Problem set 3 (Radiation Processes, Spring 2018)

Due Tuesday 2pm, Mar. 13th. Same collaboration/grading policy as previous sets.

1. **Emission of Lyα from a HII region** This involves a lot of order-of-magnitude estimates.

1) Estimate the neutral fraction for hydrogen in a HII region around an O6 star. The Stromgren sphere has a radius $r_s \sim 100$ pc and a mean density of $1/\,\mathrm{cm}^3$. Note that an ionizing photon (Ly-continuum) has a mean-free-path of order the Stromgren radius.

2) What is the mean-free-path for a Lyα photon (n=2 $\longleftrightarrow$ n=1) in the same environment? The Einstein coefficient $A_{Ly\alpha} \approx 6 \times 10^8 s^{-1}$.

3) A Lyα photon can also freely escape the HII region if it gets absorbed by a (very) fast moving atom and be re-emitted at a frequency that is optically thin. Assume the line profile is thermally broadened with a temperature $T \sim 10^4\,\mathrm{K}$ (typical of a HII region), how far away from the line centre does the new frequency has to be? Compared with diffusion by random walk (see above) through the HII region, is this process more likely?

4) What is the fraction of the neutral hydrogen that is in the $n = 2$ state? To answer this question, we split it into 2 parts. Ionized hydrogen can recombine, with roughly similar probabilities, into the $2s$ and $2p$ state. This part looks at the $2s$ state. The transition between $2s$ and $1s$ is forbidden by dipole selection rules.[1] However, an exotic process called 2-photon process[2] can happen and produces a transition probability of $A_{2\gamma} = 8.23\,\mathrm{s}^{-1}$. Argue that for our HII region, this latter process dominates over collisional processes in depopulating the $2s$ state. Table 3.12 of Osterbrock gives the collision strength between electrons and H-atoms, $\Omega \approx 0.26$. Obtain the fraction of neutral hydrogen in the $2s$ state.

5) On the other hand, the transition between $2p$ and $n = 1$ is allowed. Estimate which of the following rates are important for the $2p$ state: radiative recombination(+); electron collisional excitation from the $n = 1$ and $2s$ states (+); radiative pumping by Lyα photons (+); spontaneous emission to $n = 1$ state (-); spontaneous emission to $2s$ state (-); electron collisional deexcitation to $n = 1$ and $2s$ states (-). Take the relative numbers of Lyα and Ly-continuum photons to be their relative residence times in the HII region. What is the fraction of neutral hydrogen in the $2p$ state?

6) Estimate the Einstein coefficient for a Balmer-α photon ($n = 2 \longleftrightarrow n = 3$), and compare your result with what you obtain by querying the NIST database. What is the integrated optical depth at its line centre?

7) Compare the 2-photon process to those in sub-question 3, which one is a more likely fate for a Lyα photon? Relatedly, what do you think will be the dominant hydrogen lines from a HII region?

2. **Largest atom in space**.

1) Using L-S coupling, determine the spectroscopic terms for the ground configuration ($2s^2 2p^2$) and

[1] One can also see that it is further forbidden in quadrupole, octupole... i.e., all multiple transitions.

[2] Arising from expanding the electro-magnetic Hamiltonian to the second order in vector potential A. See Shu eq. [21.13] & Osterbrock p. 65.

excited configurations ($2s^22p3p$, $2s^22p3s$). Mark also possible J values. Which one is ground state?[3]

2) A claim of "the largest atom in space" was reported by Stepkin et al. (2007, MNRAS, 374, 852). The authors detected CI absorption (jumping from, e.g., $n = 1009$ to $n = 1013$ state). Estimate the size of such an atom (e.g., $2s^22p1009p$), the orbital period for the outer electron, and the transition frequency.

3) The presence of such atoms yields information about the local environment. If, for instance, collisional deexcitation is faster than the electron orbital period, no such atoms can exist. Let the electron density of the observed region be $n_e = 0.02\,\mathrm{cm}^{-3}$ and temperature be $T_e = 75\,\mathrm{K}$. Estimate the collisional deexcitation lifetime for this atom. The quantum mechanical correction in this case is large, and you can take the collision strength $\Omega \sim n^4$ (and $g \sim 1$). Balancing the electron orbital period with the deexcitation lifetime[4] yields the largest possible atoms in this region.

