Enhancing Quantum Diffusion Model for Complex Image Generation

Qiskit Advocate Mentorship Program: 33

Jeongbin Jo[1], Santanam Wishal, Shah Md Khalil Ullah, Shan Zeng, Dikshant Dulal (**mentor**)

[1] *Dept. of Physics, Yonsei University*, `jeongbin033@yonsei.ac.kr`

Introduction

Qiskit Advocate Mentorship Program (QAMP) is a program focused on bring new contributors into Qiskit open source software development where Qiskit advocates work on a 3-month projects under the guidance of mentors. It is an initiative within the Qiskit advocate program designed to support growth and collaboration within our vibrant community.
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Quantum Data Encoding

Basis Encoding :

$$|x\rangle = |b_1, b_2, \ldots, b_P\rangle, \quad b_i \in \{0, 1\}$$

Amplitude Encoding : $n = \lceil \log_2 N \rceil$.

$$|\psi_x\rangle = \frac{1}{\alpha} \sum_{i=1}^{N} x_i |i\rangle$$

$$\alpha = \sqrt{\sum_{i=1}^{N} |x_i|^2}$$

Angle Encoding :

$$|\vec{x}\rangle = \bigotimes_{k=1}^{N} R_Y(x_k) |0\rangle = \bigotimes_{k=1}^{N} \left(\cos\left(\frac{x_k}{2}\right) |0\rangle + \sin\left(\frac{x_k}{2}\right) |1\rangle \right)$$

Phase Encoding :

$$|\vec{x}\rangle = \bigotimes_{k=1}^{N} P(x_k) |+\rangle = \frac{1}{\sqrt{2^N}} \bigotimes_{k=1}^{N} \left(|0\rangle + e^{ix_k} |1\rangle \right)$$

Dense Angle Encoding :

$$|x_k, x_l\rangle = R_Z(x_l) R_Y(x_k) |0\rangle = \cos\left(\frac{x_k}{2}\right) |0\rangle + e^{ix_l} \sin\left(\frac{x_k}{2}\right) |1\rangle$$

$$|\vec{x}\rangle = \bigotimes_{k=1}^{N/2} \left(\cos x_{2k-1} |0\rangle + e^{ix_{2k}} \sin x_{2k-1} |1\rangle \right)$$

Parameterized Quantum Circuit

Optimal two-qubit decomposition **Vatan-Williams** decomposition [5]. The circuit structure, depicted in Fig. 1, consists of three CNOT gates interleaved with parameterized single-qubit rotations. This configuration is capable of simulating the unitary operator $U = \exp(-i(\alpha \sigma_x \otimes \sigma_x + \beta \sigma_y \otimes \sigma_y + \gamma \sigma_z \otimes \sigma_z))$ up to local basis transformations.

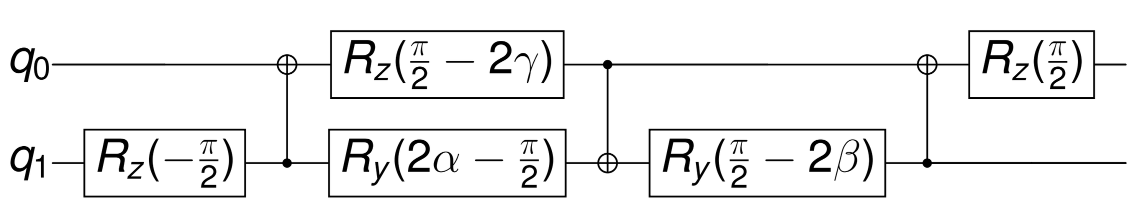

Fig. 1: This structure implements the optimal decomposition for arbitrary two-qubit entangling gates, characterized by the parameters α, β, γ. Note the alternating CNOT direction and the specific rotation angles derived from the canonical decomposition.

