CVIČENÍ 3 — 23.2.2021

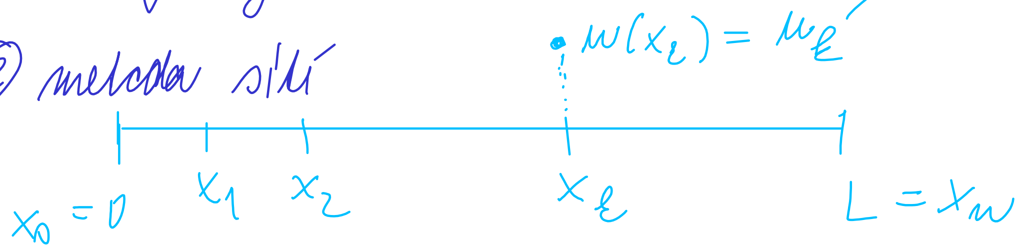
- difúze + Dirichlet + Robin ✓
- difúze + reakční člen ← Dirichlet + Neumann ✓; Dirichlet + Robin ✓
- difúze + materiálové rozhraní

$$\begin{cases} -k\, u''(x) = f & x \in (0, L) \\ u(0) = U \\ -k\, u'(L) = \alpha\,(u(L) - \hat{U}) \end{cases}$$

$\hat{U}$ … vnější teplota (Robinova / Newtonova)

① analyticky – v bodech ✓

② metoda sítí



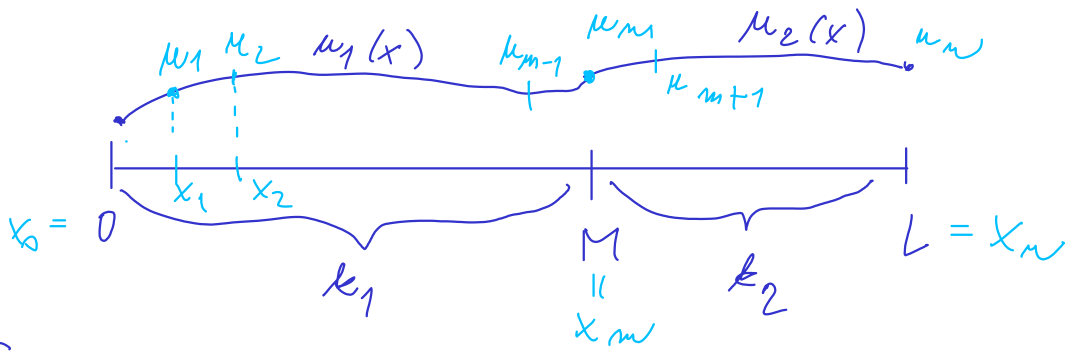
$$u'(x) \doteq \frac{u(x) - u(x-h)}{h}$$

$$u'(x_N) \doteq \frac{u_N - u_{N-1}}{h}$$

$$-k\,\frac{u_N - u_{N-1}}{h} = \alpha\,(u_N - \hat{U})$$

→ poslední rovnice soustavy:

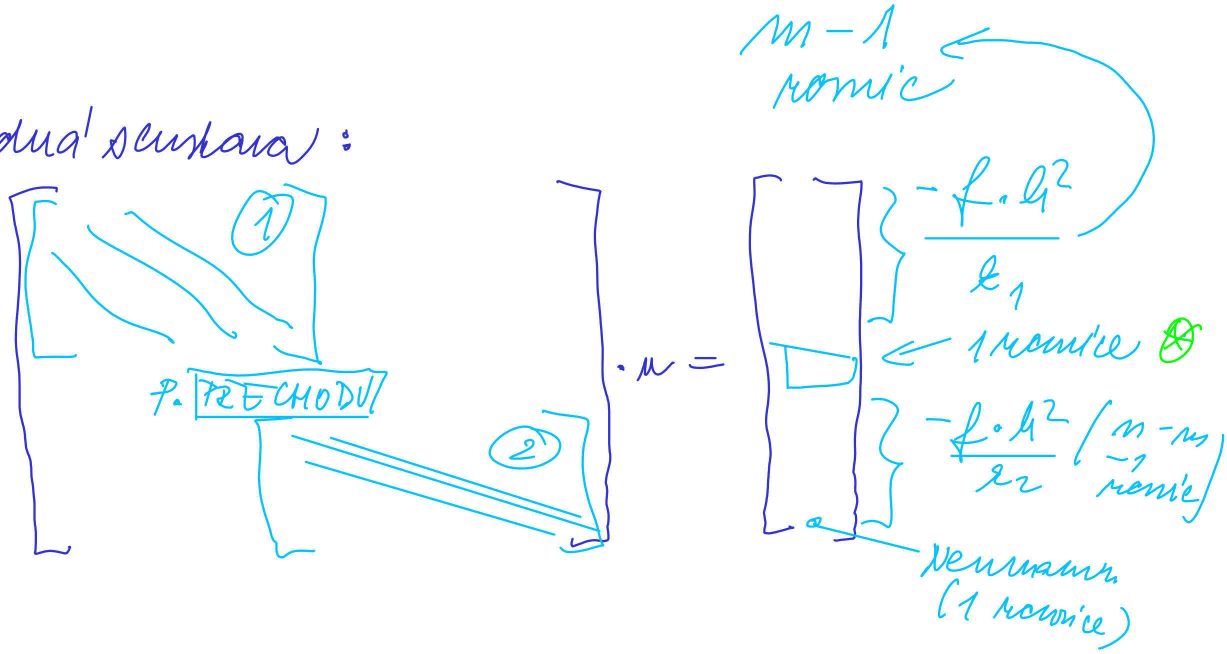
$$k\,u_{N-1} - (k + \alpha h)\,u_N = -\alpha h \hat{U}$$



