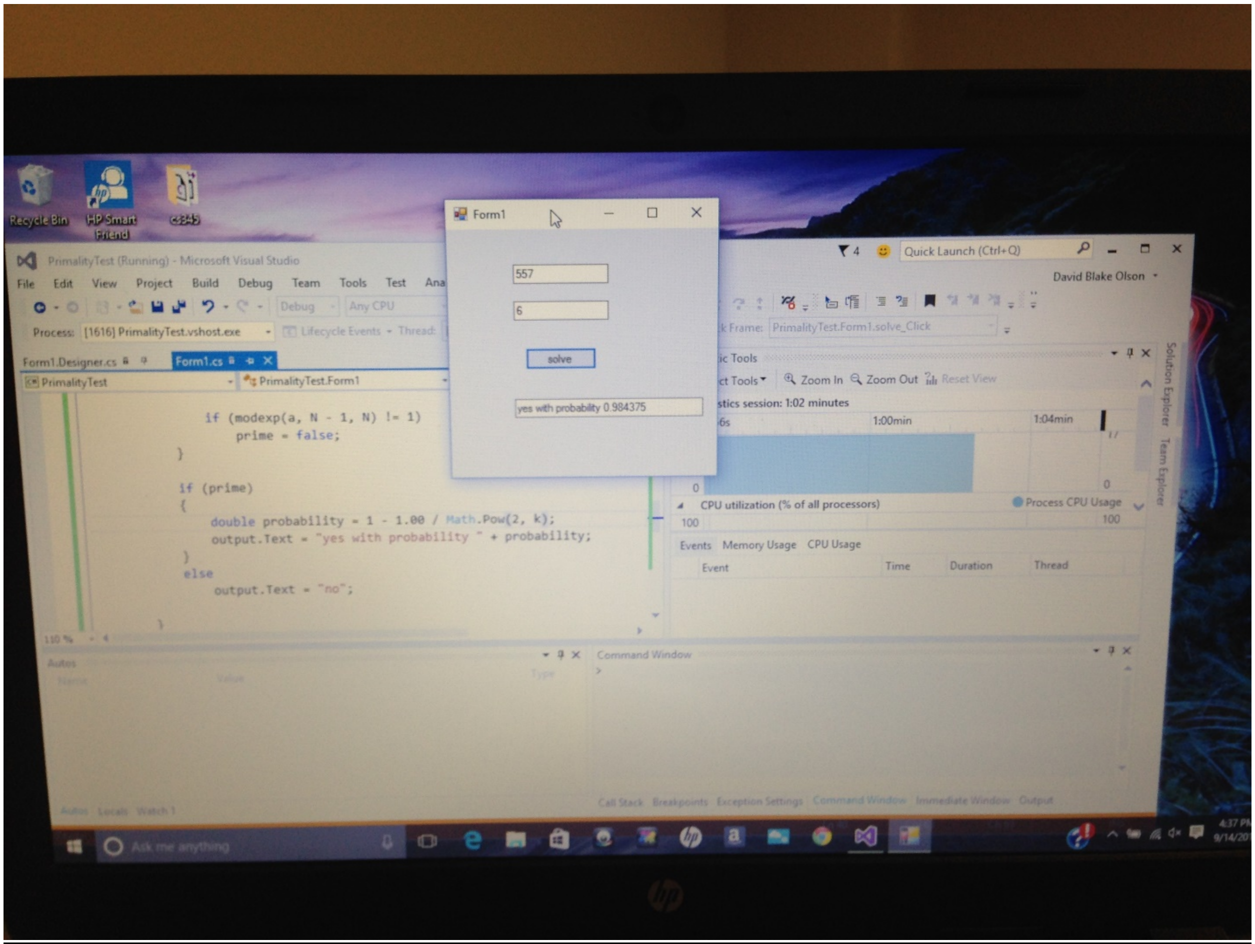
David Olson: Project 1

<u>Screenshot:</u>



<u>Code:</u>

```
 /*
    on the button click, run my main primality function.  inputs are
    the number of random values to choose (k) and the number to test
    primality on (input/N)
 */

private void solve_Click(object sender, EventArgs e)
{
   fermatPrimality(Convert.ToInt32(k.Text), Convert.ToInt32(input.Text));
}
```

```
//complexity is k(modexp complexity) so O(n^3)
private void fermatPrimality(int k, int N)
{
    bool prime = true;
    Random random = new Random();

     /*
      loop through k times
      choose a new random number between 2 and N-1 each time
      compute a^(N-1) mod N each time and if that is ever not 1, N is not prime
    */
    for(int i = 0; i < k; i++)
    {
        int a = random.Next(2, N - 1);

        if (modexp(a, N - 1, N) != 1)
            prime = false;
    }

    if (prime)
    {
        //caluculate the sureness probability and output that along with yes
        double probability = 1 - 1.00 / Math.Pow(2, k);
        output.Text = "yes with probability " + probability;
    }
    else
        output.Text = "no";

}

/*
  function used to do a^(N-1) mod N from above (x is a, y is N-1)
  this is essentially straight from the book
  function complexity is O(n^2)
        ( the program will run this n times, so overall O(n^3))
*/
private int modexp(int x, int y, int N)
{
    if (y == 0)
        return 1;

    int z = modexp(x, y / 2, N);

    //O(n^2)
    if ((y%2) == 0)
        return (z * z) % N;
    else
        return (x * z * z) % N;
}
```

Time/Space Complexity:

As was documented in my code, on an n-bit input, the modular exponentiation function is a O(n^2) function, because multiplication is involved. Since y is halved in every call, this function is called n times. The program itself, or in other words time spent in the fermat function's call to modexp, is O(n^3). The number chosen as k is just a constant, which doesn't affect the big-O class. Space complexity is the same as time complexity in this algorithm.

Probability Equation:

The equation I used was 1 - 1.00 / Math.Pow(2, k). The two zeros in 1.00 and the Math.pow are simply c# techniques to get this equation to work, but more simply the equation is 1-1/(k^2). This is the same as raising ½ to the kth power and subtracting it from one. Since there is a 50/50 chance fermat's theorem is right on each try, the chance that we are wrong halves every time k is increased by one. (1/2, 1/4, 1/8, 1/16 …)