Global Information Propagation To achieve this efficiently, we employ a *Phase Mixing* layer inspired by 1D Cluster States (or Graph States) [3].

$$U_{\text{Mix}}(\phi) = \left(\bigotimes_{j=1}^{N} R_X(\phi_j) \right) \left(\prod_{j=1}^{N-1} CZ_{(j,j+1)} \right) H^{\otimes N} \tag{1}$$

Hadamard test

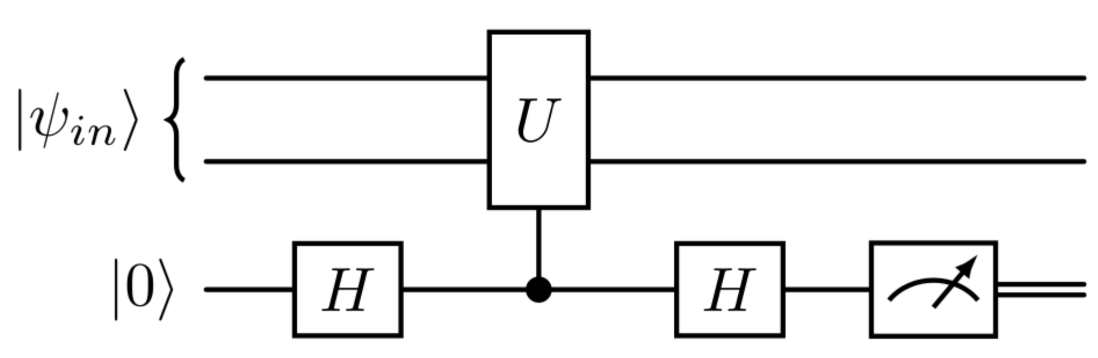

Fig. 2: Hadamard test for non linear processing

Diffusion Process

While classical diffusion models operate on probability distributions over classical data, Quantum Diffusion Models (QDMs) extend this concept to the Hilbert space of quantum states. The goal is to generate quantum states (density matrices) from a maximally mixed state.

Forward Process: Depolarizing Channel Instead of adding Gaussian noise, the forward process in QDM is typically modeled as a standard depolarizing channel acting on a density matrix ρ. For a system of n qubits with dimension $d = 2^n$, the state at step t is given by:

$$\rho_t = \mathcal{E}_t(\rho_{t-1}) = (1 - p_t)\rho_{t-1} + p_t \frac{I}{d} \tag{2}$$

where $p_t \in [0, 1]$ is the depolarization probability (noise schedule). Similar to the classical case, we can express ρ_t directly from the initial state ρ_0. Let $\alpha_t = \prod_{s=1}^{t} (1 - p_s)$, then:

$$\rho_t = \alpha_t \rho_0 + (1 - \alpha_t) \frac{I}{d} \tag{3}$$

As $t \to T$, $\alpha_T \to 0$, and the state converges to the maximally mixed state $\rho_T \approx \frac{I}{d}$, which contains no information about ρ_0.

