrices revisited summery

### 7 moria Singularity of discrete random matrices summery

### 8 shirel On the smoothed analysis of of the smallest singular value with discrete noise

### 9 dolev article1 summery of graph

### 10 shirel on graph article

### 11 Haddas last article

### 12 Results

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