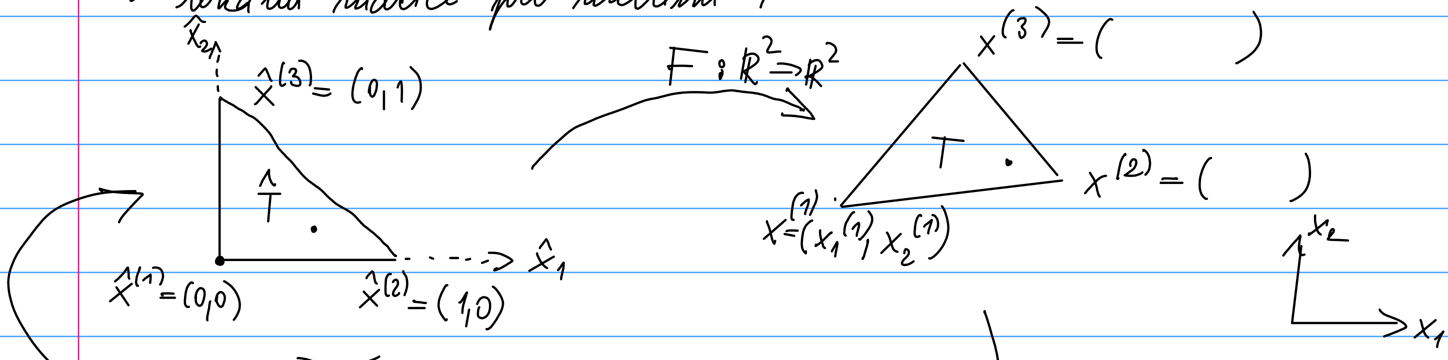
$$A_{ij} = \sum_T \int_T k(x)\, \underbrace{\nabla\varphi_j(x)\cdot\nabla\varphi_i(x)}\, dx \qquad x=(x_1,x_2)\in\mathbb{R}^2$$

← prvek rozšířené matice soustavy

→ lokální matice pro konkrétní $T$




REFERENČNÍ △:

$\hat{\phi}_1(\hat{x}) = \hat{\phi}_1(\hat{x}_1,\hat{x}_2) = 1-\hat{x}_1-\hat{x}_2$

$\hat{\phi}_2(\hat{x}) = \hat{x}_1$

$\hat{x}_3(\hat{x}) = \hat{x}_2$

$$\hat{B} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{cccc} & \phi_1 & \phi_2 & \phi_3 \\ B = & \multicolumn{3}{c}{\begin{bmatrix} \frac{\partial\phi_1}{\partial x_1} & \cdot & \cdot \\ \frac{\partial\phi_2}{\partial x_2} & \cdot & \cdot \end{bmatrix}} \end{array}$$

$k(x): \mathbb{R}^2 \to \mathbb{R}^2$

$A_{\hat{T}} = \frac{1}{2}\hat{B}^T K \hat{B}$ ... anizotropní materiál reprezentovaný na $T$ maticí $K\in\mathbb{R}^{2\times 2}$

$\square\cdot\square\cdot\square$

$A_{\hat{T}} = \frac{1}{2} k\, \hat{B}^T B$ ... izotropní materiál reprezentovaný $k\in\mathbb{R}$

Zobrazení $F: \mathbb{R}^2 \to \mathbb{R}^2 \quad (\hat{T} \to T)$

$$F(\hat{x}) = F(\hat{x}_1,\hat{x}_2) = \left(F_1(\hat{x}_1,\hat{x}_2), F_2(\hat{x}_1,\hat{x}_2)\right)$$

libovolný bod $(x_1,x_2)$ uvnitř T

$$x_1 = F_1(\hat{x}_1,\hat{x}_2) = x_1^{(1)} + \hat{x}_1(x_1^{(2)}-x_1^{(1)}) + \hat{x}_2(x_1^{(3)}-x_1^{(1)})$$

$$x_2 = F_2(\hat{x}_1,\hat{x}_2) = x_2^{(1)} + \hat{x}_1(x_2^{(2)}-x_2^{(1)}) + \hat{x}_2(x_2^{(3)}-x_2^{(1)})$$

$$DF = \begin{bmatrix} x_1^{(2)}-x_1^{(1)} & x_1^{(3)}-x_1^{(1)} \\ x_2^{(2)}-x_2^{(1)} & x_2^{(3)}-x_2^{(1)} \end{bmatrix}$$

$$\boxed{x = F(\hat{x}) = x^{(1)} + DF\,\hat{x}}$$

$$\det DF = |DF| = \left[ x_1^{(2)} - x_1^{(1)} \right] \cdot \left[ x_2^{(3)} - x_2^{(1)} \right] - \left[ x_2^{(2)} - x_2^{(1)} \right] \cdot \left[ x_1^{(3)} - x_1^{(1)} \right]$$

$$\begin{vmatrix} 1 & x_1^{(1)} & x_2^{(1)} \\ 1 & x_1^{(2)} & x_2^{(2)} \\ 1 & x_1^{(3)} & x_2^{(3)} \end{vmatrix} = \det DF = \underbrace{2 \cdot |T|}_{2 \cdot \text{obsah } T}$$

Bázové funkce: $\hat{\phi}_i = \phi_i \circ F$

$$\hat{\phi}_i(\hat{x}_1, \hat{x}_2) = \phi_i(x_1, x_2) = \phi_i(F_1(\hat{x}_1, \hat{x}_2), F_2(\hat{x}_1, \hat{x}_2))$$

$$\frac{\partial \hat{\phi}_i}{\partial \hat{x}_1} = \frac{\partial \phi_i}{\partial x_1} \cdot \frac{\partial F_1}{\partial \hat{x}_1} + \frac{\partial \phi_i}{\partial x_2} \cdot \frac{\partial F_2}{\partial \hat{x}_1}$$

$$\frac{\partial \hat{\phi}_i}{\partial \hat{x}_2} = \frac{\partial \phi_i}{\partial x_1} \cdot \frac{\partial F_1}{\partial \hat{x}_2} + \frac{\partial \phi_i}{\partial x_2} \cdot \frac{\partial F_2}{\partial \hat{x}_2}$$

$$\nabla \hat{\phi}_i = DF^T \nabla \phi_i$$

$$\begin{bmatrix} \dfrac{\partial \hat{\phi}_i}{\partial \hat{x}_1} \\ \dfrac{\partial \hat{\phi}_i}{\partial \hat{x}_2} \end{bmatrix} = DF^T \cdot \underbrace{\begin{bmatrix} \dfrac{\partial \phi_i}{\partial x_1} \\ \dfrac{\partial \phi_i}{\partial x_2} \end{bmatrix}}_{\text{sloupce matice } B}$$

$$\hat{B} = DF^T B$$

$$B = DF^{-T} \hat{B} = \underbrace{DF}_{\text{výše}}{}^{-T} \cdot \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$A_T = B^T K B \cdot |T| = \hat{B}^T DF^{-1} K \, DF^{-T} \hat{B} \cdot \frac{1}{2} |DF| =$$

$$\overset{\text{izotropní}}{=} k \, B^T B \cdot |T|$$