Problem set 2 (Radiation Processes, Spring 2018)

Due Tuesday 2pm, Feb. 20th. Same collaboration/grading policy as set I.

1. **LTE** This is preparation for Problem 2. An atom of two energy levels is immersed in a sea of electrons (of number density n_e and kinetic temperature T_K), and bathed in a black-body background radiation (of radiation temperature T_R). This atom is excited both **collisionally** and **radiatively**. Define an excitation temperature (T_s, sometimes called the spin temperature when the transition concerns spin) by

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} \exp(-\frac{\Delta E}{k_B T_s}), \tag{1}$$

where $\Delta E = E_2 - E_1 > 0$ is the transition energy and k_B the Stefan-Boltzman constant.

1) First write down the equation that describes the statistical equilibrium at which excitation and de-excitation of the upper level balances, and manipulate it to find

$$\exp(-\frac{\Delta E}{k_B T_s}) = \frac{\exp(-\frac{\Delta E}{k_B T_K}) + \frac{\xi}{\exp(\frac{\Delta E}{k_B T_R})-1}}{1 + \frac{\xi \exp(\frac{\Delta E}{k_B T_R})}{\exp(\frac{\Delta E}{k_B T_R})-1}}, \tag{2}$$

where the dimensionless number $\xi = A_{21}/(n_e q_{21}) = A_{21}/(n_e \sigma_{21} v_e) = n_{\rm crit}/n_e$. Here, $n_{\rm crit}$ is the critical density for matter LTE.

2) Simplify the above expression to obtain that in the limits of $\xi \gg 1$, $T_s \approx T_R$; and when $\xi \ll 1$, $T_s \approx T_K$. Contemplate the meaning of these results.

3) Prove that if $T_K > T_R$, indepedent of the value of ξ, we have $T_s > T_R$. In other words, there cannot be absorption of the background radiation.

2. **21cm Emission from the Early Universe** We apply results from the above problem to investigate the 21cm emission from the epoch of reionization (when all hydrogen in the intergalactic space is ionized by UV photons from, presumably, massive stars). The 21cm transition is between two energy levels (one being the hydrogen ground state) split by the nucleus spin (so called hyper-fine transitions). ΔE when expressed in temperature equals 0.068 K. This is much smaller than T_K or T_R. We ignore redshift effect in this problem.

1)For this transition, $A_{21} = 2.87 \times 10^{-15}\,\mathrm{s}^{-1}$, $g_2 = 3$ and $g_1 = 1$. Assume that σ_{21} is of order (1 Å)^2 and that $T_K = 100\,\mathrm{K}$. What is your estimated value of $n_{\rm crit}$? How does this compare with the mean baryon density of the universe $n_{\rm baryon} \sim 10^{-6}(1+z)^3$ at re-ionization (currently thought to occur at $z \sim 10$)?

2) Derive the following absorption coefficient for a clump of neutral hydrogen (number density $n_{\rm HI}$) in the 21cm wavelength,

$$\alpha_\nu \approx \frac{3c^2 A_{21} n_1}{8\pi\nu^2} \frac{0.068\,\mathrm{K}}{T_s} \phi(\nu) \approx \frac{3c^2 A_{21} n_{\rm HI}}{32\pi\nu^2} \frac{0.068\,\mathrm{K}}{T_s} \phi(\nu), \tag{3}$$

where $\phi(\nu)$ is the line profile function normalized as $\int_0^\infty \phi(\nu) d\nu = 1$. The optical depth is the absorption coefficient integrated through the cloud, $\tau_\nu = \int \alpha_\nu ds$. Typically $\tau_\nu \ll 1$.

3) Define a brightness temperature for the 21cm emission, $T_b = T_b(\nu) = c^2/(2\nu^2 k_B)\, I_\nu$. We are in the Rayleigh-Jeans limit ($h\nu = \Delta E \ll k_B T$, where T can be T_s, T_b, T_R or T_K). Show that the source function $S_\nu \approx 2\nu^2/c^2\, k_B T_s$ and that this allows the equation of radiative transport to be simplified to the following form (eq. [1.61] in RL)[1]

$$\frac{dT_b}{d\tau_\nu} = T_s - T_b. \tag{4}$$

Write down the solution for T_b in terms of variables defined above. (*Hint: the initial specific intensity originates from the radiation background.)*

4) Radio telescopes will be able to measure 21cm emission from the early universe by detecting excess temperature over the CMB,

$$\delta T_b = T_b - T_R. \tag{5}$$

Show that this excess is independent of the actual spin temperature as long as it lies sufficiently far above T_R (currently at 2.7K). This measurement allows one to obtain a good estimate of the HI column density as a function of redshift, thereby probing the processes occurring during reionization.

