Problem set 4 (Radiation Processes, Spring 2018)

Due Tuesday 2pm, Apr. 3rd, my office. Same collaboration/grading policy as previous sets. Last set and you are done!

We are covering synchrotron the week of Mar. 19th, and compton/inverse-compton the week of Mar. 26th. So if you are working on this set during these period, keep the inverse compton part (questions 3.4; 4.7) till later. Also, to reduce your load, I make questions 4.4 - 4.7 optional.

1. Thermal Bremsstrahlung and Temperature of a HII region The 3rd problem of Problem set 4 in Frank Shu's <<The physics of astrophysics>> , except for the last paragraph.

2. Synchrotron Emission (courtesy of Christopher Pfrommer)

1) A particle of mass m, charge e, moves in a plane perpendicular to a uniform, static magnetic field. Work out the total energy emitted per unit time, expressing it in terms of Thomson cross section σ_T and magnetic field energy density $U_B = B^2/8\pi$.

2)For a tangled magnetic field, this emitted radiation has to be averaged over an isotropic distribution of pitch angles. Let α denote the pitch angle between field and velocity. Take the ultra-relativistic limit and express the total emitted energy in terms of $\gamma = (1-\beta^2)^{-1/2}$.

3) Calculate the time it takes a particle to loose an energy $\Delta E = (\gamma_0 - \gamma)mc^2$ due to synchrotron radiation. How is this expression changed if there are other radiation fields present, e.g., the CMB relic radiation field?

4) Why can proton synchrotron radiation be ignored compared to electron synchrotron? Where does the mass dependence you find physically arise?

3. Intergalactic magnetic field Synchrotron radiation at sub-GHz radio frequencies is currently the most sensitive direct way to trace widespread intergalactic magnetic fields down to about 0.1 μG. The first successful attempt to detect the presence of such a field is at 326 MHz (Kim, Kronberg, Giovannini & Venturi, 1989, Nature 341, 720, the first two authors were from Toronto).

1) To obtain a magnetic field strength using the observed flux, it is common to adopt an assumption called 'minimum energy' assumption (thus the associated 'minimum energy field'). Assume cosmic ray electrons have a single γ factor but isotropic pitch angles (see last question). Write down the sum of the electron kinetic energy and magnetic field energy. For a given synchrotron luminosity, this sum reaches a minimum at $B = B_{\min}$.[1] Determine $B_{\min}$. How different is this field strength from 'equipartition field'?

2) A somewhat more complicated derivation is given in P. 179 of Shu where he accounts for a power-law distribution of electrons. We will adopt his results here. The measured flux at 326 MHz is 760 milli-Jansky, while at 430 MHz is 600 mJy. This is roughly consistent with a spectral index $\alpha_\nu = 1.5$ where $F_\nu \propto \nu^{-\alpha_\nu}$. Follow the authors in assuming a geometry for the

[1] Admittedly, this is a somewhat arbitrary choice. But it 'sounds' very physical.

radiating region: a 1900 kpc long of cylinder with 800 kps in diameter. What do you obtain for the minimum energy field?

3) For the electrons responsible for the 326 MHz radiation, what is their synchrotron cooling time?

4) Inverse-compton scattering of CMB photons can also lead to cooling. Calculate the energy density of a 3K background glow. Now what is the inverse-compton cooling time for the same electrons? At what wavelength band should we expect to observe the 'cooling' radiation?

4. **Radio Emission from Normal Galaxies (non-AGN)** One of the tightest correlation in astronomy is the FIR/radio correlation of normal galaxies (see Fig. 8 of Condon, Annual Review of Astronomy & Astrophysics, 1992, whereas Fig. 1 shows the the spectral energy distribution for a typical galaxy FIR/radio band). The far infrared luminosities of galaxies scale roughly linearly with their radio luminosities over four orders of magnitude. This is surprising in more than one ways. We will try to appreciate the physics behind this correlation, following the narratives of Condon (1992).