4) Consider atoms at states $n \sim 1000$. Estimate the lifetime for radiative decay (to a similar level), and the lifetime for collisional deexcitation. Are the atoms in LTE with respect to each other? Are they in LTE with atoms at states $n \sim 1$? Can the equivalent width in the observed absorption lines be used to infer the total carbon column density?

5) How do these atoms get to such high levels? Substantiate your conclusion by estimating the relevant rates.

References:

- On the interpretation of radio recombination line observations, Brocklehurst, M.; Seaton, M. J., 1972, MNRAS.157..179

- Stark Broadening by Electron and Ion Impacts of $n\alpha$ Hydrogen Lines of Large Principal Quantum Number, Griem, Hans R., 1967ApJ...148..547G

3. **Oxygen Lines and Density of a HII region** We consider two emission lines ([OII]λ3728.80Å, [OII]3726.04Å, both forbidden lines) that arise from two finely spaced upper states (call them a and b) decaying into the same ground state of OII (call it 1) from a HII region. Obtain the Einstein coefficients for these transitions, and the requisite g-factors, from the following website:

`http://physics.nist.gov/PhysRefData/ASD/lines_form.html`

The critical electron densities for the two transitions are, respectively, $n_{\mathrm{crit,a1}} = 1.6 \times 10^4 \mathrm{cm}^{-3}$ and $n_{\mathrm{crit,b1}} = 3 \times 10^3 \mathrm{cm}^{-3}$.

Consider the flux ratio of these two emission lines, received at Earth. Let the HII region be optically thin to these photons. Ignore the photon pumping.

1) Derive, analytically, values for the line ratios when the electron density is extremely low, and when it is exremely high.

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[3] You would need to do some self-teaching to understand terms like 'configuration', 'spectroscopic notation', 'Gotrian diagram', etc.

[4] In other words, the Stark broadening is comparable to the line spacing.

2) How does the flux ratio vary when the density varies from, say, $1\mathrm{cm}^{-3}$ to $10^5\mathrm{cm}$? How does your result depend on the temperature of the HII region?

4. **Spectral Energy Distribution of a Proto-planetary disk** We produce a theoretical prediction for the spectral energy distribution (SED) of a minimum-mass-solar-nebula (one that is needed to account for all planet masses) around a sun-like star. The (gas+dust) surface mass density (measured perpendicular to the disk plane) of such a disk goes with radius as $\Sigma = 1700(r/AU)^{-3/2}\,\mathrm{g/cm}^2$, out of which 1% is dust. Dust grains have a size distribution of $dN/ds = c(r)s^{-3.5}$ everywhere, with a minimum size $s_{\rm min}$ and a maximum size $s_{\rm max}$. You can assume $s_{\rm min} \sim 0.01\mu m$ and $s_{\rm max}$ sitting somewhere between $100\mu m$ and $100\,\mathrm{km}$.

1) Consider the mass opacity ($\kappa_\nu = n\sigma_\nu/\rho$) in the disk. Derive its dependence on $s_{\rm max}$ and ν for the two limits: $s_{\rm max} \gg \lambda$ and $s_{\rm max} \ll \lambda$. *(Hint: for the former case you have to consider opacity in the $s < \lambda$ and $s > \lambda$ ranges seperately. λ is the photon wavelength.)*

2) Outside of what radius is the disk optically thin (vertically) to optical light, and thin to submm (say, 0.1cm) radiation, respectively? Typical disks have sizes < 1000 AU.

3) Write down the thermal emission from a ring at radius r in the optically thick limit. Integrate this over the disk ($r_{\rm min}$ to $r_{\rm max}$) to obtain the frequency slope of the integrated flux F_ν, at the submm wavelength. Are you in the Rayleigh-Jeans tail? Do your results depend on dust mass and/or dust sizes? For disk temperature, assume a simple law of $T \propto 250\,\mathrm{K}(r/AU)^{-1/2}$, independent of grain size.

4) Now repeat the exercise in the optically thin limit. Obtain the slope of the integrated spectrum, F_ν as well as any dependence on $s_{\rm max}$. *(Hint: when you integrate the flux over radius, be sure to include the radius dependence of $c(r)$. Also, differentiate between the case where $s_{\rm max} \gg \lambda$ and $s_{\rm max} \ll \lambda$.)*

5) Grains in the disk can grow by conglomeration. What happens to the SED when $s_{\rm max}$ increases from $100\mu m$ to $100\,\mathrm{km}$?

Now you are ready to observe (as Andrews & Williams 2007 did).