$$
\begin{cases}
-k_1\, u_1''(x) = f & x\in(0,M) \quad \text{(1)} \rightarrow \text{3diag. matice} \\
-k_2\, u_2''(x) = f & x\in(M,L) \quad \text{(2)} \rightarrow \text{3diag. matice} \\
u_1(0) = U \\
-k_2 u_2'(L) = T \\
u_1(M) = u_2(M) \; \checkmark \\
+k_1 u_1'(M) = +k_2\, u_2'(M) \quad (\circledast)
\end{cases}
$$

$u_1(M) = u_2(M)$ – splněno volbou sítě

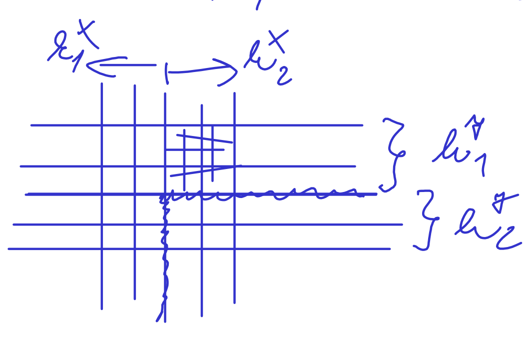
výsledná soustava:



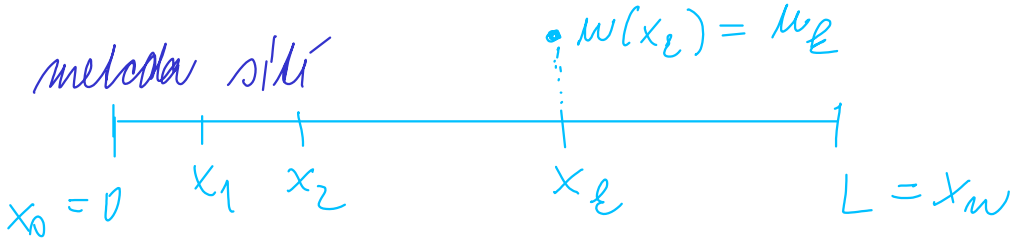
⊛ FMD: $k_1 \dfrac{u_m - u_{m-1}}{h} = k_2 \dfrac{u_{m+1} - u_m}{h}$

$$k_1 u_m - k_1 u_{m-1} = k_2 u_{m+1} - k_2 u_m$$

$$-k_1 u_{m-1} + (k_1 + k_2) u_m - k_2 u_{m+1} = 0$$



$$\begin{cases} -k u''(x) + \overbrace{k_0 u(x)}^{?} = g \\ u(0) = U \\ -k u'(L) = T \end{cases}$$



$n-1$ rovnic (ℓ diferenciálky) pro x_ℓ $(\ell \in \{1, \dots, n-1\})$

$$-k \frac{u_{\ell-1} - 2u_\ell + u_{\ell+1}}{h^2} + k_0 u_\ell = g$$

$$u_{\ell-1} \underbrace{-2}\, u_\ell + u_{\ell+1} \underbrace{- \frac{k_0}{k} h^2}\, u_\ell = - \frac{g h^2}{k}$$

$$\begin{cases} -k u''(x) + k_0 u(x) = g & x \in (0, L) \\ u(0) = U \\ -k u'(L) = \alpha (u(L) - \hat{U}) \end{cases}$$

Řešení metodou sítí
← $n-1$ rovnic pro x_ℓ, $\ell \in \{1, \dots, n-1\}$
poslední rovnice – z Robinovy o.p.