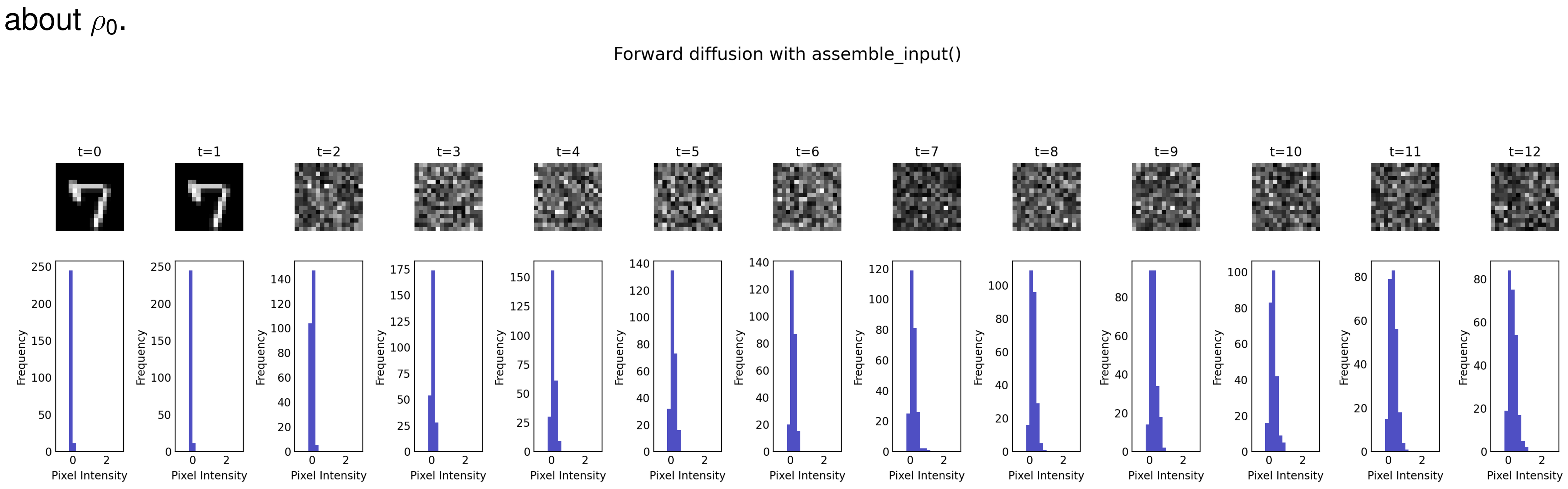

Fig. 3: Forward process

Denoising Process

Reverse Process: Quantum Denoising The reverse process aims to restore the quantum state from the noise. This is modeled by a parameterized quantum circuit (PQC), denoted as a unitary operator $U(\theta)$. The discrete reverse step can be approximated as:

$$\rho_{t-1} \approx \mathcal{D}_\theta(\rho_t, t) = U(\theta_t) \rho_t U^\dagger(\theta_t) \tag{4}$$

For more complex generative tasks, the reverse process may involve ancillary qubits and measurements to simulate non-unitary maps.

Loss Function: Fidelity or Trace Distance Quantum models often use Fidelity or Hilbert-Schmidt distance to measure the closeness between the generated state and the target state. A common objective is to maximize the overlap with the target pure state $|\psi\rangle$:

$$\mathcal{L}(\theta) = 1 - \mathbb{E}\left[\langle\psi| \rho_{\text{gen}}(\theta) |\psi\rangle \right] \tag{5}$$

Alternatively, for density matrices, we minimize the trace distance or maximize the quantum fidelity $F(\rho, \sigma) = (\text{tr}\sqrt{\sqrt{\rho}\sigma\sqrt{\rho}})^2$.

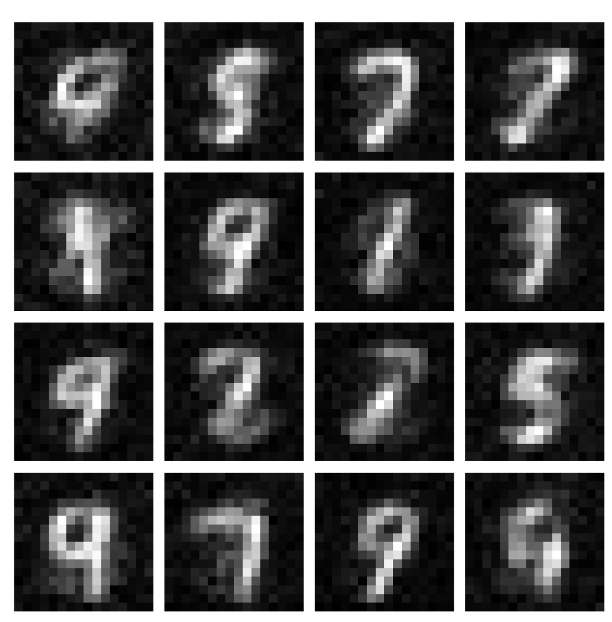

Fig. 4: Sample of generation process in random time t.

Architecture

We adopted bottleneck architecture Fig. 5, that commonly used for convolutional neural network(CNN) in machine learning.

