CVIČENÍ 13 — 4. 5. 2021

## Úloha s konvektivním členem

Uvažujme např. $-\mathrm{div}(k\nabla u) + q\frac{\partial u}{\partial x_1} = f$ na $(0,L_1)\times(0,L_2)$

+ okrajové podmínky; $q$ konstantní

variační formulace

$$\underbrace{\int_\Omega k\,\nabla u\,\nabla v\,dx + \overbrace{\int_\Omega q\,\frac{\partial u}{\partial x_1}\,v\,dx}^{\text{konvektivní člen}}}_{a(u,v)} = \int_\Omega f v\,dx + \int_{\Gamma_N} \hat{T} v\,dS$$

$$A_{ij} = \sum_T \int_T k(x)\,\nabla\varphi_j(x)\,\nabla\varphi_i(x)\,dx + \underbrace{\sum_T \int_T q(x)\,\frac{\partial\varphi_j}{\partial x_1}(x)\,\varphi_i(x)\,dx}_{C_{ij}\ \dots\ \text{nesymetrická matice pro konvektivní člen}}$$

prvek lokální matice $\mathbb{P}_T$ pro $T$: $\quad p_{ij,T} = \int_T q(x)\,\frac{\partial\varphi_j}{\partial x_1}(x)\,\varphi_i(x)\,dx$

na referenčním prvku, $q$ konstantní: $\quad \hat{p}_{ij,\hat T} = q\int_{\hat T} \frac{\partial\hat\varphi_j}{\partial\hat x_1}(\hat x)\,\hat\varphi_i(\hat x)\,d\hat x$

Chceme vyjádřit $p_{ij,T}$:

$$\rightarrow\ p_{ij,T} = q\int_{\underset{F(\hat T)}{T}} \frac{\partial\varphi_j}{\partial x_1}(x)\,\varphi_i(x)\,dx = \left[\begin{array}{l}\text{SUBSTITUCE}\\ x = F(\hat x)\\ dx = |DF|\,d\hat x\end{array}\right] = q\int_{\hat T}\frac{\partial\varphi_j}{\partial x_1}(F(\hat x))\,\underbrace{\varphi_i(F(\hat x))}_{\hat\varphi_i(\hat x)}\,|DF|\,d\hat x$$

$$= q\,\underbrace{\frac{\partial\varphi_j}{\partial x_1}(F(\hat x))}_{\text{viz odvození v minulu}}\,|DF|\int_{\hat T}\hat\varphi_i(\hat x)\,d\hat x$$

$$DF = \begin{bmatrix} x_1^{(2)} - x_1^{(1)} & x_1^{(3)} - x_1^{(1)} \\ x_2^{(2)} - x_2^{(1)} & x_2^{(3)} - x_2^{(1)} \end{bmatrix}$$

$$\underbrace{\begin{bmatrix}\frac{\partial\varphi_j}{\partial x_1}\\[4pt] \frac{\partial\varphi_j}{\partial x_2}\end{bmatrix}}_{j\text{-tý sloupec matice } B} = \underline{\underline{(DF)^{-T}}}\ \underbrace{\begin{bmatrix}\frac{\partial\hat\varphi_j}{\partial\hat x_1}\\[4pt] \frac{\partial\hat\varphi_j}{\partial\hat x_2}\end{bmatrix}}_{j\text{-tý sloupec matice } \hat B}$$

$$\hat B = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$B = (DF)^{-T} \hat{B}$$

označíme: $B_1$ ... první řádek $B$ ZDE PRO $\frac{\partial u}{\partial x_1}$

$B_2$ ... 2. řádek $B$

lokální matice $$P_{T,1} = q \cdot B_1^T \cdot |DF| \cdot \underbrace{\left[ \int_{\hat T} \hat\varphi_1 \, d\hat x \quad \int_{\hat T} \hat\varphi_2 \, d\hat x \quad \int_{\hat T} \hat\varphi_3 \, d\hat x \right]}_{= [1 \;\; 1 \;\; 1] \cdot \frac{1}{6}} \in \mathbb{R}^{3\times 3}$$

podobně pro $$-\mathrm{div}(k \nabla u) + \underbrace{q \frac{\partial u}{\partial x_2}} = f$$