1) Far-infrared light from a galaxy can dominate its total energy output. This arises from dust reprocessing of star light, therefore depends on the dust temperature and the dust size distribution. Assume that grains of size $\sim 0.05\mu$m are responsible, that at FIR the galaxy is optically thin, that dust is embedded in a radiation field with energy density of $U_r \sim 10^{-12}\,\mathrm{erg/\,cm^3}$ (the Milky Way value) with typical photons in the optical. Obtain the dust temperature and show that most of the dust radiation indeed occurs in FIR ($\sim 100\mu$m). Then follow the lead of the article to obtain his Eq. 26. This relates $L_{\rm FIR}$ with the current star formation rate.

2) In contrast, the radio luminosity is only some 10^{-4} of the bolometric luminosity. However, this luminosity is also related to the star formation rate. Show that the thermal radio luminosity, arising from free-free radiation from electrons in HII regions around massive stars, is as expressed in his Eq. 2 (you may need to refer to Shu for the relevant Gaunt factor, which roughly scales as $\nu^{-0.1}$). Depending on the values of order-unity coefficients that you adopt, you may find a slightly different expression as Condon does. After this, follow the lead of the article to obtain Eq. 23, where L_T (thermal radio luminosity) is also related to the current star formation rate.

3) Obtain the value of q (eq. 15) based on the above $L_{\rm FIR}$ and L_T. You should find that while you get the observed linear relationship, the normalization seems off.

– Questions below will be counted as bonus points–

4) Stars can further produce radio emission when they explode into supernova and the supernova shock accelerates electrons to relativistic speeds. These electrons later escape the supernova remnants and make up the cosmic ray ecosystem in a galaxy. Derive the synchrotron power of a galaxy, as a function of radio frequency, for a number density of electron cosmic ray: $n_0 = \int_1^\infty n_\gamma d\gamma = \int_1^\infty c\gamma^p d\gamma$ (the article uses index γ for our p here, while we use γ to indicate the Lorentz factor, likely $p \sim 2.0$), a galactic magnetic field strength of B, and a galaxy volume V.

5) Relate the above expression to the galactic supernova rate, assuming each supernova remnant

puts in a fixed fraction (f) of its 10^{50} ergs energy into accelerating cosmic ray electrons. The supernova rate is related to the star formation rate as in Eq. 20. This exercise highlights a problem to explain the observed FIR/radio correlation: magnetic field energy density (U_m) varies widely across different galaxies. Your result differs from Eq. 18 in that article, adopted ad hoc, to explain the observed correlation.

6) One way to reconcile your result to Eq. 21 is to calculate the synchrotron luminosity from a continuously injected (with power slope p), but continuously cooled cosmic ray population. This has a better success since the total luminosity radiated over the cooling lifetime of the electron is independent of the magnetic field strength. Write down an evolution equation for the number of electrons with γ, changing due to injection and cooling. Show that at steady state, $n_\gamma \propto \frac{1}{U_m}\gamma^{-(p+1)}$. Now compute the equilibrium state synchrotron luminosity, and argue that it now scales with only the rate of supernova (or star formation), but not with U_m. We have finally explained the FIR/radio correlation.

7) But have we? Inverse compton is another process that the cosmic ray electrons can cool down (and emit non-radio photons). This cooling rate, relative to the synchrotron cooling rate, is the ratio of radiation energy density and magnetic field energy density, U_r/U_m. If U_r/U_m varies across different galaxies, the radio luminosity will not scale nicely with the FIR. Volk (1989) argued that U_r/U_m is kept constant in all galaxies since U_r is contributed by the stars (at redshift 0), and U_m is related to the turbulence stirred by the stars (see p. 600 in the article). This would then explain the observed FIR/radio correlation. Now if we follow galaxies across different redshifts, where U_r is instead dominated by CMB photons with $U_r \propto (1+z)^4$, argue how the q value in Eq. 15 will vary with redshift.

Even though we start from rigorous physics, we have to make a large number of estimates, hand-waving arguments to arrive at our final results. This second part, where 'common sense' is important, makes astrophysics fun.