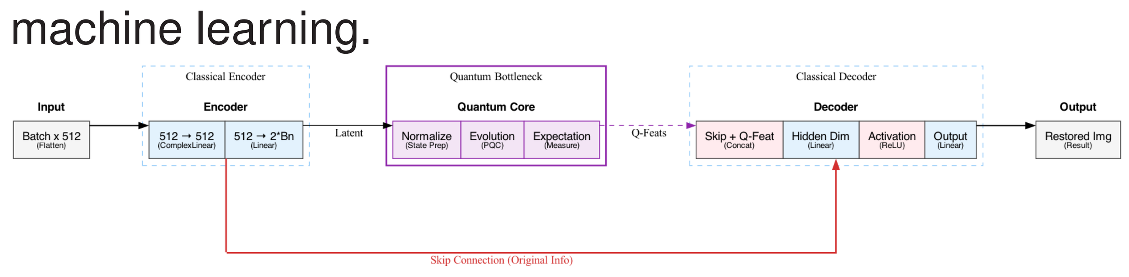

Fig. 5: Quantum Unet architecture

Measurement

- **ANO Measurement**: *Adaptive Non-Local Observable (ANO)* framework [1]. Instead of fixed operators, we employ a set of trainable Hermitian observables $\{H_k\}_{k=1}^{K}$ that act on the entire qubit register.
- **Expressibility based on Meyer-Wallach values** [2]
  For a given state $|\psi\rangle$, the MW measure is defined as the average purity of the single-qubit reduced density matrices:

$$Q(|\psi\rangle) = \frac{4}{n} \sum_{k=1}^{n} D(\iota_k(|\psi\rangle)) = \frac{4}{n} \sum_{k=1}^{n} \frac{1}{2} \left(1 - \text{Tr}(\rho_k^2)\right) \tag{6}$$

where $\rho_k = \text{Tr}_{\neg k}(|\psi\rangle\langle\psi|)$ is the reduced density matrix of the k-th qubit. The value Q ranges from 0 (product states, unentangled) to 1 (maximally entangled, e.g., GHZ states).
We estimate the *Entangling Capability* [4] of our ansatz by averaging Q over an ensemble of states sampled with uniformly random parameters:

$$\bar{Q} = \frac{1}{S} \sum_{i=1}^{S} Q(|\psi(\theta_i)\rangle) \tag{7}$$

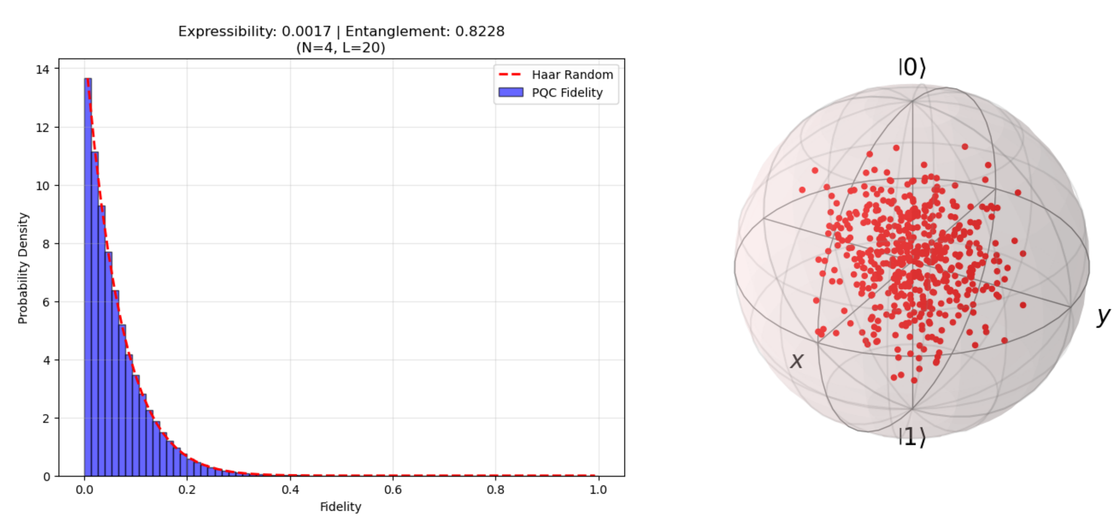

Fig. 6: Visualization of the single-qubit reduced state distribution on the Bloch sphere generated by our ansatz with ANO.

References

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