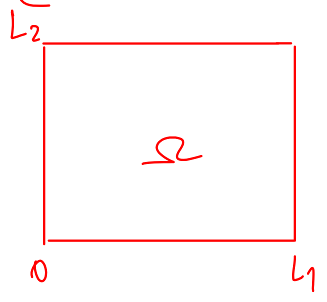
lokální matice $P_{T,2} =$

$$= q \cdot B_2^T \cdot |DF| \cdot [1 \;\; 1 \;\; 1] \cdot \frac{1}{6}$$

$\Rightarrow$ kód pro:

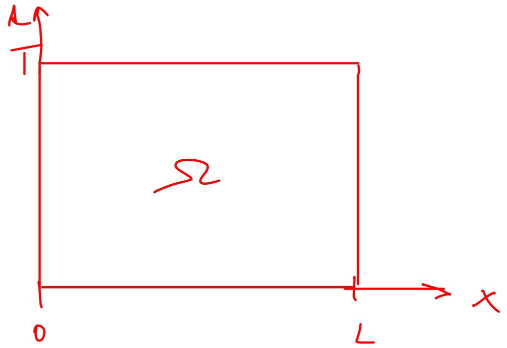
$$\left\{ -\mathrm{div}(\varepsilon \nabla u) + k_0 u + \overbrace{q_1 \frac{\partial u}{\partial x_1} + q_2 \frac{\partial u}{\partial x_2}}^{\text{konvekce}} = f \quad \text{v } \Omega \right.$$

$+ \sigma \cdot u$



## 1d difúze v čase

$$\begin{cases} c \dfrac{\partial u}{\partial t}(x,t) - \dfrac{\partial^2 u}{\partial x^2}(x,t) = 0 & \text{v } \Omega = \overbrace{(0,L)}^{x\in} \times \overbrace{(0,T)}^{t\in} \\ u(x,0) = u_0(x) & x\in(0,L) \\ u(0,t) = f_0(t) & t\in(0,T) \\ u(L,t) = f_L(t) & t\in(0,T) \end{cases}$$




$$\int_\Omega c \frac{\partial u}{\partial t} \cdot v \, dx\,dt - \underbrace{\int_\Omega \frac{\partial^2 u}{\partial x^2} \cdot v \, dx\,dt}_{[P.P.]} = 0$$

$$= \int_\Omega \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} \, dx\,dt - \int_0^T \overbrace{\left[ \frac{\partial u}{\partial x} v \right]_0^L}^{0}$$

$$(V) \begin{cases} \text{Hledáme } u \in U : \\ a(u,v) = 0 \qquad \forall v \in V \end{cases}$$

$$a(u,v) = \int_\Omega c \frac{\partial u}{\partial t} v \, dx\,dt + \int_\Omega \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} \, dx\,dt$$

prvek rozšířené matice soustavy:

$$A_{ij} = \underbrace{\sum_T \int_T c \frac{\partial \varphi_j}{\partial t} \cdot \varphi_i \, dx\,dt}_{\substack{\text{ZNÁMÉ} \\ \text{(VIZ KONVEKCE)}}} + \underbrace{\sum_T \int_T \frac{\partial \varphi_j}{\partial x} \frac{\partial \varphi_i}{\partial x} \, dx\,dt}_{\to \text{ povede na lokální matici } M_{xx,T}}$$

prvek lokální matice $M_{xx,T}$:

$$\int_T \frac{\partial \varphi_j}{\partial x} \frac{\partial \varphi_i}{\partial x} \, dx\,dt = \frac{\partial \varphi_j}{\partial x} \cdot \frac{\partial \varphi_i}{\partial x} \cdot |T|$$

→ řádek matice $B$ (1. řádek) $= B_1$

$$\boxed{M_{xx,T} = B_1 B_1^T \cdot |T|}$$