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Assignment-3

5) a) update Rule ~~for~~ rosenblatt

$$w_i' = w_i + \alpha\,(y - y_i')\,x_i \quad \text{---} ①$$

update rule for sigmoid

$$\text{sigmoid fn} \rightarrow \frac{1}{1+e^{-x}}$$

cross entropy loss

$$L = y\log(y) + (1-y)\log(1-y)$$

$$\therefore \frac{\partial loss}{\partial w_i} = \left[y\log\frac{1}{1+e^{-x}} + (1-y)\log\left(1-\frac{1}{1+e^{-x}}\right)\right]$$

$$\cancel{\Rightarrow y\frac{\partial[\log e^{x} - \log(1+e^{x})]}{\partial w_i} + (y-1)\frac{\partial[\log(1+e^{x})]}{\partial w_i} - \frac{\partial[\log(1+e^{x})]}{\partial w_i}}$$

$$\Rightarrow y\frac{\partial[\log e^{x} - \log(1+e^{x})]}{\partial w_i} + \frac{(y-1)\partial[\log(1+e^{x})]}{\partial w_i}$$

$$\Rightarrow \frac{y\,\partial x}{\partial w_i} - \frac{y\,\partial(\log(1+e^{x}))}{\partial w_i} + \frac{y\,\partial[\log(1+e^{x})]}{\partial w_i} - \frac{\partial[\log(1+e^{x})]}{\partial w_i}$$

$$\Rightarrow y x_i - \frac{e^{x}}{1+e^{x}}\,x_i$$

$$\Rightarrow \left[y - \frac{1}{(1+e^{-x})}\right]x_i$$

$$\text{update rule} \rightarrow w_i' = w_i + \alpha\left[y - \frac{1}{(1+e^{-x})}\right]x_i$$

$$\Rightarrow \boxed{w_i' = w_i + \alpha[y - y_i]\,x_i} \quad \text{---} ②$$

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Therefore, ① = ②
the update rule doesn't change.

b) given,

$$\phi(\beta, \beta_0) = -\sum_{i=1}^{n} y_i (\beta^T x_i + \beta_0) \quad \text{for } \beta^T\beta = 1$$

$$g(\beta) = \beta^T \beta - 1$$

$$\alpha(\beta, \beta_0) = \phi(\beta, \beta_0) + \lambda g(\beta)$$

$$= -\sum_{i=1}^{n} y_i (\beta^T x_i + \beta_0) + \lambda(\beta^T \beta + 1)$$

$$= \frac{-y_i x_i \beta_{12} \cdot e^{-v_i}}{(1+e^{v_i})^2}$$

$$\frac{\partial d_i}{\partial \beta_{01}} = -y_i \frac{(\beta_{12} \partial y_{i1})}{\partial \beta_{01}}$$

$$\frac{\partial y_{i1}}{\partial \beta_{01}} = \sigma'(\underbrace{\beta_1 x_i + \beta_{01}}_{v_i})$$

$$= \frac{e^{-v_i}}{(1+e^{-v_i})^2}$$

$$\frac{\partial d_i}{\partial \beta_{01}} = \frac{-y_i \beta_{12} e^{-v_i}}{(1+e^{-v_i})^2}$$

$$\frac{\partial d_i}{\partial \beta_{12}} = -y_i (y_{i1})$$

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$\Sigma - y_i \left(\sigma \left(\beta_{11} x + \beta_{01} \right) \right)$

$\Sigma - y_i$

$\therefore \beta_{01} \leftarrow \beta_{01} - \eta \dfrac{\partial d_i}{\partial \beta_{01}}$

$\beta_{12} \leftarrow \beta_{12} - \eta \dfrac{\partial d_i}{\partial \beta_{12}}$

$\beta_{02} \leftarrow \beta_{02} - \eta \dfrac{\partial d_i}{\partial \beta_{02}}$

$\beta_{11} \leftarrow \beta_{11} - \eta \dfrac{\partial d_i}{\beta_{11}}$

6) Given $x \longrightarrow \bigcirc \xrightarrow{\;y_1\;} \bigcirc \longrightarrow y_2$

perceptron

sigmoid fn.

$y_1 = \sigma(\beta_{11} x + \beta_{01})$

$y_2 = \text{sign}(\beta_{12} y_1 + \beta_{02})$

for misclassified samples

$err_i = -y_i(\beta_{12} y_{1i} + \beta_{02})$

$y_i \rightarrow$ expected output

$y_{1i} \rightarrow$ ~~output of~~

perceptron

activation fn = $\sigma'(x) = \sigma(x)[1 - \sigma(x)]$

(derivative of sigmoid)

$d_i = -y_i(\beta_{12} y_{1i} + \beta_{02})$

$$\frac{\partial d_i}{\partial \beta_{11}} = -y_i\left(\beta_{12} \frac{\partial y_{1i}}{\partial \beta_{11}} + 0\right)$$

$$= -y_i\left(\beta_{12}\, \sigma'(\beta_{11} x_i + \beta_{01}) \cdot x_i\right)$$

$$= -y_i\left(\beta_{12}\left[\sigma(\underbrace{\beta_{11} x_i + \beta_{01}}_{v_i})(1 - \sigma(\beta_{11} x_i + \beta_{01}))\right] x_i\right)$$

$$= \frac{-y_i x_i \beta_{12} \cdot e^{-v_i}}{(1 + e^{-v_i})^2}$$

$$\beta_{11} \leftarrow \beta_{11} - \eta \frac{\partial d_i}{\partial \beta_{11}} \qquad \frac{\partial d_i}{\partial \beta_{01}} = -y_i\left(\beta_{12} \frac{\partial y_{1i}}{\partial \beta_{01}}\right)$$

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Now,

$$\frac{\partial y_{1i}}{\partial \beta_{01}} = \sigma'(\underbrace{\beta_{11}x_i + \beta_{01}}_{v_i}) = \frac{e^{-v_i}}{(1+e^{-v_i})^2}$$

$$\frac{\partial d_i}{\partial \beta_{01}} = \frac{-y_i \beta_{12} e^{-v_i}}{(1+e^{-v_i})^2}$$

$$\frac{\partial d_i}{\partial \beta_{12}} = -y_i(y_{1i})$$

$$= -y_i[\sigma(\beta_{11}x + \beta_{01})]$$

$$\frac{\partial d_i}{\partial \beta_{02}} = -y_i$$

$$\beta_{01} \leftarrow \beta_{01} - \eta \frac{\partial d_i}{\partial \beta_{01}}$$

$$\beta_{12} \leftarrow \beta_{12} - \eta \frac{\partial d_i}{\partial \beta_{12}}$$

$$\beta_{02} \leftarrow \beta_{02} - \eta \frac{\partial d_i}{\partial \beta_{02}}$$