References for Problems 1 & 2

- Field, 1958, Proc. IRE, 46, 240 (original, no electronic version available)
- Field, 1959, ApJ, 129,525 (visionary work)
- Zaldarriaga et al 2004, ApJ, 608, 622 (recent analytical development)

4. **Emission lines fluxes** Atomic and molecular infrared transitions are important for cooling the interstellar gas. One example for this is the cooling of metallic gas in the debris disk around stars like β Pictoris and others. The gas is largely ionized (except for OI which has a first ionization potential 16 eV) and is hydrogen-depleted. The gas is mostly composed of oxygen and carbon and has a mean ion/electron density of $\sim 10/\,\mathrm{cm}^3$. Ignore carbon here.

1) The ground-state of OI (which has a configuration of $2s^2 2p^4$) is split by spin-orbit coupling to three levels. Transitions between these produce the OI 44 μm, 63.2 μm and 145.6 μm lines. Assuming LTE, which transition is most important for cooling the gas when gas temperature $T \sim 300\,\mathrm{K}$? What about when $T \sim 50\,\mathrm{K}$? For an OI number density of $1/\,\mathrm{cm}^3$ and LTE, a gas temperature of 300 K, calculate the cooling luminosity from individual lines radiated by a cubic centimeter of material?

2) Perform the same calculations but without assuming LTE. Ignore radiative pumping. Table 8 of Hollenbach & McKee (1989, ApJ, 342, 30) lists relevant numbers for all three transitions (Oxygen is an important ISM coolant). (*Hint: for this question, you may not have to solve for the population occupations. Cross off the unimportant terms.*)

3) Simplify the debris disk to a homogeneous sphere of radius 100 AU, lying at a distance of 20 pc. Will the line fluxes you obtain be measurable by the current Herschel mission (a typical sensitivity of $3 \times 10^{-18}\,\mathrm{W/\,m^2}$ over 1 hour of integration) ?

[1] Scattering of photons can be safely ignored for this line radiation.

4) Assuming that the two strongest lines are detectable, what can we learn about disk density and temperature?

5) Riviere-Marichalar et al (2012, A&A, 546, 8) reported detection of OI 63μm line from star HD 172555, using the Herschel telescope (http://arxiv.org/abs/1210.0089). They used the observed flux to derive the mass of oxygen around that star (appendix B). Can you spot any inappropriate assumption? How would it change their result on the oxygen mass?

5. Photoevaporating a close-in planet Stellar XUV (100-1000 Å) photons can ionize hydrogen in the upper atmosphere of a planet. The dissociated electrons typically have an energy of $\sim$ eV and heats the ionized gas to a temperature of $T \sim 10^4$ K. The vertical pressure scale height associated with such a temperature is so large (of order the planet radius) that the gas readily gets lost from the planet. Estimate the mass loss rate from a Jupiter-like planet at 0.05 AU, based on the following simplifying assumptions:

1) If the XUV photon can ionize the atmosphere down to a layer with base density n (unit cm^{-3}), we will obtain a mass loss (Parker wind, a hydrodynamical outflow) rate of $\dot{M} \sim 4\pi R_p^2 n m_H v_{\rm th}$, where m_H is the proto mass, R_p is the planet radius, and $v_{\rm th}$ is the thermal velocity of the gas.

2) Recombination is important in determining the extent of the ionized region.

3) A star similar to the Sun in mass and age has a XUV luminosity of $L_{XUV} \sim 10^{-6} L_\odot$.

Compare your results with papers calculating precisely this process. Lammer et al (2003,ApJ,598,L121) produces $\dot{M} \sim 10^{12}$ g/s, Yelle (2004, Icarus, 170,167) gives $\dot{M} \sim 5 \times 10^{10}$ g/s. The only observation concerns the transiting planet HD209468b and claims a detection of mass-loss with a rate $> 10^{10}$ g/s (Vidal-Madjar et al 2003, Nature, 422, 143, but see Holmstrom et al 2008, Nature 451, 970).